Show that the following Hoare triple is satisfied under partial correctness:

$$\{T\} \text{ if } (i < j)\{m := i\} \text{ else } \{m := j\} \; \{(m \le i) \wedge (m \le j)\}$$

Proof. We first prove the following Hoare triples

$$\{T \wedge (i < j)\}\; m := i \;\{(m \le i) \wedge (m \le j)\} \tag{1}$$
$$\{T \wedge \neg(i < j)\}\; m := j \;\{(m \le i) \wedge (m \le j)\} \tag{2}$$

starting with (1). We have

$$\begin{aligned}
\{(i \le i) \wedge (i \le j)\}\; m := i \;\{(m \le i) \wedge (m \le j)\} & \qquad \text{(Assignment)} \\
i \le i \iff T & \qquad \text{(Logic)} \\
T \wedge (i \le j) \iff (i \le i) \wedge (i \le j) & \qquad \text{(Logic)} \\
i \le j \iff T \wedge (i \le j) & \qquad \text{(Logic)} \\
\{i \le j\}\; m := i \;\{(m \le i) \wedge (m \le j)\} & \qquad \text{(PreConEq)} \\
i < j \implies i \le j & \qquad \text{(Logic)} \\
T \wedge (i < j) \iff i < j & \qquad \text{(Logic)} \\
\{T \wedge (i < j)\}\; m := i \;\{(m \le i) \wedge (m \le j)\} & \qquad \text{(PreConStr)}
\end{aligned}$$

For (2), we have

$$\begin{aligned}
\{(j \le i) \wedge (j \le j)\}\; m := j \;\{(m \le i) \wedge (m \le j)\} & \qquad \text{(Assignment)} \\
j \le j \iff T & \qquad \text{(Logic)} \\
(j \le i) \wedge T \iff (j \le i) \wedge (j \le j) & \qquad \text{(Logic)} \\
j \le i \iff (j \le i) \wedge T & \qquad \text{(Logic)} \\
j \le i \iff \neg(i < j) & \qquad \text{(Logic)} \\
T \wedge \neg(i < j) \iff \neg(i < j) & \qquad \text{(Logic)} \\
\{T \wedge \neg(i < j)\}\; m := j \;\{(m \le i) \wedge (m \le j)\} & \qquad \text{(PreConEq)}
\end{aligned}$$

Hence we have

$$\frac{\begin{array}{l}\{T \wedge (i < j)\}\; m := i \;\{(m \le i) \wedge (m \le j)\} \\ \{T \wedge \neg(i < j)\}\; m := j \;\{(m \le i) \wedge (m \le j)\}\end{array}}{\{T\} \text{ if } (i < j)\{m := i\} \text{ else } \{m := j\} \; \{(m \le i) \wedge (m \le j)\}}$$

by Conditionals, as desired. $\square$

Find an invariant for the following Hoare triple and prove partial correctness.
$\{x = 0 \land y = 1 \land z = 1 \land n \geq 1\}$ while $(z < n)\{y := x + y; x := y - x; z := z + 1\}$ $\{y = f(n)\}$