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Agents and Multi-Agent Systems

Multi-Agent Decision Making

Game Theory

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Multi-Agent Decision Making

- How do we make collective decisions in societies where agents are self-interested?
- That is, how do we reach agreements?

- Game theory: strategic interactions among rational agents
- Social choice theory: group decisions on possible outcomes
- Mechanism design: effective protocols for multi-agent systems
- Negotiation: finding mutually beneficial deals in the presence of conflicting objectives

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Multi-Agent Decision-Making

GAME THEORY

Game Theory

- **Game theory** is the mathematical study of *strategic decision making* among independent, *self-interested* agents
  - Given the rules of the game, game theory studies strategic behavior of the agents in the form of a strategy

- The tools and techniques of game theory have found many applications in computational multi-agent systems research

- Types of games
  - Cooperative/non-cooperative, symmetric/asymmetric, zero-sum/win-win, simultaneous/sequential, perfect/imperfect information, complete/incomplete information, …

Utilities

- **Utility theory** aims to quantify an agent's degree of preference across a set of available alternatives
- A utility function is a mapping from world states to real numbers
- World states (outcomes): $\Omega = \{\omega_1, \omega_2, \dots\}$
- Utility function for agent i: $u_i\colon \Omega \to \mathbb{R}$

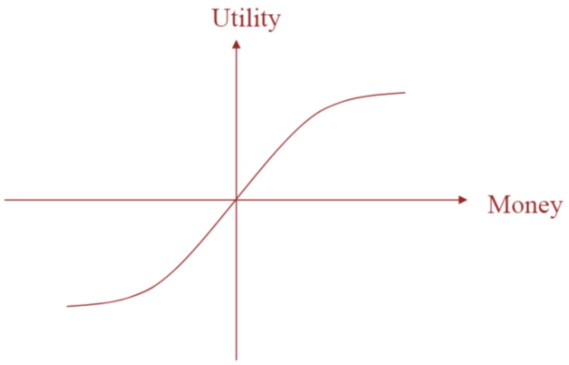
Preferences

- Preference ordering:
  - $u_i(\omega_1) > u_i(\omega_2)$ or $\omega_1 \succ_i \omega_2$
    - agent i prefers outcome ω_1 to outcome ω_2
  - $u_i(\omega_1) \geq u_i(\omega_2)$ or $\omega_1 \succcurlyeq_i \omega_2$

- Properties:
  - Reflexivity: $\forall_{\omega \in \Omega}\ \omega \succcurlyeq_i \omega$
  - Transitivity: If $\omega_1 \succcurlyeq_i \omega_2$ and $\omega_2 \succcurlyeq_i \omega_3$ then $\omega_1 \succcurlyeq_i \omega_3$
  - Comparability: $\forall_{\omega_1, \omega_2 \in \Omega}$ either $\omega_1 \succcurlyeq_i \omega_2$ or $\omega_2 \succcurlyeq_i \omega_1$

Utility is not Money

- The utility of money depends on how much money one already has
  - Linear utility for smaller amounts of money, logarithmic for larger amounts
    - *marginal utility*: utility for the *next* million dollars



- Utility function depends on agent's risk aversion attitude
  - Different curves for different people
- People are not entirely rational when making choices about money
  - Should agents behave like their human owners?
  - Rational agents would be better negotiators

Multi-Agent Encounter

- Simplified setting
  - Two agents i and j perform simultaneous actions from set Ac
  - Environment behavior (a state transformer function): $\tau: Ac \times Ac \rightarrow \Omega$
    - Actual outcome depends on the combination of actions chosen by all agents
- Consider two actions: $Ac = \{C, D\}$
  - $\tau(D, D) = \omega_1,\ \tau(D, C) = \omega_2,\ \tau(C, D) = \omega_3,\ \tau(C, C) = \omega_4$
  - Depending on their preferences over these outcomes, agents may need to think strategically (consider what the other agent will do)
  - In other settings, the agent can decide regardless of what the other will do

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Multi-Agent Encounter

- $\tau(D,D) = \omega_1,\ \tau(D,C) = \omega_2,\ \tau(C,D) = \omega_3,\ \tau(C,C) = \omega_4$
  - If $\omega_1 \succcurlyeq_i \omega_3$, $\omega_1 \succcurlyeq_i \omega_4$, $\omega_2 \succcurlyeq_i \omega_3$, $\omega_2 \succcurlyeq_i \omega_4$: for agent i, it is always better to execute action D, regardless of what agent j will do

- $\tau(D,D) = \omega_1,\ \tau(D,C) = \omega_1,\ \tau(C,D) = \omega_1,\ \tau(C,C) = \omega_1$
  - Neither agent has any influence – the outcome will be the same no matter what the agents do

- $\tau(D,D) = \omega_1,\ \tau(D,C) = \omega_2,\ \tau(C,D) = \omega_1,\ \tau(C,C) = \omega_2$
  - The outcome depends solely on the actions performed by agent j – it does not matter what agent i does

Normal-form Game

- A **normal-form game** is a tuple $\langle N, A, u\rangle$
  - N is a finite set of *n* players, indexed by i
  - $A = A_1 \times \cdots \times A_n$, where A_i is a finite set of actions available to player i
    - Each vector $a = (a_1, \dots, a_n) \in A$ is called an *action profile*
  - $u = (u_1, \dots, u_n)$, where $u_i\colon A \to \mathbb{R}$ is a real-valued utility (or *payoff*) function for player i
    - For simplification, u_i maps directly from actions (instead of outcomes)

- Representation in a *payoff matrix* (2 agents, 2 actions)

		i: D	i: C
j	D	$u_j(D,D)$, $u_i(D,D)$	$u_j(C,D)$, $u_i(C,D)$
j	C	$u_j(D,C)$, $u_i(D,C)$	$u_j(C,C)$, $u_i(C,C)$

Example: the Prisoner's Dilemma

- 2 prisoners suspected of a crime are taken to separate interrogation rooms
  - If both cooperate by not confessing, each is jailed for one year
  - If both defect by confessing, each is jailed for two years
  - If only one confesses, that one is freed and the other one gets jailed for three years

		i	
		D	C
j	D	2 2	0 5
	C	5 0	3 3

Example: Rock-Paper-Scissors

		Player 2		
		Rock	Paper	Scissor
Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissor	-1, 1	1, -1	0, 0

Dominant Strategy

- Strategy (or action) s_i is **dominant** for player i if, no matter what strategy s_j agent j chooses, i will do *at least as well* playing s_i as it would doing anything else
  - s_i is *dominant* if it is i's best response to all of agent j's strategies

		i	
		D	C
j	D	2 2	0 5
	C	5 0	3 3

- Generalizing to *n* agents:
  - Strategy $s \in S_i$ dominates strategy $s' \in S_i$ iff $\forall_{s_{-i} \in S_{-i}} u_i(s, s_{-i}) \geq u_i(s', s_{-i})$
    - where $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$

Nash Equilibria

- Nash (1951) defined one of the most important concepts in game-theory

Two strategies s_i and s_j are in (pure-strategy) **Nash equilibrium** if:
- assuming that agent i plays s_i, agent j can do no better than play s_j, and
- assuming that agent j plays s_j, agent i can do no better than play s_i

- Strategies s_i and s_j , which together form a strategy profile (s_i, s_j), are the *best* response to each other
- Neither agent has any incentive to deviate from a Nash equilibrium

→ Not every interaction scenario has a pure strategy Nash equilibrium
→ Some scenarios have more than one pure strategy Nash equilibrium

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Nash Equilibria

		i	
		D	*C*
j	*D*	2, 2	5, 0
	C	0, 5	3, 3

Coordination games

	I go home	I work late
She goes home	(3,3)	(1,2)
She works late	(2,1)	(2,2)

	Attack East City	Attack West City
Defend East City	(0,1)	(1,0)
Defend West City	(1,0)	(0,1)

Mixed Strategies

- A pure strategy determines the exact action to play
- A **mixed strategy** is an assignment of a probability to each pure strategy
  - Randomizes over the set of available actions according to some probability distribution

- Given a normal-form game $\langle N, A, u\rangle$, a **mixed strategy** for player i is a probability distribution $\rho_i: A_i \rightarrow [0,1]$, with $\sum_{a \in A_i} \rho_i(a) = 1$
  - $\rho = (\rho_1, \dots, \rho_n)$ is a *mixed strategy profile*

→ *Every game in which every player has a finite set of possible strategies has a Nash equilibrium in mixed strategies*

Mixed Strategy Nash Equilibrium

Player 1 \ Player 2	Rock	Paper	Scissor
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissor	-1, 1	1, -1	0, 0

- Example: Rock-Paper-Scissors
  - No dominant strategy
  - No pure-strategy Nash equilibrium
  - Best strategy: play an action at random – a *mixed strategy*
    - $\rho_i(rock) = 1/3$, $\rho_i(paper) = 1/3$, $\rho_i(scissors) = 1/3$
  - This mixed strategy is in Nash equilibrium with itself
    - It is the best response if the opponent is using the same strategy
    - However: if we notice that the opponent favors an action (e.g. scissors) then we should favor another action (e.g. rock)

→ In general, it is tricky (and computationally expensive) to compute a game's mixed-strategy Nash equilibria

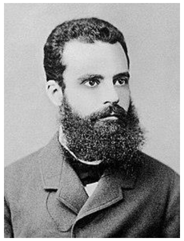
ϵ-Nash Equilibrium

- Players might not care about changing their strategies to a best response when the amount of utility that they could gain by doing so is very small

A strategy profile $s = (s_1, \dots, s_n)$ is an **ϵ-Nash Equilibrium** if, for all agents i and for all strategies $s_i' \neq s_i$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) - \epsilon$.

- ϵ-Nash equilibria always exist
  - Every Nash equilibrium is surrounded by a region of ϵ-Nash equilibria, for any $\epsilon > 0$
- The argument that agents are indifferent to sufficiently small gains is convincing
- The concept of ϵ-Nash equilibria is computationally useful

Pareto Optimality



Italian polymath, 1848-1923

- Strategy profile s **Pareto dominates** strategy profile s' if for all $i \in N$, $u_i(s) \geq u_i(s')$, and there exists some $j \in N$ for which $u_j(s) > u_j(s')$
  - In a *Pareto-dominated strategy profile* some player can be made better off without making any other player worse off

- Strategy profile s is **Pareto optimal** (or *Pareto efficient*) if there does not exist another strategy profile s' that Pareto dominates s
  - There is no other outcome that improves one player's utility without making someone worse off

- A Pareto inefficient strategy profile 'wastes' some utility
  - Some other outcome would make someone better off to no one's expense!

Pareto Optimality

		i	
		D	***C***
j	***D***	2 2	0 5
	C	5 0	3 3

- Every game must have at least one Pareto optimum
- Some games will have multiple optima

Social Welfare Maximization

- Social welfare = total utility
- Measure how much utility, in total, is created by an outcome
  - Social welfare $sw(\omega) = \sum_{i \in Ag} u_i(\omega)$
  - We would like to maximize this value
- But this conflicts with individual points of view: different payoffs, thus different preferences regarding possible outcomes
- Maximizing total welfare is relevant in common-payoff games
  - The utility of the outcome is divided among the players
  - Example: climate change (or so we wish…)

		i	
		D	***C***
j	***D***	2 2	0 5
	C	5 0	$\boxed{3 \quad 3}$

		i	
		D	***C***
j	***D***	2 2	1 1
	C	3 3	$\boxed{7 \quad 0}$

Extensive-form Games

- Normal-form assumes players act simultaneously
- **Extensive-form** accommodates situations in which actions are played sequentially

- A perfect-information game in extensive form is a tree
  - Each node represents a choice point of one of the players
  - Each edge represents a possible action
  - Leaves represent final outcomes

→ In two-player, zero-sum games: the *minimax algorithm* (with *alpha-beta pruning*) can be used to compute the value of the game, i.e., a player’s payoff in equilibrium (both agents using their best response strategies)

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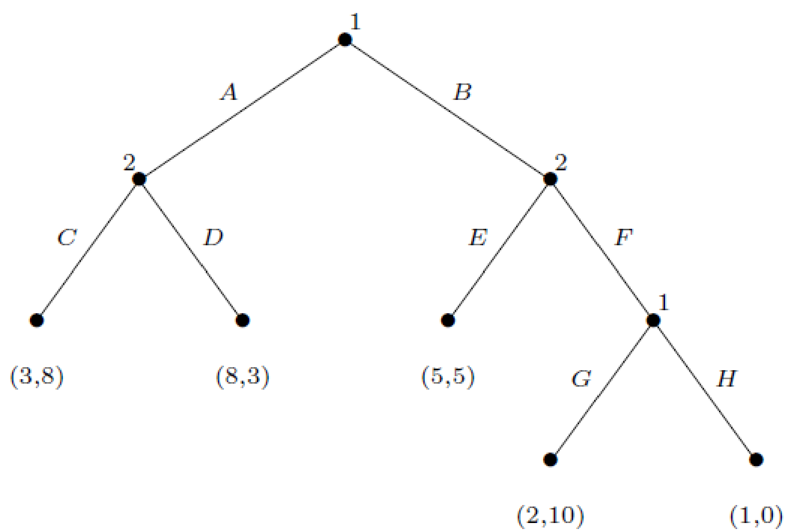
Extensive vs. Normal-form

Extensive-form



- Pure strategies:
  $S_1 = \{(A,G),(A,H),(B,G),(B,H)\}$
  $S_2 = \{(C,E),(C,F),(D,E),(D,F)\}$

Normal-form

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3, 8	3, 8	8, 3	8, 3
(A,H)	3, 8	3, 8	8, 3	8, 3
(B,G)	5, 5	2, 10	5, 5	2, 10
(B,H)	5, 5	1, 0	5, 5	1, 0

➢ Every perfect-information game can be converted to an equivalent normal-form game

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Extensive-form Games

- Every (finite) perfect-information game in extensive form has a pure-strategy Nash equilibrium
  - Players take turns – everyone gets to see everything that happened before making a move
  - It is never necessary to introduce randomness into action selection to find an equilibrium

	(C, E)	(C, F)	(D, E)	(D, F)
(A, G)	3, 8	$\boxed{3, 8}$	8, 3	8, 3
(A, H)	3, 8	$\boxed{3, 8}$	8, 3	8, 3
(B, G)	5, 5	2, 10	5, 5	2, 10
(B, H)	$\boxed{5, 5}$	1, 0	5, 5	1, 0

- This theorem does not hold for imperfect-information games

Uncertainty

- Imperfect-information games (partially observable environment)
  - Need to act with partial knowledge
    - Partial or no knowledge of the actions taken by others
    - Limited memory of their own past actions
  - Choice nodes are partitioned into *information sets*
    - If two choice nodes are in the same information set, the agent cannot distinguish between them

- Incomplete-information games (unknown environment)
  - Players are uncertain about the game being played
    - Players might have private information that affects their payoffs
      - Probabilistic information about other agents payoffs
  - What moves are possible, how outcomes depend on actions

- Bounded rationality
  - Need to reason under computational limitations

Imperfect-information games*

- An imperfect-information game is an extensive-form game in which each agent's choice nodes are partitioned into information sets
  - An information set = {all the nodes you might be at}
    - The nodes in an information set are indistinguishable to the agent
    - The set of actions in those nodes is also the same
  - Agent *i*'s information sets are $I_{i1}, \ldots, I_{im}$ for some *m*, where
    - $I_{i1} \cup \cdots \cup I_{im} = \{all\ nodes\ where\ it\ is\ agent\ i's\ move\}$
    - $I_{ij} \cap I_{ik} = \emptyset$ for all j $\neq k$
    - $X(h) = X(h')$ for all histories ´$\mathrm{h}, \mathrm{h}' \in I_{ij}$,

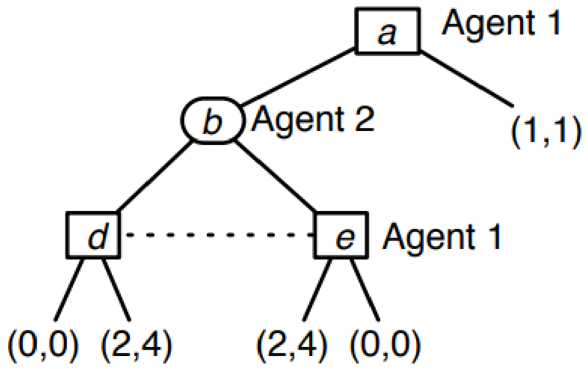
      where $X(h) = \{all\ available\ actions\ at\ h\}$
- A perfect-information game is a special case in which each I_{ij} contains just one node *h*

Imperfect-information games*

Example

- Agent 1 has two information sets:
  - $I_{11} = \{a\}$
  - $I_{12} = \{d, e\}$
  - in I_{12}, agent 1 doesn´t know whether agent 2 moved to *d* or *e*

- Agent 2 has just one information set:
  - $I_{21} = \{b\}$



- Examples: bridge, poker, battleship

Incomplete-information games*

- So far, we have assumed that:
  - everything relevant about the game being played is common knowledge to all the players:
    - the number of players,
    - the actions available to each , and
    - the payoff associated with each action
  - True even for imperfect-information games
    - the actual moves aren't common knowledge, but the game is
  - Thus "complete information" games
- We will now consider games of incomplete information
  - Players are uncertain about the game being played:
    - payoffs, who the other players are, what moves are possible, how outcome depends on the action, what opponent knows, and what he knows I know....

Incomplete-information games*

- A **Bayesian Game** is a class of games G that satisfies two fundamental conditions:
  - *Condition 1*: the games in G have the same number of agents, and the same strategy space for each agent. The only difference is in the payoffs of the strategies.

This condition isn't very restrictive

Incomplete-information games*

Example:

- Suppose we don't know whether player 2 only has strategies *L* and *R*, or also an additional strategy *C*:

Game G_1

	L	R
U	1, 1	1, 3
D	0, 5	1, 13

Game G_2

	L	C	R
U	1, 1	0, 2	1, 3
D	0, 5	2, 8	1, 13

- If player 2 doesn't have strategy *C*, this is equivalent to having a strategy *C* that's dominated by the other strategies:

Game G_1'

	L	C	R
U	1, 1	0, −100	1, 3
D	0, 5	2, −100	1, 13

- The Nash equilibria for G_1' are the same as the Nash equilibria for G_1
- We've reduced the problem to whether C's payoffs are those of G_1' or G_2

Incomplete-information games*

  - *Condition 2 (common prior)*: the probability distribution over the games in G is common knowledge (i.e., known to all the agents)

- So a Bayesian game defines:
  - the uncertainties of agents about the game being played,
  - what each agent believes the other agents believe about the game being played
  - the concept of a Nash equilibrium can be extended to Bayesian games: **Bayes-Nash equilibrium**
    - The details are complicated, and we will skip them

- Example: Auctions
  - several kinds of auctions are incomplete-information, and can be modeled as Bayesian games

Cooperative Game Theory

- Problems with non-cooperative game theory
  - *Binding agreements* are not possible
    - *E.g.* binding commitment to cooperate in the Prisoner's dilemma
  - Utility is assigned to *individuals* as a result of individual action
    - Each agent is assumed to be an individual utility maximizer

- In many real-world situations, these assumptions do not hold – **cooperative game theory** addresses these limitations

→ In *non-cooperative game theory*, the basic modeling unit is the *individual*

→ In *cooperative (or coalitional) game theory*, the modeling unit is the *group*

Further Reading

- Wooldridge, M. (2009). *An Introduction to MultiAgent Systems*, 2nd ed., John Wiley & Sons: Chap. 11
- Shoham, Y. and Leyton-Brown, K. (2008). *Multiagent systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press: Chap. 3, 5