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Agents and Multi-Agent Systems

Multi-Agent Decision Making
Social Choice Theory

2023/2024

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Social Choice

- Social choice theory is concerned with making *group decisions*: given the preferences of different agents, how do we aggregate them to reflect the wishes of the population as a whole?
  - Formally, the issue is *combining preferences* to derive a *social outcome*
  - Typical example: *voting* procedures

- Strategic flavor: how should an agent vote to bring about its most preferred outcome?
  - Take into account its own preferences and those of others
  - Strategic manipulation is the possibility for agents to benefit from strategically misrepresenting their preferences (declared preferences $\neq$ true preferences)

Social Choice Model

- Assume a set $\mathrm{Ag} = \{1, \ldots, n\}$ of *voters*.

  These are the entities who will be expressing preferences

- *Voters* make group decisions with respect to a set of *outcomes*

  $$\Omega = \{\omega_1, \omega_2, \ldots\}$$

  - In an election, these can be seen as the *candidates*
    - If $|\Omega| = 2$, we have a *pairwise election*
    - If $|\Omega| > 2$, we have a *general voting scenario*

- Each voter has preferences over Ω : an *ordering* over the set of possible outcomes Ω
  - As a group choice, we may want to simply choose one
  - In other cases, we may want to rank the outcomes (candidates)

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Preference Aggregation

- Different voters typically have different preference orders

The fundamental problem of social choice theory

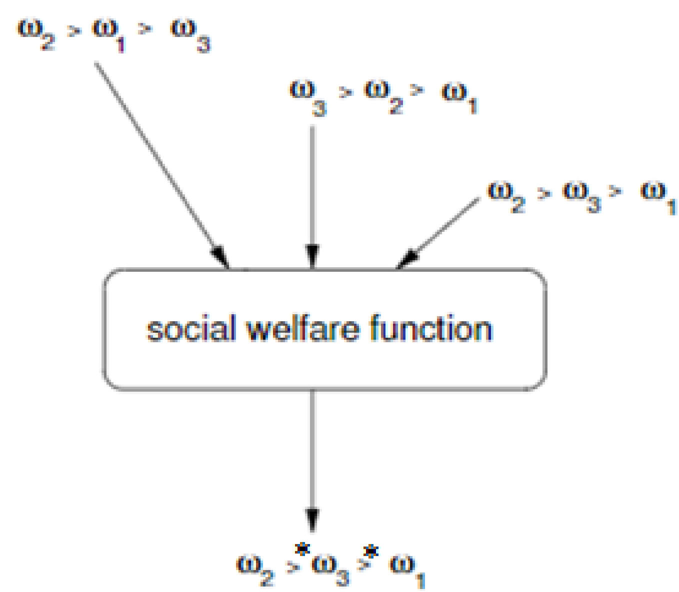
> *Given a collection of preference orders, one for each voter, how do we combine these to derive a group decision, that reflects as closely as possible the preferences of voters?*

- Two variants of preference aggregation:
  - social welfare functions produce a social preference order
  - social choice functions produce a single choice

Social Welfare Functions

- A *social welfare function* takes the voter preferences and produces a social preference order:

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{\text{n times}} \mapsto \Pi(\Omega)$$



- example: beauty contest

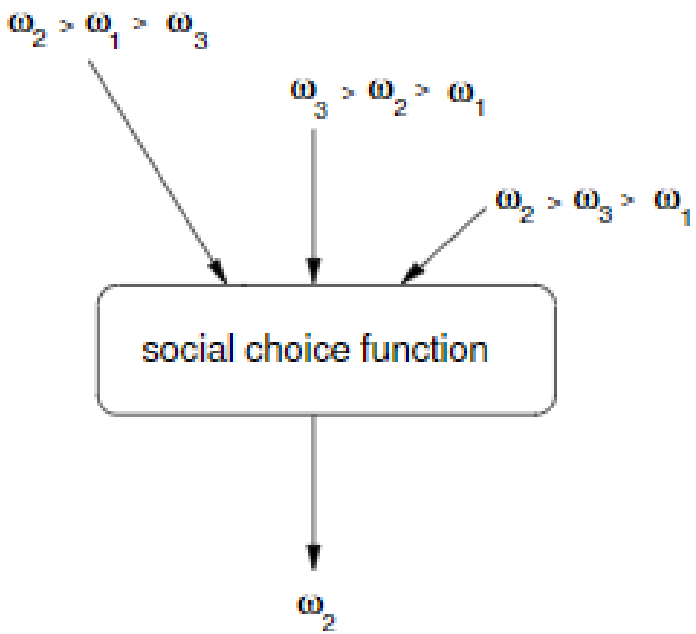
$\succ^*$ indicates the outcome of a social welfare function: $\omega \succ^* \omega'$

– which indicates that ω is ranked above ω' in the social ordering

Social Choice Functions

- *A social choice function* takes the voter preferences and produces a single choice.

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{\text{n times}} \rightarrow \Omega$$



- example: presidential election

Voting procedures

- Plurality Voting
- Sequential Majority
- Borda Count
- … others (not mentioned here)

Voting procedure: Plurality

- Most commonly used to select a single outcome (but idea generalizes)
  - *Social choice function*

- Each voter submits its preference ordering.
- Each candidate gets one point for every preference order that ranks it first
- Winner is the one with largest number of points.

Paradox

Suppose $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and:

40% voters voting for ω_1

30% of voters voting for ω_2

30% of voters voting for ω_3

ω_1 wins, even though a *clear majority (60%)* prefer another candidate!

- With only two candidates, then plurality is a *simple majority election*.

Plurality: strategic manipulation by tactical voting

- Suppose your preferences are:

$$\omega_1 \succ_i \omega_2 \succ_i \omega_3$$

you believe:

- 49% of voters have preferences $\omega_2 \succ_i \omega_1 \succ_i \omega_3$
- 49% of voters have preferences $\omega_3 \succ_i \omega_2 \succ_i \omega_1$

- You may do better voting for ω_2 , even *though this is not your true preference* profile

- *Tactical voting*: strategically misrepresenting your preferences to bring about a more preferred outcome

Plurality: Condorcet's paradox

- Suppose $Ag = \{1, 2, 3\}$ and $\Omega = \{\omega_1, \omega_2, \omega_3\}$ with:
  - $\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$
  - $\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$
  - $\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$

- For every possible candidate, 2/3 (a majority) of the voters prefer another outcome

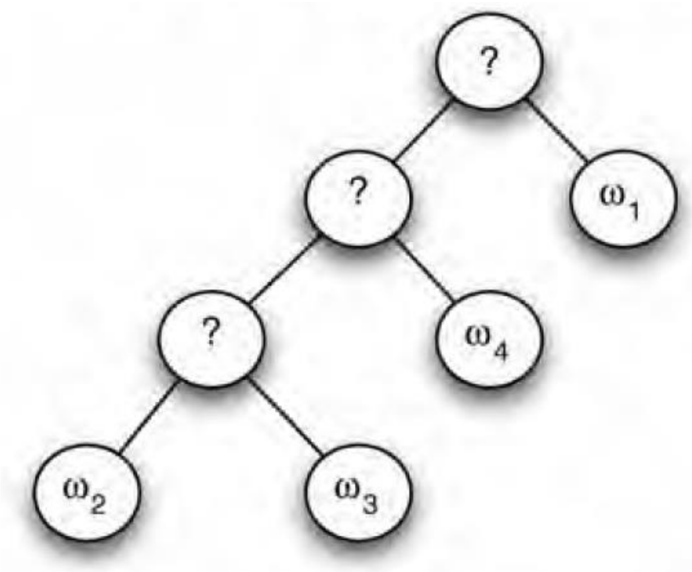
Nicolas de Caritat, marquis de Condorcet (1743-1794)

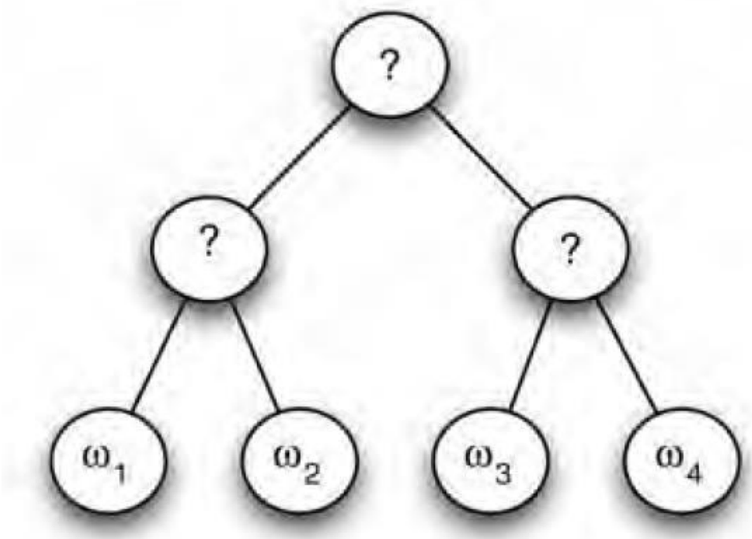
- In a democracy, it seems inevitable that we can't choose an outcome that will make everyone happy.

> *Condorcet's paradox: there are situations in which, no matter which outcome we choose, a majority of voters will be unhappy with the outcome chosen*

Voting procedure: Sequential Majority

- A variant of plurality, in which players play in a series of rounds: either a *linear sequence* or a *tree* (knockout tournament).





Sequential Majority: Agendas

- Need to pick an ordering of the outcomes – the *agenda* – which determines who plays against whom
  - The final outcome depends also on the agenda (besides voter preferences)

- Problems:
  - Selecting a random order: does a democratic process depend on chance?
  - Selecting with some criteria: open to manipulation

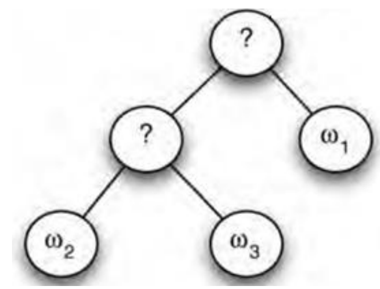
This idea is easiest to illustrate using a majority graph

Sequential Majority: Agendas

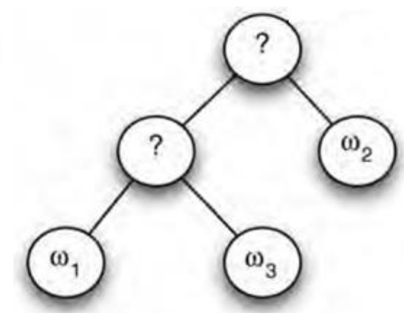
- If an equal number of voters prefer each of

$$\omega_1 \succ \omega_2 \succ \omega_3$$
$$\omega_3 \succ \omega_1 \succ \omega_2$$
$$\omega_2 \succ \omega_3 \succ \omega_1$$

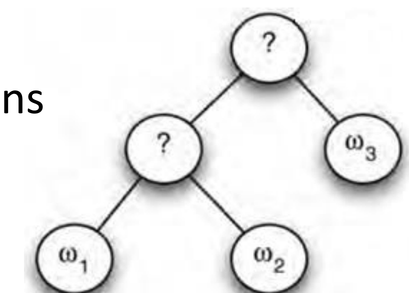
- Then, for every candidate, we can fix an agenda for that candidate to win in a sequential pairwise election!
  - With agenda $(\omega_3, \omega_2, \omega_1)$, ω_1 wins



  - With agenda $(\omega_1, \omega_3, \omega_2)$, ω_2 wins



  - With agenda $(\omega_1, \omega_2, \omega_3)$, ω_3 wins



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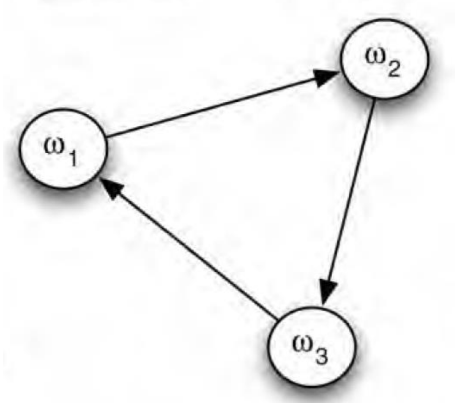
Sequential Majority: Majority Graphs

- *Majority Graph*: a compact representation of voter preferences

  a directed graph with:
  - nodes = candidates
  - an edge (ω, ω') if ω would beat ω' in a simple majority election, that is, a majority of voters rank ω above ω'.



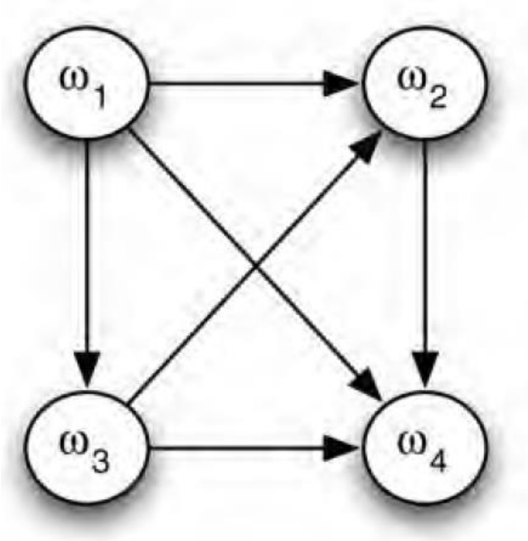
Example:
- with agenda $(\omega_3, \omega_2, \omega_1)$, ω_1 wins
- with agenda $(\omega_1, \omega_3, \omega_2)$, ω_2 wins
- with agenda $(\omega_1, \omega_2, \omega_3)$, ω_3 wins

To determine if ω_i is a possible winner, we have to find, for every other ω_j , if there is a path from ω_i to ω_j in the majority graph.

Sequential Majority: Condorcet Winners

- A *Condorcet winner* is a candidate that would beat every other candidate in a pairwise election (winner for every possible agenda)

Example:



ω_1 is a Condorcet winner: there is an edge from ω_1 to every other node

Voting procedure: Borda Count

- Plurality and Sequential Majority have many anomalies:
  - We're ignoring most information in a voter's preference order
  - We're only considering the top-ranked candidates

- The Borda Count takes the *whole preference order* into account:
  - For each candidate, we count the strength of opinion in favor for it
  - Suppose we have k candidates:
    - If ω_i appears first in a preference order, then we increment its counter by ω_i by $k-1$;
    - For the next candidate in a preference order, the counter is incremented by $k-2, \ldots,$
    - the last candidate in a preference order is not incremented $(k-k=0)$.
  - After considering all voters, we order the outcomes by their count

Borda Count: example

- Suppose:

  $\Omega = \{\omega_L, \omega_D, \omega_C\}$

  – 43% of $|Ag|$: $\omega_L \succ \omega_D \succ \omega_C$

  – 12% of $|Ag|$: $\omega_D \succ \omega_L \succ \omega_C$

  – 45% of $|Ag|$: $\omega_C \succ \omega_D \succ \omega_L$

- Result:

  – ω_L: $43 \times (3-1) + 12 \times (3-2) + 45 \times (3-3) = 98$

  – ω_D: $43 \times (3-2) + 12 \times (3-1) + 45 \times (3-2) = 112$

  – ω_C: $43 \times (3-3) + 12 \times (3-3) + 45 \times (3-1) = 90$

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Desirable Properties of Voting Procedures

Can we classify the properties of a “good” voting procedure?

Three key properties:

- The Pareto condition

  If everybody prefers ω_i over ω_j , then ω_i should be ranked over ω_j in the social outcome.

- The Condorcet winner condition

  If ω_i is a Condorcet winner, then ω_i should always be ranked first.

- Independence of Irrelevant Alternatives (IIA)

  Whether ω_i is ranked above ω_j in the social outcome should depend only on the relative orderings of ω_i and ω_j in voters’ preference profile

The Pareto Condition

- Recall the notion of Pareto efficiency
  - An outcome is Pareto efficient if there is no other outcome that makes one agent better off without making another worse off

- Pareto condition: if every voter ranks ω_i above ω_j, then the voting method should not choose ω_j , that is, we should have $\omega_i \succ^* \omega_j$

- *Plurality* and *Borda Count* satisfy this criterion

- *Sequential Majority* violates this criterion

The Condorcet winner condition

- Recall the notion of Condorcet winner
  - The Condorcet winner is an outcome that would beat every other in a pairwise election
  - If there is a Condorcet winner, the voting method should choose it

- *Sequential Majority* satisfies this criterion
- *Plurality* and *Borda Count* violate this criterion

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Independence of Irrelevant Alternatives

- Suppose there are a number of candidates including ω_i and ω_j and voter preferences make $\omega_i \succ^* \omega_j$
  - Now assume one voter k changes preferences, but still ranks make $\omega_i \succ_k \omega_j$
  - The independence of irrelevant alternatives condition says that however $\succ^*$ changes, we should still have $\omega_i \succ^* \omega_j$
  - In other words, if the relative ranking of ω_i and ω_j is not changed, the outcome should still rank ω_i and ω_j in the same way.
- *Plurality, Sequential Majority* and *Borda Count* do not satisfy this criterion

Arrow's Theorem

Are there any voting procedures that satisfy the Pareto condition and the Independence of Irrelevant Alternatives condition?

Arrow's theorem

For elections with more than 2 candidates, the only voting procedure satisfying these conditions is a dictatorship, in which the social outcome is in fact simply selected by one of the voters

$$f(\varpi_1, \dots, \varpi_n) = \varpi_i$$

Gibbard-Satterthwaite Theorem


- Strategic manipulation
  - Each voter has its own ‘true’ preferences (private information)
  - Each voter is free to declare *any* preference profile
  - A voting procedure is manipulable if a voter can obtain a better outcome by unilaterally changing its announced preference profile

- Is there a voting procedure that is immune to such manipulation?
  - Yes! A dictatorship: the only non-manipulable voting method satisfying the Pareto property for elections with more than 2 candidates

Gibbard-Satterthwaite theorem

Only tells us that manipulation is possible in principle

- it does not give any indication of how to misrepresent preferences
- there are voting procedures whose manipulation is computationally complex

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Further Reading

- Wooldridge, M. (2009). *An Introduction to MultiAgent Systems*, 2nd ed., John Wiley & Sons: Chap. 12
- Shoham, Y. and Leyton-Brown, K. (2008). *Multiagent systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press: Chap. 9