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Agents and Multi-Agent Systems

Multi-Agent Decision Making
Mechanism Design

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Mechanism Design

- **Mechanism design** is the strategic version of social choice theory
  - Also known as *implementation theory*, or *inverse game theory*
  - Assumes that agents will behave so as to maximize their individual (private) payoffs

- It addresses the design of effective protocols for multi-agent systems
  - What game design might give rise to certain desired behaviors, even when agent preferences are unknown?
  - We want to select a mechanism whose equilibria have desirable properties

- Typical application: *auction theory*

Auctions

- **Auctions** are mechanisms used to reach agreements on how to allocate *scarce resources* to agents
  - Paintings in auction houses
  - Items in B2B, B2C or C2C e-commerce (e.g. eBay)
  - Mineral resources or water exploitation rights
  - Electromagnetic spectrum usage rights
  - Ad space in search engines
  - …
- In general, resources are scarce and are desired by more than one agent
- Auctions allow allocating resources *efficiently* – to those that value them the most

Auction Elements

- Participants: one *auctioneer* and a collection of *bidders*
- Values: *private* value vs *public/common* value
  - A 'typical' dollar bill is worth exactly $1 for everyone (a *common* value)
  - The last dollar bill spent by John Lennon may have different *private* valuations
  - *Correlated* value: depends partly on private factors, partly on other agents' valuations
    - *E.g.* bidding for an item to sell it later
- The auctioneer chooses an appropriate auction protocol
- Bidders use bidding strategies

Auction Protocols

- Winner determination: who gets the good and at what price?
  - *first-price* vs *second-price*
- Bid disclosure: are bids known to other agents?
  - *open cry* vs *sealed-bid*
- Bidders: who bids?
  - *single-sided* vs *two-sided*
- Bidding mechanism: how many rounds?
  - *one-shot* vs *ascending/descending*

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Auctions for Single Items

- English auctions
  - *first-price, open cry, ascending*
  - Auctioneer starts with a reservation price
  - Bidders must bid more (subject to a minimum bid change) than the current highest bid, which is public
  - Good is allocated to the highest bidder when no agent is willing to raise

  - Dominant strategy: bid a small amount more than the current highest bid until bid price reaches private evaluation
  - What if the value of the good is uncertain?
    - Winner's curse: no other agent has valued the item as high

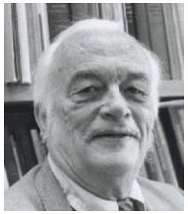
Auctions for Single Items

- Dutch auctions
  - *open cry*, *descending*
  - Auctioneer starts with an artificially high price
  - Auctioneer lowers the price by some amount, until some bidder takes it

- Japanese auctions
  - *open cry*, *ascending*
  - Auctioneer starts with a *reservation price*
  - Auctioneer increases the price by some amount
  - In each round, bidders choose to be ‘in’ or ‘out’ (for good)
  - When a single bidder is ‘in’, it gets the item for the current price

Auctions for Single Items

- First-price sealed-bid auctions
  - *one-shot*
  - Best strategy: bid less (how much less?) than true valuation, given that it would be enough to bid slightly more than the second highest bid



Canadian-American economist, 1914-1996

- Vickrey auctions
  - *second-price*, *sealed-bid*
  - Price to pay by highest bidder is the second highest bid
  - Dominant strategy: bid true valuation (the mechanism is *incentive compatible*)
    - If bid more, risk paying more than private valuation
    - If bid less, lower chances of winning
    - In case of win, amount is not affected by own bid
  - Vickrey auctions are not prone to strategic manipulation

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Combinatorial Auctions

- Many goods: *identical* (**multiunit**) or *different* (**combinatorial**)

$$Z = \{z_1, \dots, z_m\}$$

- Bidders have preferences over possible *bundles* of goods

$$v_i : 2^Z \longrightarrow \mathbb{R}$$

  - Usually, these valuation functions are non-additive
    - substituability
    - complementarity

- Bidders bid on *bundles* of goods ("*all or nothing*")

Non-Additive Valuation Functions

- v_i exhibits **substitutability** if there exist two sets of goods $Z_1, Z_2 \subseteq Z$, such that

$$Z_1 \cap Z_2 = \emptyset \quad \text{and}$$

$$v_i(Z_1 \cup Z_2) < v_i(Z_1) + v_i(Z_2)$$

  – Valuation function v_i is *subadditive*: combined value is lower than sum
  – Partial substitutes

  – Strict substitutes: combined value is the same as one of the goods
    - E.g., multiple units of the same good

Non-Additive Valuation Functions

- v_i exhibits **complementarity** if there exist two sets of goods $Z_1, Z_2 \subseteq Z$, such that

$$Z_1 \cap Z_2 = \emptyset \quad \text{and}$$

$$v_i(Z_1 \cup Z_2) > v_i(Z_1) + v_i(Z_2)$$

  – Valuation function v_i is *superadditive*: combined value is higher than sum

Winner Determination in Combinatorial Auctions

- An outcome of a combinatorial auction is an *allocation of (some of) the goods being auctioned among the agents*
  - An **allocation** is a list of sets $Z_1, \ldots, Z_n \subseteq Z$, one for each agent, such that for all $i \neq j$ we have $Z_i \cap Z_j = \emptyset$ (no good is allocated to more than one agent)

- **Winner determination problem**: what properties should be satisfied?
  - *Social welfare maximization*:

$$\max_{(Z_1,\ldots,Z_n)} \sum_{i=1}^{n} v_i(Z_i)$$

    - a NP-hard combinatorial optimization problem
    - but we only know the *declared* valuations $\hat{v}_i(Z_i)$

Example

Bidder 1	Bidder 2	Bidder 3
$v_1(x,y) = 100$	$v_2(x) = 75$	$v_3(y) = 40$
$v_1(x) = v_1(y) = 0$	$v_2(x,y) = v_2(y) = 0$	$v_3(x,y) = v_3(x) = 0$

- $\max_{(Z_1,Z_2,Z_3)} \sum_{i=1}^{3} \hat{v}_i(Z_i) = 115$, with $(Z_1, Z_2, Z_3) = (\emptyset, \{x\}, \{y\})$

- Charging winners for their bids:
  - If agents 1 and 2 bid truthfully, agent 3 is better off declaring, for example, $\hat{v}_3(y) = 26$

  → Not *incentive compatible*

Vickrey-Clarke-Groves (VCG)

- If truth-telling is rational, then we will know the *true* valuations $v_i(Z_i)$, and can thus use them to maximize social welfare

- Given a set $\hat{v}$ of declared preferences: $\chi(\hat{v}) = \arg\max_{(Z_1,\ldots,Z_n)} \sum_i \hat{v}_i(Z_i)$,

  in a VCG mechanism payment by i contains <u>two components</u>:

  - Every other agent's utility for the choice that would have been made had i not participated: $\sum_{j \neq i} \hat{v}_j\left(\chi(\hat{v}_{-i})\right)$
    - Each agent is made to pay his *social cost* – a 'compensation' for the other agents

  - Every other agent's utility for the mechanism's choice: $\sum_{j \neq i} \hat{v}_j\left(\chi(\hat{v})\right)$

Vickrey-Clarke-Groves (VCG)

Payment by i:

$$\wp_i = \sum_{j \neq i} \hat{v}_j\big(\chi(\hat{v}_{-i})\big) - \sum_{j \neq i} \hat{v}_j\big(\chi(\hat{v})\big)$$

- VCG is incentive compatible: payment does not depend on declaration $\hat{v}_i$

- VCG is a generalization of the Vickrey auction
  - If there is a single good:
    - $\sum_{j \neq i} \hat{v}_j\big(\chi(\hat{v}_{-i})\big)$ is the valuation of the second highest bidder (the only one that would benefit from i not participating)
    - $\sum_{j \neq i} \hat{v}_j\big(\chi(\hat{v})\big)$ is equal to 0, since no other agent gets the single good
    - Thus, $\wp_i$ is the second highest bid

Vickrey-Clarke-Groves (VCG)

Bidder 1	Bidder 2	Bidder 3
$v_1(x,y) = 100$ $v_1(x) = v_1(y) = 0$	$v_2(x) = 75$ $v_2(x,y) = v_2(y) = 0$	$v_3(y) = 40$ $v_3(x,y) = v_3(x) = 0$

$Z_1 = \emptyset$ $\quad$ $Z_2 = \{x\}$ $\quad$ $Z_3 = \{y\}$

$\wp_1 = 0$ $\quad$ $\wp_2 = 100 - 40 = 60$ $\quad$ $\wp_3 = 100 - 75 = 25$

- If, for example, $\hat{v}_3(y) = 26$, agent 3 would still pay 25 (although agent 2 would pay more)

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Two-sided Auctions

- Many buyers and sellers bidding *simultaneously* in the same auction
  Typical example: stock market

- Double Auction
  - Agents bid as many times as they want: price and quantity to buy/sell
  - Bids are put in an *order book*

  - *Continuous double auction (CDA):* match bids in order book as soon as a new bid is received
  - *Call market*, trade is attempted at predefined intervals (*clearing*):
    - sell/buy bids are ranked in ascending/descending order and then matched

  - Clearing price is somewhere within the *bid-ask* spread (buy and sell bids)

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Two-sided Auctions

- Call market example

before	Sell:	5@$1	3@$2	6@$4	2@$6	4@$9	
	Buy:	6@$9	4@$5	6@$4	3@$3	5@$2	2@$1

$\Downarrow$

after	Sell:	2@$6	4@$9		
	Buy:	2@$4	3@$3	5@$2	2@$1

Further Reading

- Wooldridge, M. (2009). *An Introduction to MultiAgent Systems*, 2nd ed., John Wiley & Sons: Chap. 14
- Shoham, Y. and Leyton-Brown, K. (2008). *Multiagent systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press: Chap. 10-11