

Compilers

Design and Implementation

Data-Flow Analysis

Overview

Available Expressions Problem

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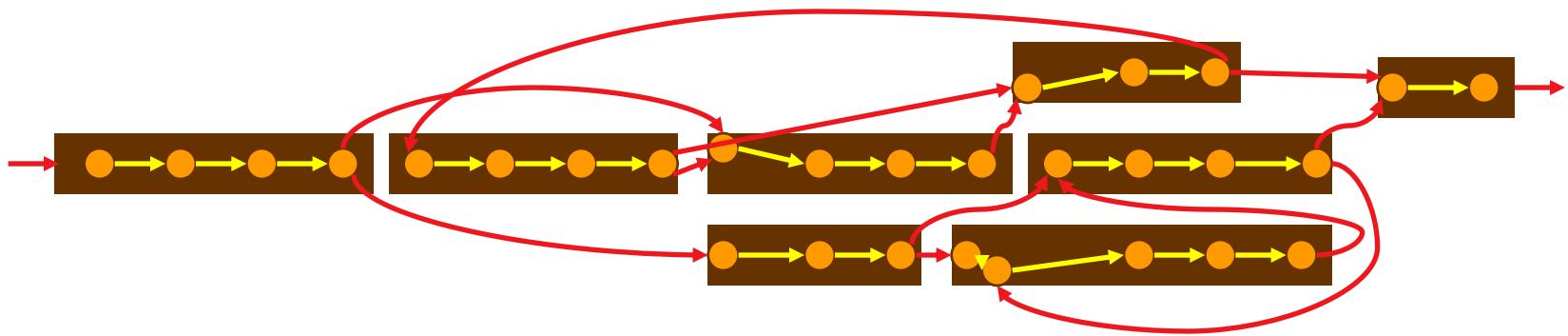
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Outline

- Overview of Control-Flow Analysis
- Available Expressions Data-Flow Analysis Problem
- Algorithm for Computing Available Expressions
- Practical Issues: Bit Sets

Control Flow of a Program

- Forms a Graph



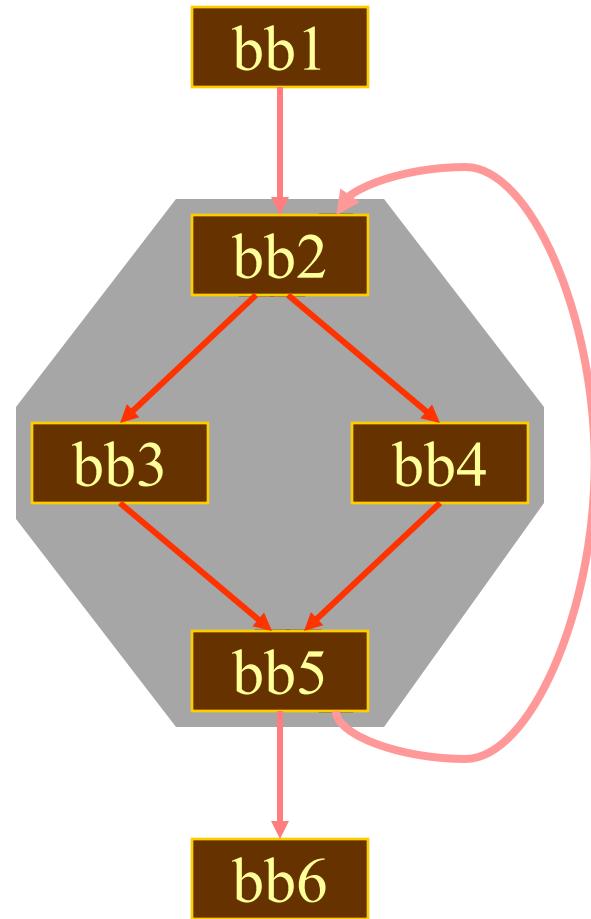
- A Very Large Graph
- Create Basic Blocks
- A Control-Flow Graph (CFG) connects basic blocks

Control Flow Graph (CFG)

- Control-Flow Graph $G = \langle N, E \rangle$
- Nodes(N): Basic Blocks
- Edges(E): $(x,y) \in E$ iff first instruction in the basic block y follows the last instruction in the basic block x

Identifying Recursive Structures Loops

- Identify Back Edges
- Find the nodes and edges in the loop given by the back edge
- Other than the Back Edge
 - Incoming edges only to the basic block with the back edge head
 - one outgoing edge from the basic block with the tail of the back edge
- How do I find the Back Edges?

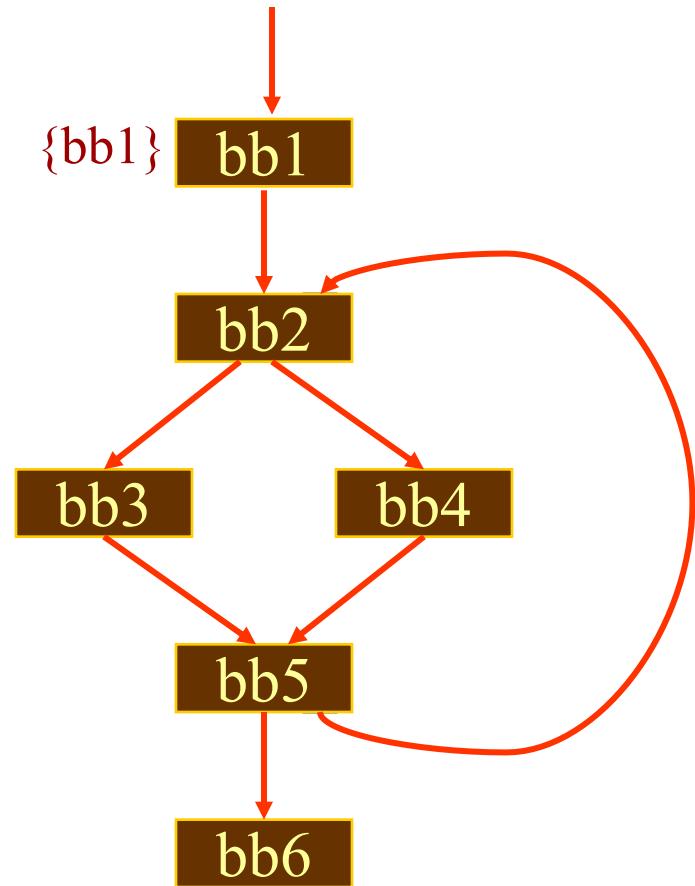


Computing Dominators

- Algorithm
 - Make dominator set of the entry node as itself
 - Make dominator set of the remainder nodes include all the nodes
 - Visit the nodes in any order
 - Make dominator set of the current node as the intersection of the dominator sets of the predecessor nodes and the current node
 - Repeat until no change

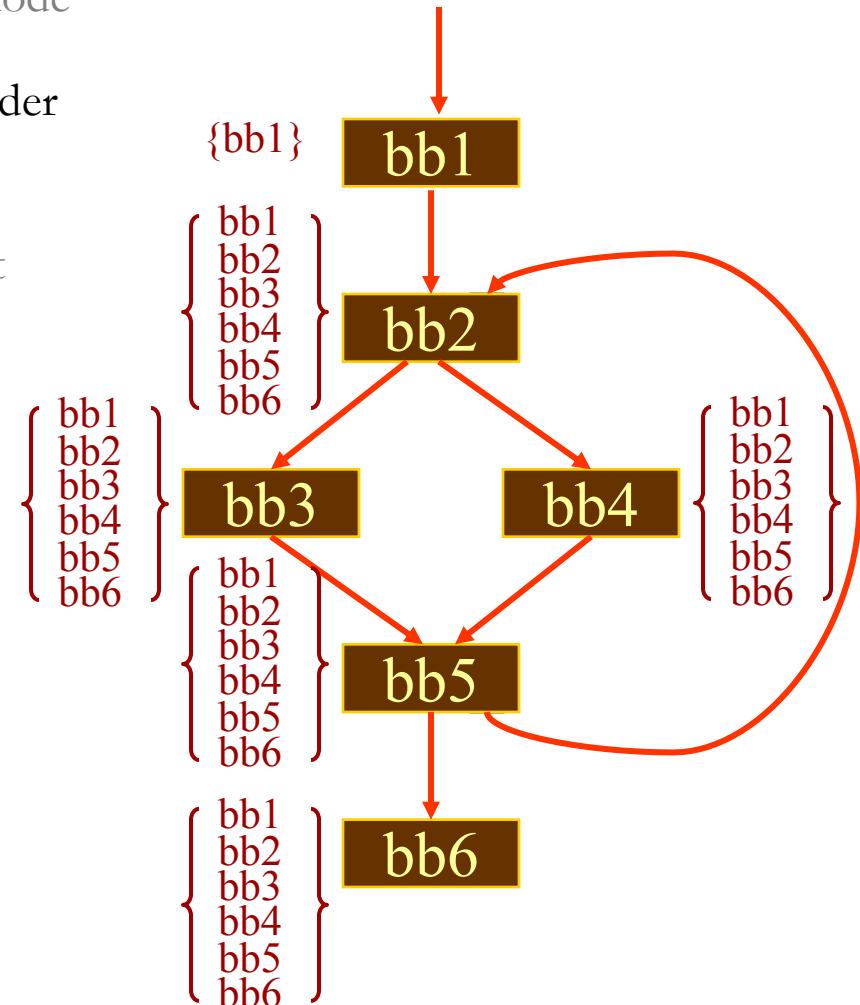
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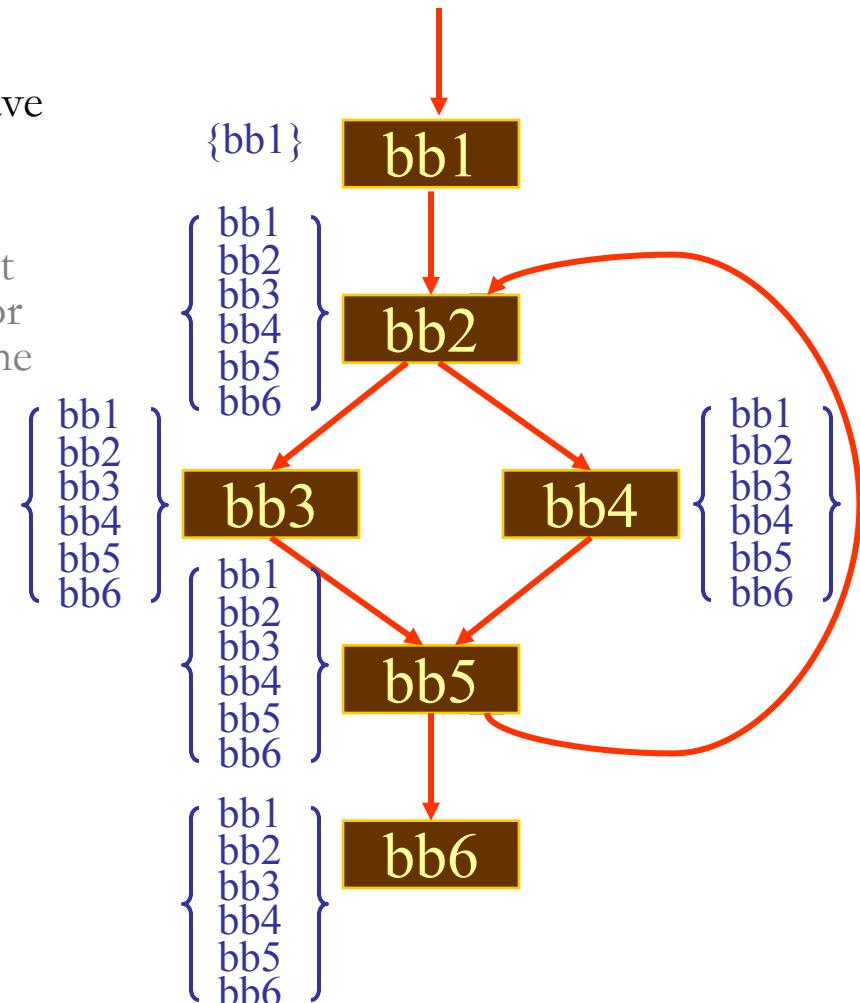
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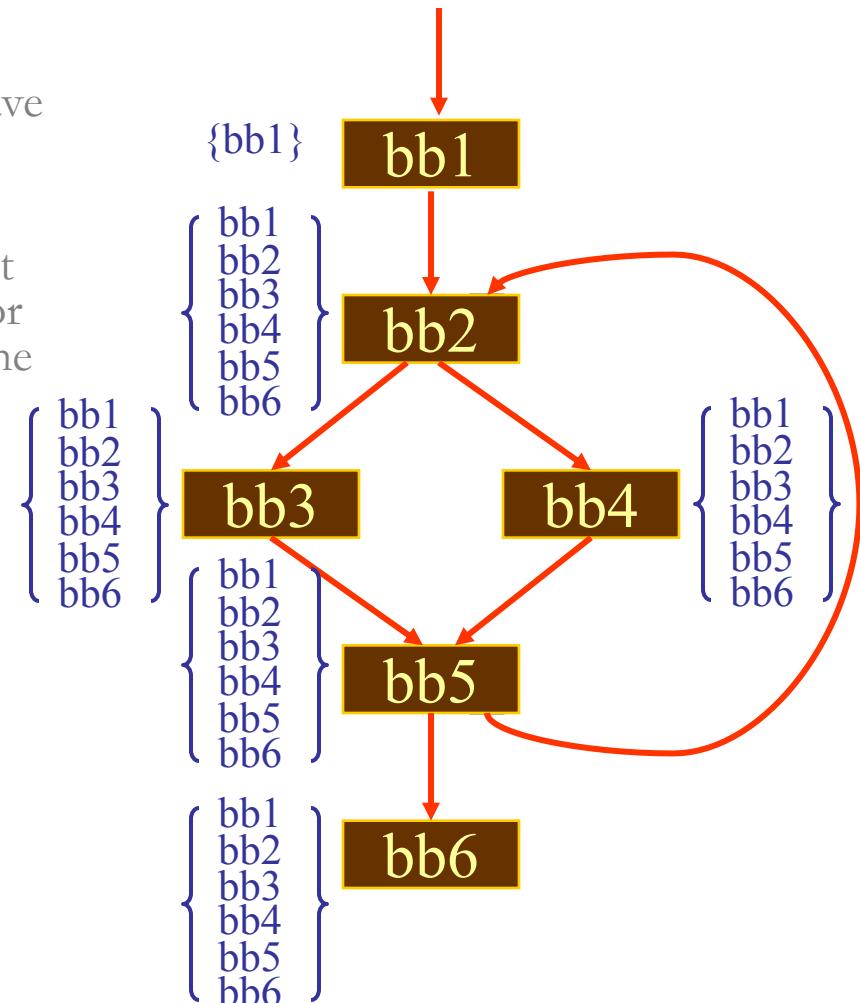
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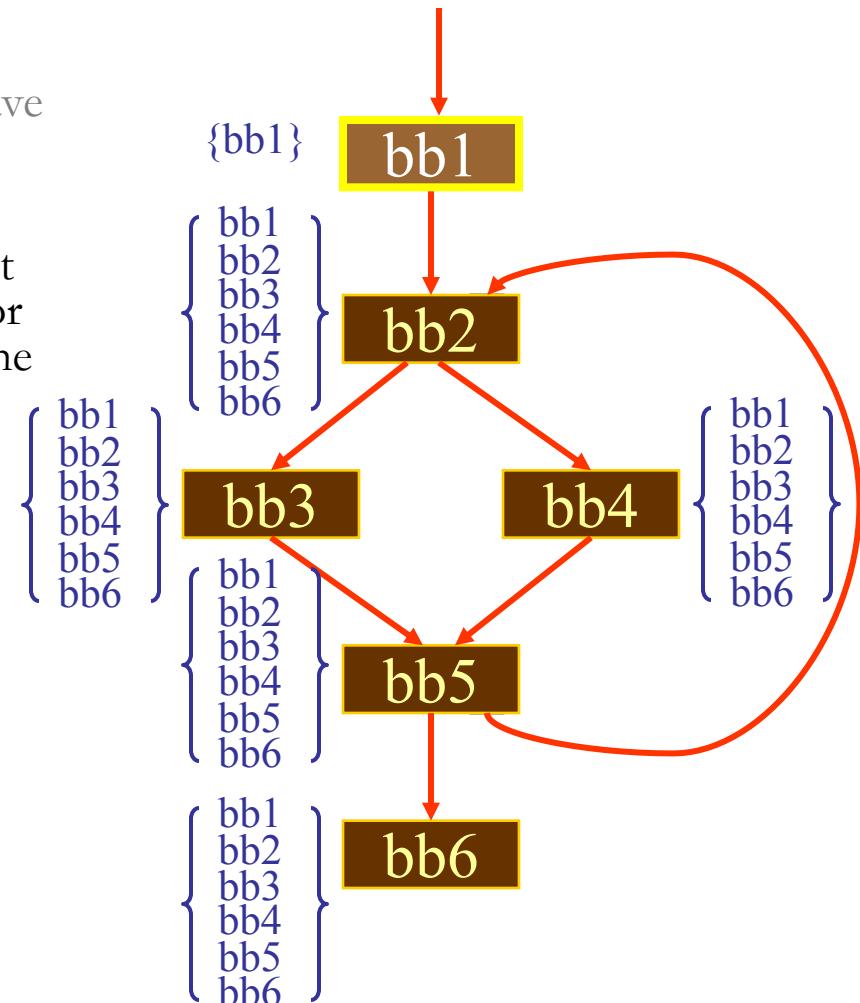
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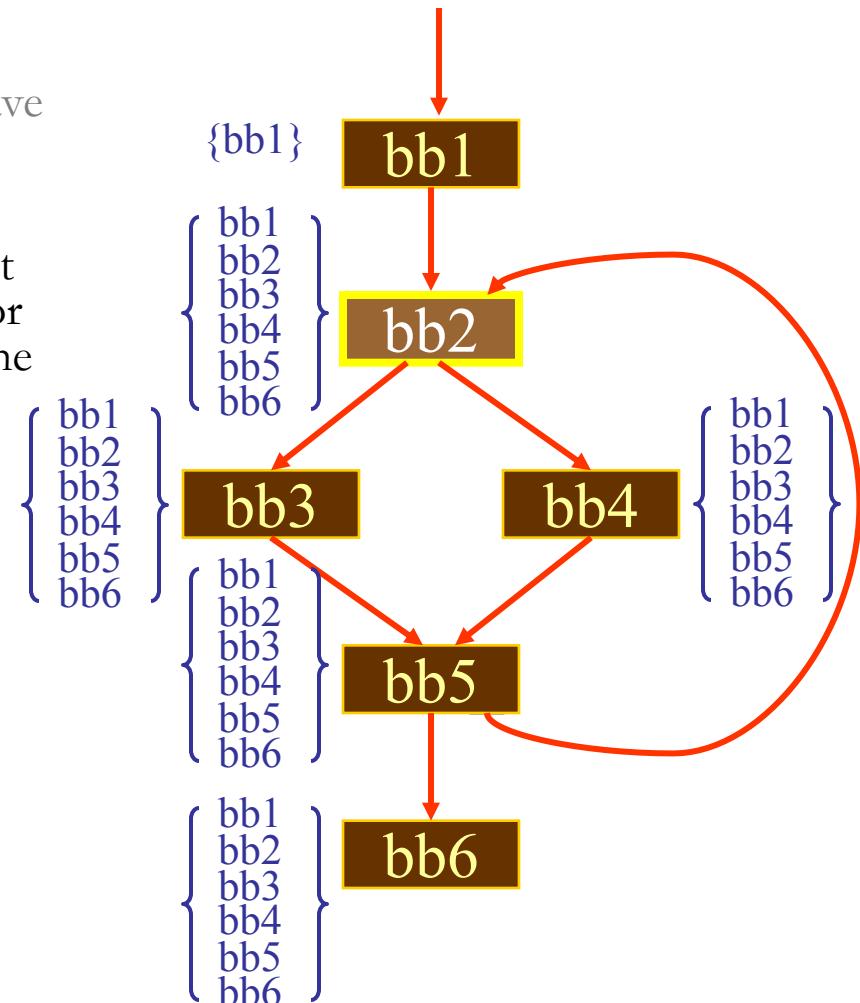
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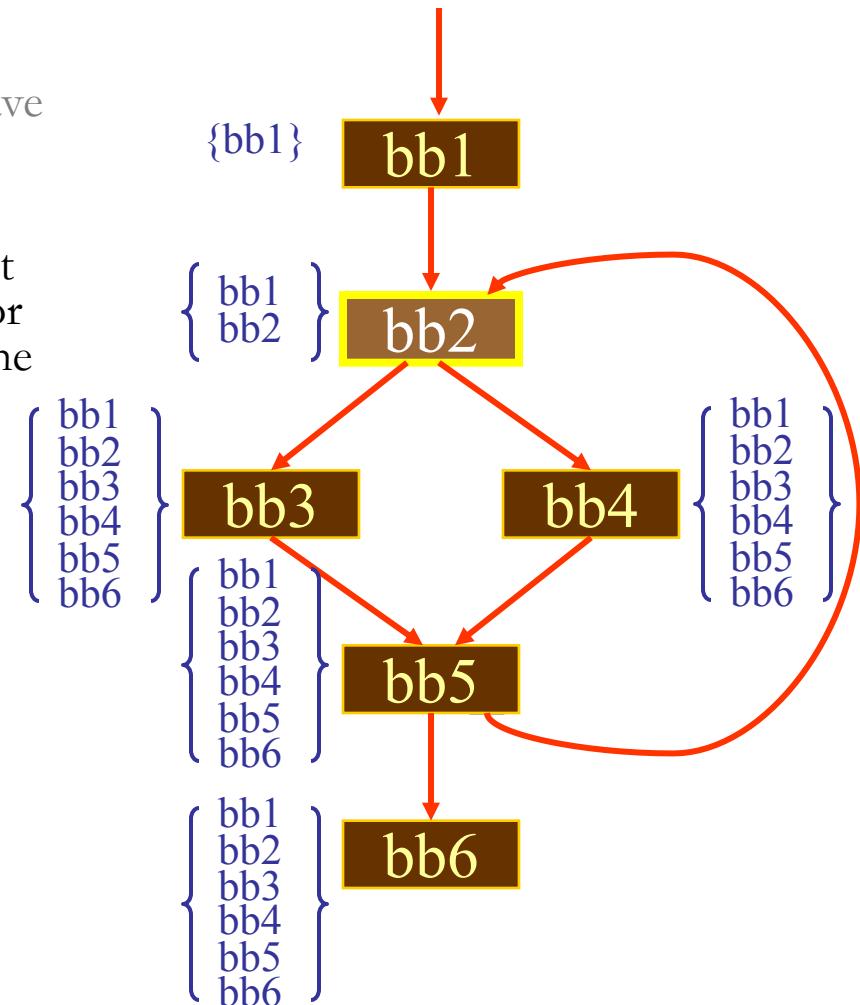
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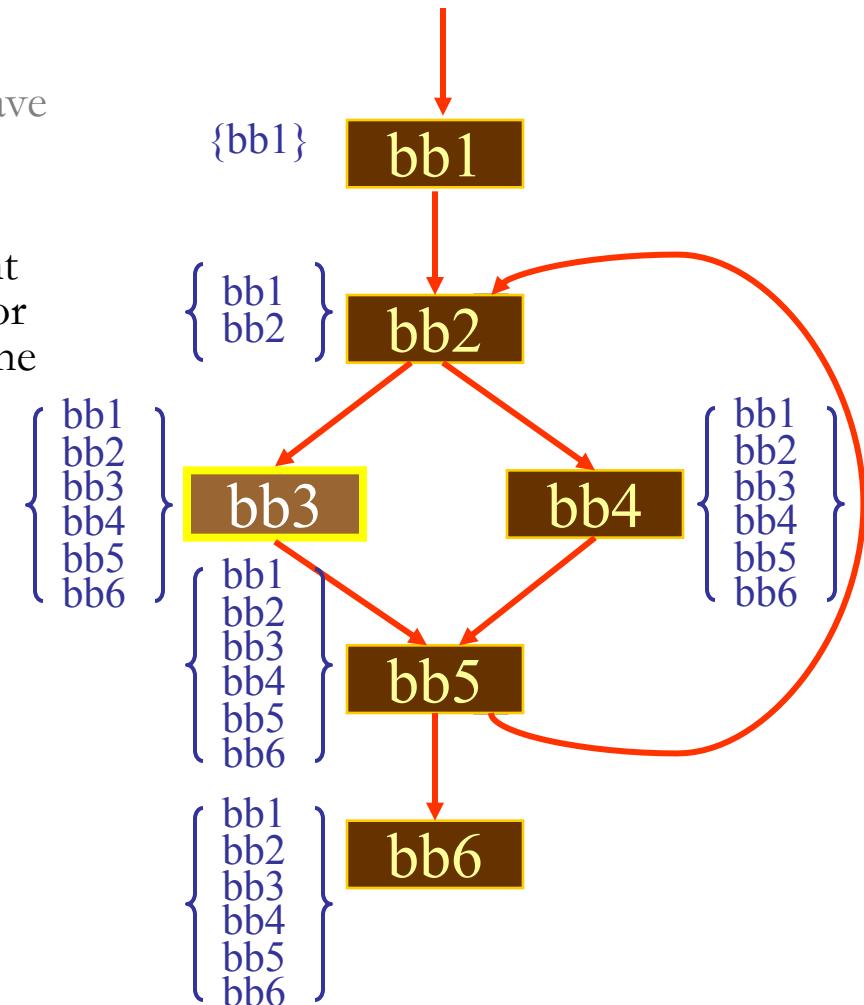
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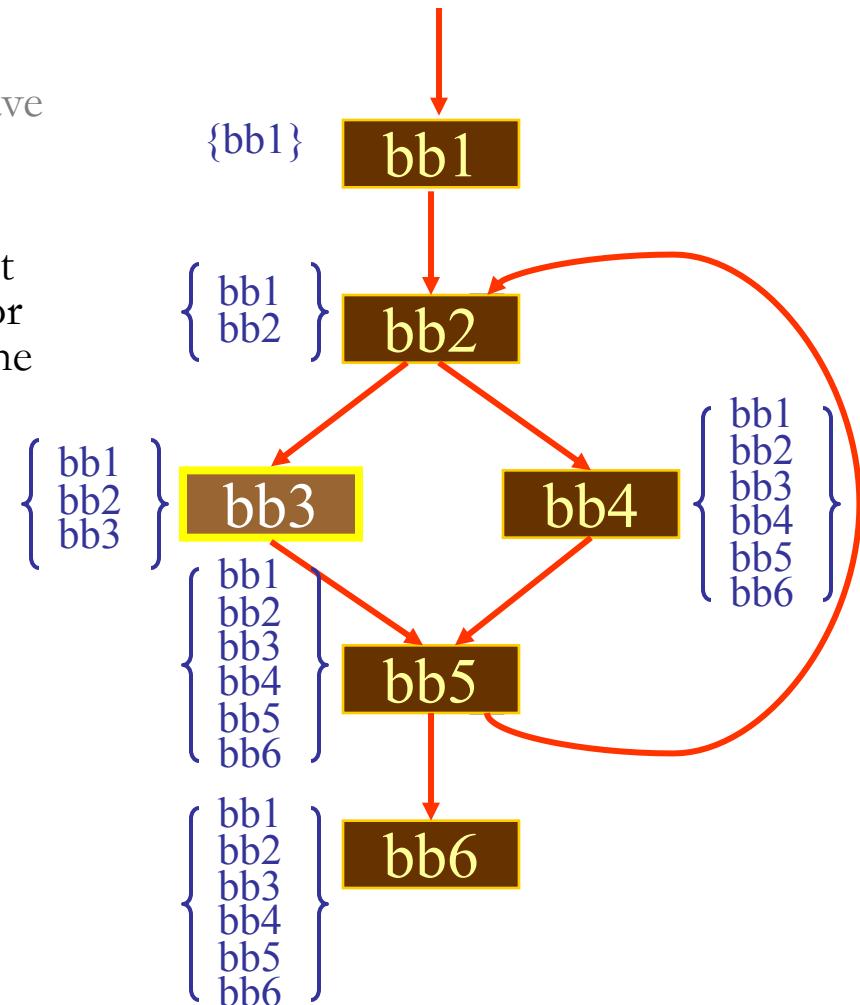
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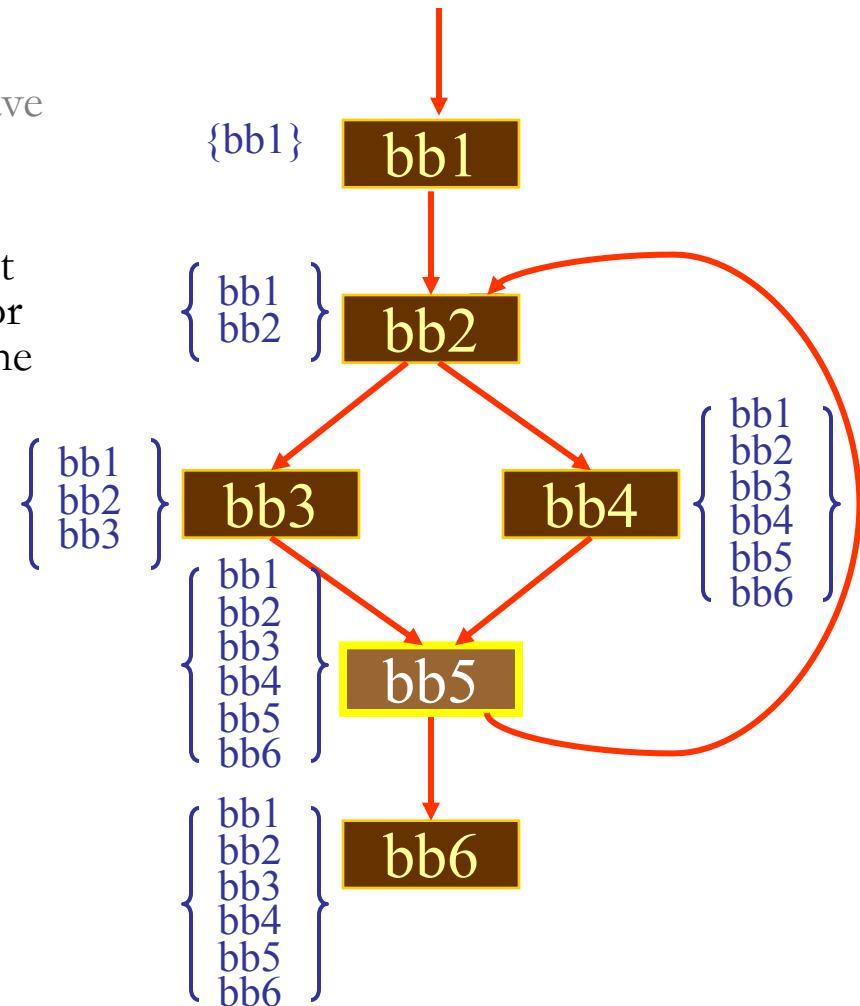
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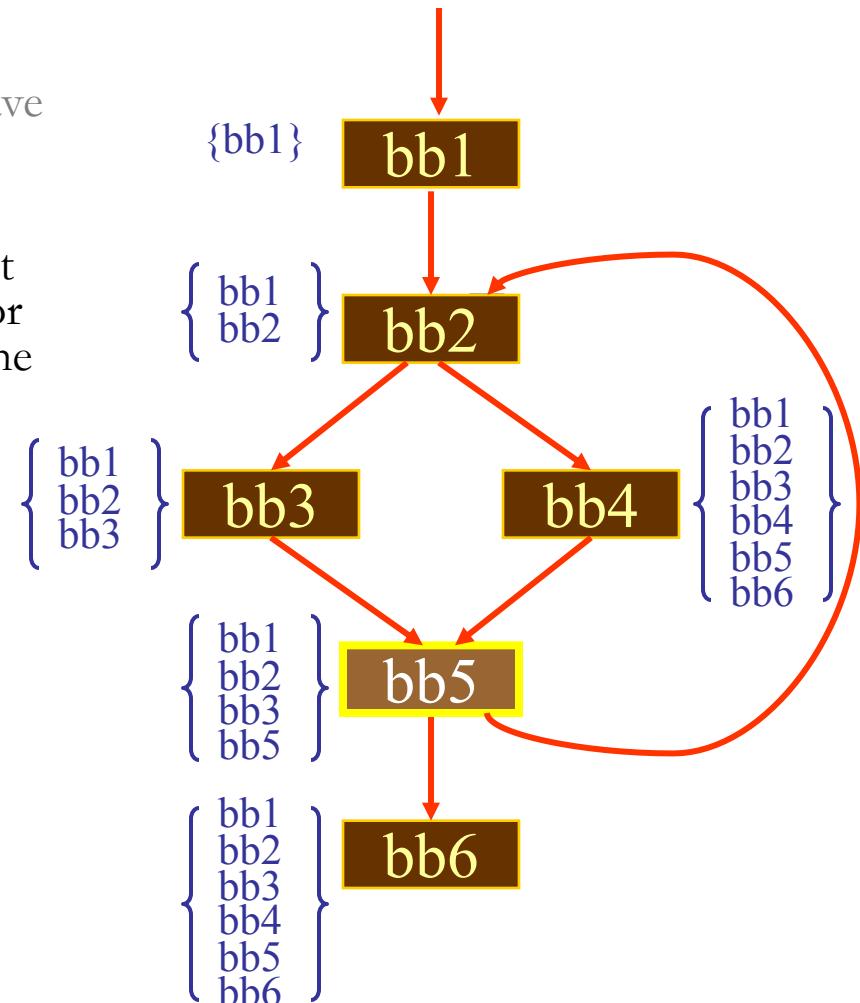
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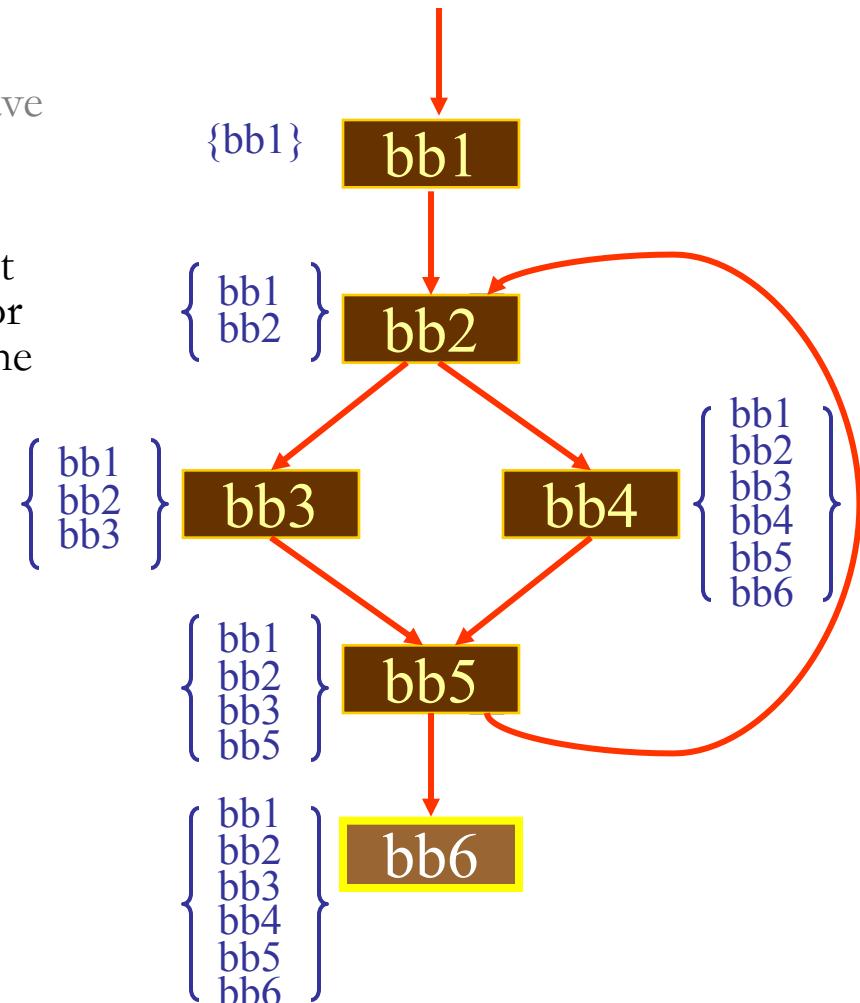
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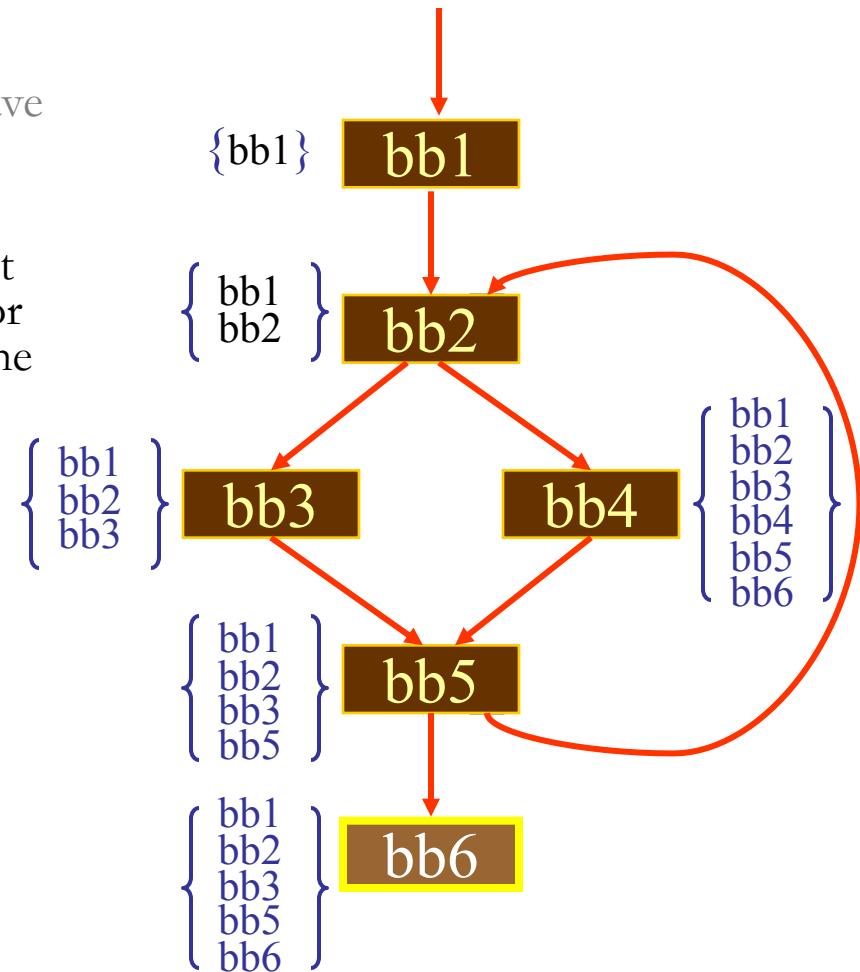
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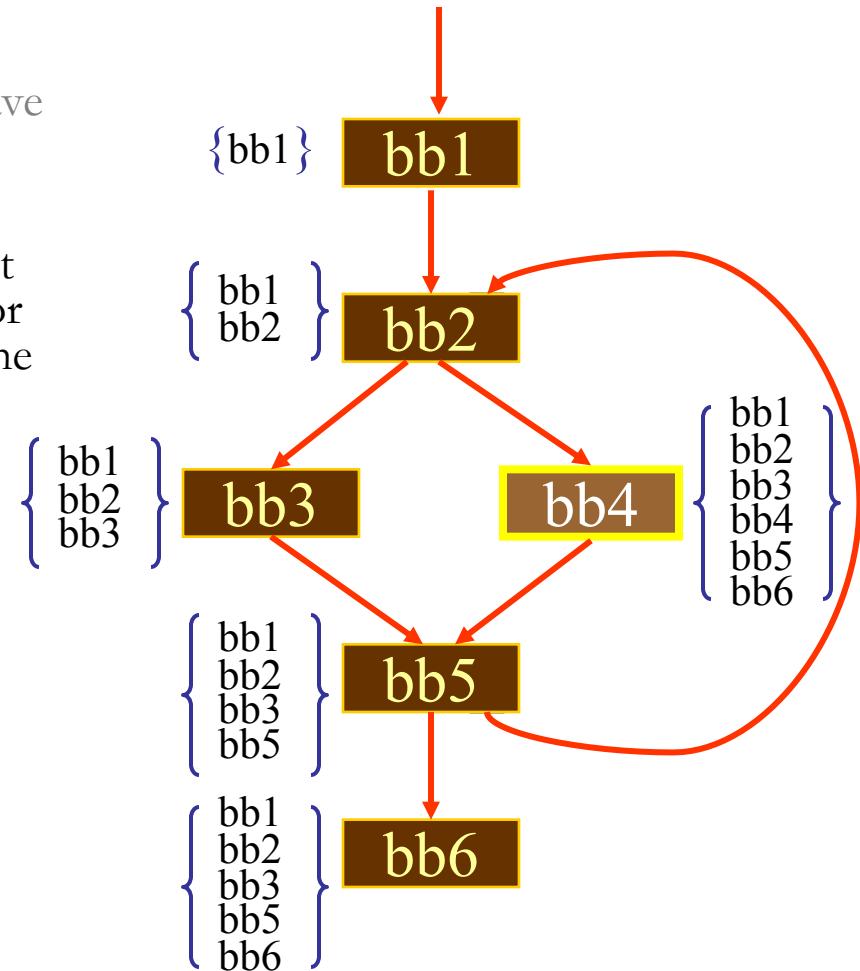
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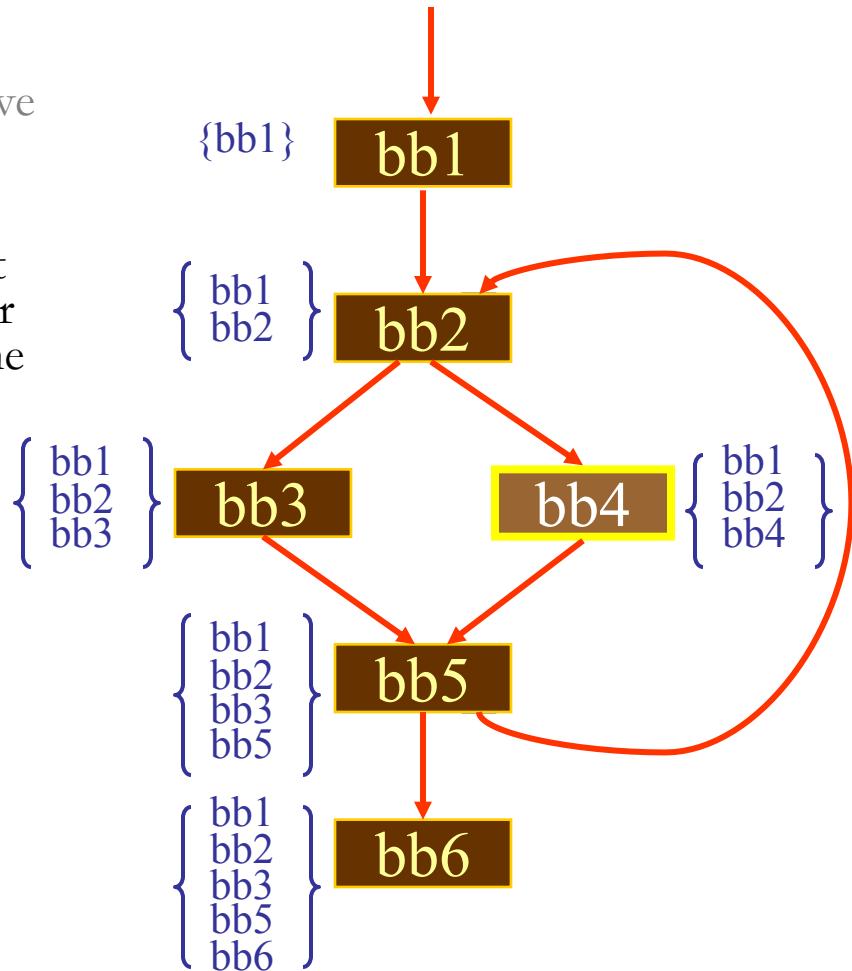
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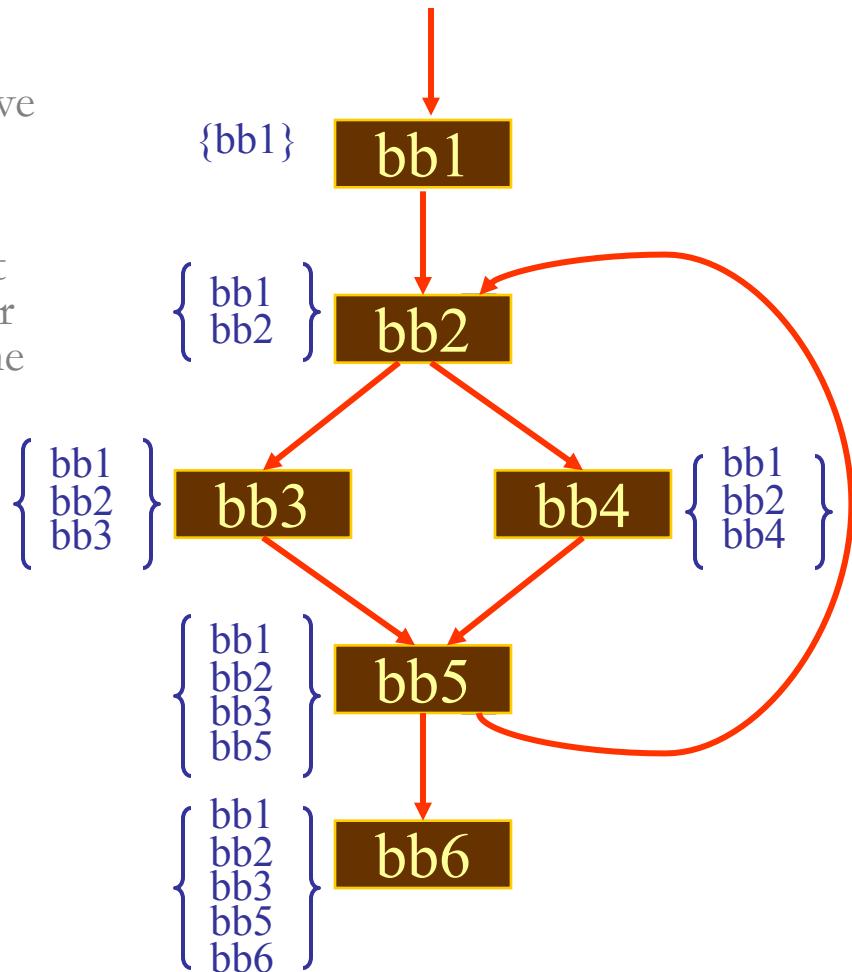
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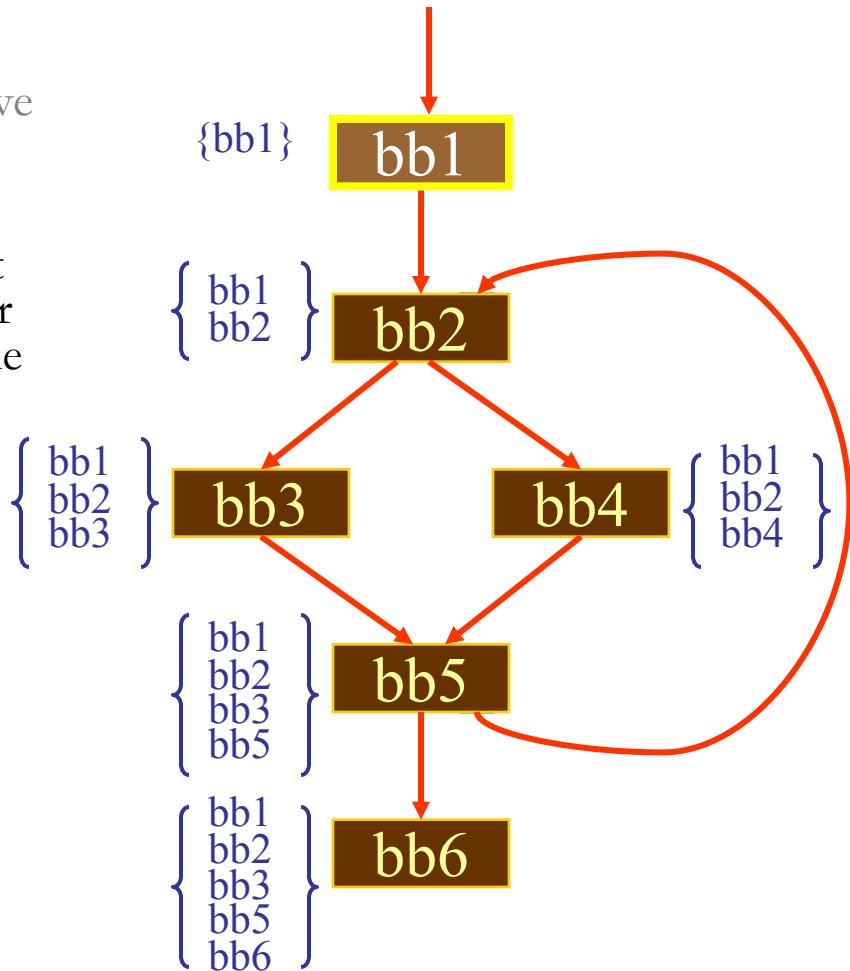
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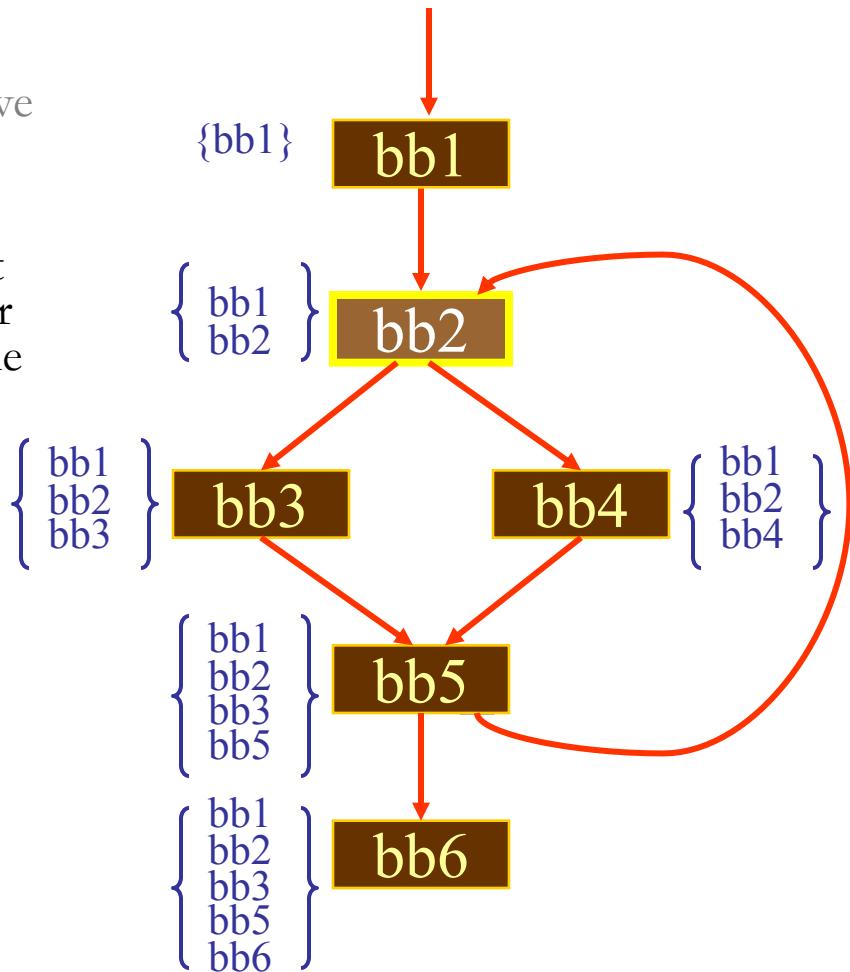
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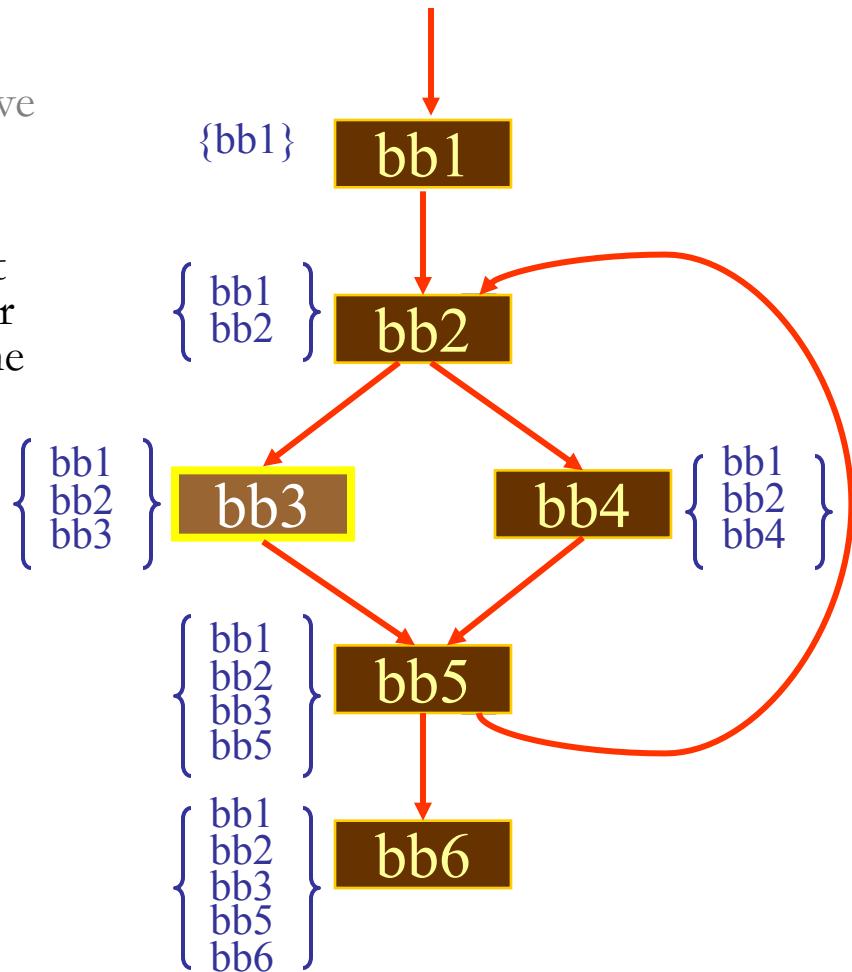
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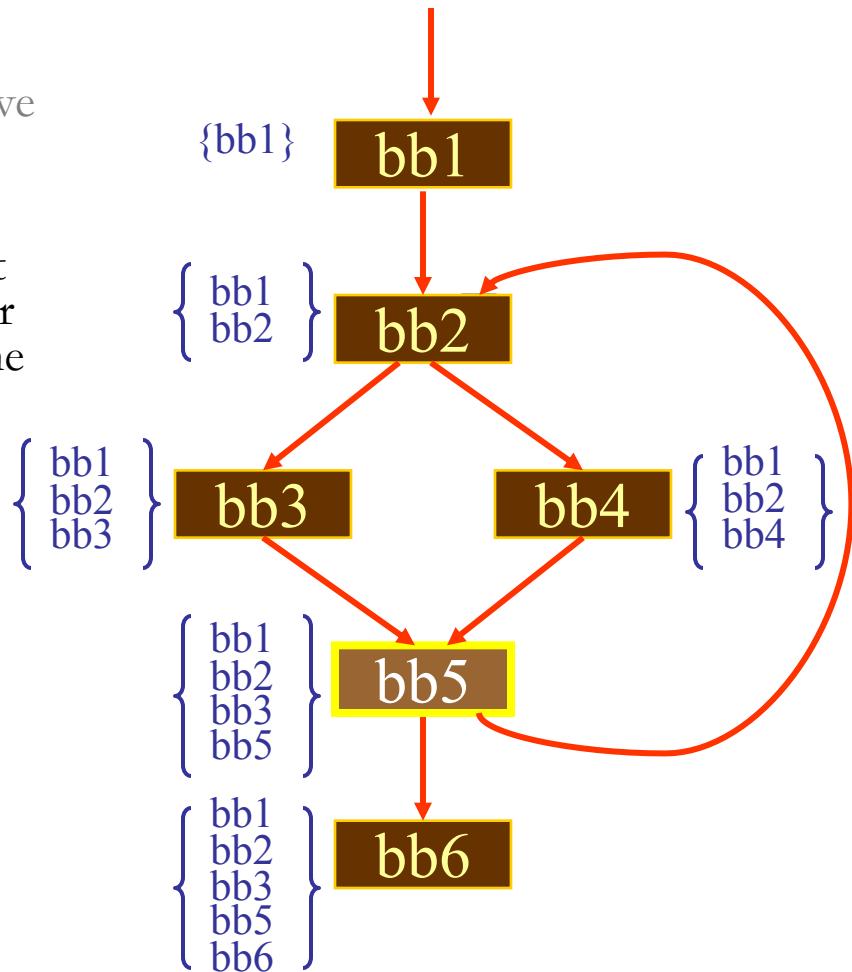
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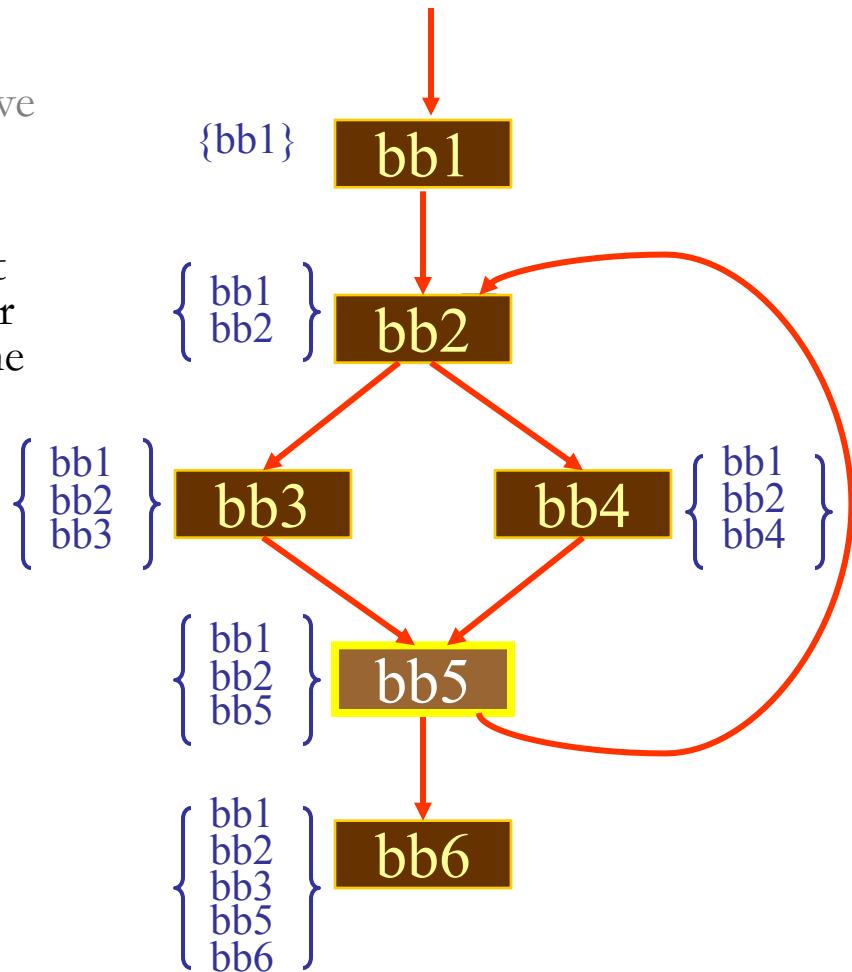
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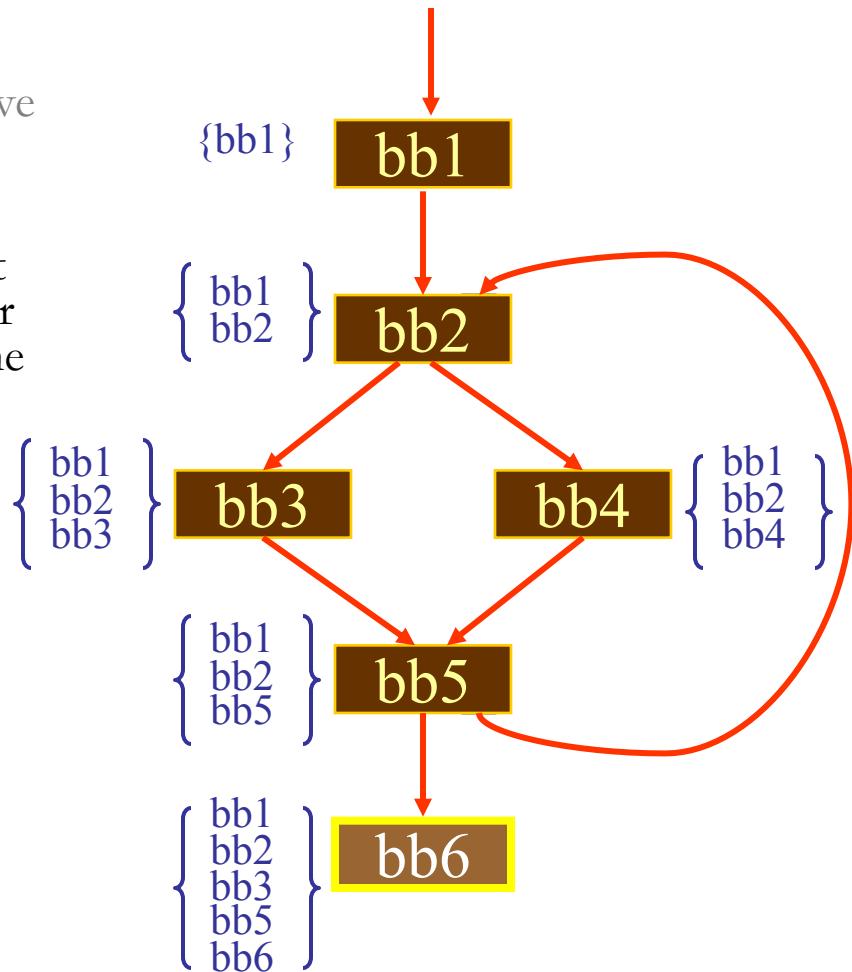
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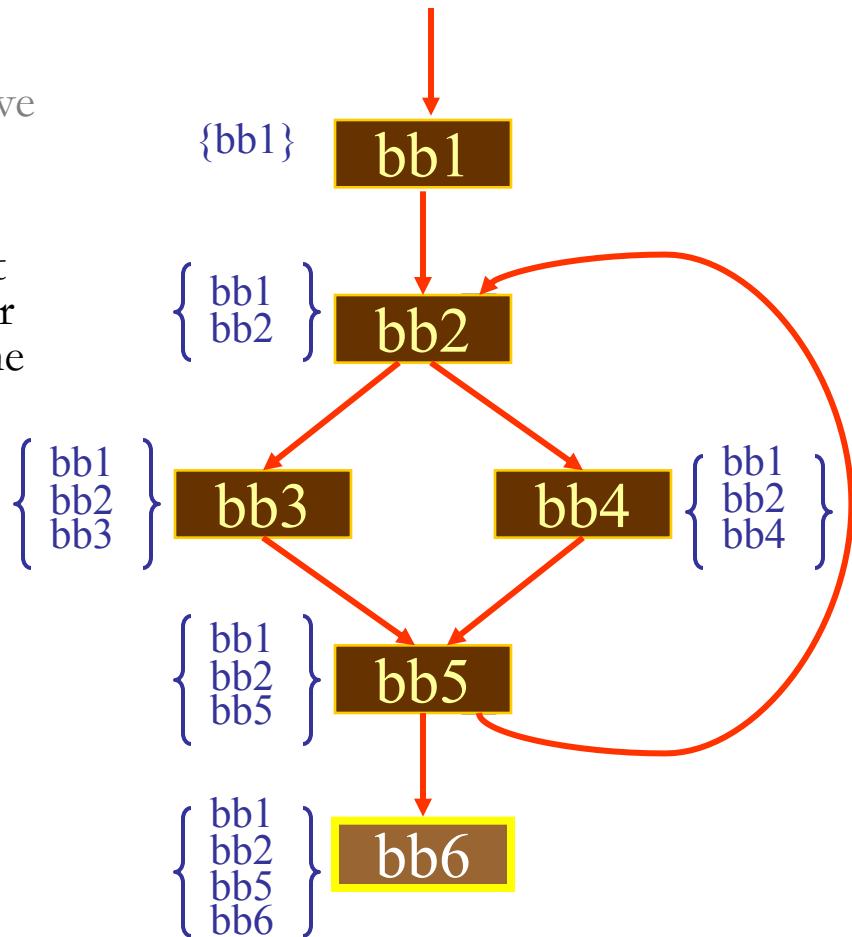
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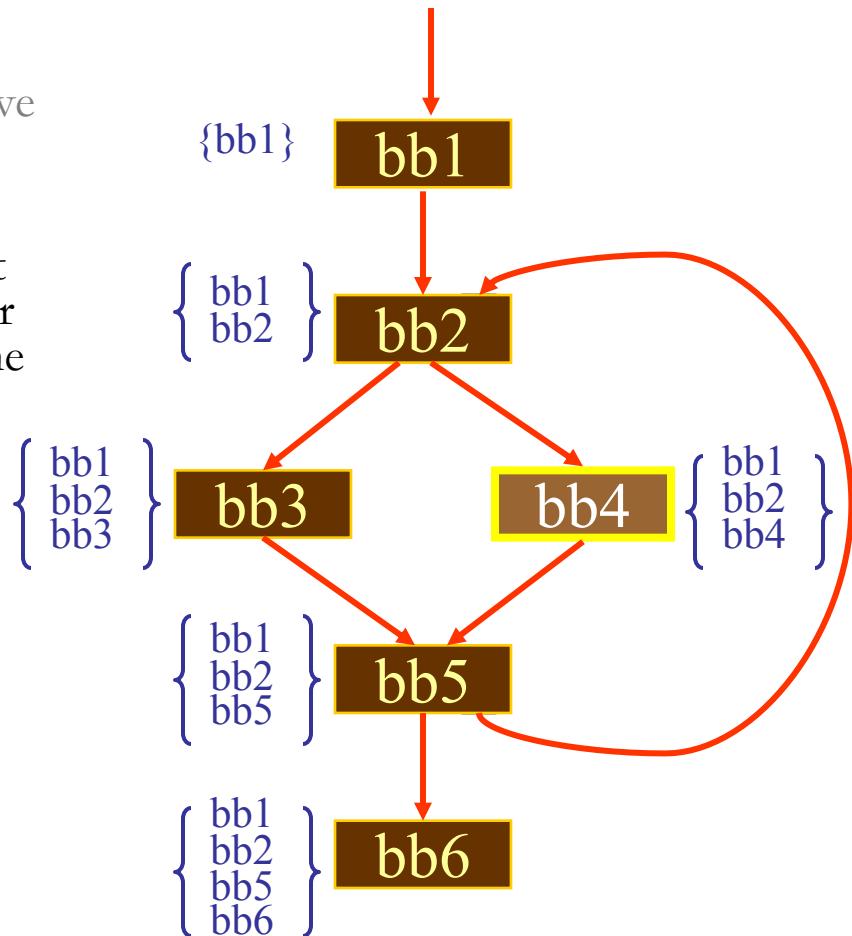
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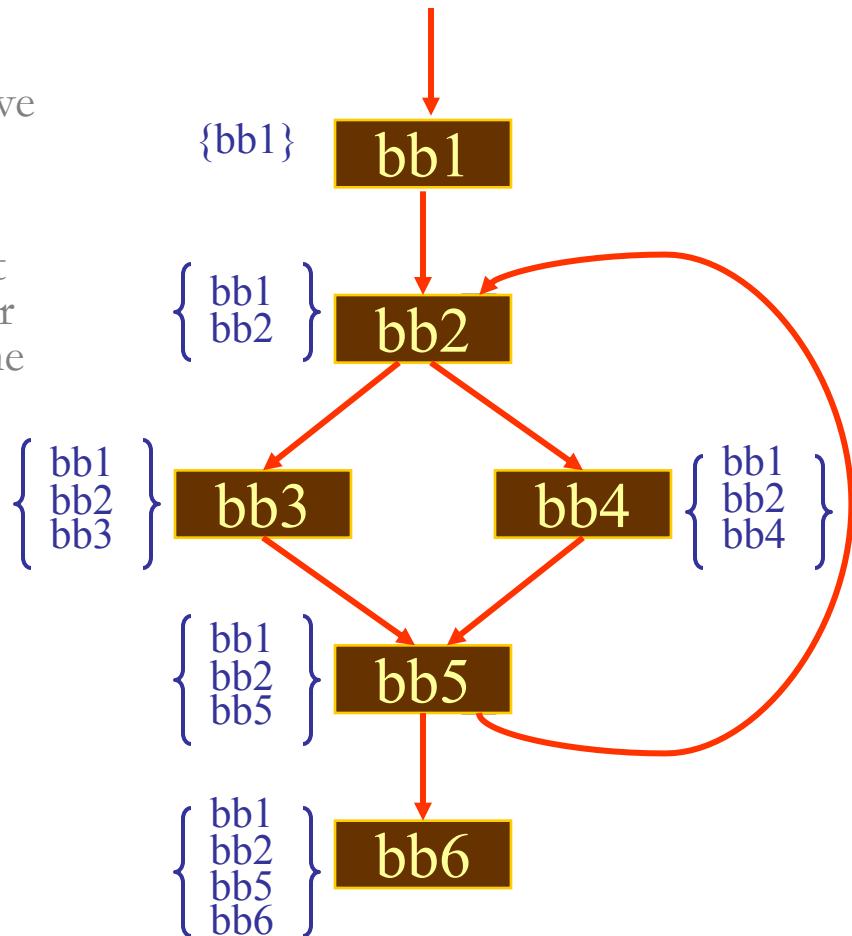
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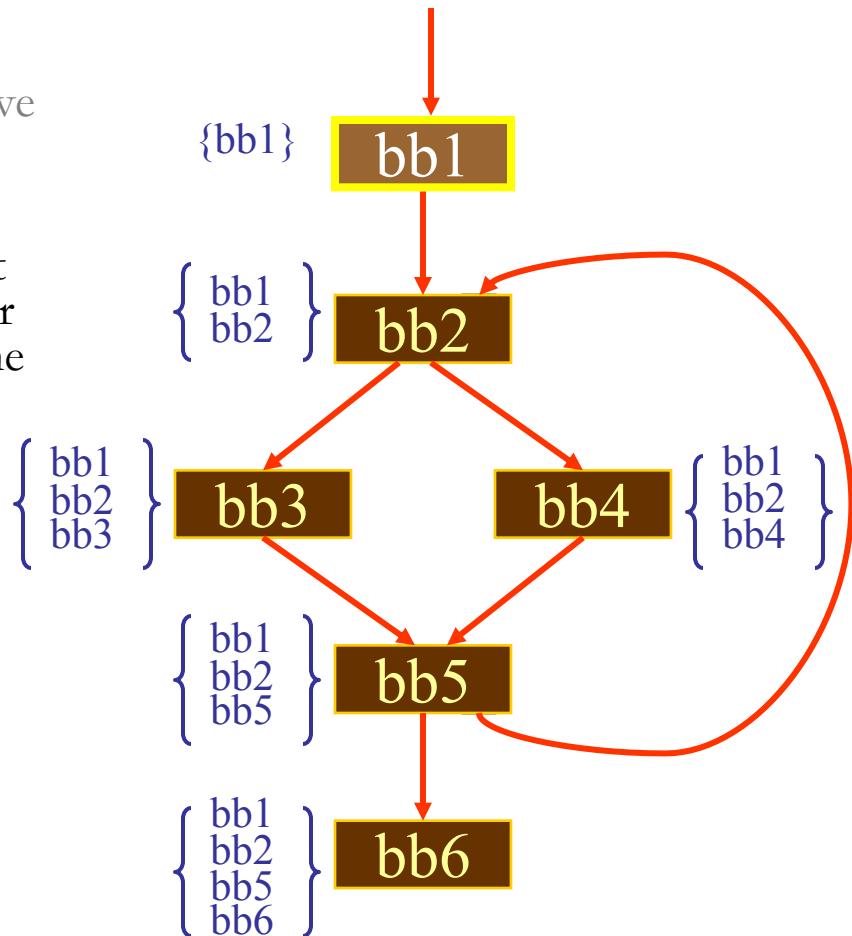
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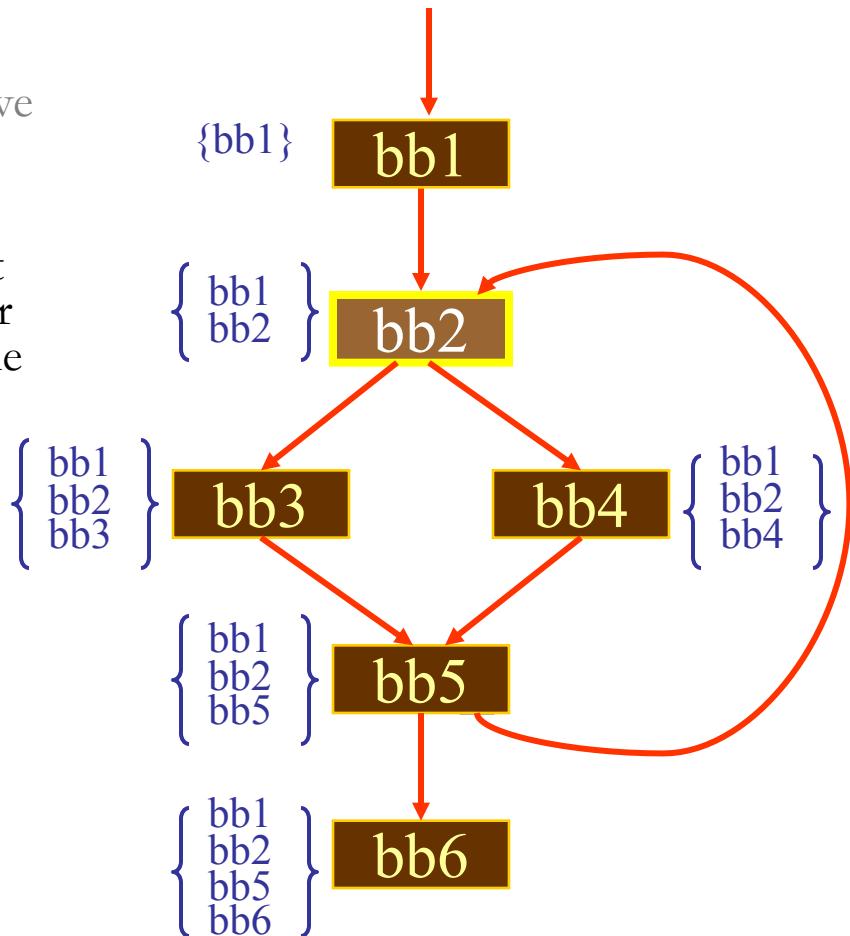
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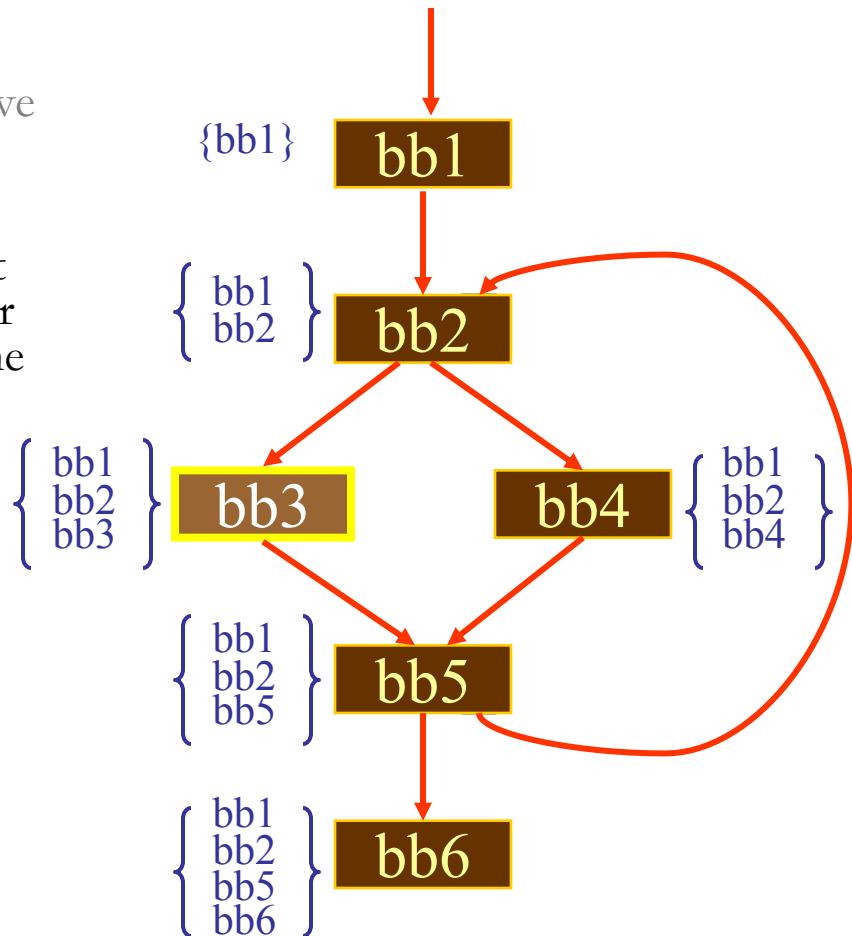
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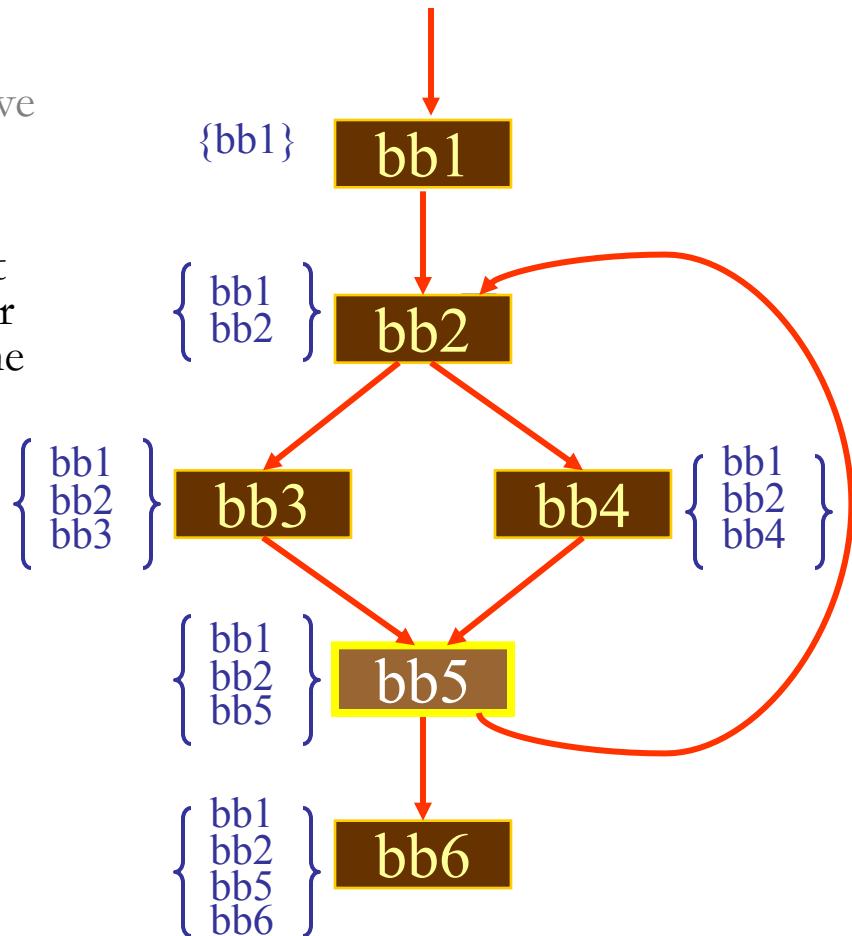
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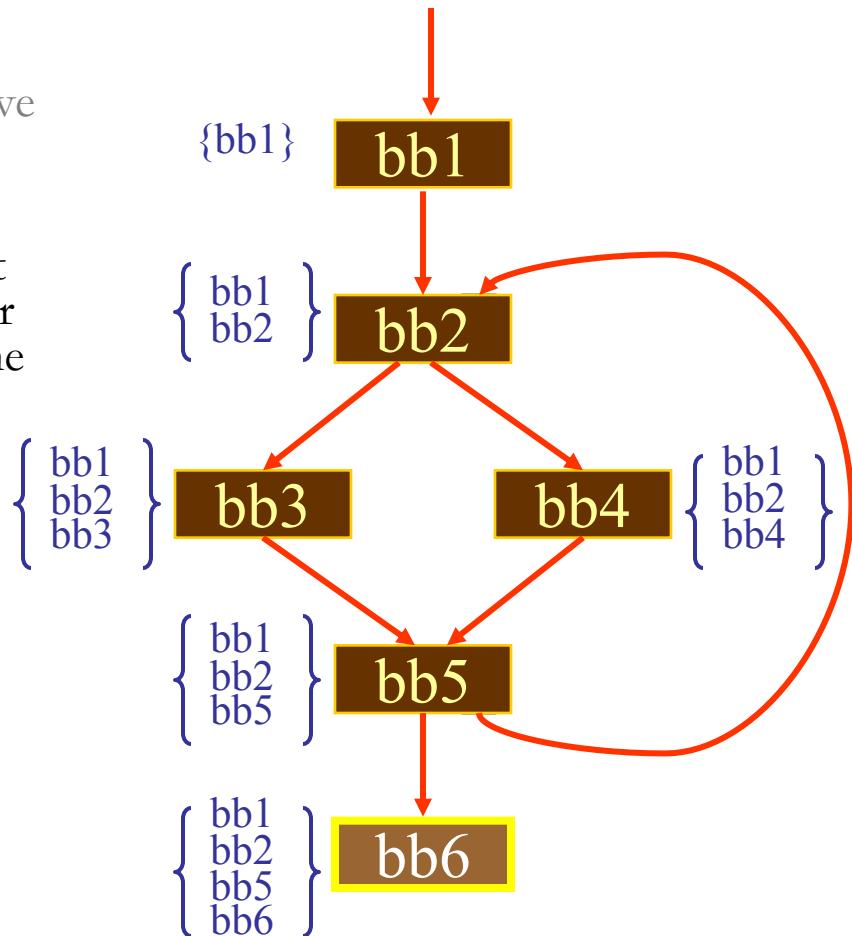
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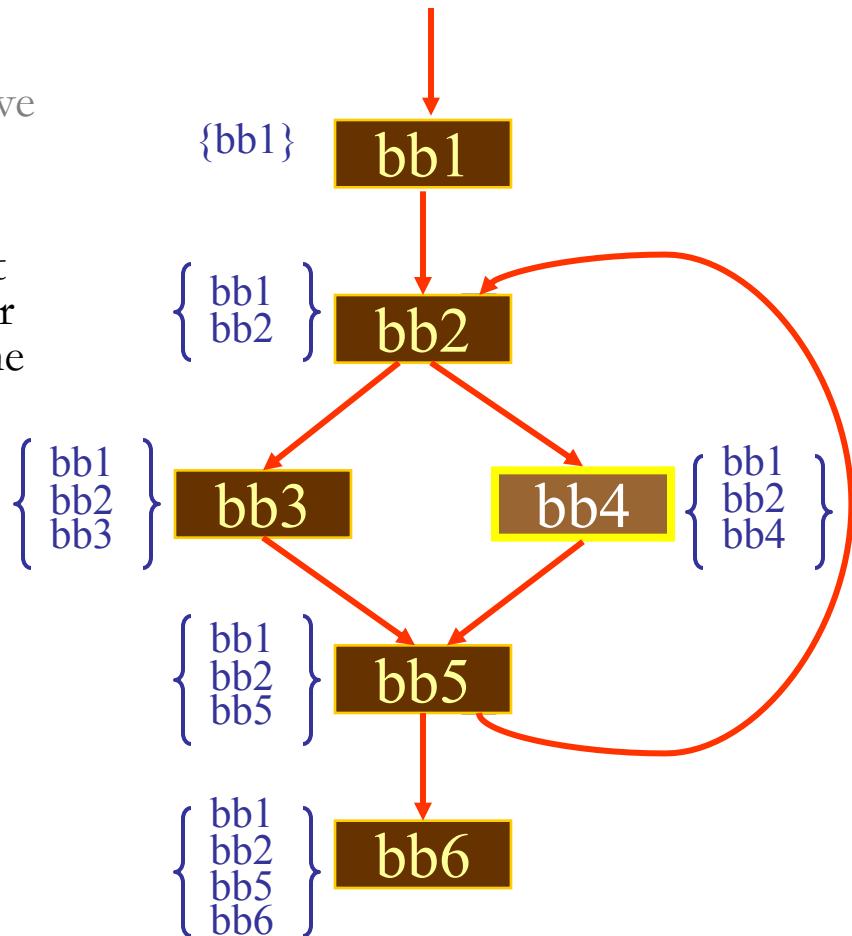
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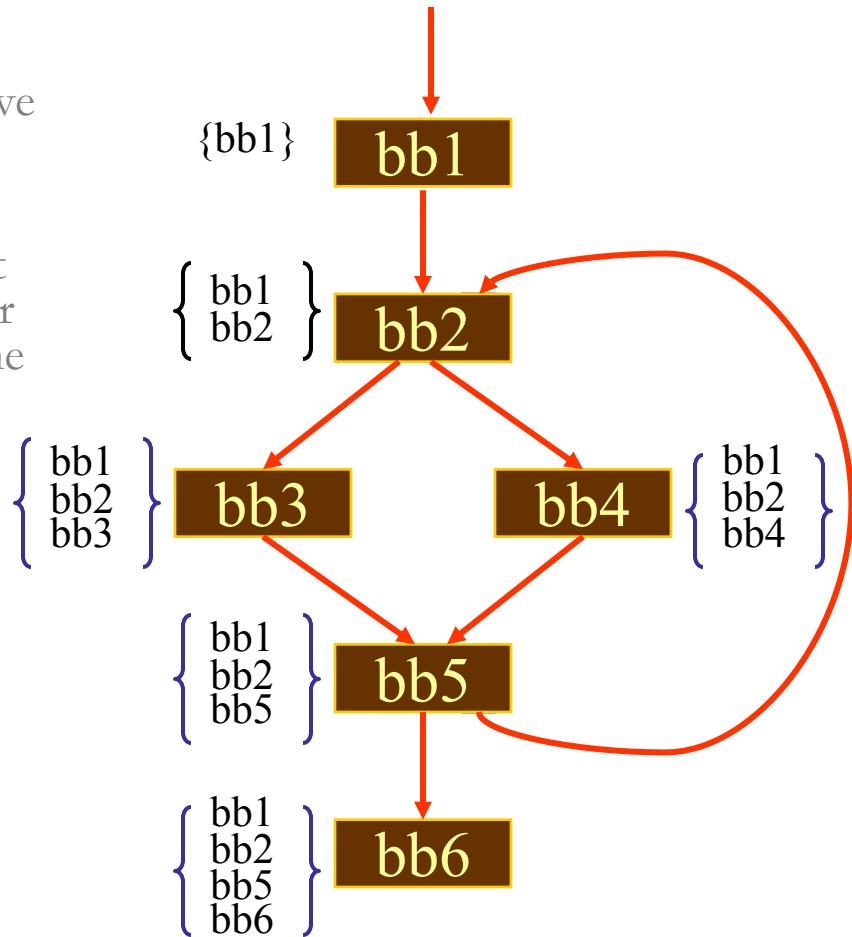
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Computing Dominators

- What we just witness was an Iterative Data-Flow Analysis Algorithm in Action
 - Initialize all the nodes to a given value
 - Visit nodes in some order
 - Calculate the node's value
 - Repeat until no value changes (fixed-point computation)

Data-Flow Analysis

A collection of techniques for compile-time reasoning about the runtime flow of values in a program

- Local Analysis
 - Analyze the “effect” of each Instruction in each Basic Block
 - Compose “effects” of instructions to derive information from beginning of basic block to each instruction
- Data-Flow Analysis
 - Iteratively propagate basic block information over the control-flow graph until no changes
 - Calculate the final value(s) at the beginning/end of the Basic Block
- Local Propagation
 - Propagate the information from the beginning/end of the Basic Block to each instruction

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Example: Available Expression

- An Expression is *Available* at point p if and only if
 - All paths of execution reaching the current point p pass through the point where the expression was defined (the definition point)
 - No variable used in the expression was modified between the definition point and the current point p

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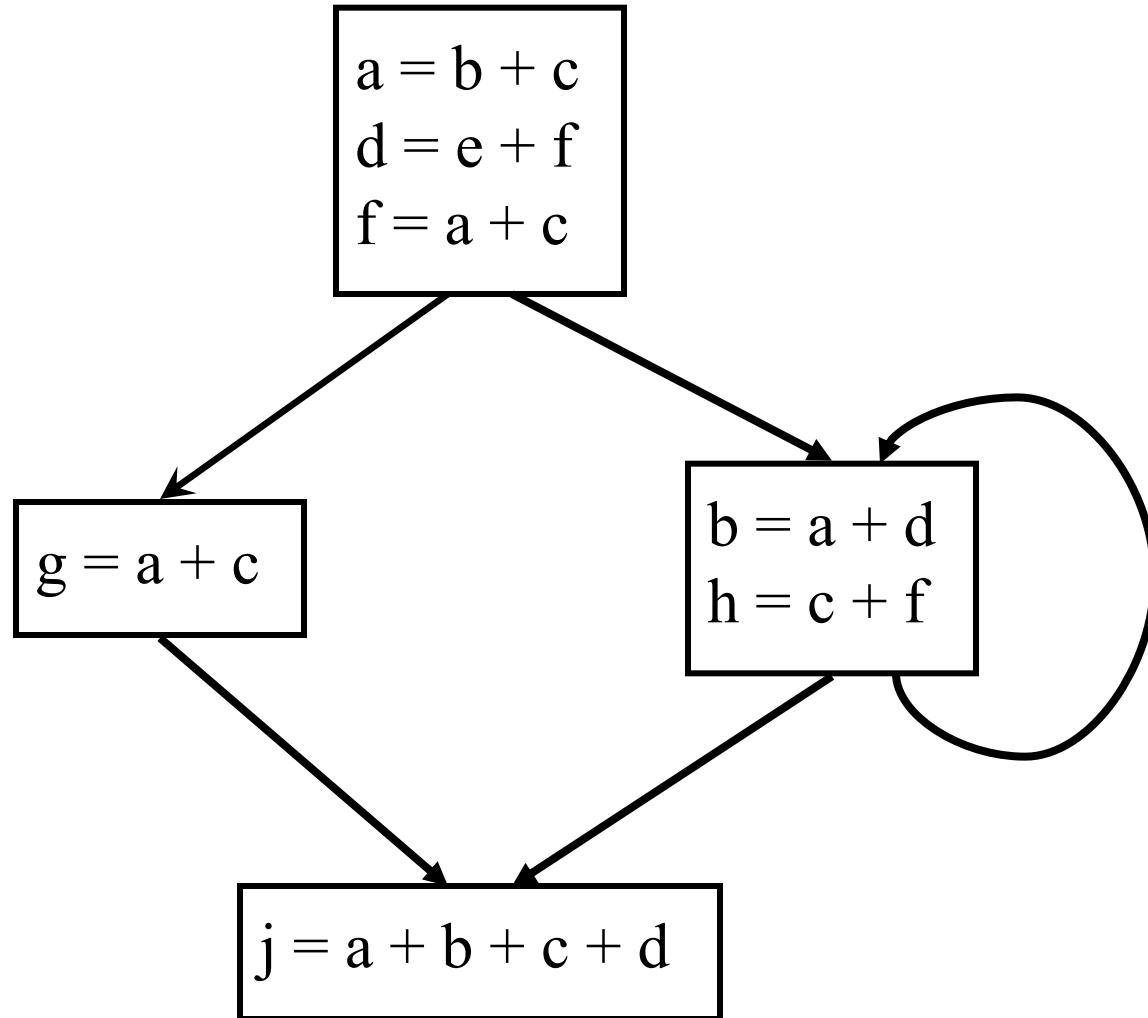
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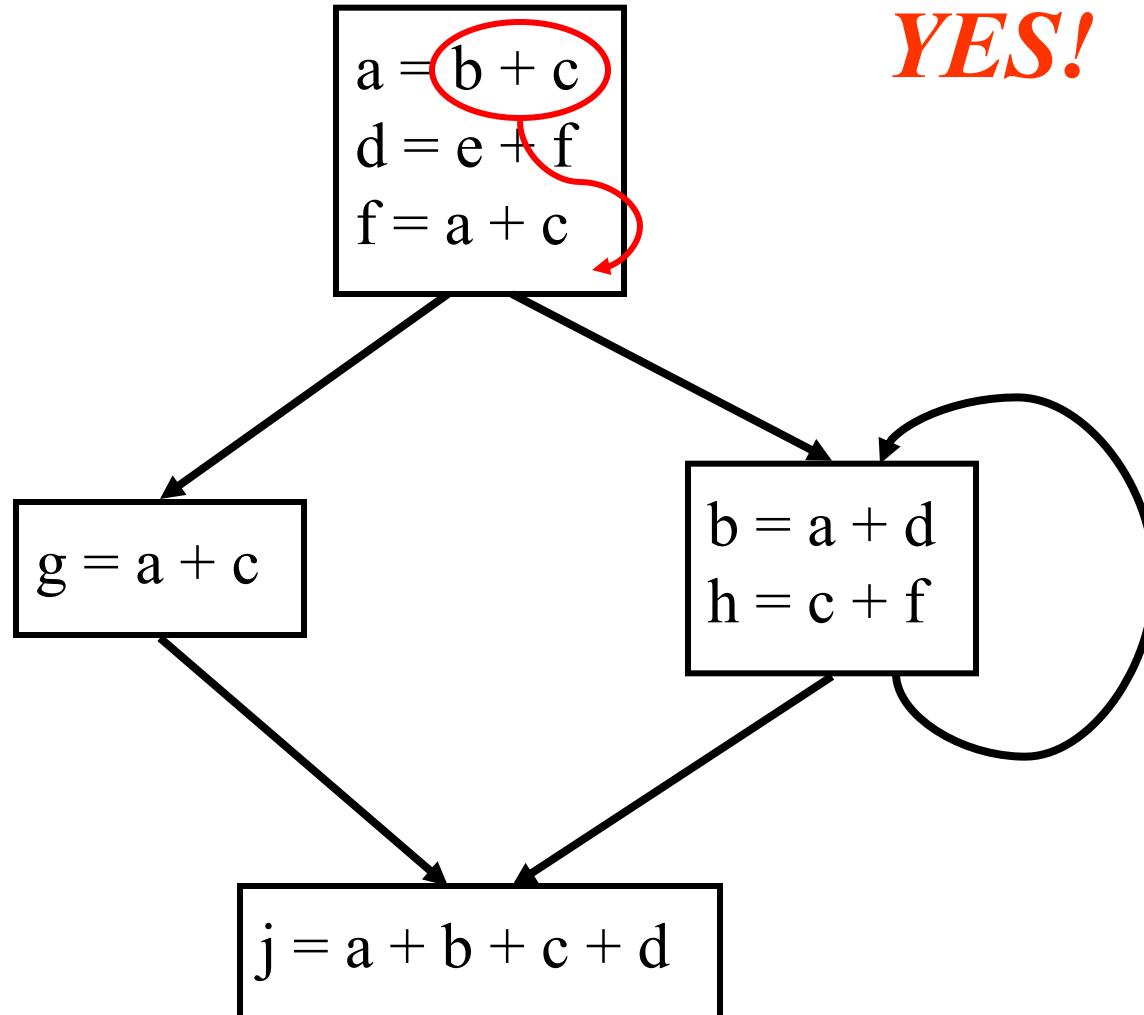
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- Why is this a Data-Flow Problem?
 - We have to “know” a property about the program’s execution that depends on the control-flow of the program!
 - All-Paths or At-Least-One-Path Issue.

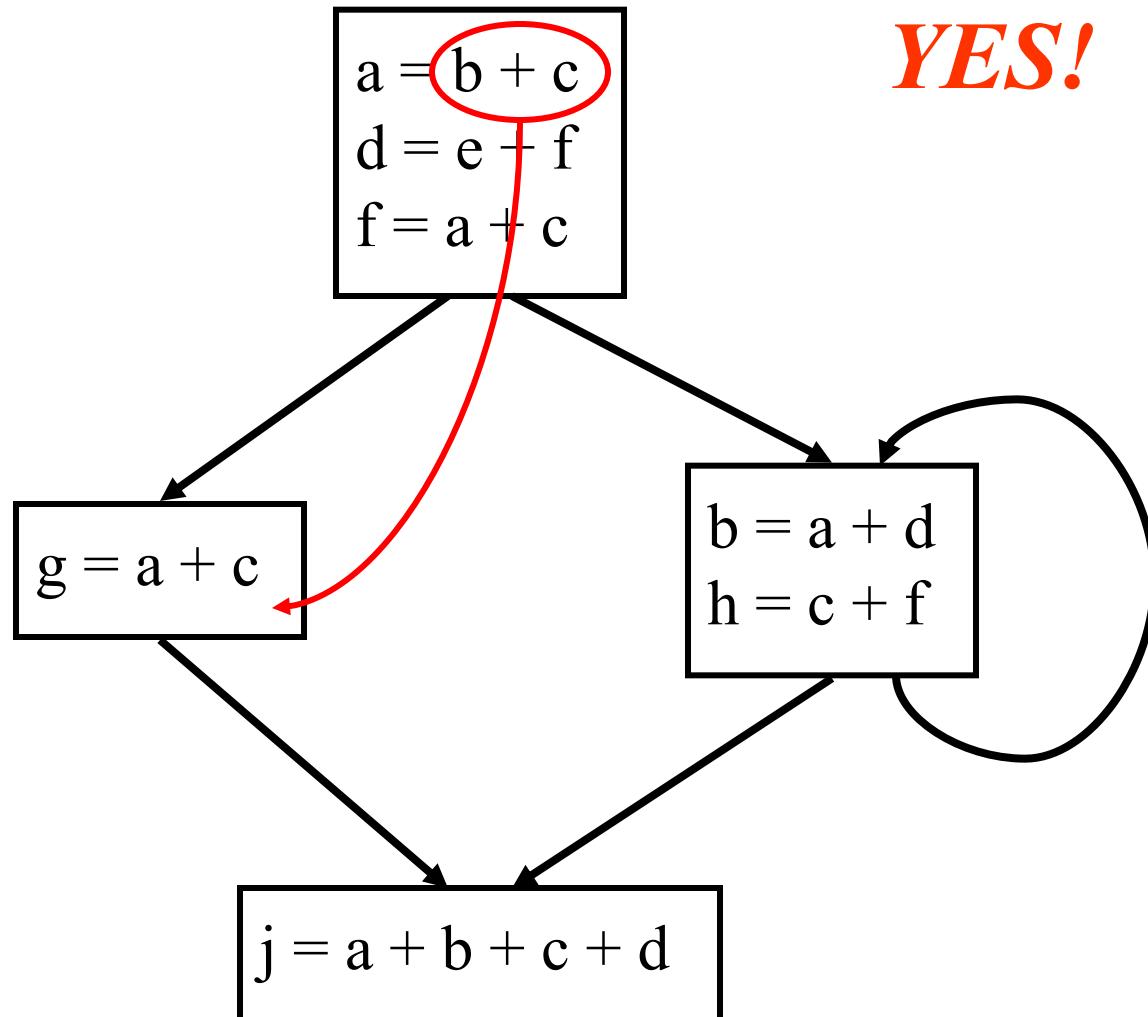
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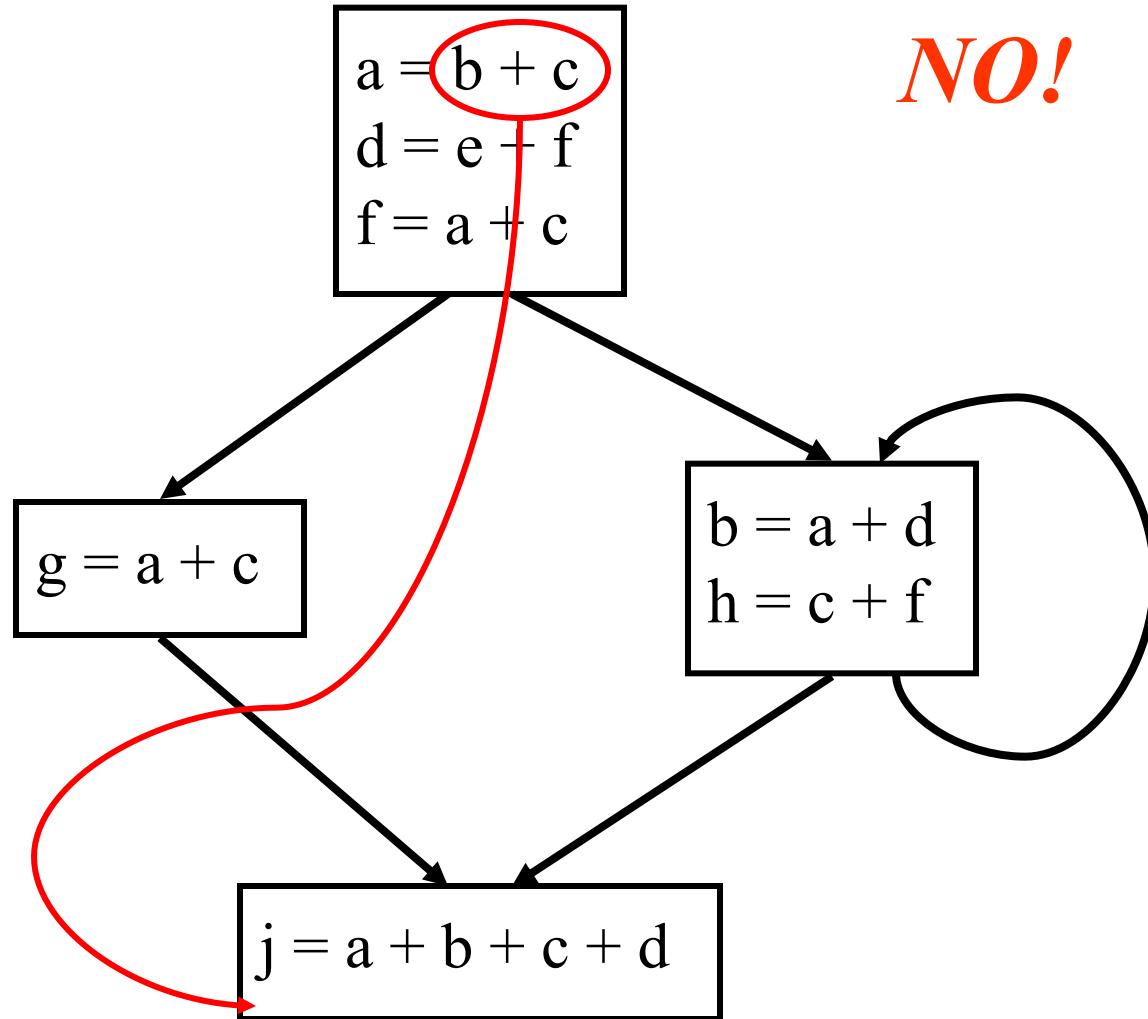
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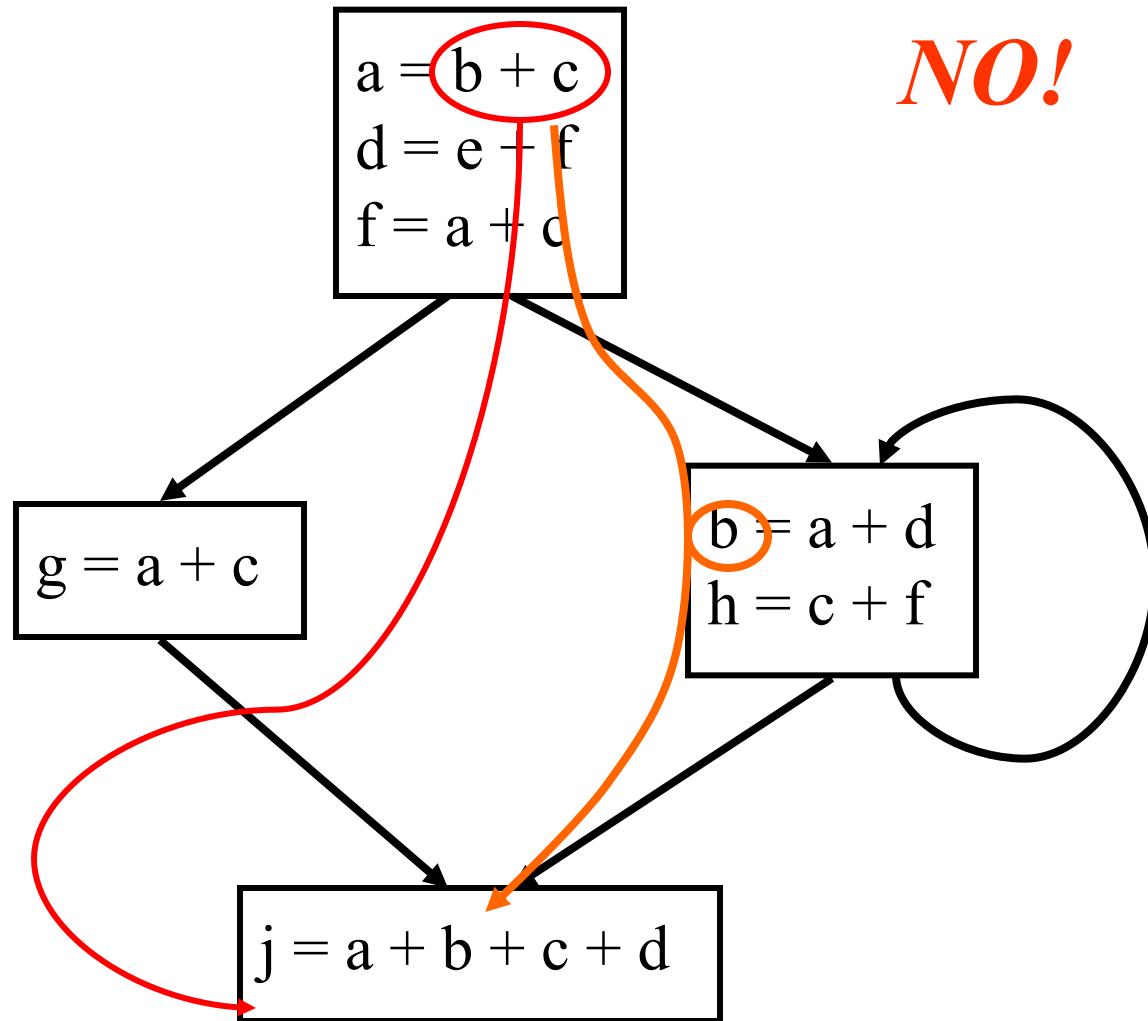
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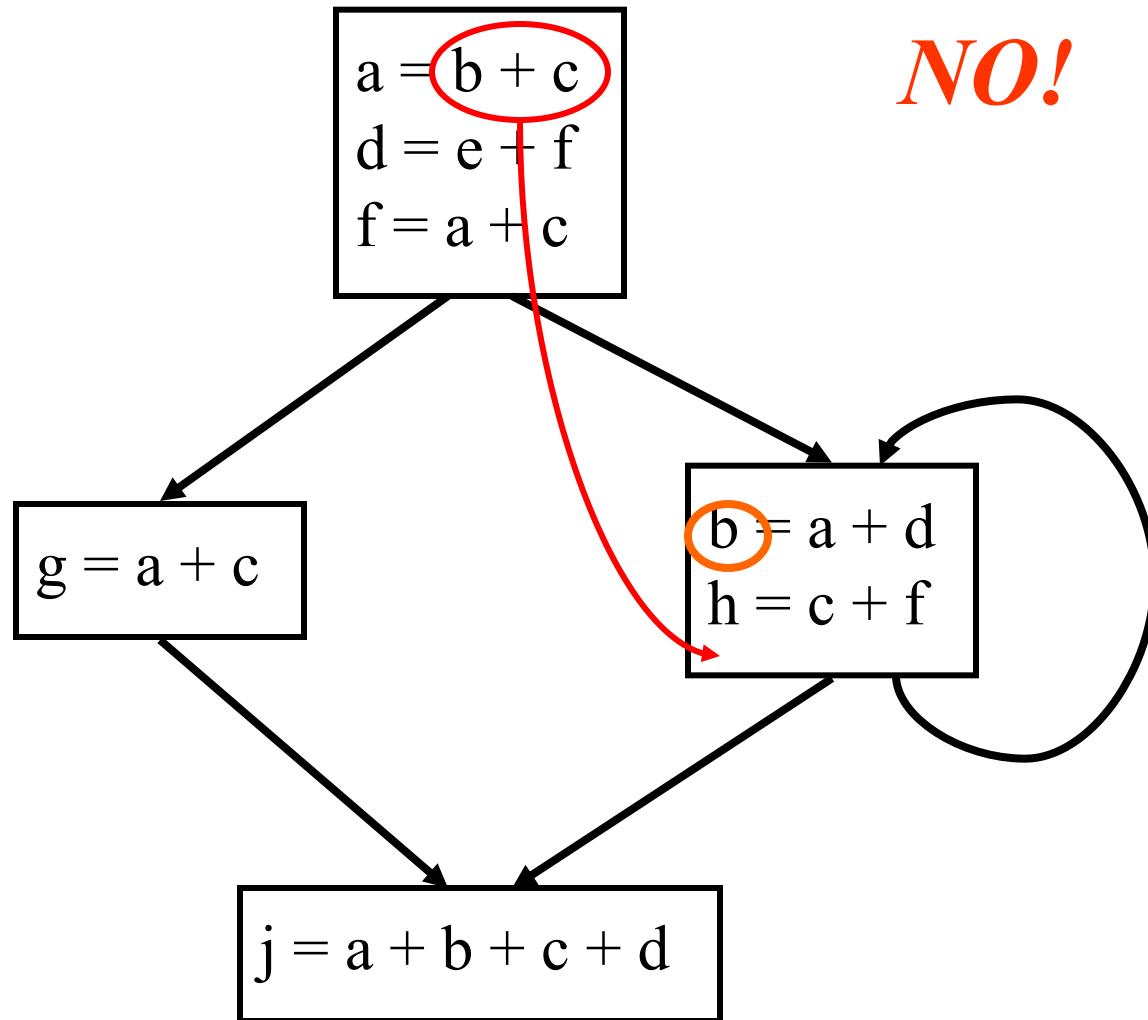
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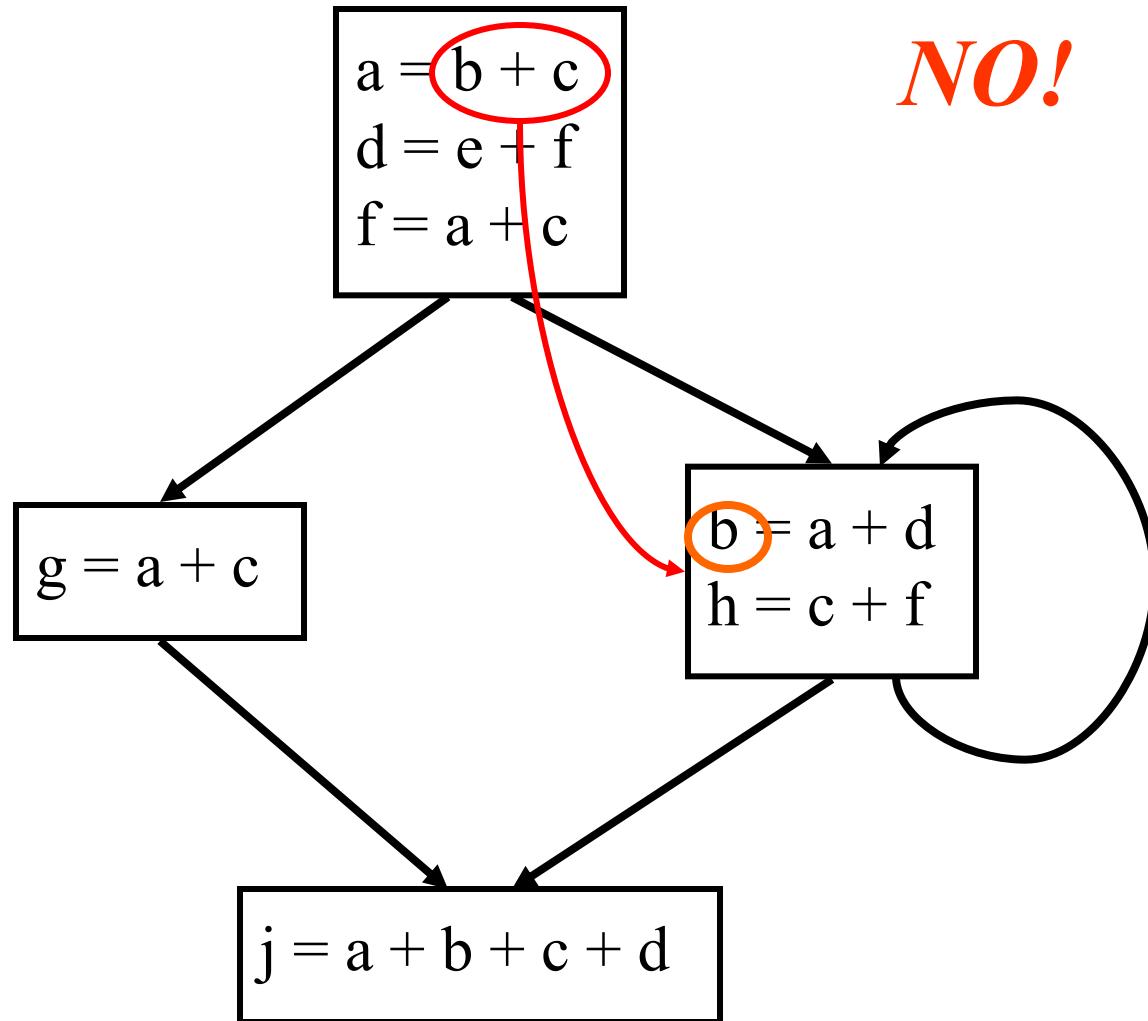
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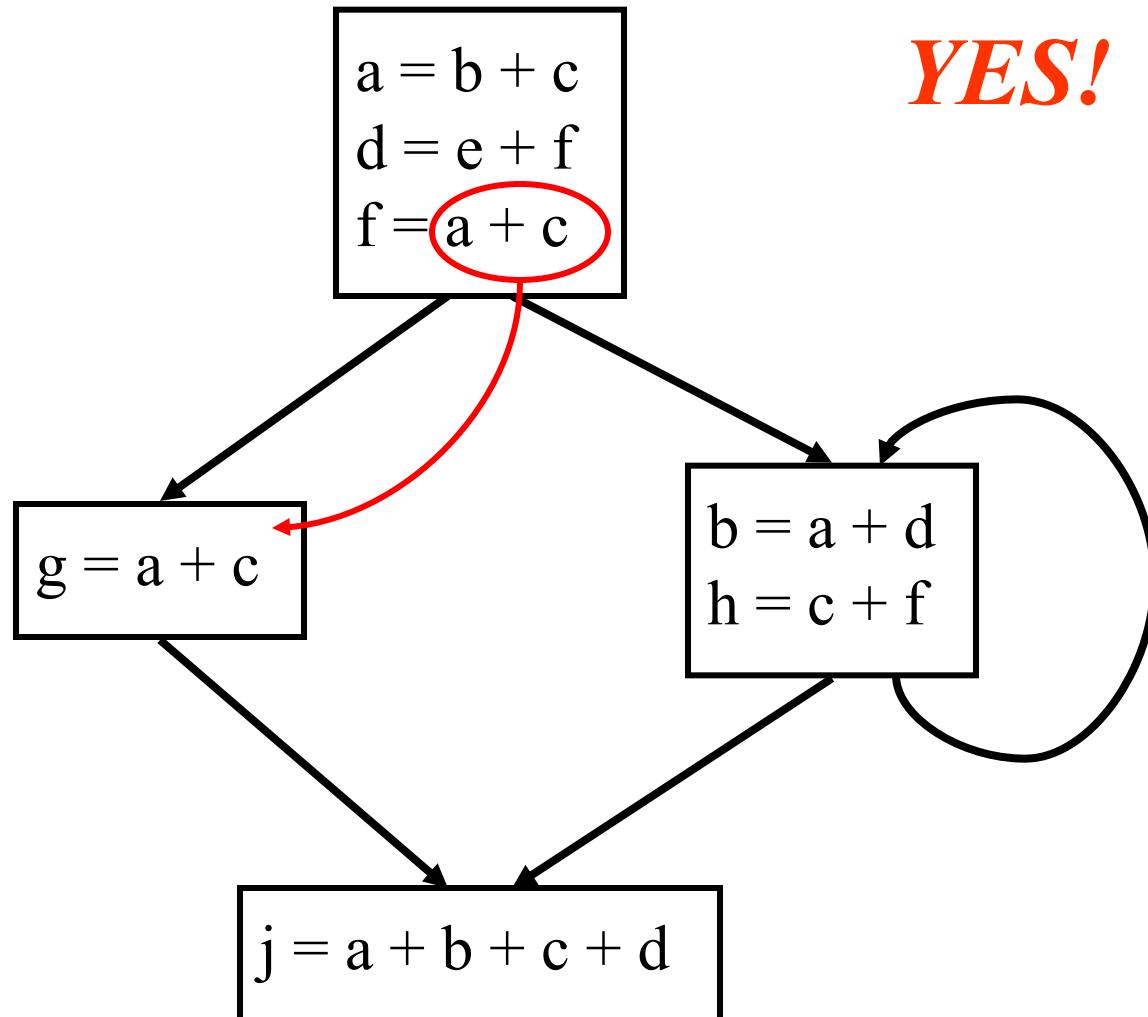
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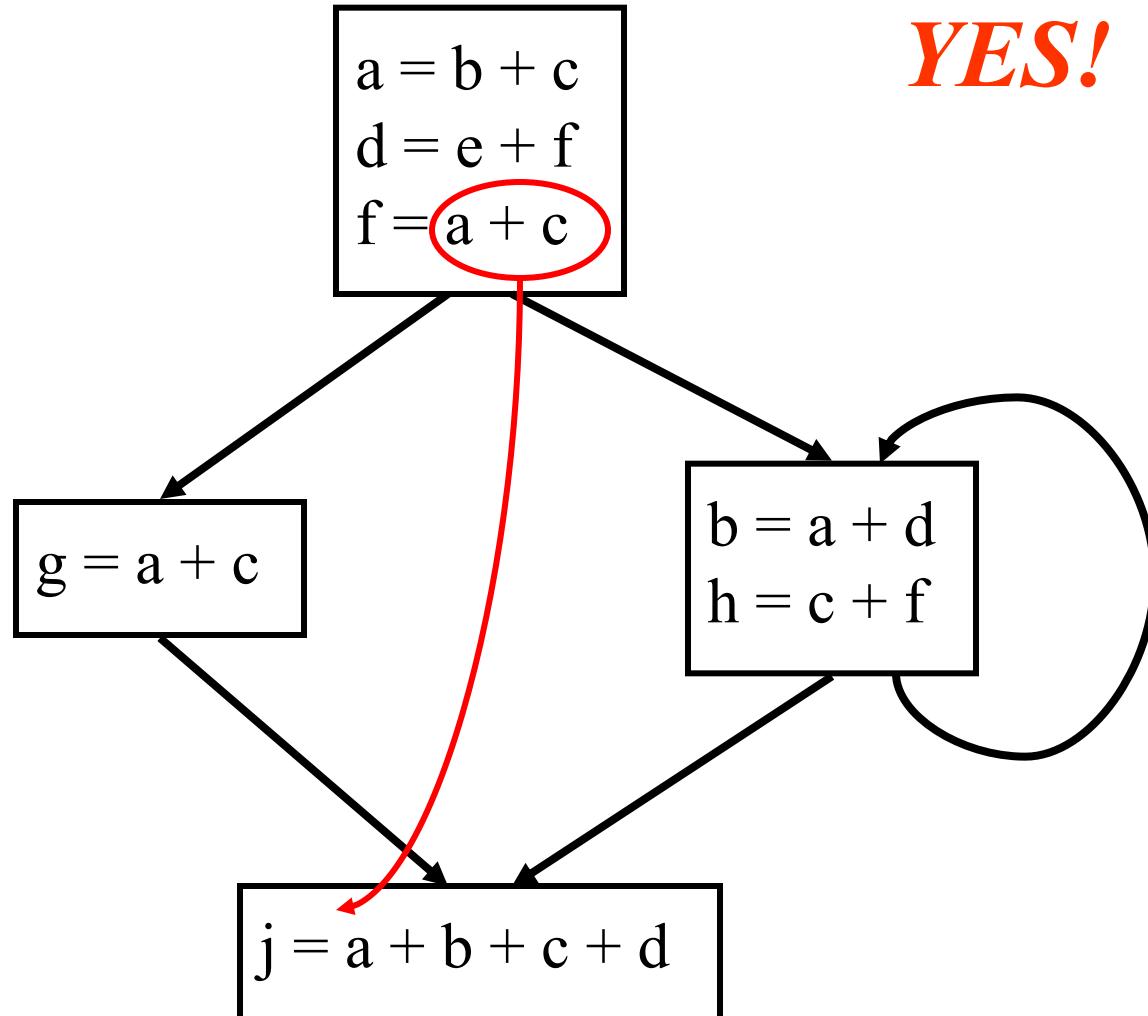
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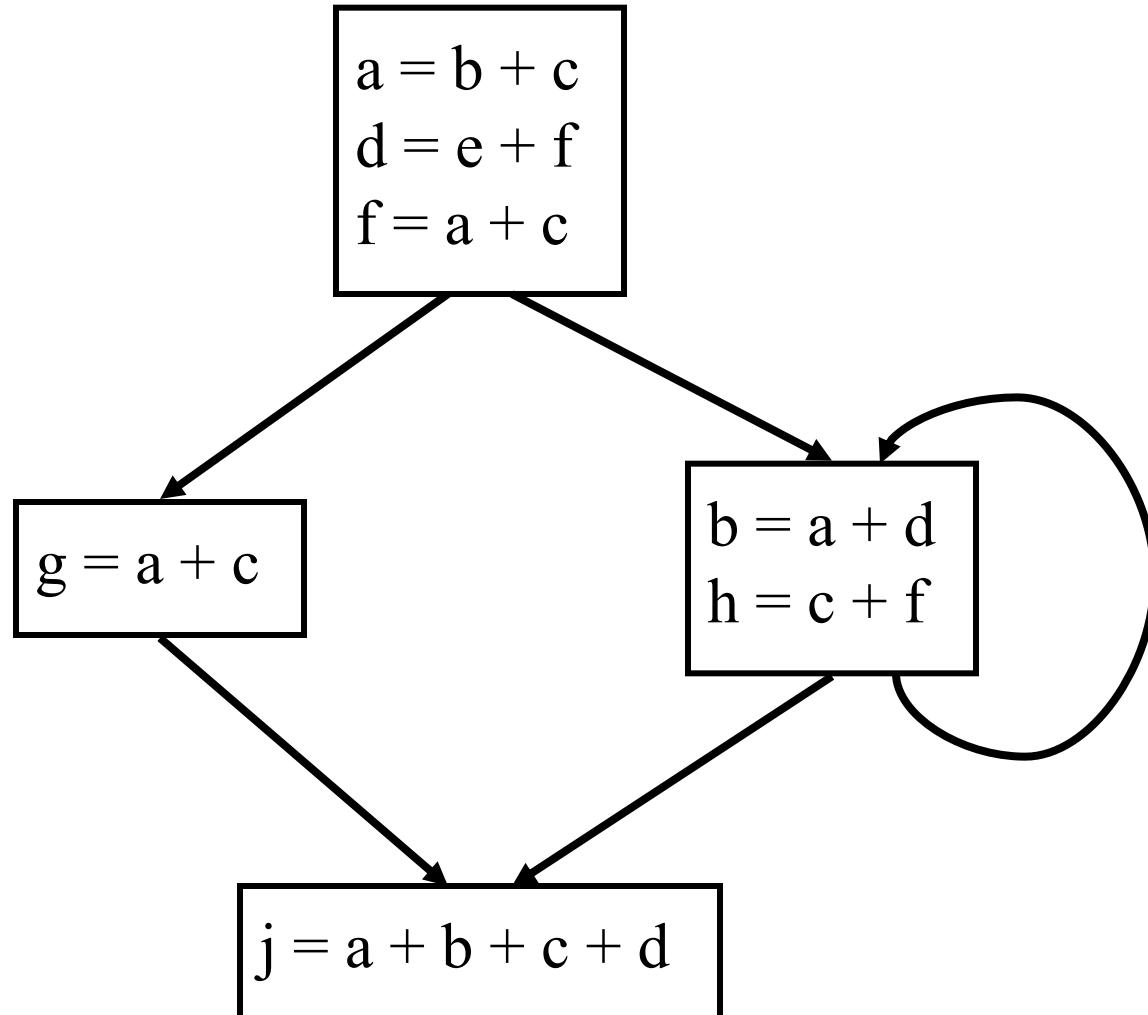
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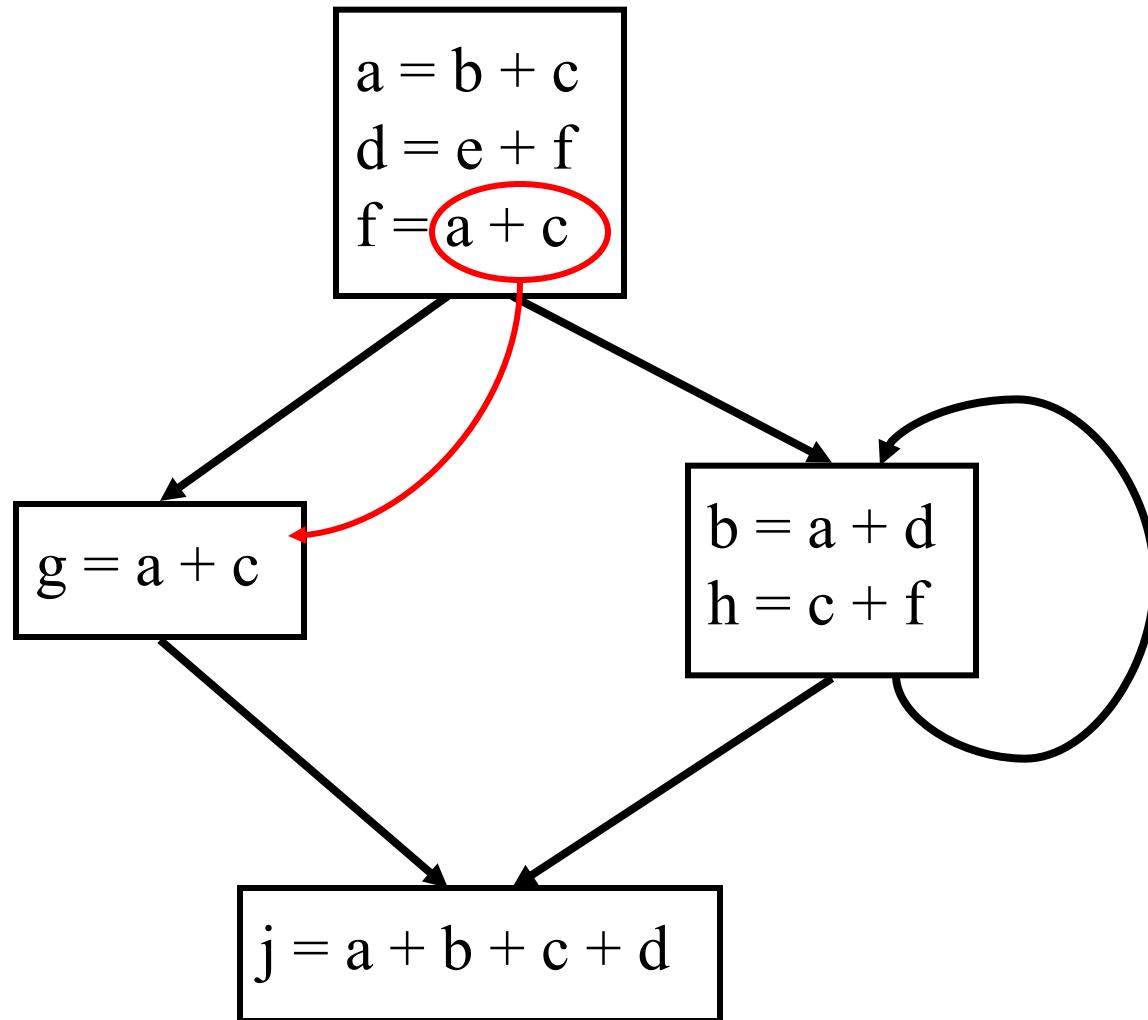
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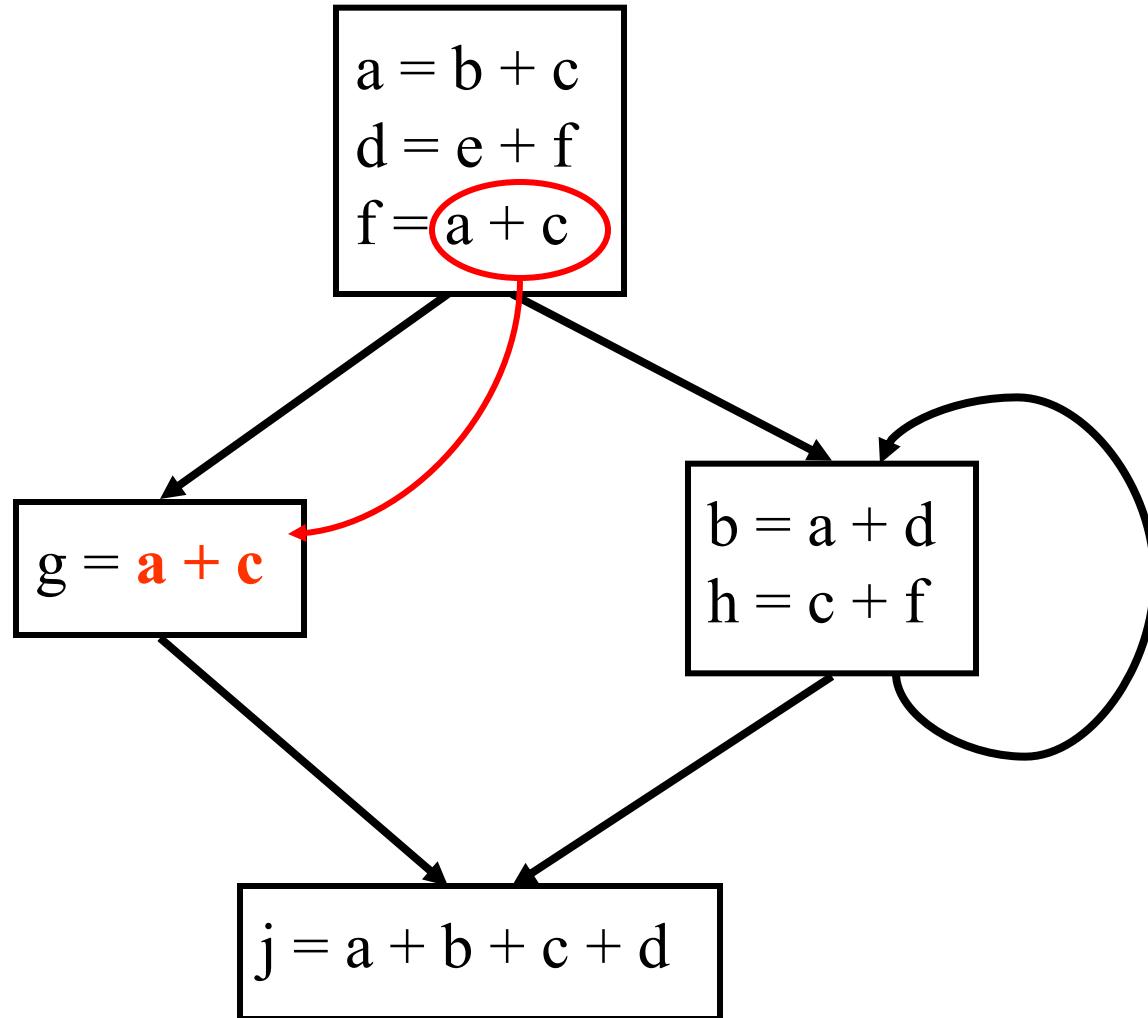
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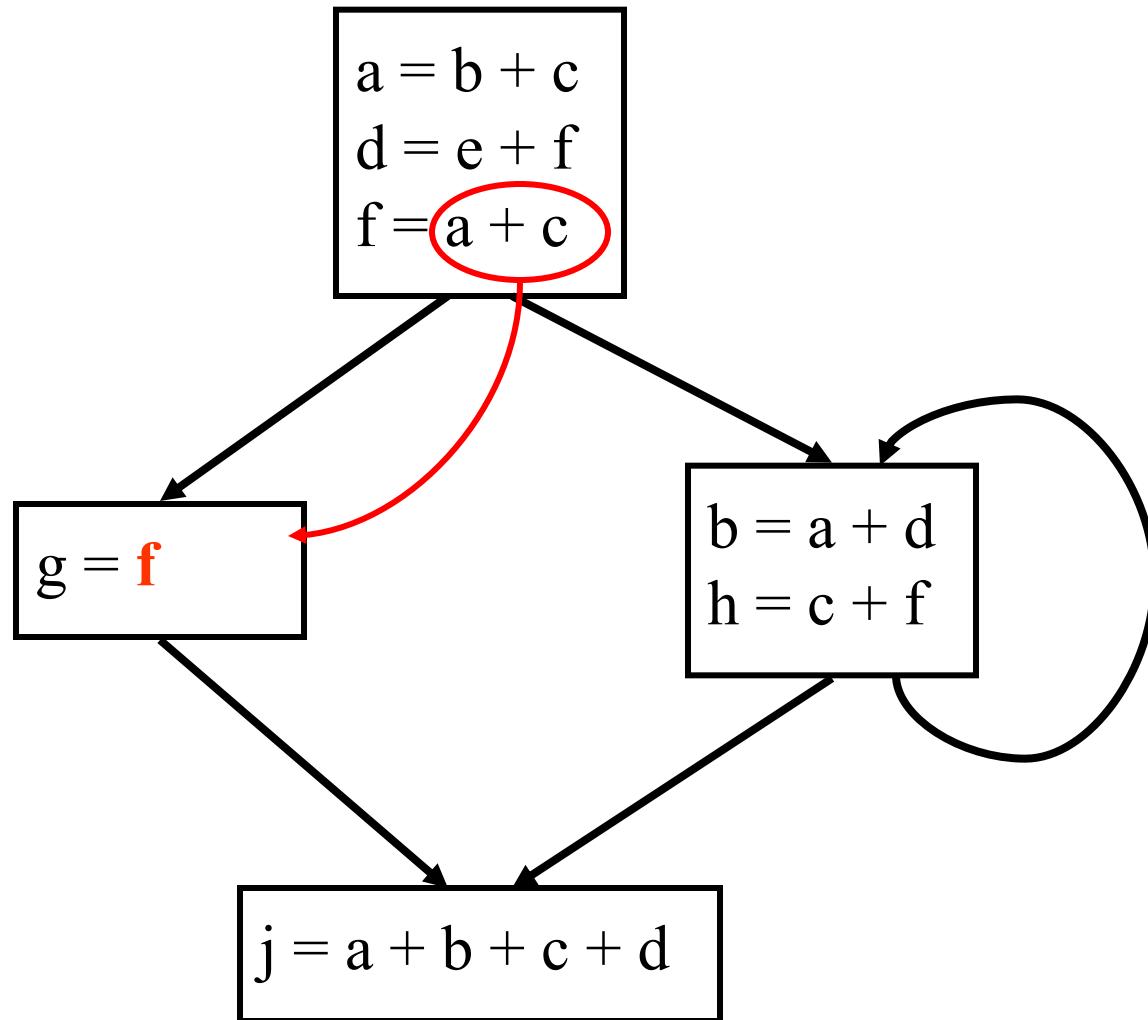
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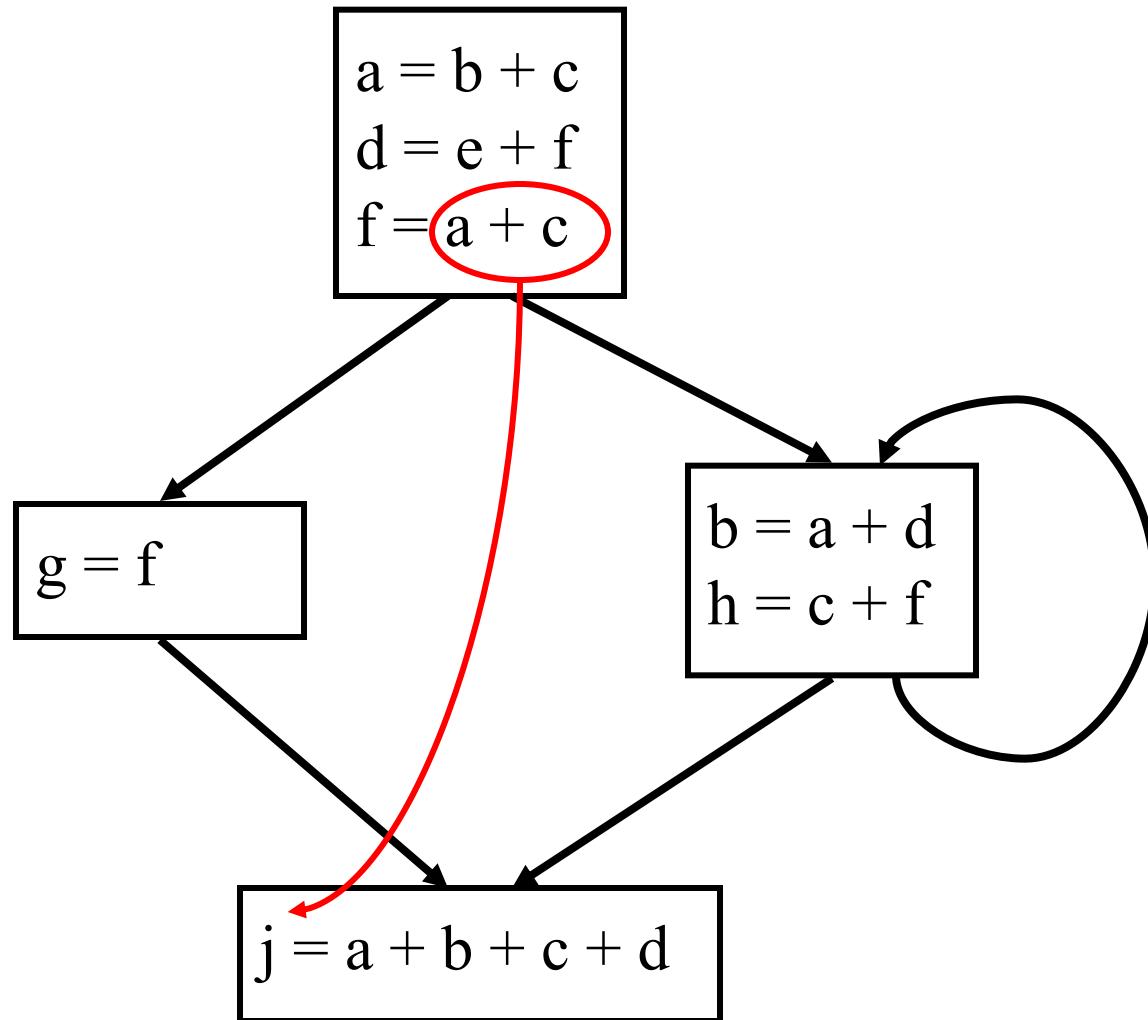
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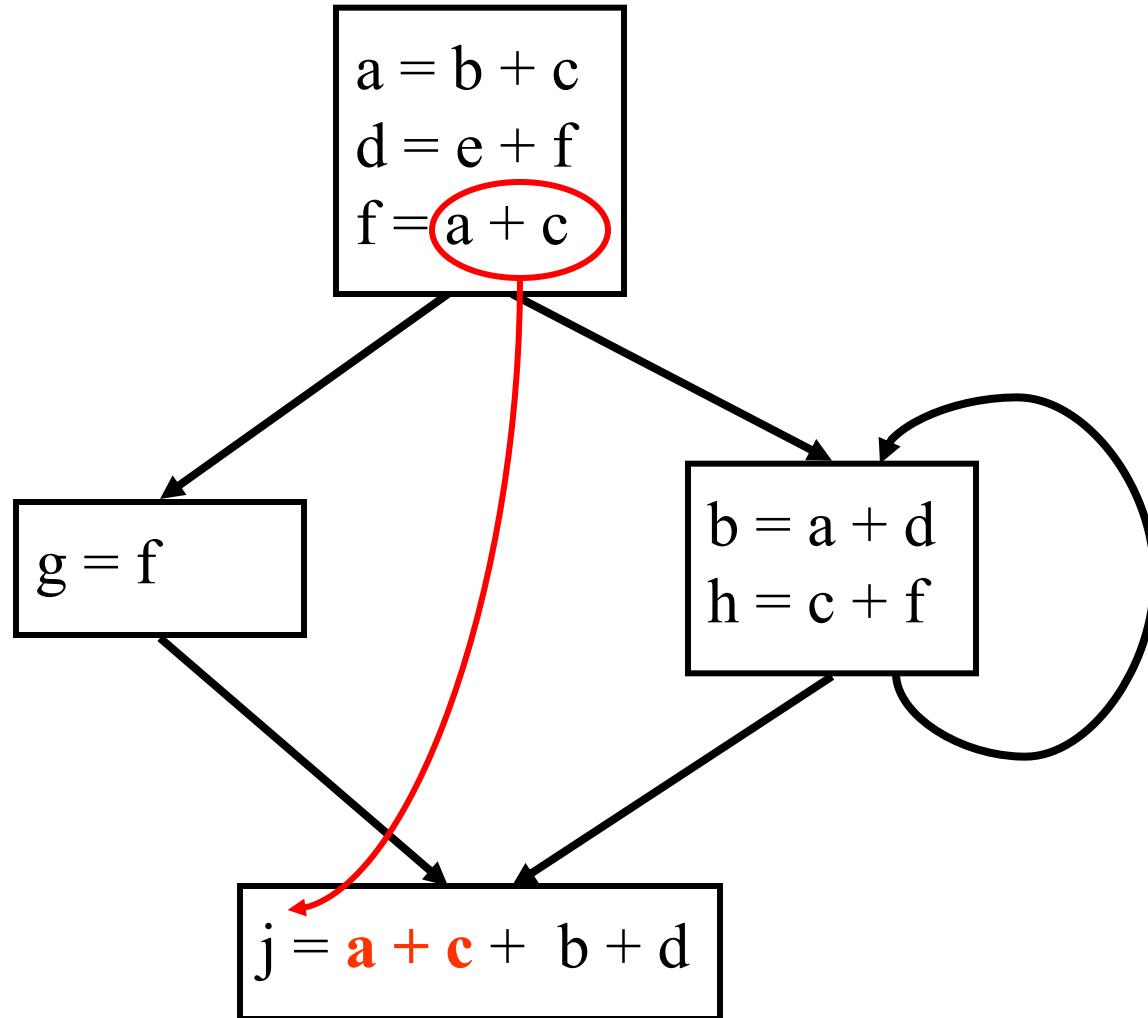
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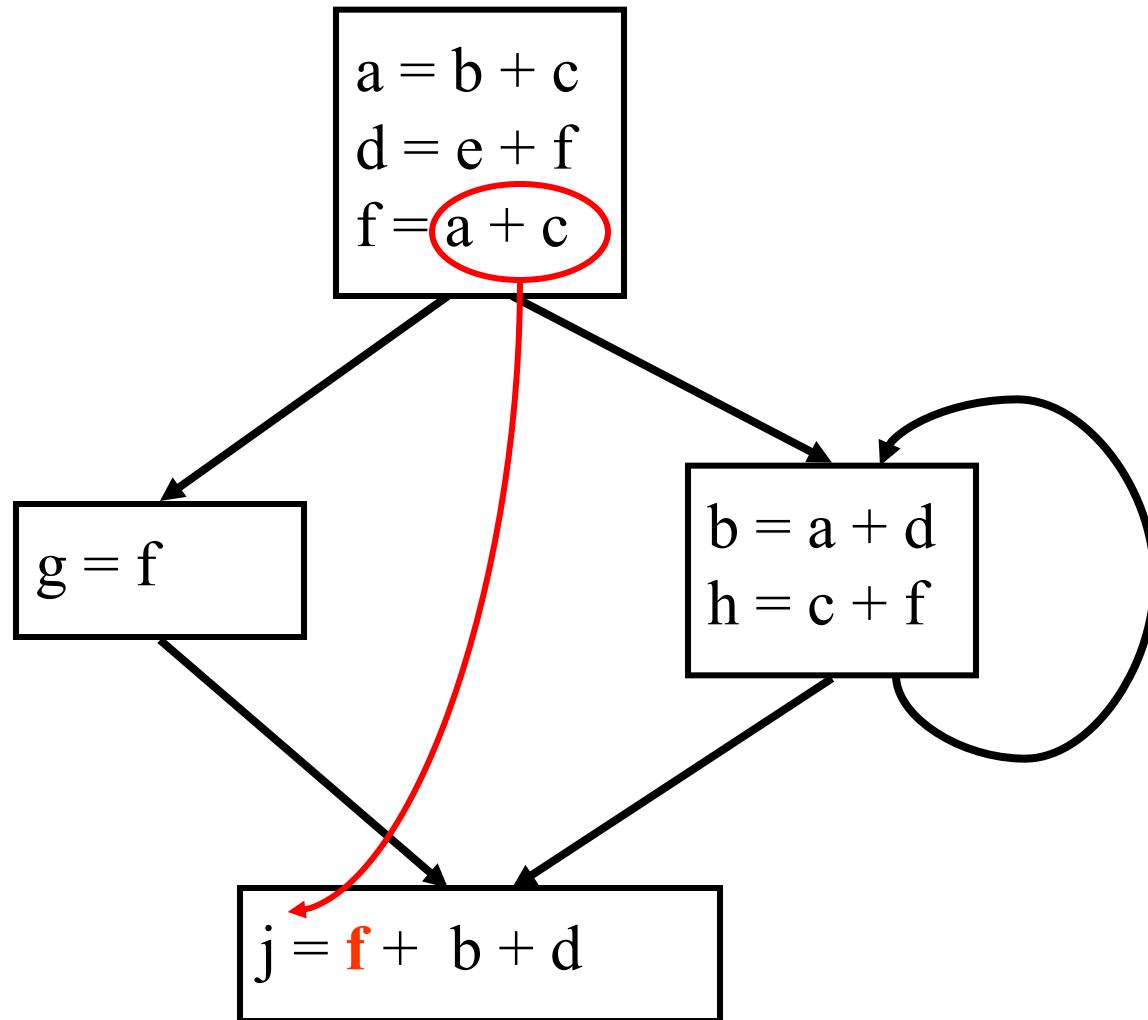
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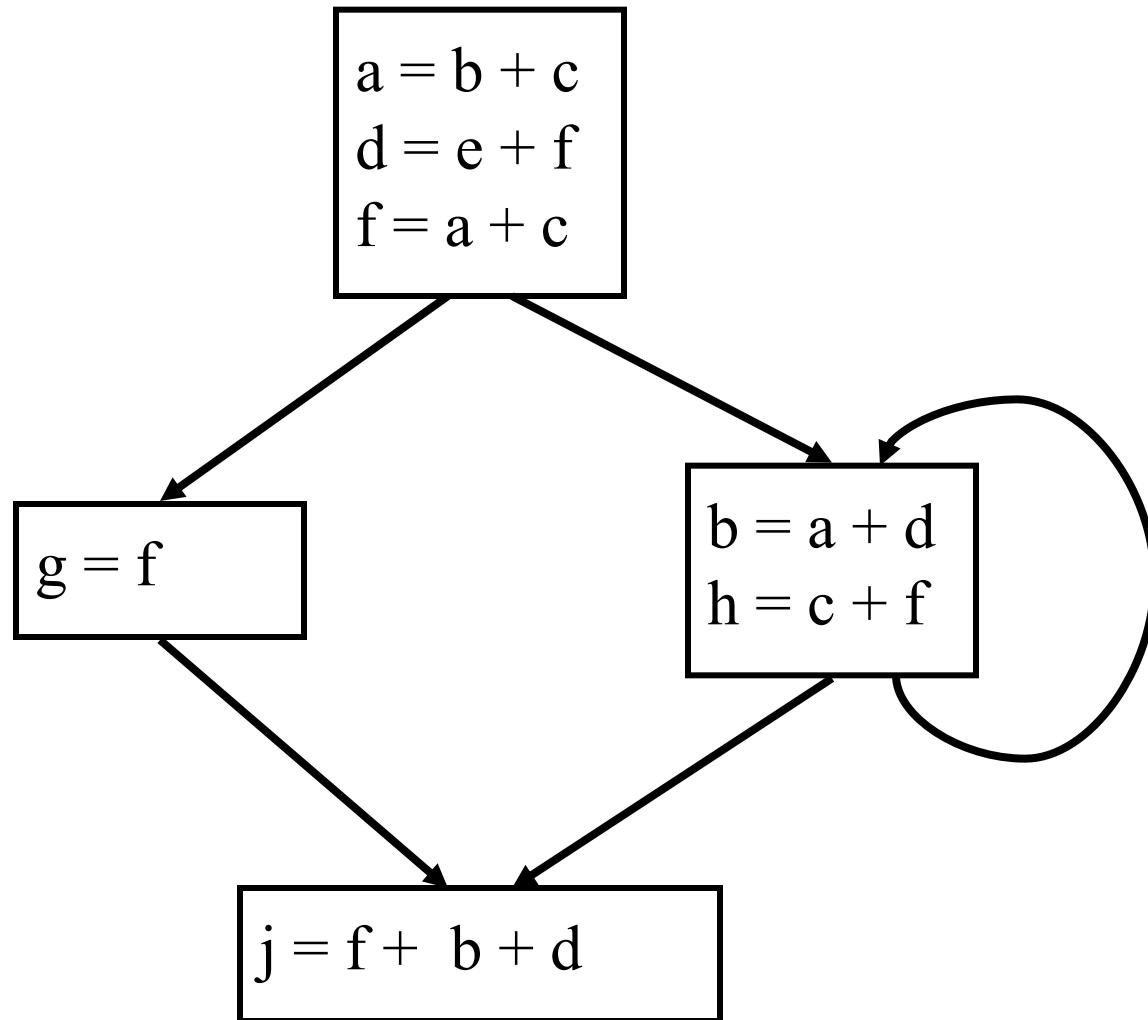
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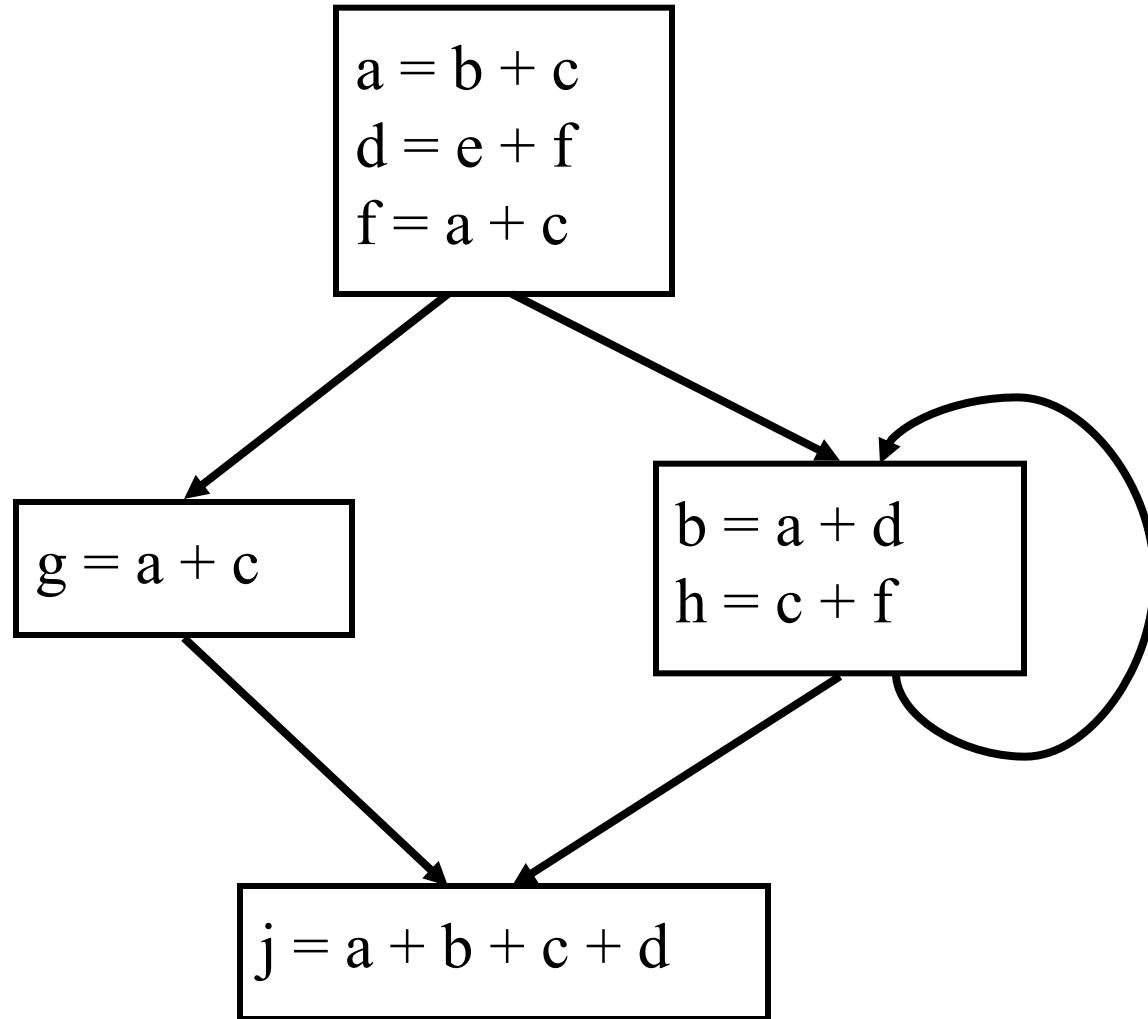
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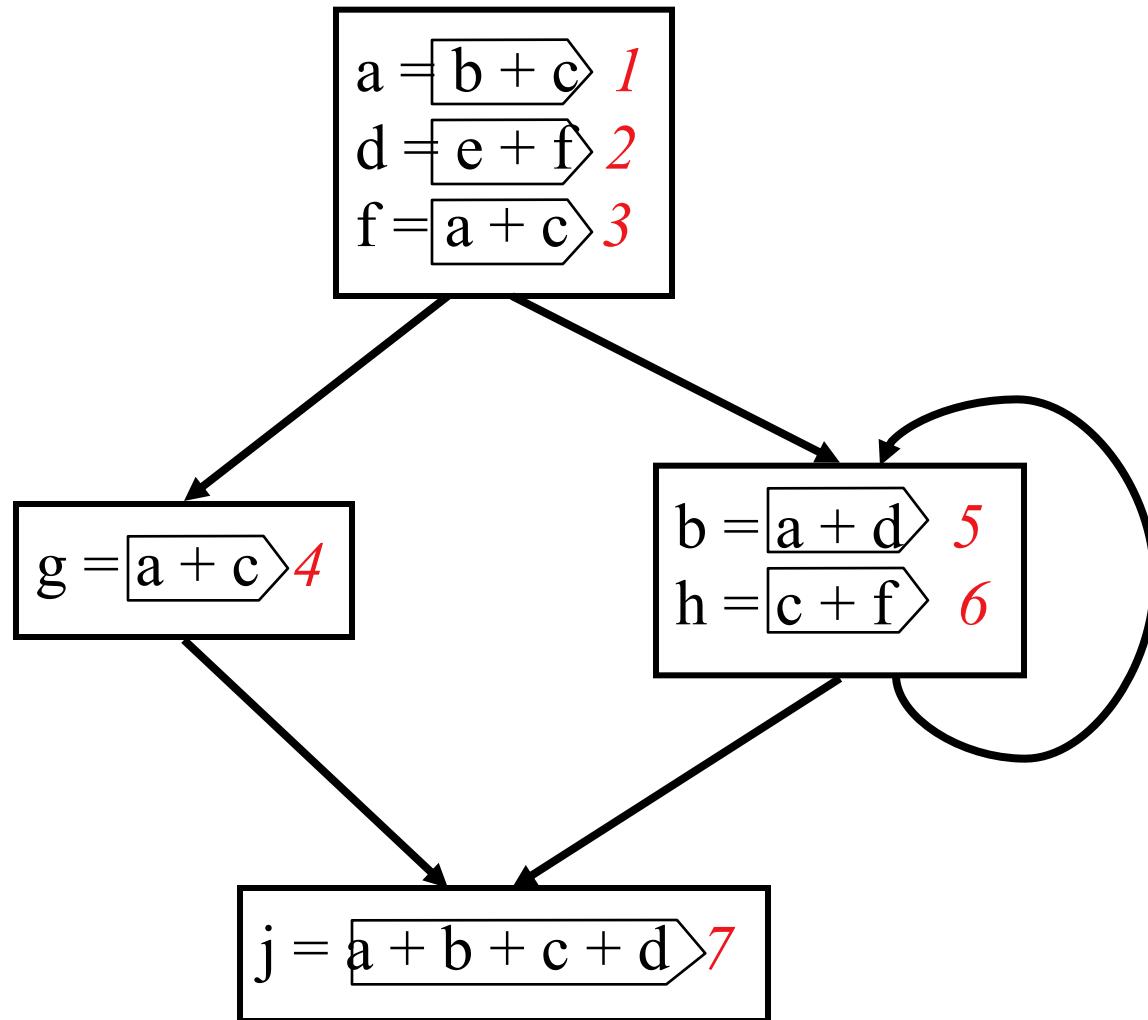
Algorithm for Available Expression

- Assign a Number to each Expression in the Program

Example: Available Expression



Example: Available Expression



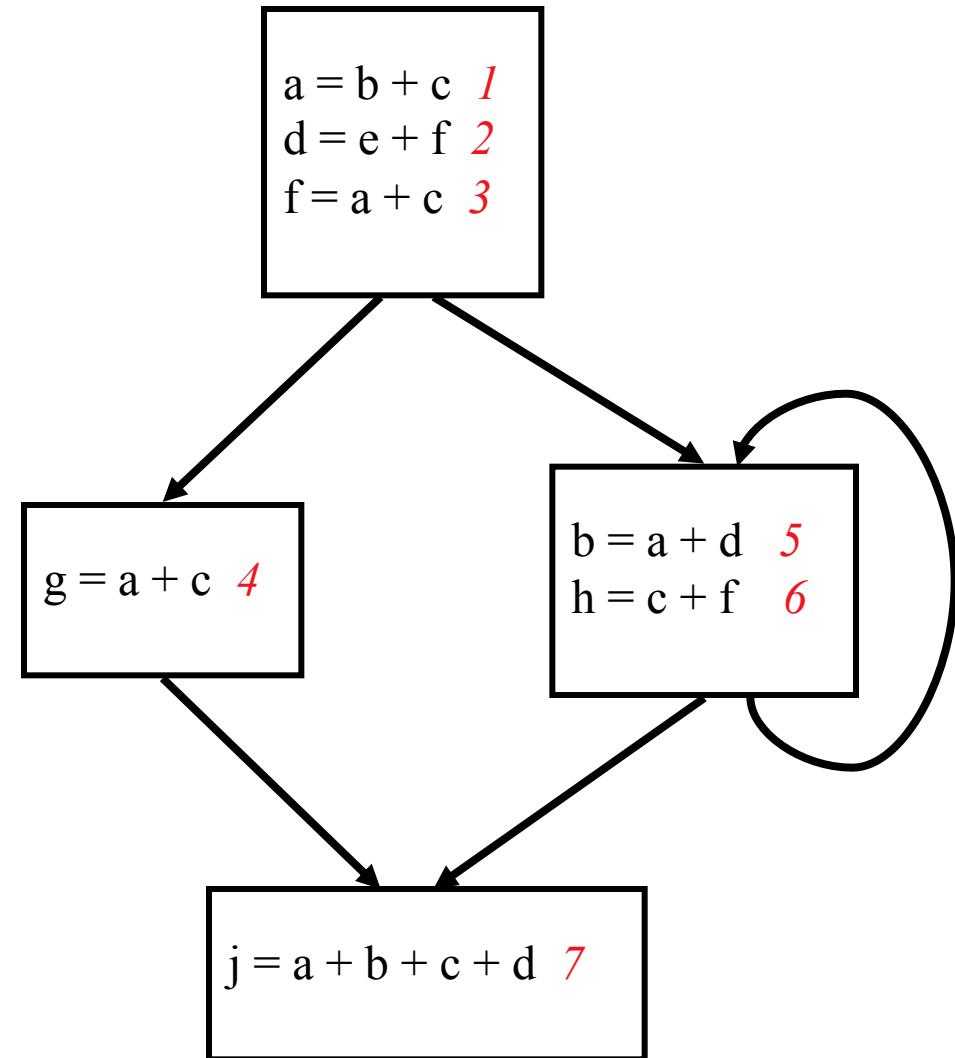
Gen and Kill Sets

- Gen Set
 - If a Basic Block (or instruction) defines the expression then the expression number is in the Gen Set for that Basic Block (or instruction)
- Kill Set
 - If a Basic Block (or instruction) (re)defines a variable in the expression then that expression number is in the Kill Set for that Basic Block (or instruction)
 - Expression is thus not valid after that Basic Block (or instruction)

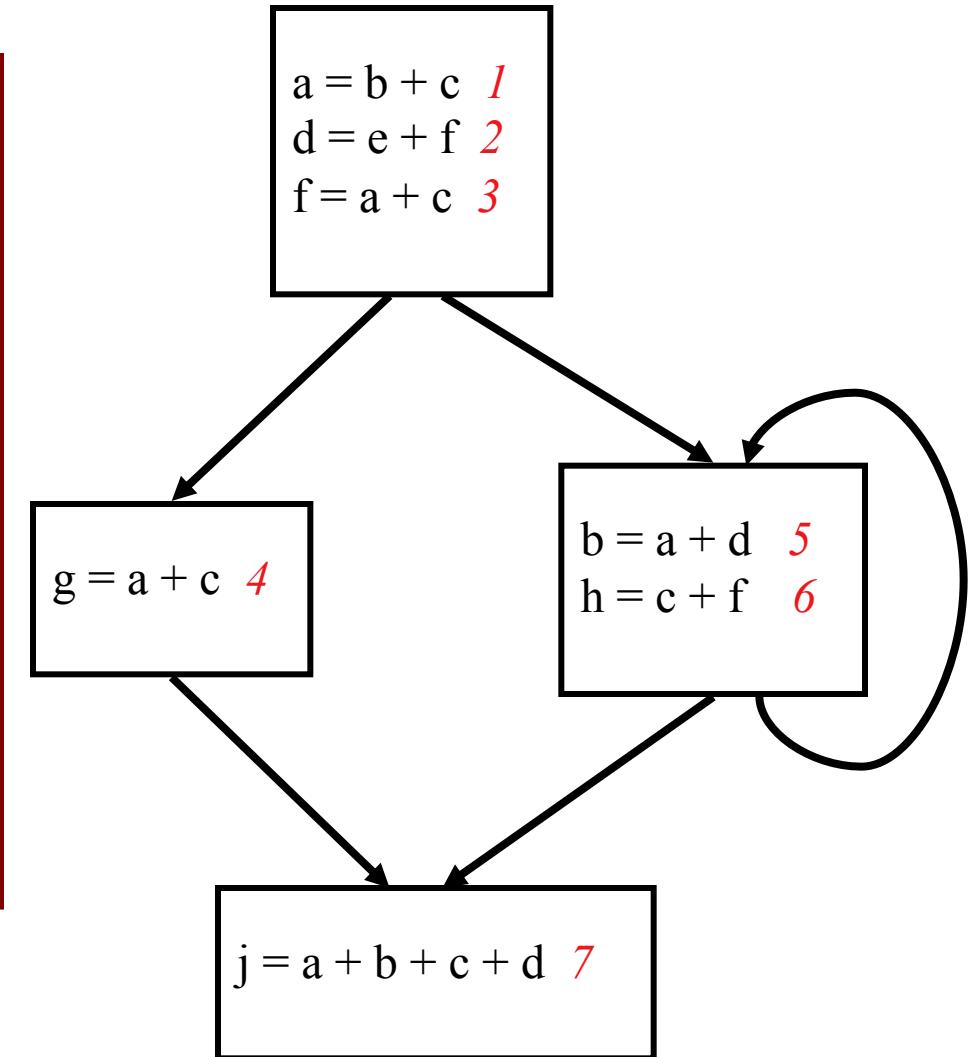
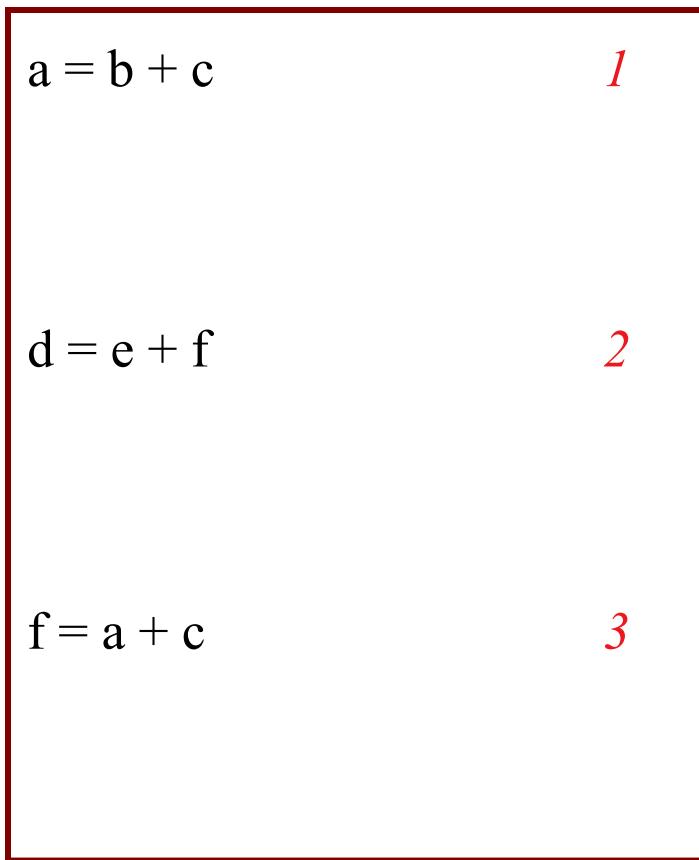
Algorithm for Available Expression

- Assign a Number to each Expression in the Program
- Compute Gen Set and Kill Set for each Basic Block (or instruction)
 - Compute Gen Set and Kill Set for each Instruction in Basic Block

Gen and Kill Sets



Gen and Kill Sets



Gen and Kill Sets

 $a = b + c$

1

 $\text{gen} = \{ b + c \}$ $\text{kill} = \{ \text{any expr with } a \}$ $d = e + f$

2

 $\text{gen} = \{ e + f \}$ $\text{kill} = \{ \text{any expr with } d \}$ $f = a + c$

3

 $\text{gen} = \{ a + c \}$ $\text{kill} = \{ \text{any expr with } f \}$ $a = b + c$ 1 $d = e + f$ 2 $f = a + c$ 3 $g = a + c$ 4 $b = a + d$ 5 $h = c + f$ 6 $j = a + b + c + d$ 7

Gen and Kill Sets

$$a = b + c \quad 1$$

gen = { 1 }

kill = { 3, 4, 5, 7 }

$$d = e + f \quad 2$$

gen = { 2 }

kill = { 5, 7 }

$$f = a + c \quad 3$$

gen = { 3 }

kill = { 2, 6 }

$$a = b + c \quad 1$$

$$d = e + f \quad 2$$

$$f = a + c \quad 3$$

$$g = a + c \quad 4$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$j = a + b + c + d \quad 7$$

Algorithm for Available Expression

- Assign a Number to each Expression in the Program
- Compute Gen Set and Kill Set for each Basic Block (or instruction)
 - Compute Gen Set and Kill Set for each Instruction in Basic Block
 - Compose them to create Basic Block Gen and Kill Sets

Aggregate Gen and Kill Sets

 $a = b + c$

1

 $\text{gen} = \{ 1 \}$ $\text{kill} = \{ 3, 4, 5, 7 \}$ $d = e + f$

2

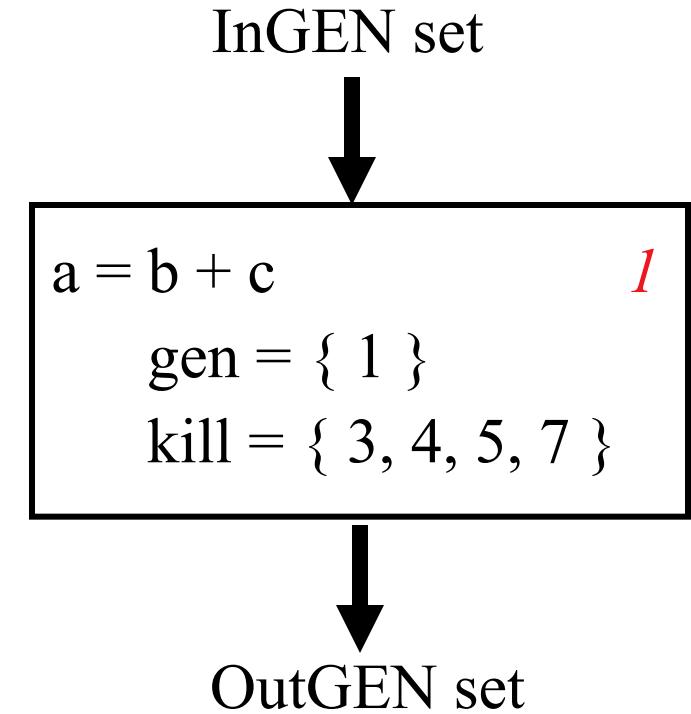
 $\text{gen} = \{ 2 \}$ $\text{kill} = \{ 5, 7 \}$ $f = a + c$

3

 $\text{gen} = \{ 3 \}$ $\text{kill} = \{ 2, 6 \}$

- Propagate all the Gen Sets and Kill Sets from top of the basic block to the bottom of the basic block
- How?

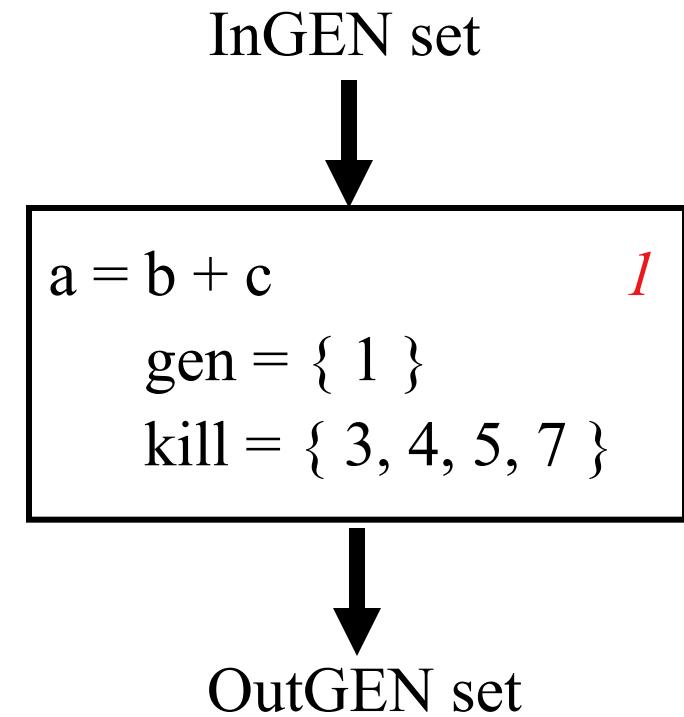
Aggregate Gen Set



OutGEN =

Aggregate Gen Set

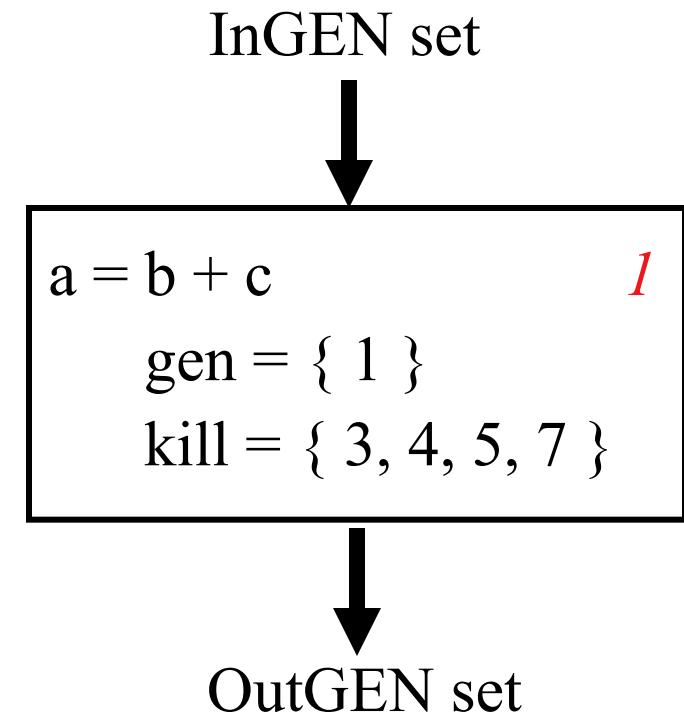
- An expression in the Gen Set in the current instruction should be in the OutGEN Set



$\text{OutGEN} = \text{gen}$

Aggregate Gen Set

- An expression in the Gen Set in the current instruction should be in the OutGEN Set
- Any expression in the InGEN Set that is not killed should be in the OutGEN Set



$$\text{OutGEN} = \text{gen} \cup (\text{InGEN} - \text{kill})$$

Aggregate Gen Set

$$\begin{aligned} a &= b + c & 1 \\ \text{gen} &= \{ 1 \} \\ \text{kill} &= \{ 3, 4, 5, 7 \} \end{aligned}$$
$$\begin{aligned} d &= e + f & 2 \\ \text{gen} &= \{ 2 \} \\ \text{kill} &= \{ 5, 7 \} \end{aligned}$$
$$\begin{aligned} f &= a + c & 3 \\ \text{gen} &= \{ 3 \} \\ \text{kill} &= \{ 2, 6 \} \end{aligned}$$

Aggregate Gen Set

InGEN = { }

a = b + c 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = gen \cup (InGEN - kill)

d = e + f 2

gen = { 2 }

kill = { 5, 7 }

f = a + c 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

a = b + c 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 } \cup ({ } - { 3, 4, 5, 7 })

d = e + f 2

gen = { 2 }

kill = { 5, 7 }

f = a + c 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

a = b + c 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

d = e + f 2

gen = { 2 }

kill = { 5, 7 }

f = a + c 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

OutGEN = gen \cup (InGEN - kill)

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

OutGEN = { 2 } \cup ({ 1 } - { 5, 7 })

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

OutGEN = { 1, 2 }

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

OutGEN = { 1, 2 }

InGEN = { 1, 2 }

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

OutGEN = gen \cup (InGEN - kill)

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

OutGEN = { 1, 2 }

InGEN = { 1, 2 }

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

OutGEN = { 3 } \cup ({1, 2} - {2, 6})

Aggregate Gen Set

InGEN = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutGEN = { 1, 2 }

InGEN = { 1, 2 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

OutGEN = { 1, 3 }

GEN = { 1, 3 }

InGEN = { }

a = b + c 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutGEN = { 1 }

InGEN = { 1 }

d = e + f 2
gen = { 2 }
kill = { 5, 7 }

OutGEN = { 1, 2 }

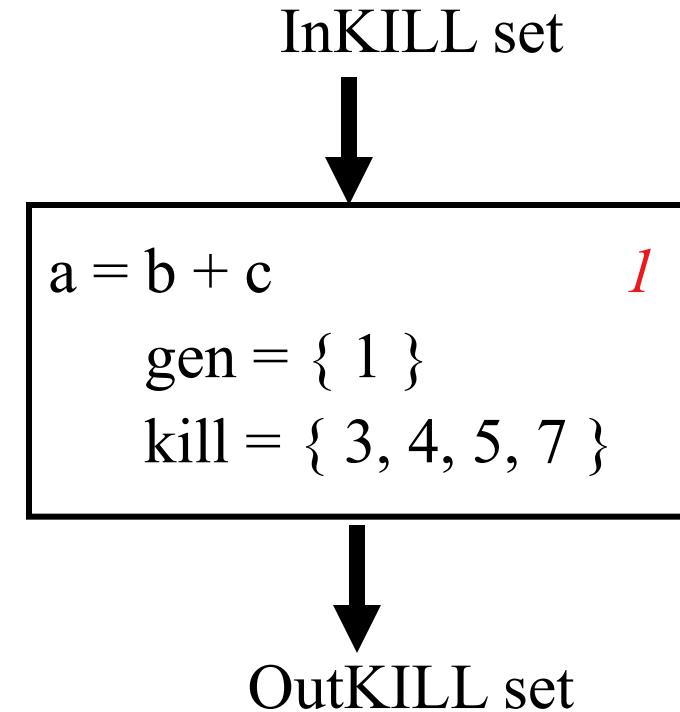
InGEN = { 1, 2 }

f = a + c 3
gen = { 3 }
kill = { 2, 6 }

OutGEN = { 1, 3 }

Aggregate Kill Set

- An expression in the Kill Set in the current instruction should be in the OutKILL set
- Note that different instruction define differently numbered expressions
⇒ expressions are unique



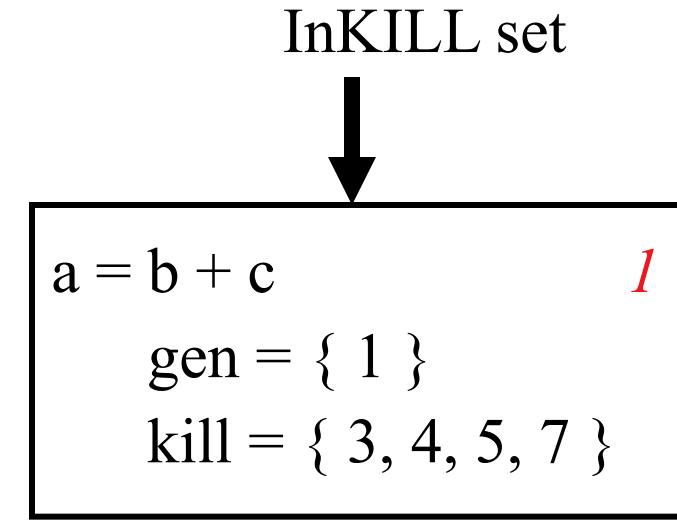
OutKILL =

Aggregate Kill Set

- An expression in the Kill Set in the current instruction should be in the OutKILL set
- Note that different instruction define differently numbered expressions

⇒ expressions are unique

- Any expression in the InKILL set should be in OutKILL



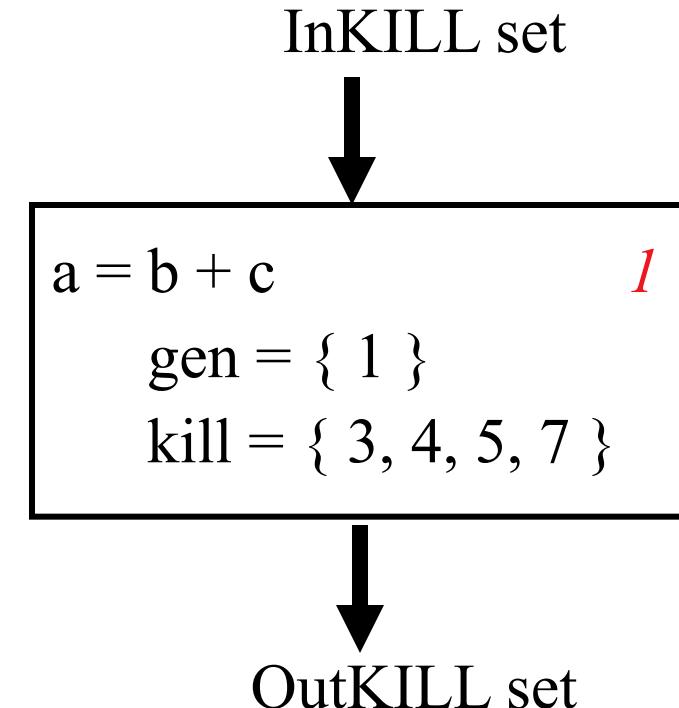
$$\text{OutKILL} = \text{kill}$$

Aggregate Kill Set

- An expression in the Kill Set in the current instruction should be in the OutKILL set
- Note that different instruction define differently numbered expressions

⇒ expressions are unique

- Any expression in the InKILL set should be in OutKILL



$$\text{OutKILL} = \text{kill} \cup \text{InKILL}$$

Aggregate Kill Set

$$\begin{aligned} a &= b + c & 1 \\ \text{gen} &= \{ 1 \} \\ \text{kill} &= \{ 3, 4, 5, 7 \} \end{aligned}$$
$$\begin{aligned} d &= e + f & 2 \\ \text{gen} &= \{ 2 \} \\ \text{kill} &= \{ 5, 7 \} \end{aligned}$$
$$\begin{aligned} f &= a + c & 3 \\ \text{gen} &= \{ 3 \} \\ \text{kill} &= \{ 2, 6 \} \end{aligned}$$

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1

gen = { 1 }

kill = { 3, 4, 5, 7 }

OutKILL = kill \cup InKILL

$d = e + f$ 2

gen = { 2 }

kill = { 5, 7 }

$f = a + c$ 3

gen = { 3 }

kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 } \cup { }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = kill \cup InKILL

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = { 5, 7 } \cup { 3, 4, 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = { 3, 4, 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

OutKILL = kill \cup InKILL

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

OutKILL = { 2, 6 } \cup { 3, 4, 5, 7 }

Aggregate Kill Set

InKILL = { }

$a = b + c$ 1
gen = { 1 }
kill = { 3, 4, 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$d = e + f$ 2
gen = { 2 }
kill = { 5, 7 }

OutKILL = { 3, 4, 5, 7 }

InKILL = { 3, 4, 5, 7 }

$f = a + c$ 3
gen = { 3 }
kill = { 2, 6 }

OutKILL = { 2, 3, 4, 5, 6, 7 }

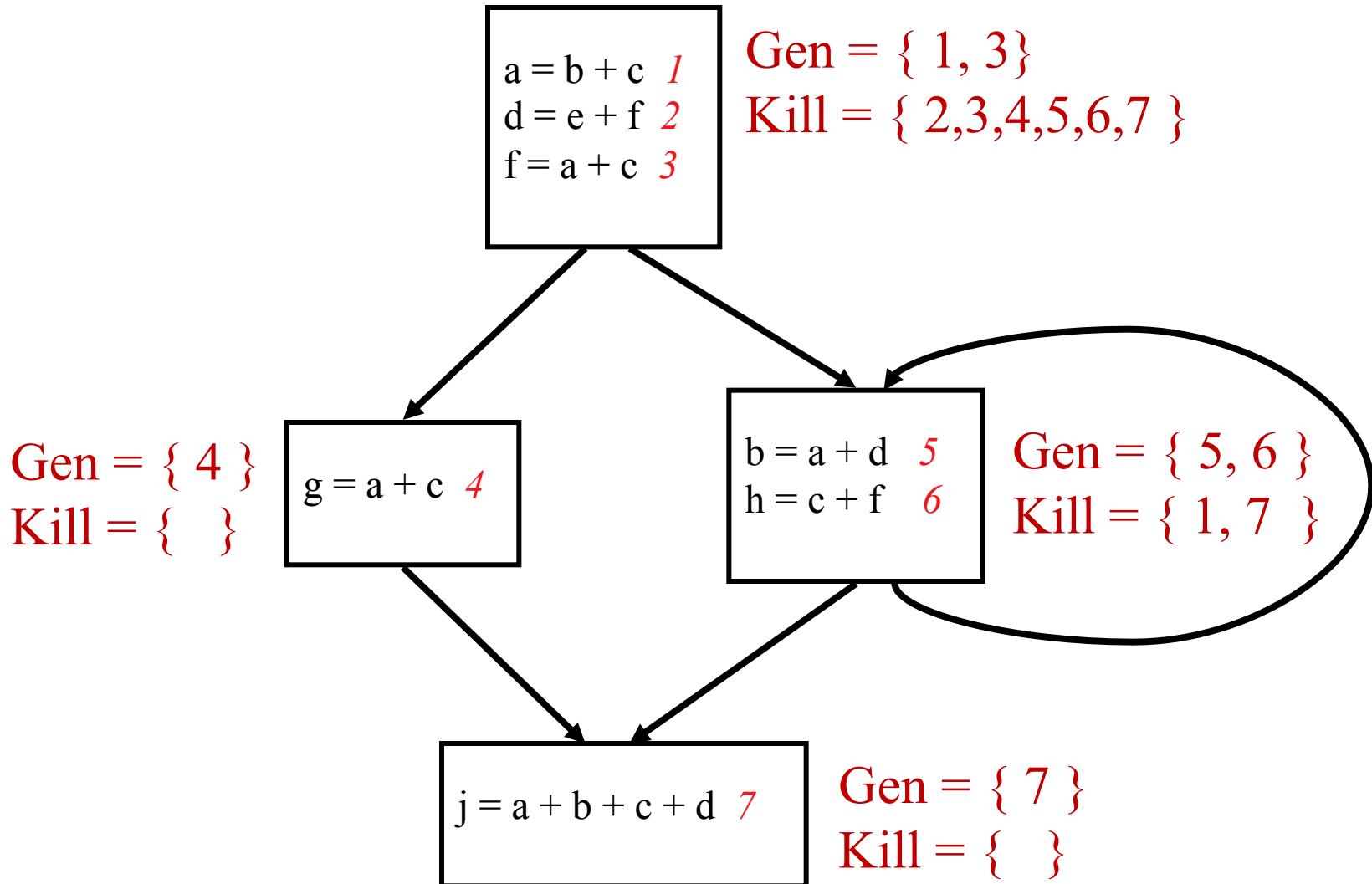
KILL = { 2, 3, 4, 5, 6, 7 }

$$\begin{aligned} a &= b + c && 1 \\ \text{gen} &= \{ 1 \} \\ \text{kill} &= \{ 3, 4, 5, 7 \} \end{aligned}$$

$$\begin{aligned} d &= e + f && 2 \\ \text{gen} &= \{ 2 \} \\ \text{kill} &= \{ 5, 7 \} \end{aligned}$$

$$\begin{aligned} f &= a + c && 3 \\ \text{gen} &= \{ 3 \} \\ \text{kill} &= \{ 2, 6 \} \end{aligned}$$

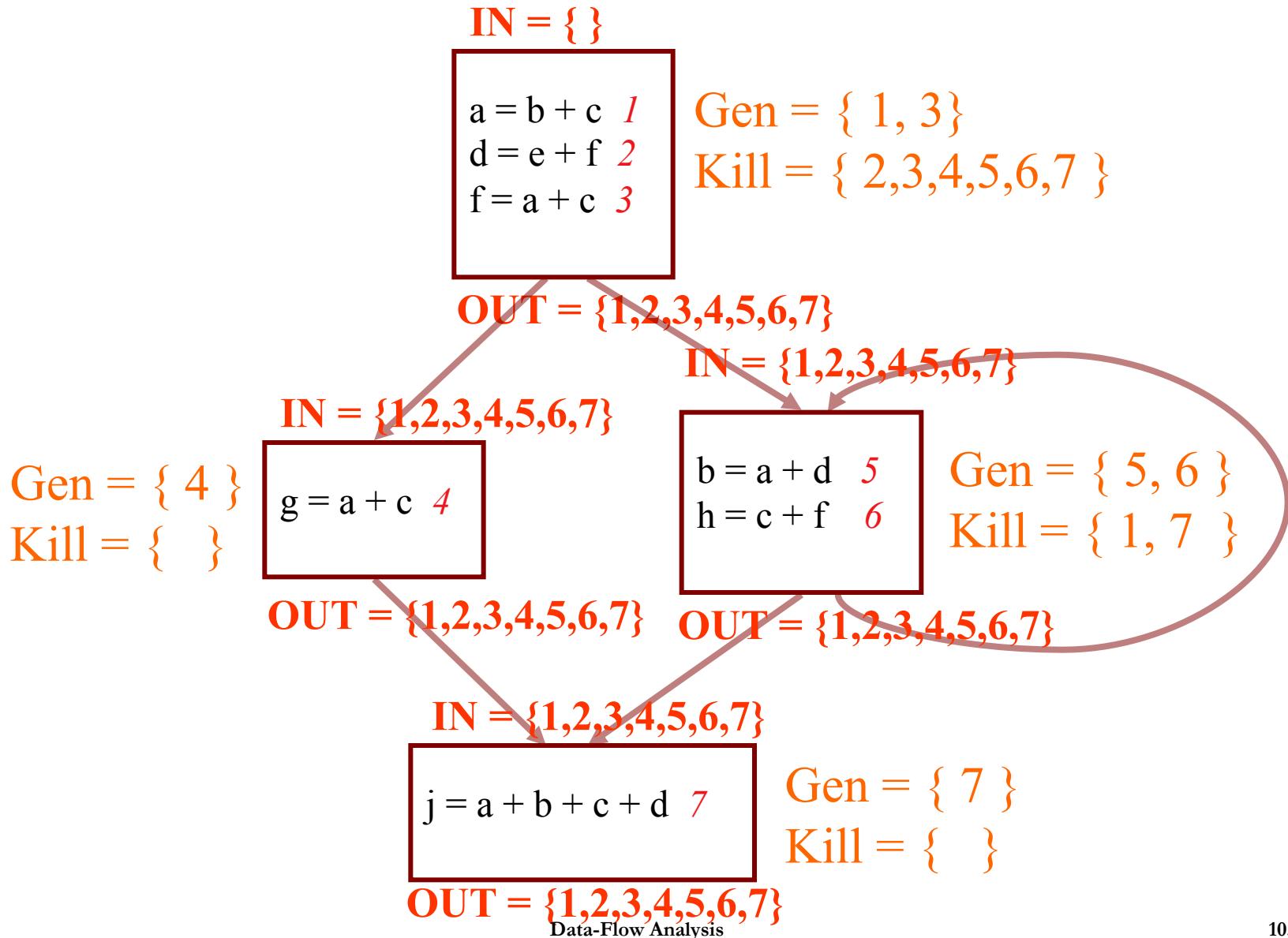
Aggregate Gen and Kill Sets



Algorithm for Available Expression

- Assign a Number to each Expression in the Program
- Compute Gen and Kill Sets for each Instruction
- Compute **Aggregate** Gen and Kill Sets for each Basic Block
- Initialize Available Set at each Basic Block as follows:
 - IN and OUT as the Entire Set (Universe of the set of expressions)
 - Exception: $\text{IN} = \emptyset$ for first Basic Block

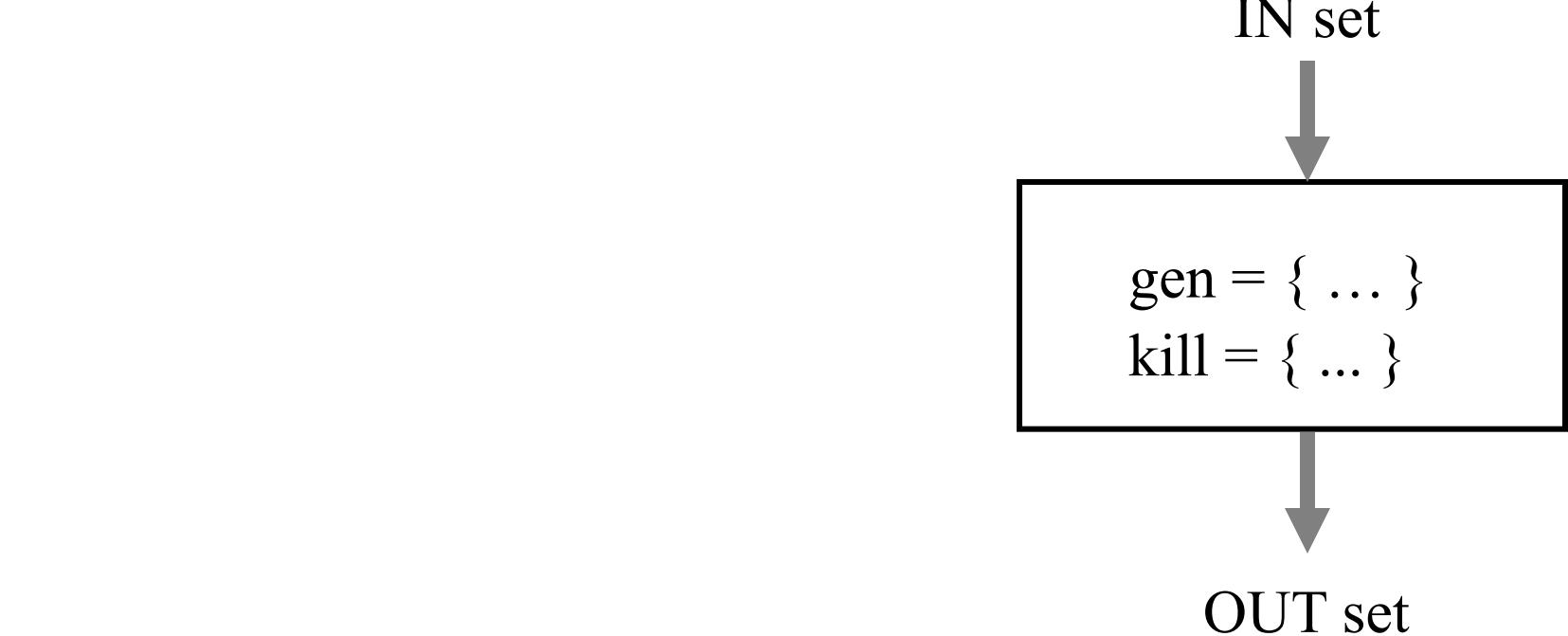
Aggregate Gen and Kill Sets



Algorithm for Available Expression

- Assign a Number to each Expression in the Program
- Compute Gen and Kill sets for each Instruction
- Compute Aggregate Gen and Kill sets for each Basic Block
- Initialize Available Set at each Basic Block as follows:
 - IN and OUT as the Entire Set (Universe of the set of expressions)
 - Exception: $\text{IN} = \emptyset$ for first Basic Block
- Iteratively propagate available expression set over the CFG

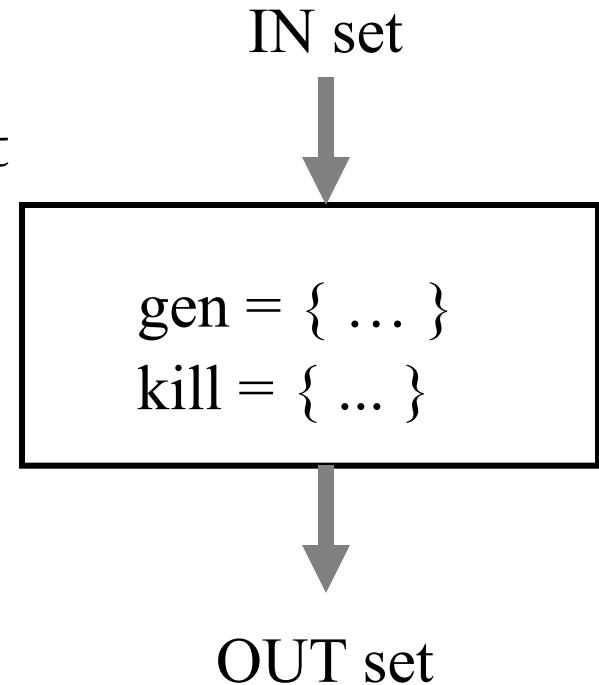
Propagate Available Expression Set



OUT =

Propagate Available Expression Set

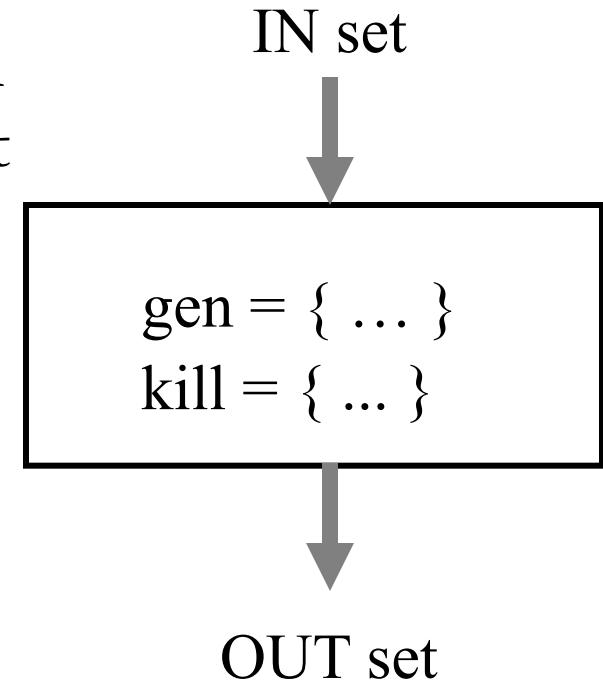
- If the expression is generated (in the Gen set) then it is available at the end
 - should be in the OUT set



$$\text{OUT} = \text{gen}$$

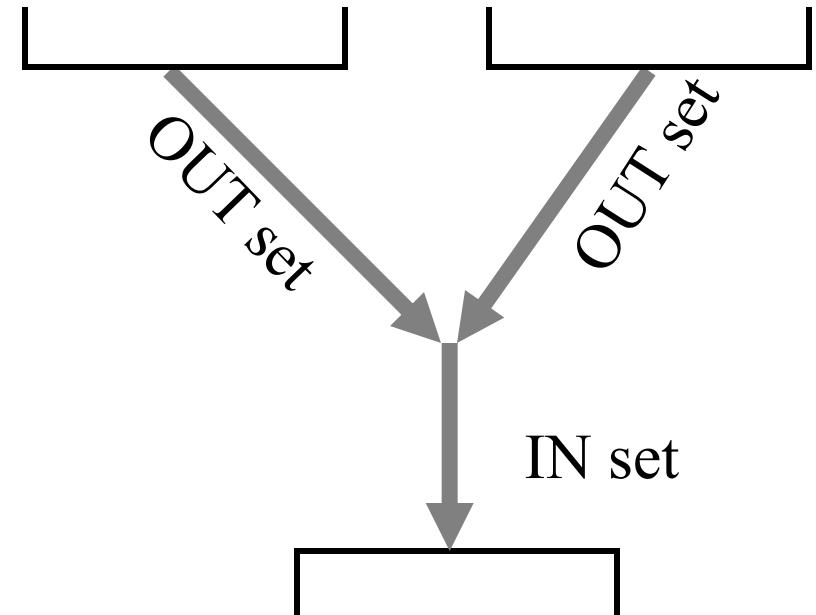
Propagate Available Expression Set

- If the expression is generated (in the Gen set) then it is available at the end
 - should be in the OUT set
- Any expression available at the input (in the IN set) and not killed should be available at the end



$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

Propagate Available Expression Set

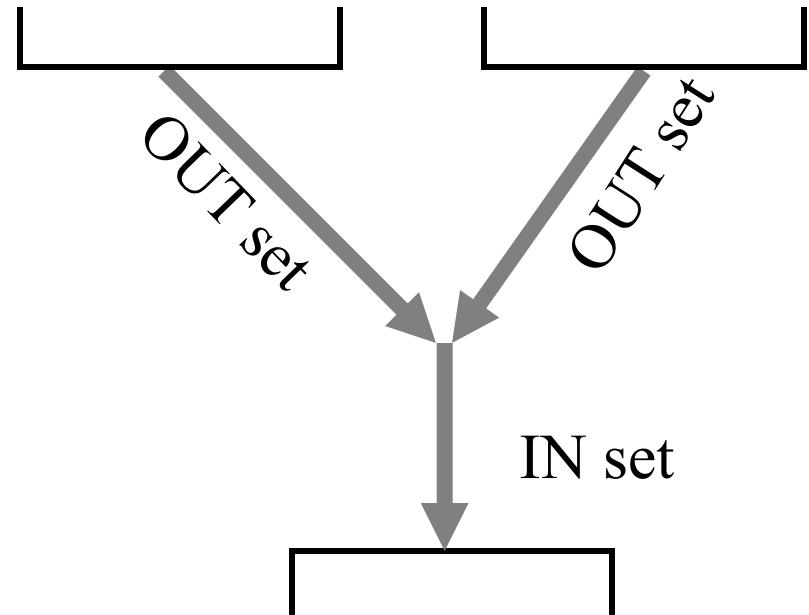


$IN =$

$OUT = \text{gen} \cup (IN - \text{kill})$

Propagate Available Expression Set

- Expression is available only if it is available in *All Input Paths*



$$\text{IN} = \bigcap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2,3,4,5,6,7 \}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 4 \}$$

$$\text{Kill} = \{ \}$$

$$g = a + c \quad 4$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$\text{Gen} = \{ 5, 6 \}$$

$$\text{Kill} = \{ 1, 7 \}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 7 \}$$

$$\text{Kill} = \{ \}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2,3,4,5,6,7 \}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 4 \}$$

$$\text{Kill} = \{ \}$$

$$g = a + c \quad 4$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$\text{Gen} = \{ 5, 6 \}$$

$$\text{Kill} = \{ 1, 7 \}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 7 \}$$

$$\text{Kill} = \{ \}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2,3,4,5,6,7 \}$$

$$\text{OUT} = \{ 1, 3 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 4 \}$$

$$\text{Kill} = \{ \}$$

$$g = a + c \quad 4$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$\begin{aligned} \text{Gen} &= \{ 5, 6 \} \\ \text{Kill} &= \{ 1, 7 \} \end{aligned}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{Gen} = \{ 7 \}$$

$$\text{Kill} = \{ \}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{\}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2,3,4,5,6,7 \}$$

$$\text{OUT} = \{1, 3\}$$

$$\text{IN} = \{1,2,3,4,5,6,7\}$$

$$\begin{aligned} \text{Gen} &= \{ 4 \} \\ \text{Kill} &= \{ \quad \} \end{aligned}$$

$$\text{IN} = \{1,2,3,4,5,6,7\}$$

$$g = a + c \quad 4$$

$$\text{OUT} = \{1,2,3,4,5,6,7\}$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$\begin{aligned} \text{Gen} &= \{ 5, 6 \} \\ \text{Kill} &= \{ 1, 7 \} \end{aligned}$$

$$\text{IN} = \{1,2,3,4,5,6,7\}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{1,2,3,4,5,6,7\}$$

$$\begin{aligned} \text{Gen} &= \{ 7 \} \\ \text{Kill} &= \{ \quad \} \end{aligned}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2,3,4,5,6,7 \}$$

$$\text{OUT}_{\Gamma} = \{ 1, 3 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1, 3 \}$$

$$g = a + c \quad 4$$

$$\begin{aligned} \text{Gen} &= \{ 4 \} \\ \text{Kill} &= \{ \} \end{aligned}$$

$$\begin{aligned} \text{Gen} &= \{ 5, 6 \} \\ \text{Kill} &= \{ 1, 7 \} \end{aligned}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

$$\text{IN} = \{ 1,2,3,4,5,6,7 \}$$

$$j = a + b + c + d \quad 7$$

$$\begin{aligned} \text{Gen} &= \{ 7 \} \\ \text{Kill} &= \{ \} \end{aligned}$$

$$\text{OUT} = \{ 1,2,3,4,5,6,7 \}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

$$\begin{aligned} a &= b + c \quad 1 \\ d &= e + f \quad 2 \\ f &= a + c \quad 3 \end{aligned}$$

$$\text{Gen} = \{ 1, 3 \}$$

$$\text{Kill} = \{ 2, 3, 4, 5, 6, 7 \}$$

$$\text{OUT} = \{ 1, 3 \}$$

$$\text{IN} = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$\text{IN} = \{ 1, 3 \}$$

$$g = a + c \quad 4$$

$$\begin{aligned} \text{Gen} &= \{ 4 \} \\ \text{Kill} &= \{ \} \end{aligned}$$

$$\text{OUT} = \{ 1, 3, 4 \}$$

$$\begin{aligned} b &= a + d \quad 5 \\ h &= c + f \quad 6 \end{aligned}$$

$$\begin{aligned} \text{Gen} &= \{ 5, 6 \} \\ \text{Kill} &= \{ 1, 7 \} \end{aligned}$$

$$\text{IN} = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$j = a + b + c + d \quad 7$$

$$\begin{aligned} \text{Gen} &= \{ 7 \} \\ \text{Kill} &= \{ \} \end{aligned}$$

$$\text{OUT} = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

Available Expressions

$$\text{IN} = \cap \text{OUT}$$

$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

$$\text{IN} = \{ \}$$

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$$\text{Kill} = \{ 2, 3, 4, 5, 6, 7 \}$$

$$\text{OUT}_{\Gamma} = \{ 1, 3 \}$$

$$\text{IN} = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$\text{IN} = \{ 1, 3 \}$$

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$$\text{OUT} = \{ 3, 5, 6 \}$$

$$\begin{aligned} \text{Gen} &= \{ 5, 6 \} \\ \text{Kill} &= \{ 1, 7 \} \end{aligned}$$

$$\text{IN} = \{ 1, 3, 4 \}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{ 1, 3, 4, 7 \}$$

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$$j = a + b + c + d \quad 7$$

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Available Expressions

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$$\text{IN} = \{ 3 \}$$

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Available Expressions

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$$\text{OUT} = \{ 1, 3, 4 \}$$

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$$\text{IN} = \{ 3 \}$$

$$j = a + b + c + d \quad 7$$

$$\text{OUT} = \{ 3, 7 \}$$

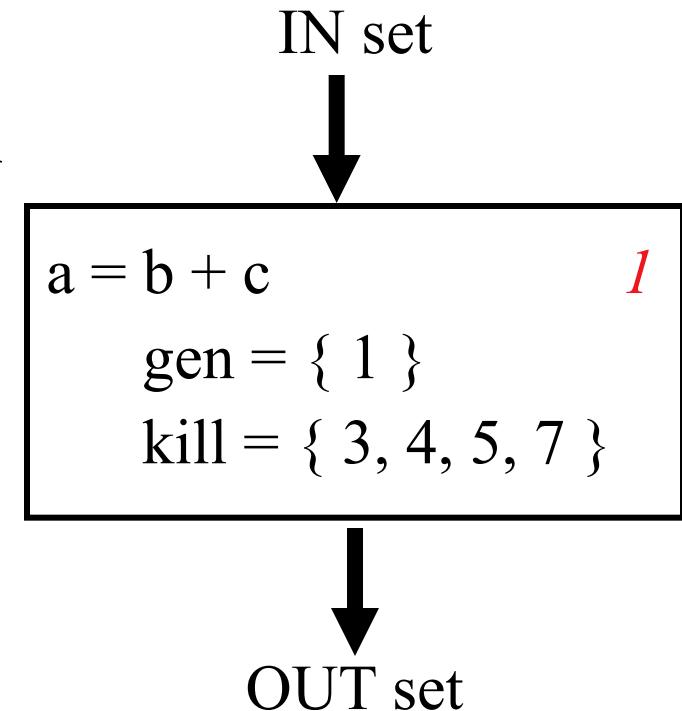
$$\begin{aligned} \text{Gen} &= \{ 7 \} \\ \text{Kill} &= \{ \} \end{aligned}$$

Algorithm for Available Expression

- Assign a Number to each Expression in the Program
- Calculate Gen and Kill Sets for each Instruction
- Derive **aggregate** Gen/Kill Sets for each Basic Block
- Initialize Available Expression Sets at each Basic Block to be the entire Set (Universe)
- Iteratively propagate Available Expression Sets over the CFG until it reaches a Solution
- Propagate Solution within the Basic Block

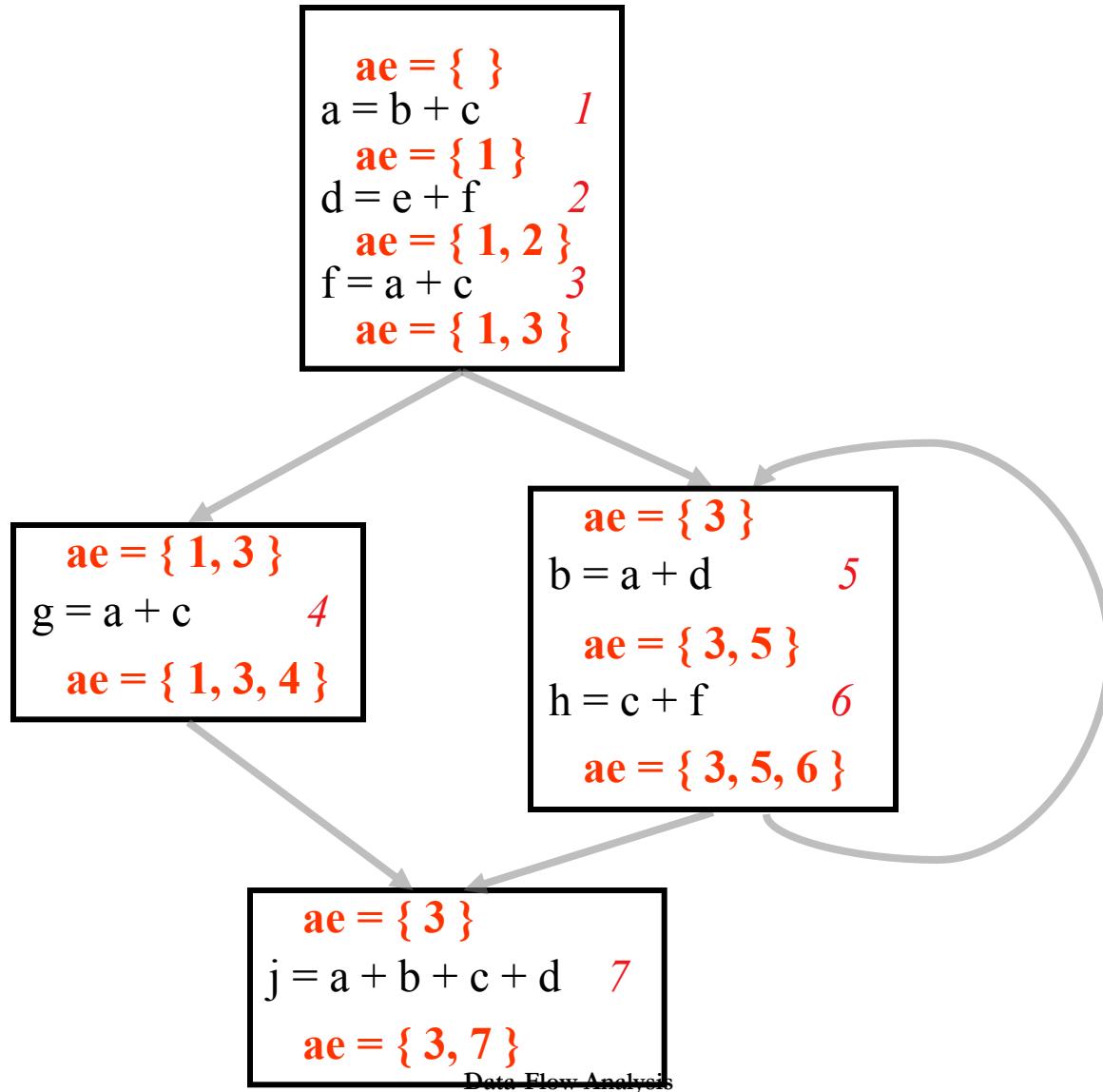
Propagate within the Basic Block

- Start with the IN set of available expressions
- Linearly Propagate Information down the basic block
 - same as a data-flow step
 - single pass since no back edges



$$\text{OUT} = \text{gen} \cup (\text{IN} - \text{kill})$$

Available Expressions



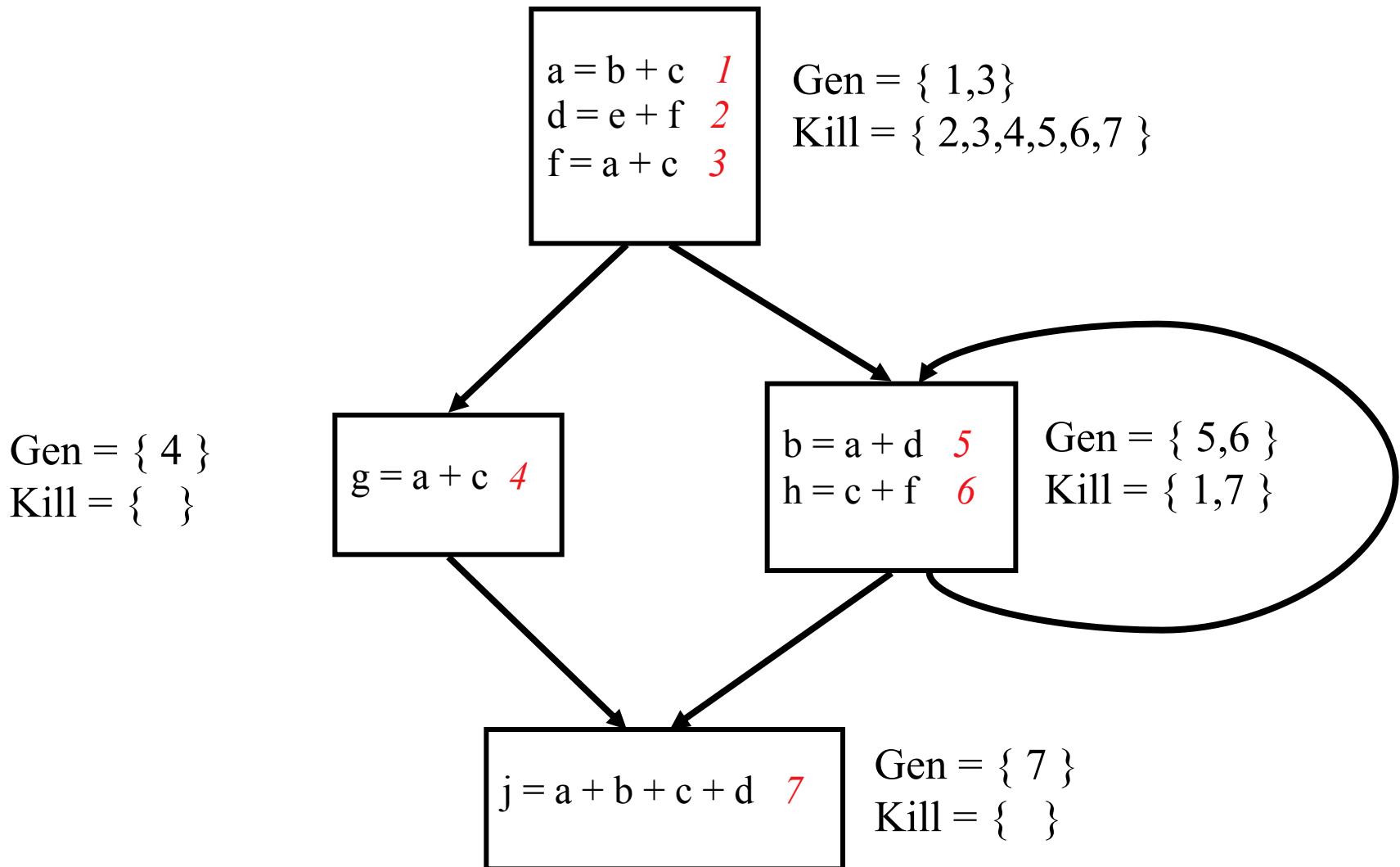
Outline

- Overview of Control-Flow Analysis
- Available Expressions Data-Flow Analysis Problem
- Algorithm for Computing Available Expressions
- Practical Issues: Bit Sets

Practical Issues: Bit Sets

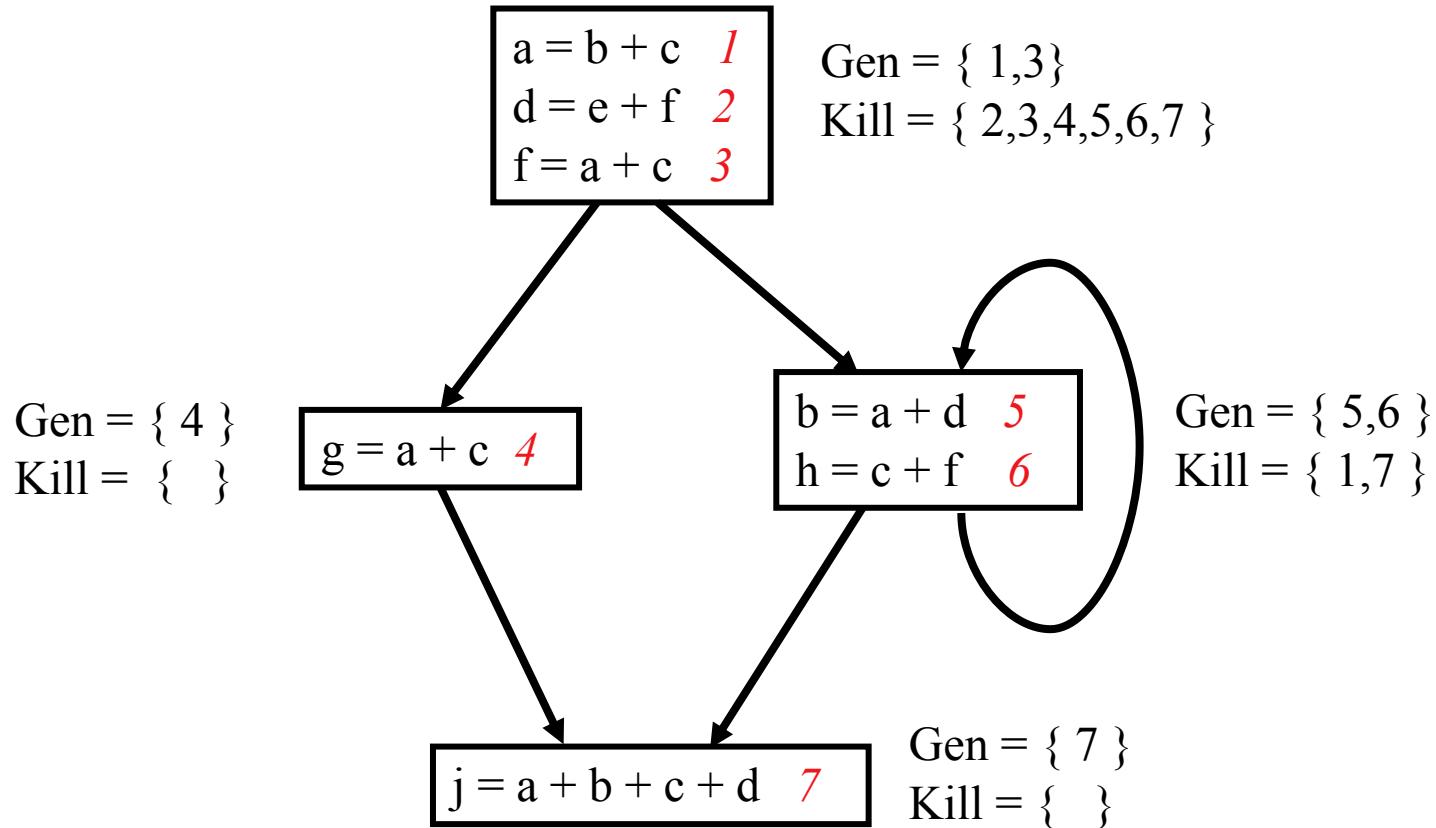
- Assign a bit to each element of the set
 - Union \Rightarrow bit OR
 - Intersection \Rightarrow bit AND
 - Subtraction \Rightarrow bit NEGATE and AND
 - Fast implementation
 - 32 elements packed to each word
 - AND and OR are single instructions

Available Expressions



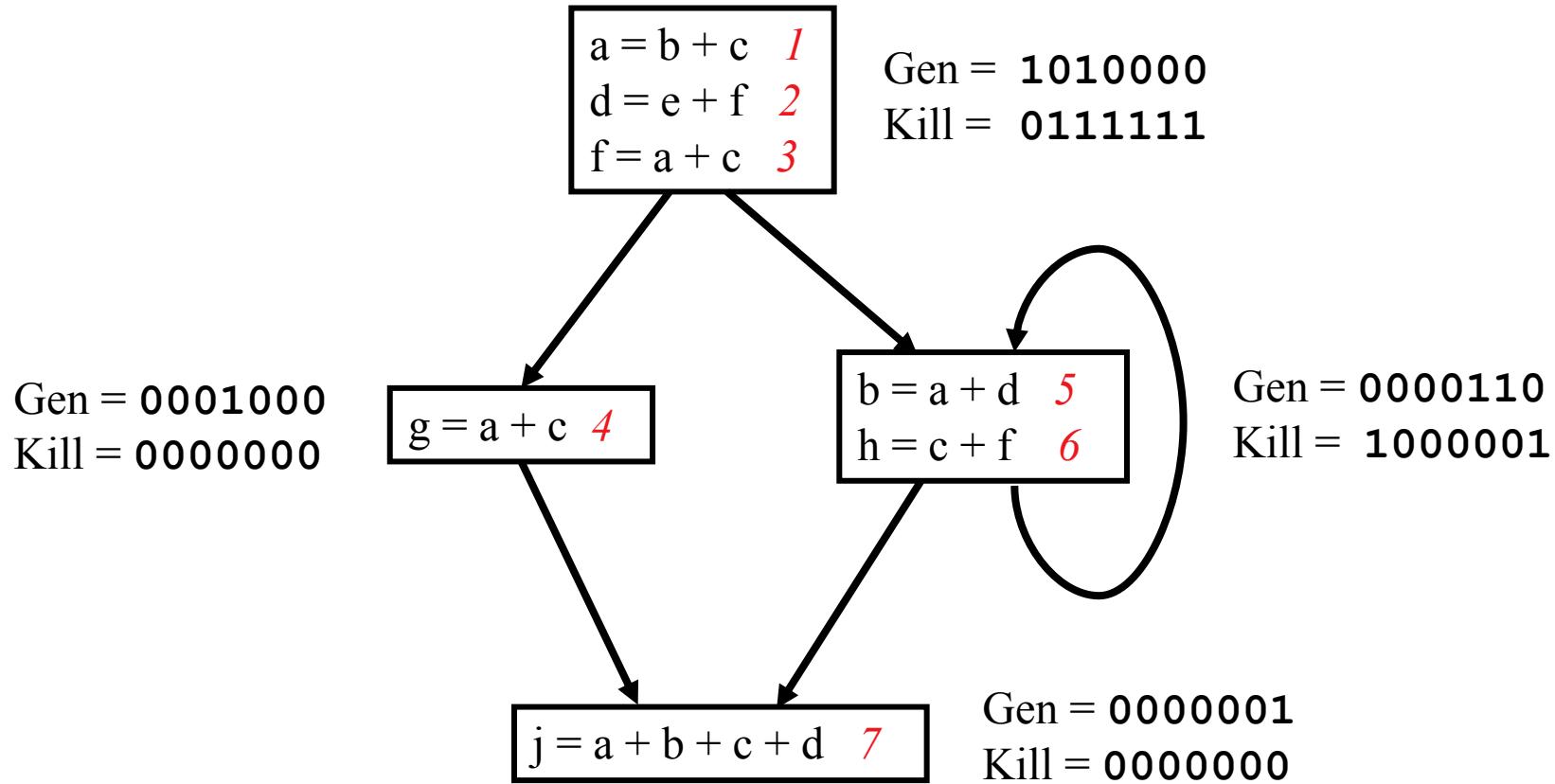
Available Expressions

7 bits per set required



Available Expressions

7 bits per set required



Summary

- Overview of Control-Flow Analysis
- Available Expressions Data-Flow Analysis Problem
- Algorithm for Computing Available Expressions
- Practical Issues: Bit Sets