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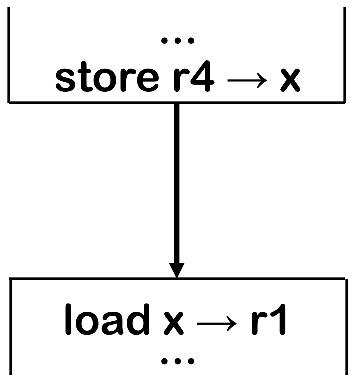
Compilers Design and Implementation

Global Register Allocation
Webs and Graph Coloring
Node Splitting and Other Transformations

What a Smart Allocator Needs to Do

- Determine ranges for each variable can benefit from using a register (Webs)
- Determine which of these ranges overlap (Interference)
- Find the benefit of keeping each web in a register (spill cost)
- Decide which webs gets a register (Allocation)
- Split webs if needed (spilling and splitting)
- Assign hard registers to Webs (Assignment)
- Generate code including spills (code gen)

Global Register Allocation

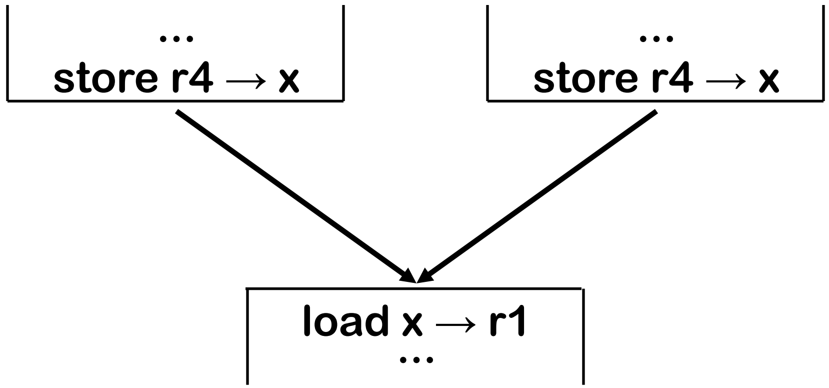


This is an assignment problem, not an allocation problem !

What's harder across Multiple Blocks?

- Could replace a load with a move
- Good assignment would obviate the move
- Must build a control-flow graph to understand inter-block flow
- Can spend an inordinate amount of time adjusting the allocation

Global Register Allocation



What if one block has x in a register, but the other does not?

A more complex scenario

- Block with multiple predecessors in the control-flow graph
- Must get the "right" values in the "right" registers in each predecessor
- In a loop, a block can be its own predecessors

This adds tremendous complications

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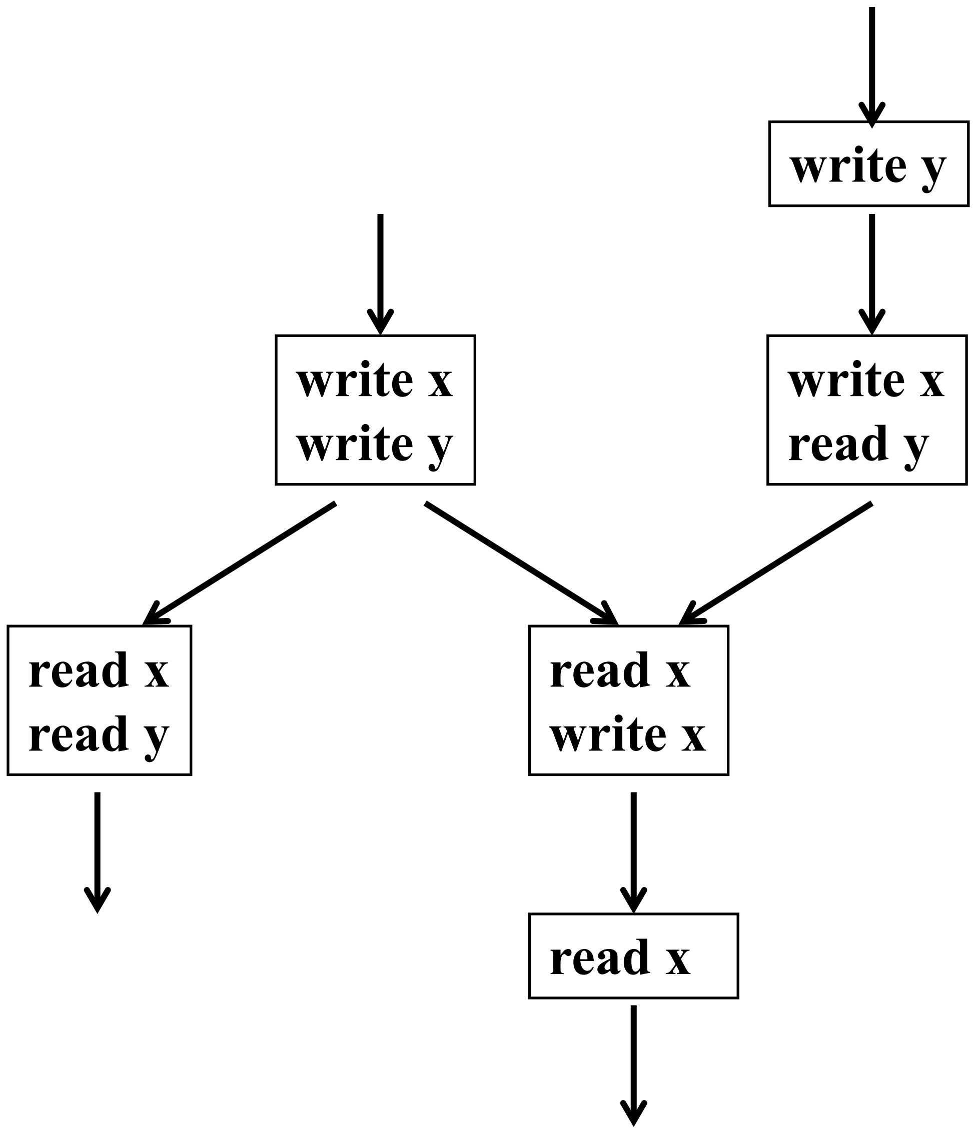
Outline

- What is Register allocation and Its Importance
- Simple Register Allocators
- Webs
- Interference Graphs
- Graph Coloring
- Splitting
- More Optimizations

Webs

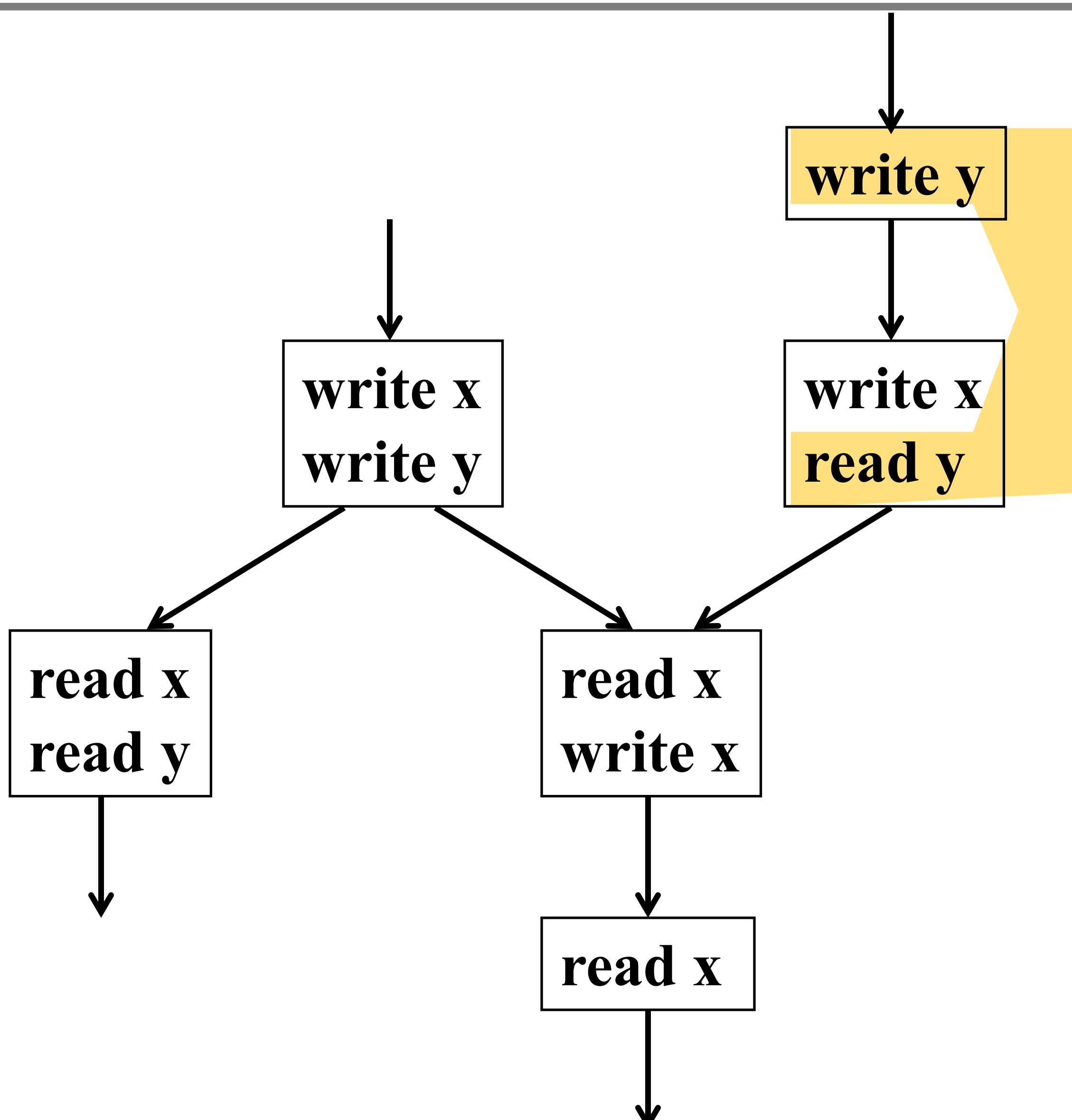
- What needs to Gets Memorized is the Value
- Divide Accesses to a Variable into Multiple Webs
  - All Definitions that Reach a Use are in the same Web
  - All Uses of a Definition are in the same Web
  - Divide the Variable into Live Ranges
- Implementation: use DU (Def-Use) chains
  - A DU-chain connects a definition to all uses reached by each definition
  - A web combines DU-chains containing a common use

Example



write y
write x
write y
write x
read y
read x
read y
read x
write x
read x

Example



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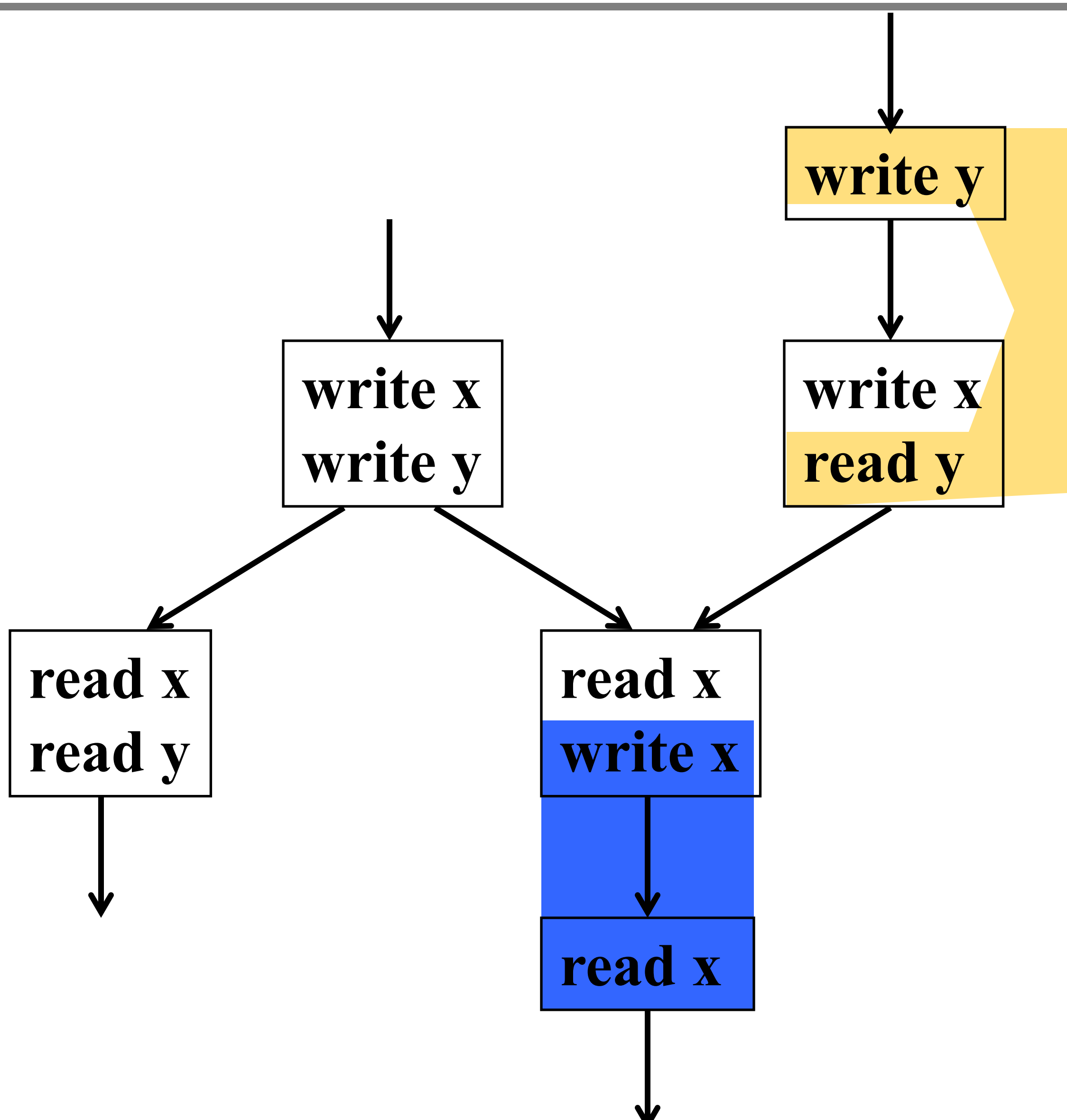
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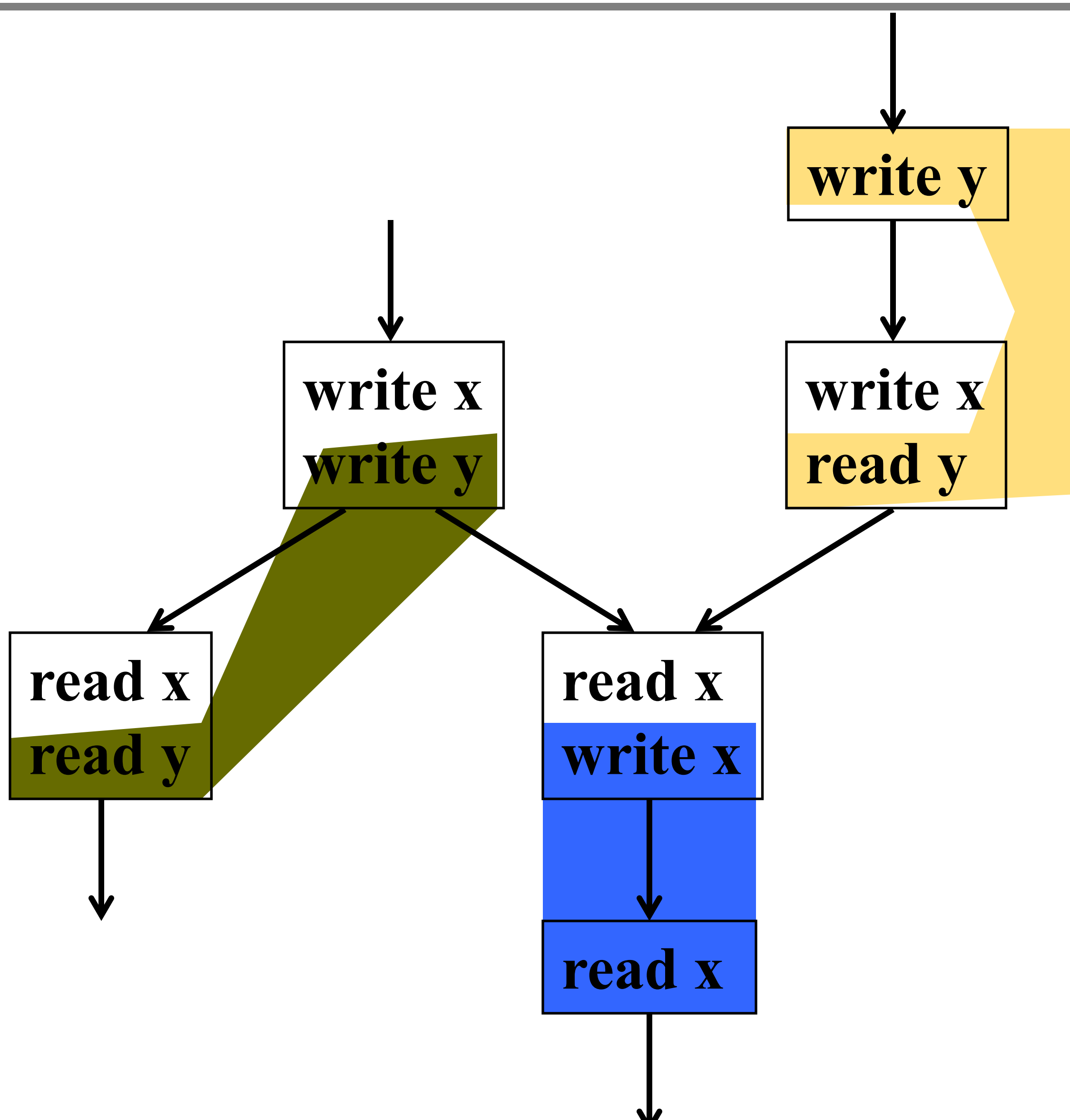
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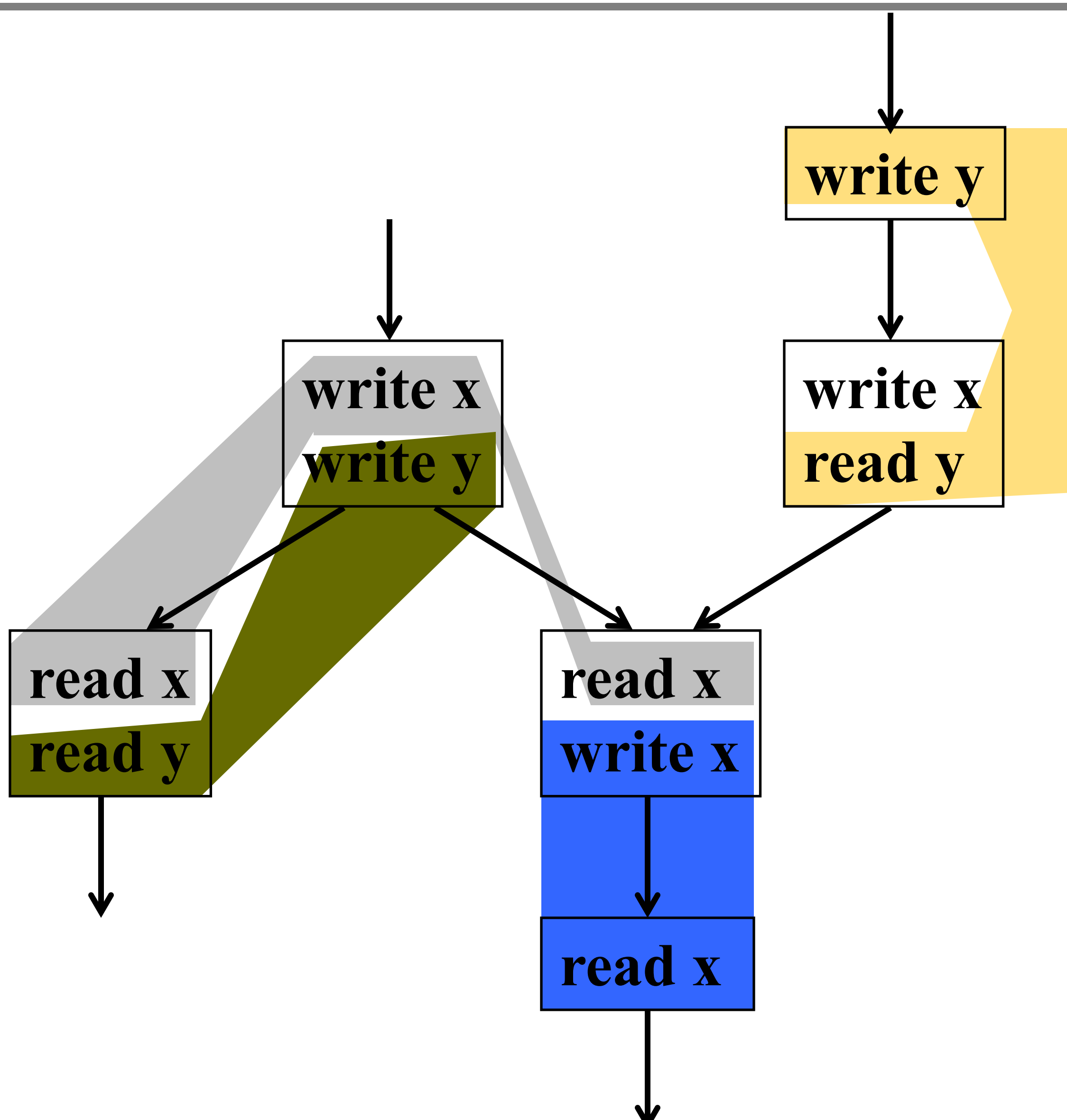
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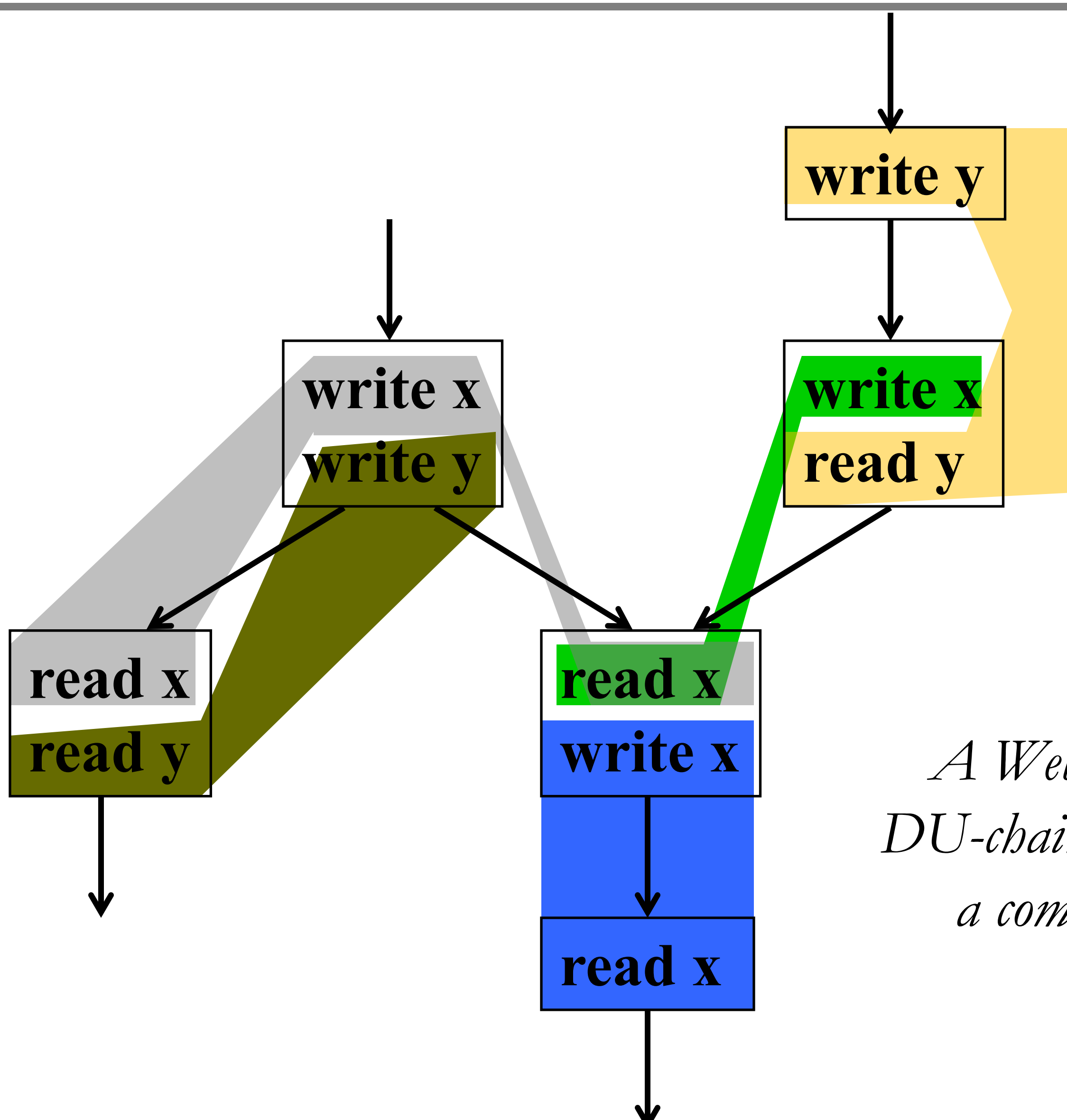
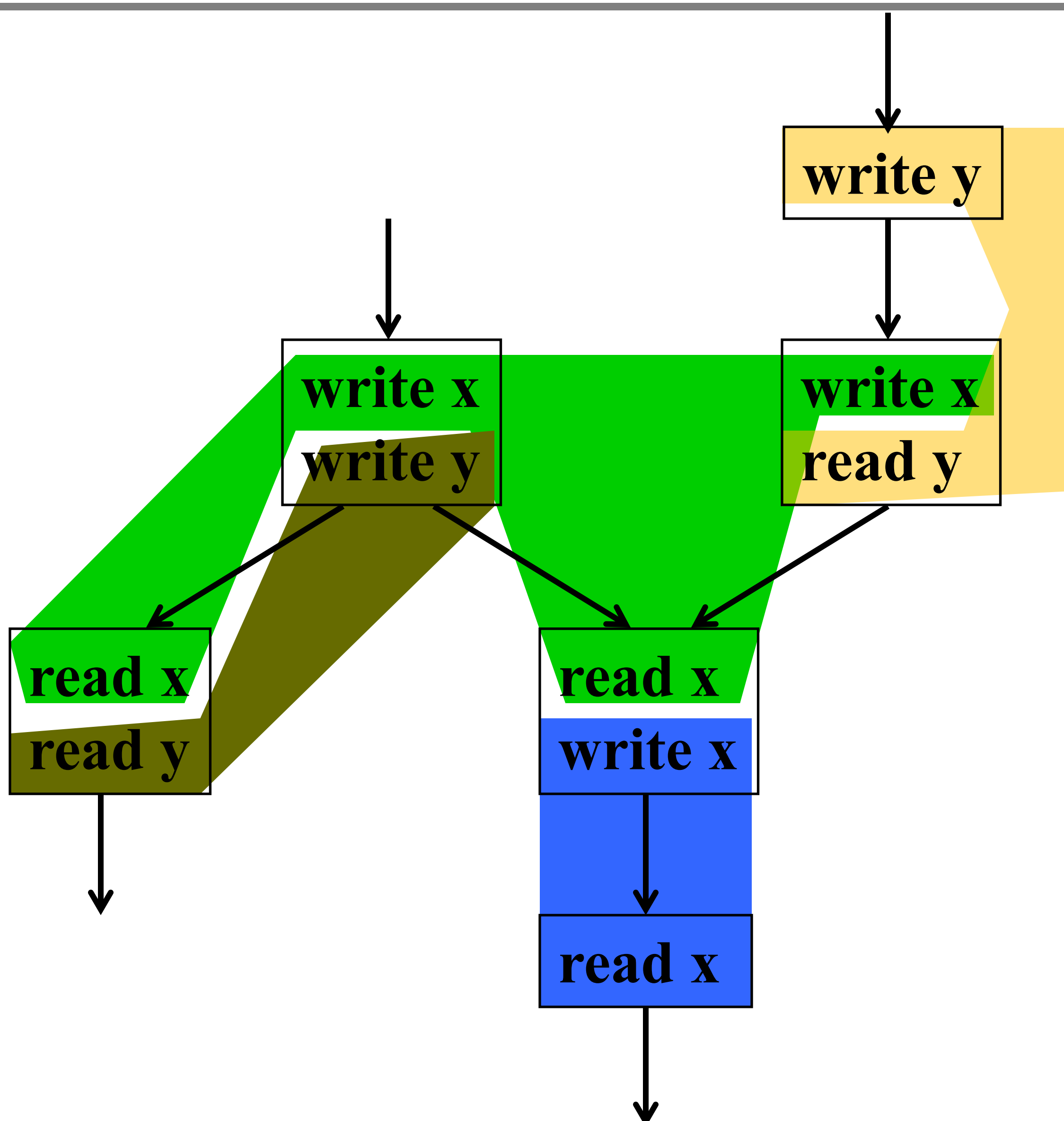


A Web combines DU-chains containing a common Use

Example



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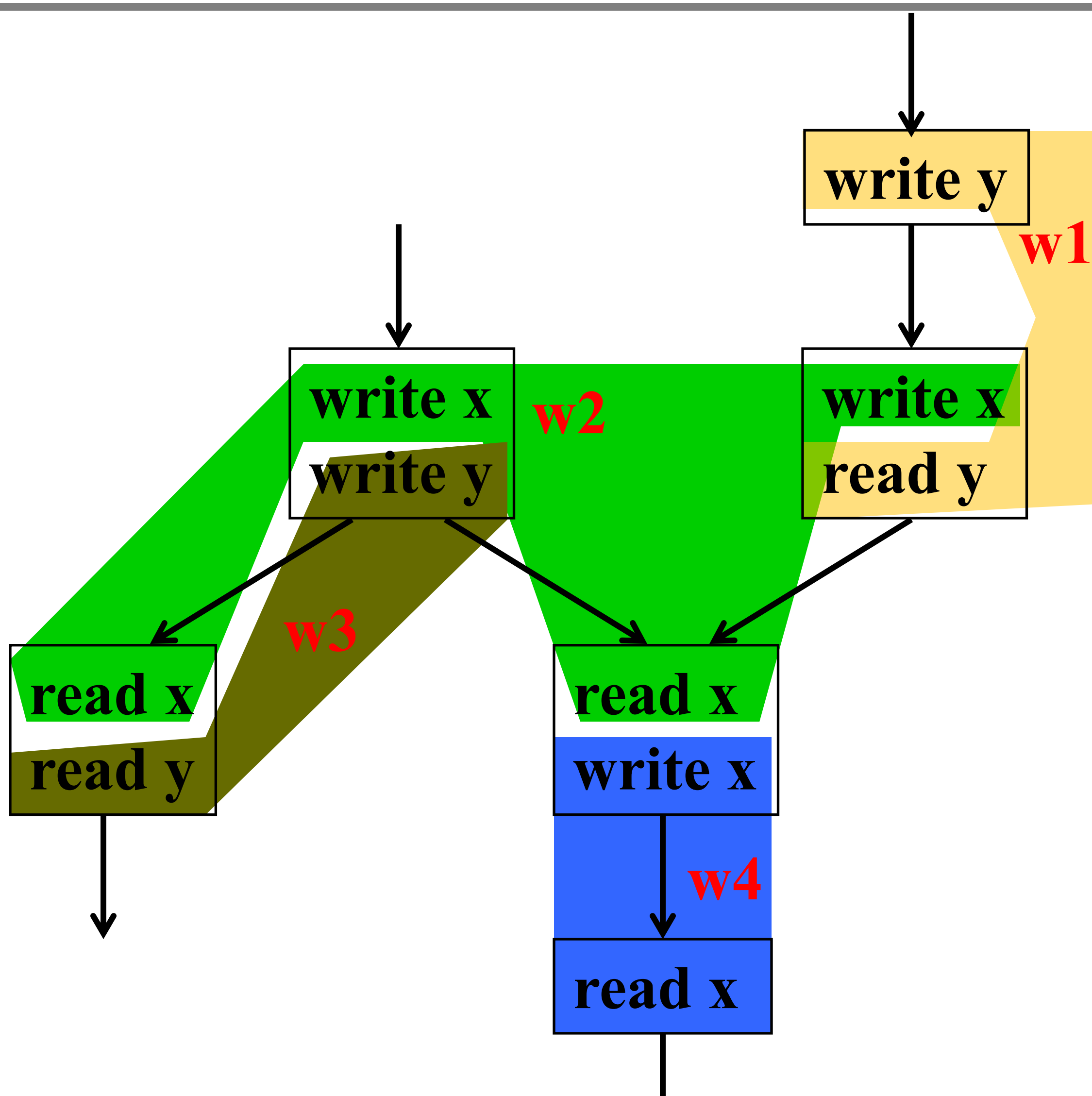
write x
write y
write x
read y
read x
read y
read x
write x
read x

Example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w1
write x
read y
write x
write y
w2
w3
read x
read y
read x
write x
w4
read x

Webs (*continued*)

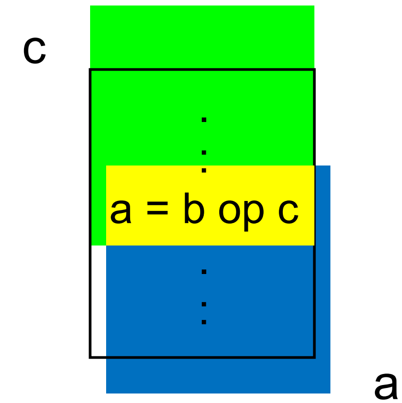
- In two Webs of the same Variable:
  - No use in one web will ever use a value defined by the other web
  - Thus, no value need to be carried between webs
  - Each web can be treated independently as values are independent
- Web is used as the Unit of Register Allocation
  - If a web is allocated to a register, all the uses and definitions within that web don't need to load and store from memory
  - Solves the issue of cross Basic Block register assignment issue
  - Different webs may be assigned to different registers or one to register and one to memory

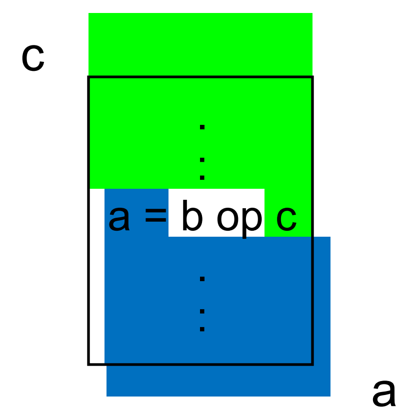
Outline

- What is Register Allocation
- A Simple Register Allocator
- Webs
- Interference Graphs
- Graph Coloring
- Splitting
- More Optimizations

Interference

- Two webs interfere if their live ranges overlap in Execution Time
  - What does time Mean, precisely?
  - There exists an instruction common to both ranges where
    - They variable values of webs are operands of the instruction
    - If there is a single instruction in the overlap
      - and the variable for the web that ends at that instruction is an operands and
      - the variable for the web that starts at the instruction is the destination of the instruction
    - then the webs do not interfere
- Non-interfering webs can be assigned to the same register





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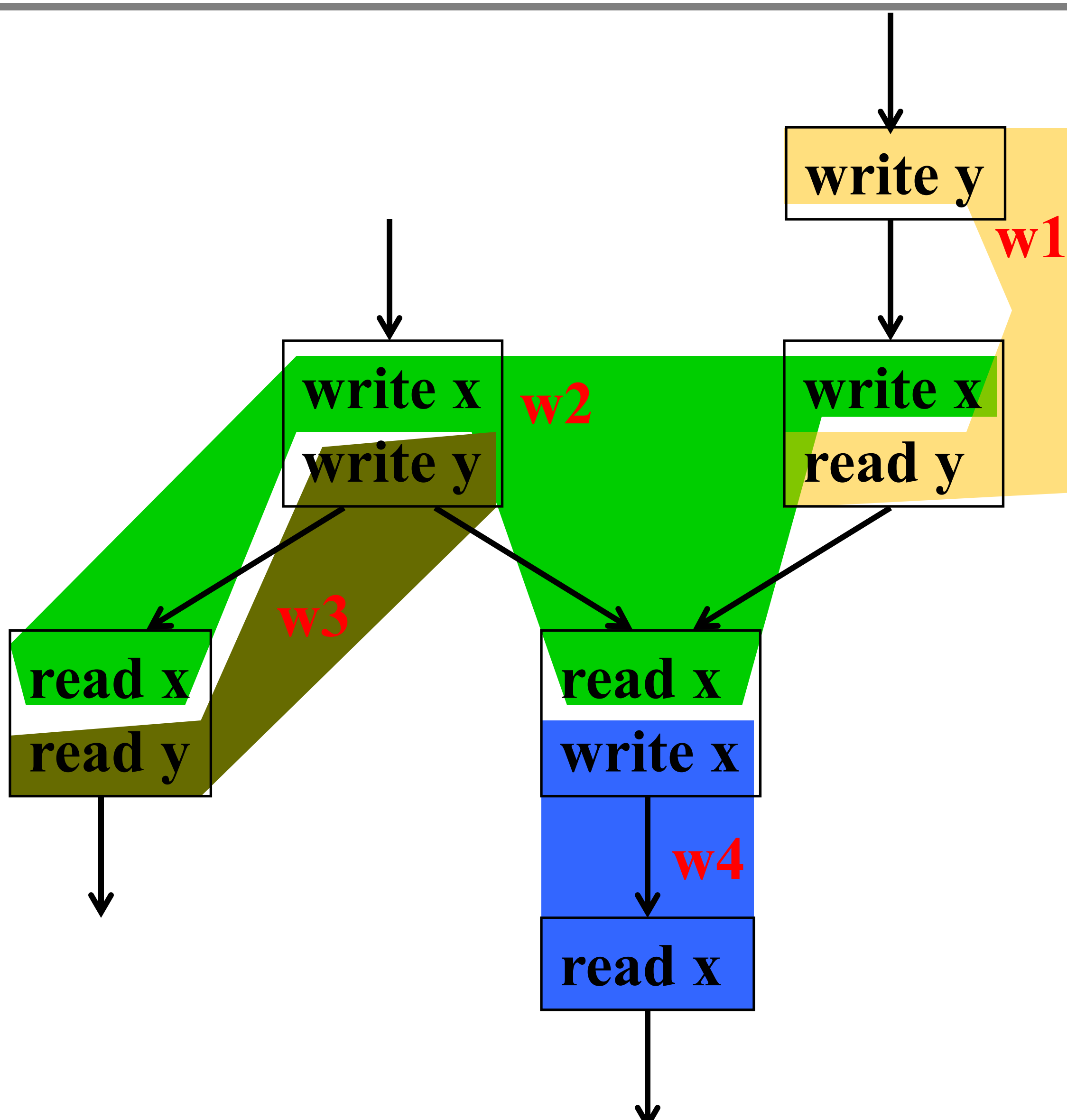
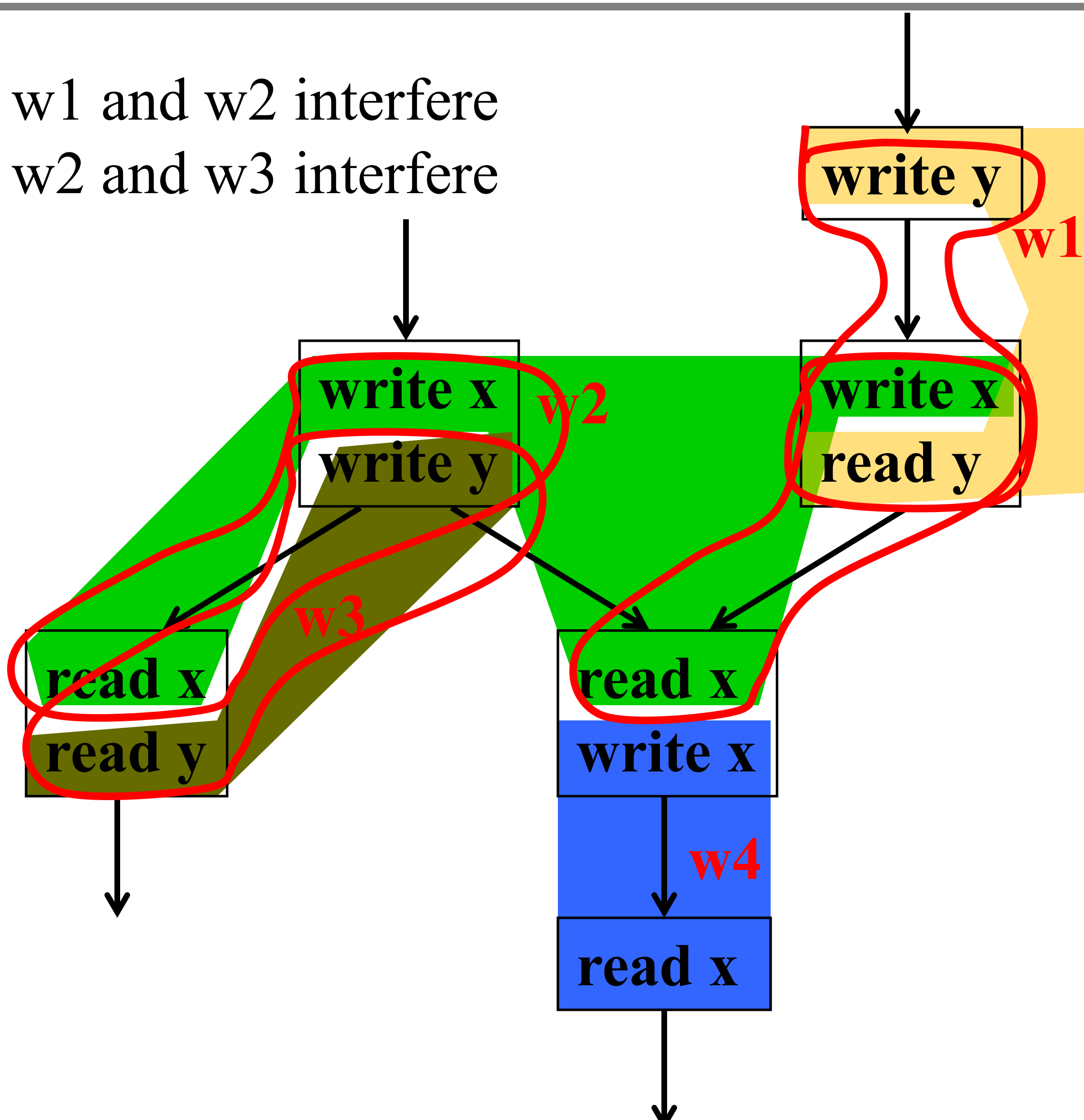


write y
w1
write x
write y
w2
write x
read y
w3
read x
read y
read x
write x
w4
read x

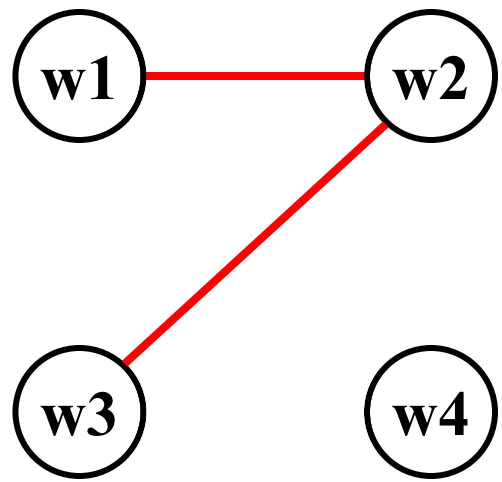
Example

Webs w1 and w2 interfere
Webs w2 and w3 interfere

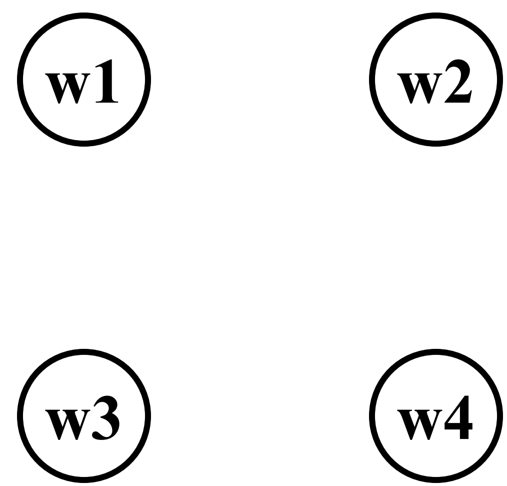


Interference Graph

- Representation of Webs & their Interference
  - Nodes are the webs
  - An edge exists between two nodes if they interfere

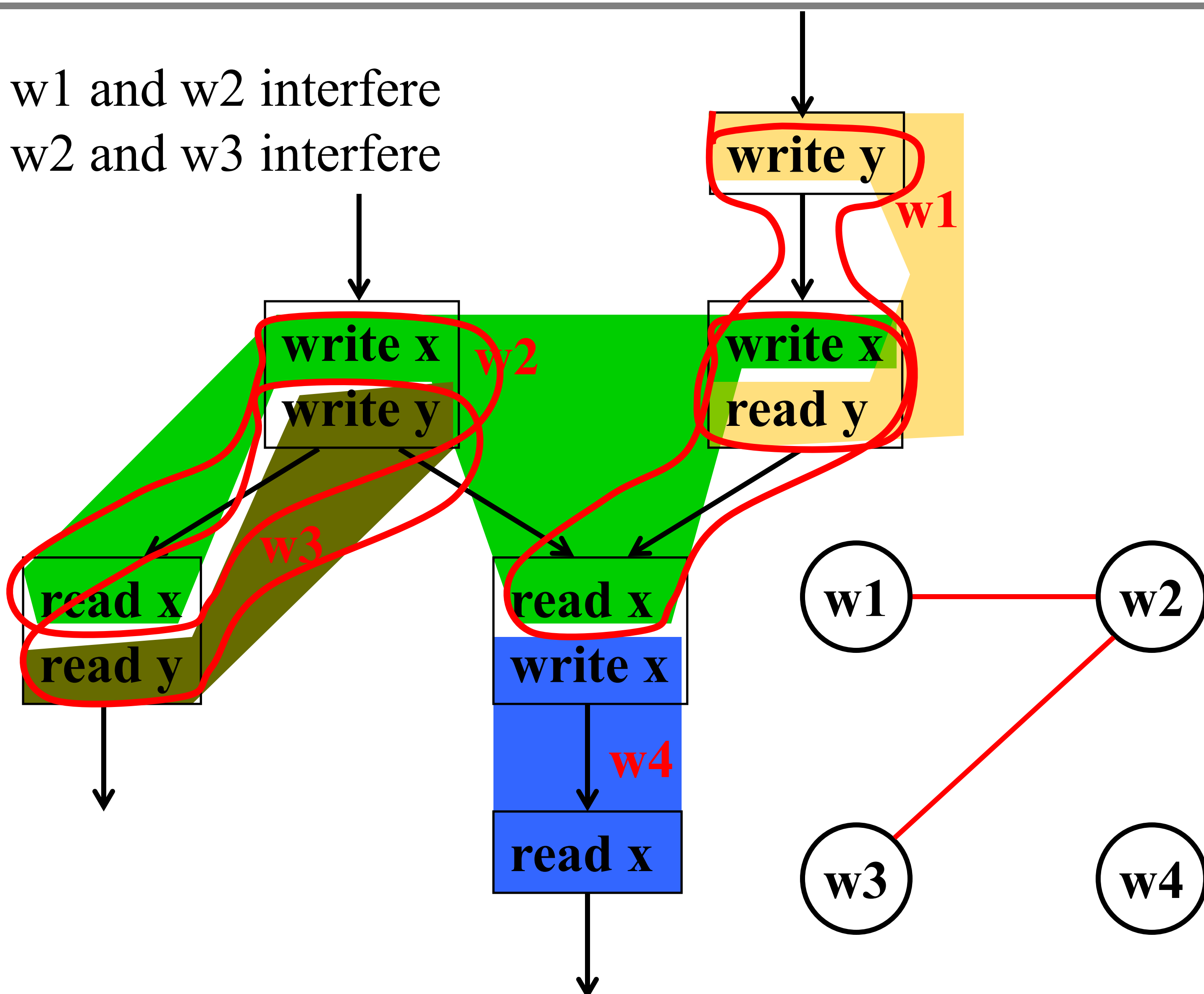


Example



Example

Webs w1 and w2 interfere
Webs w2 and w3 interfere



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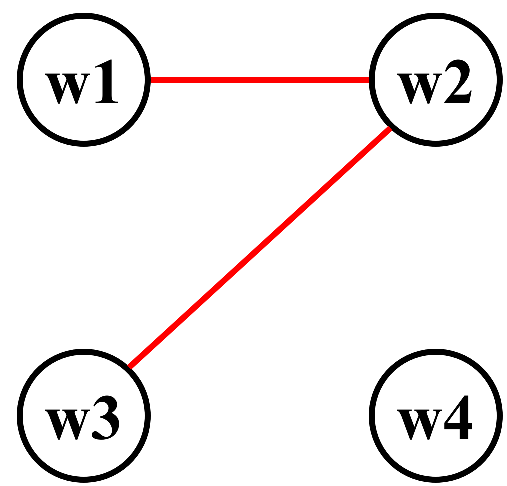
Reg. Allocation Using Graph Coloring

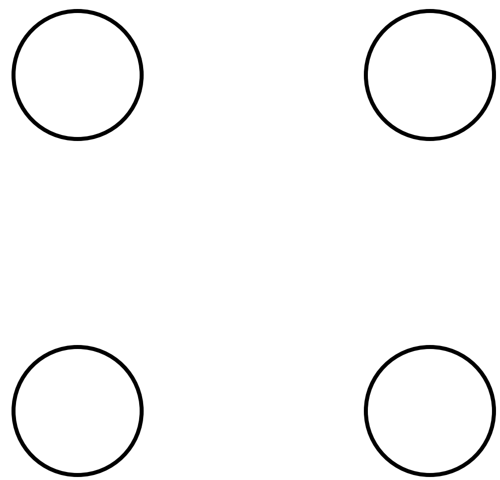
- Each Web is Allocated a Register
  - each node gets a register (color)
- If two webs interfere they cannot use the same register
  - if two nodes have an edge between them, they cannot have the same color



Graph Coloring

- What is the minimum number of colors that takes to color the nodes of the graph such that any nodes connected with an edge does not have the same color?
- Classic Problem in Graph Theory

Graph Coloring Example



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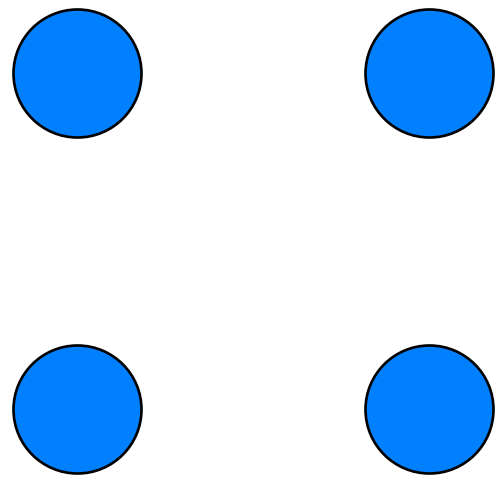
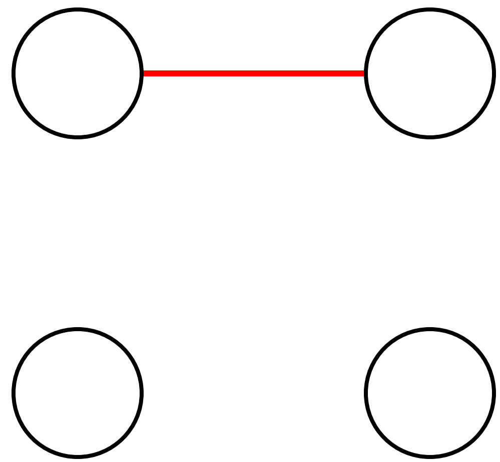
Graph Coloring Example

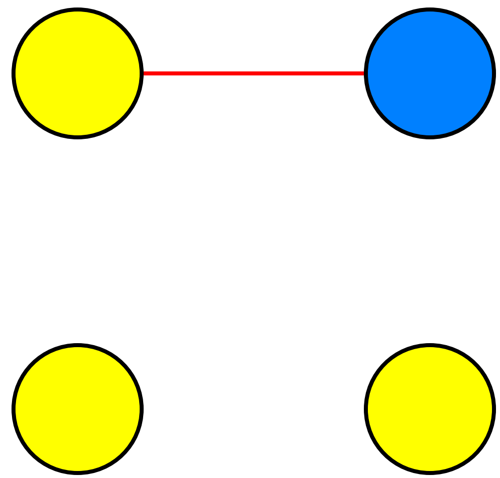


- **1 Color**

Graph Coloring Example

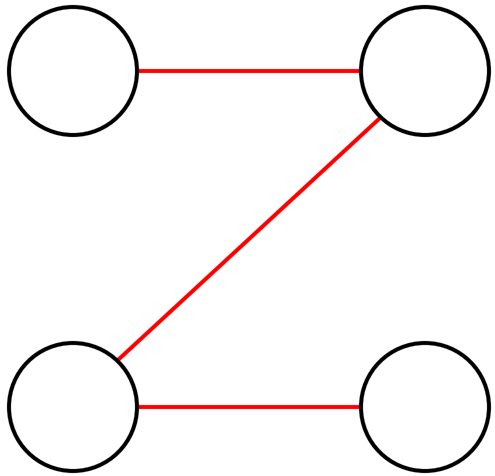


Graph Coloring Example



- **2 Colors**

Graph Coloring Example



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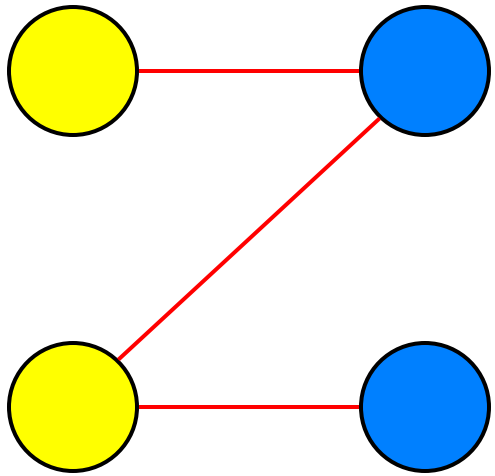
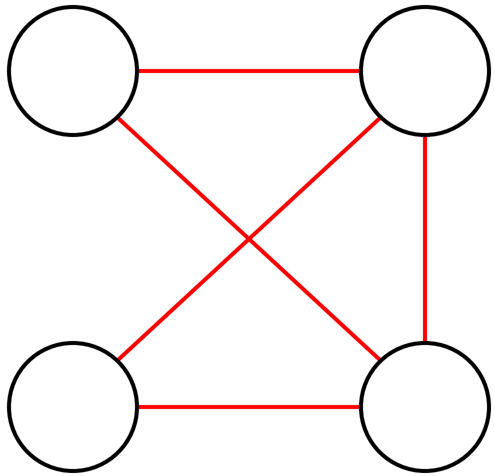
Graph Coloring Example

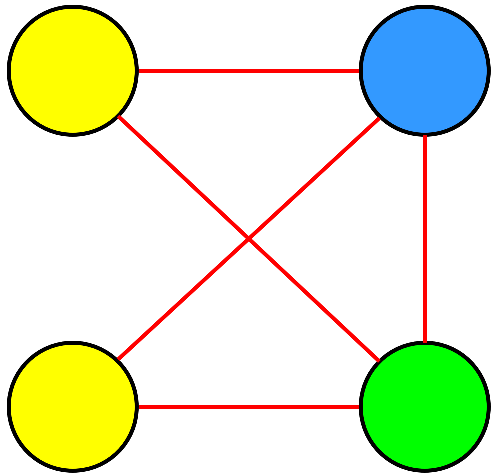


- **Still 2 Colors**

Graph Coloring Example



Graph Coloring Example



- **3 Colors**

Heuristics for Register Coloring

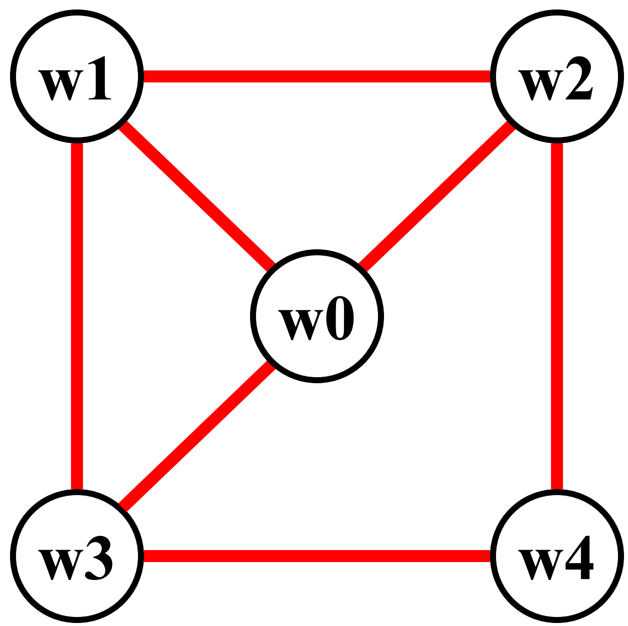
- Coloring a Graph with N colors
- If degree < N (degree of a node = # of edges)
  - Node can always be colored
  - After coloring the rest of the nodes, you'll have at least one color left to color the current node
- If degree $\geq$ N
  - still may be colorable with N colors
  - exact solution is NP complete

Heuristics for Register Coloring

- Remove nodes that have degree < N
  - Push the removed nodes onto a stack
- If all the nodes have degree ≥ N
  - Find a node to spill (no color for that node)
  - Remove that node
- When empty, start the coloring step
  - pop a node from stack back
  - Assign it a color that is different from its connected nodes (since degree < N, a color should exist)

Coloring Example

N = 3



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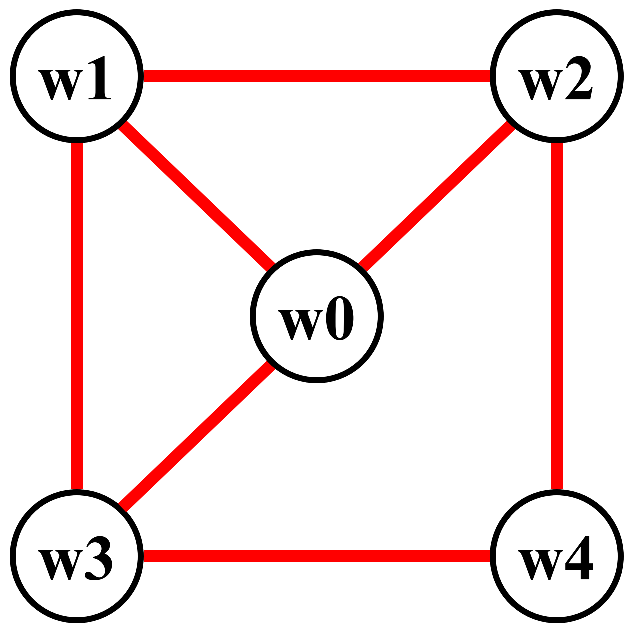

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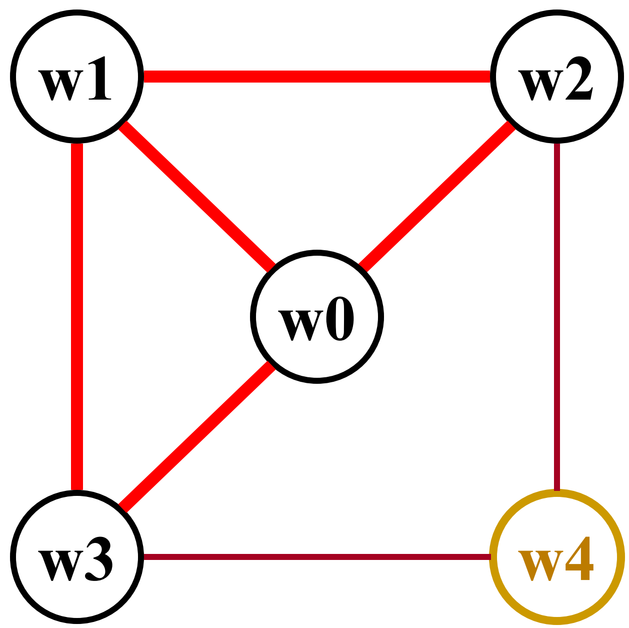

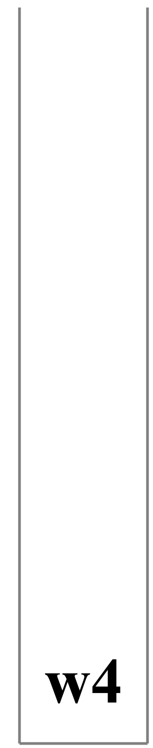

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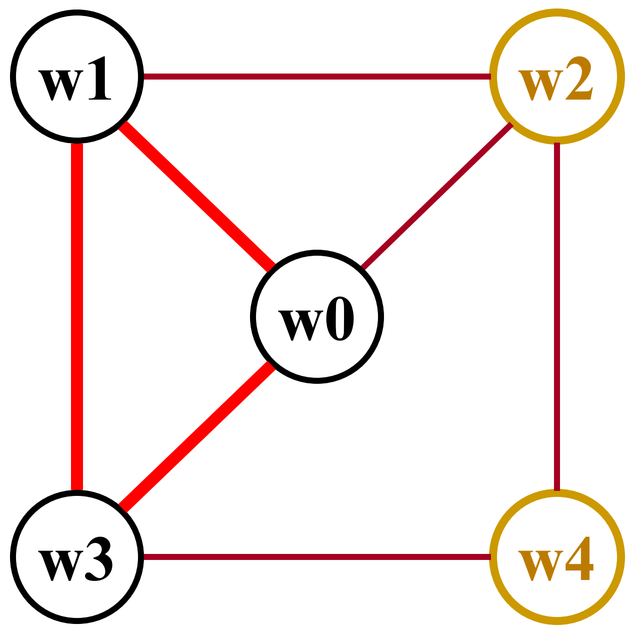

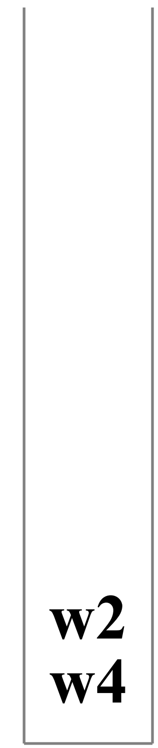

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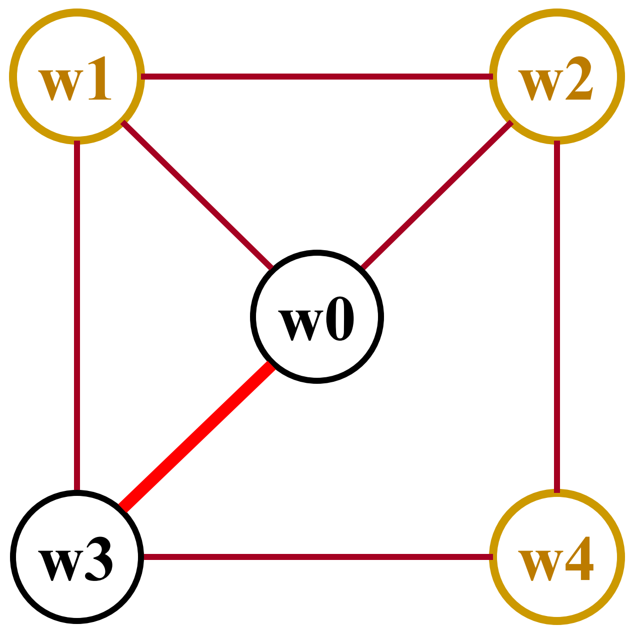

w1
w2
w4

Coloring Example

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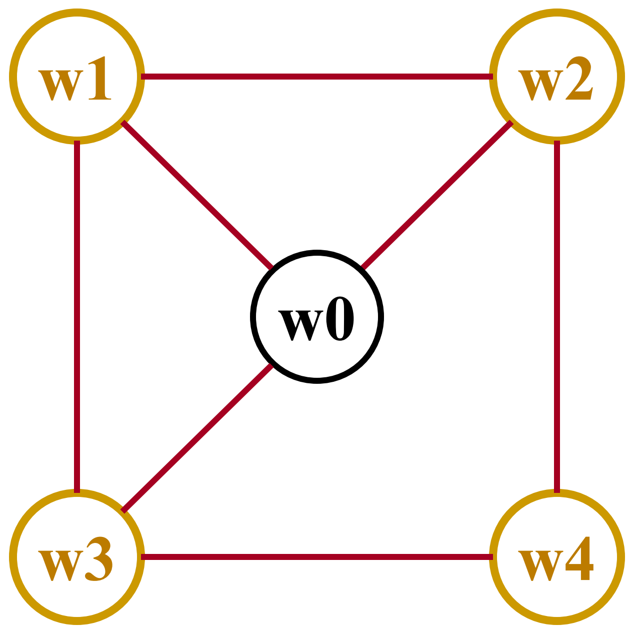

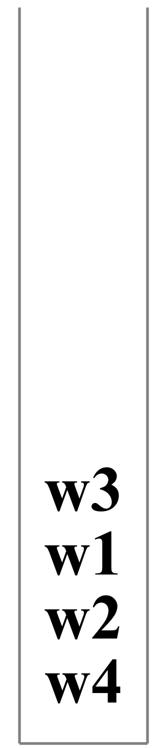

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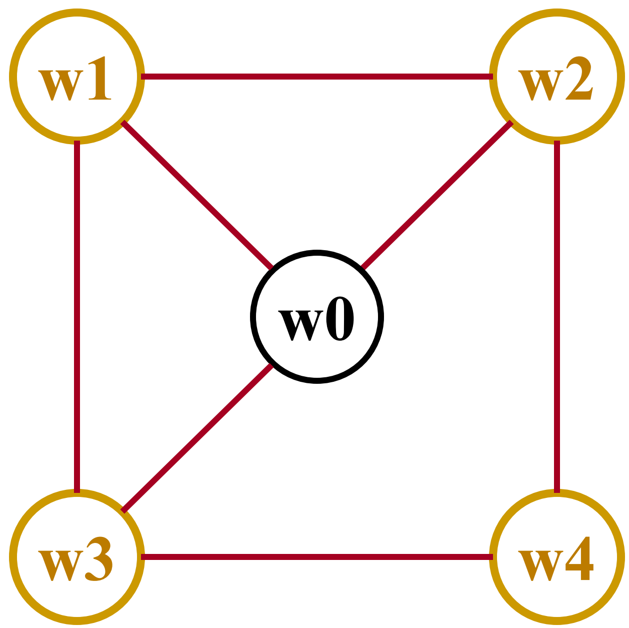

w3
w1
w2
w4

Coloring Example

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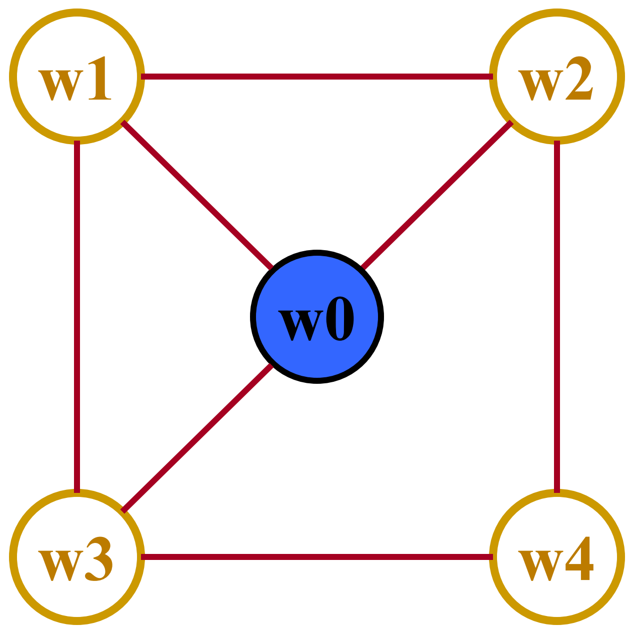

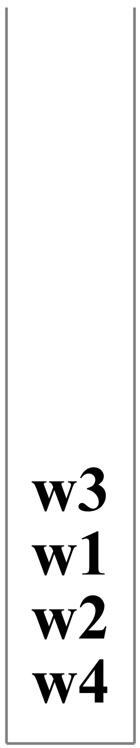

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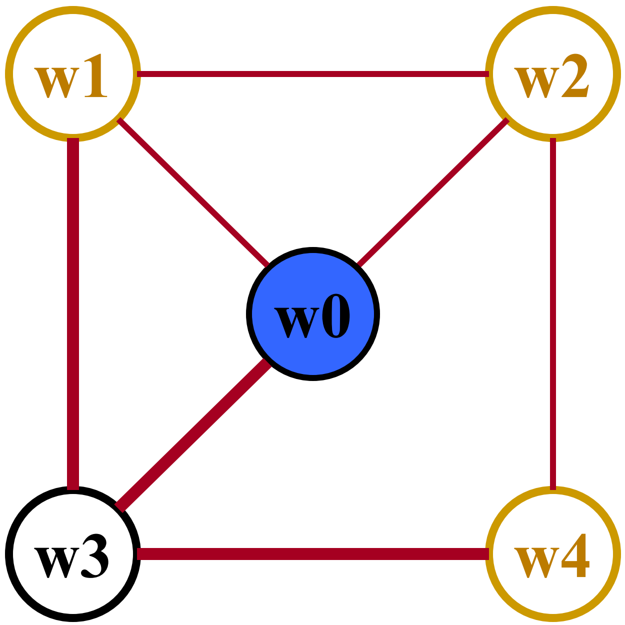

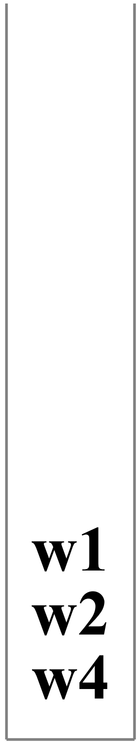

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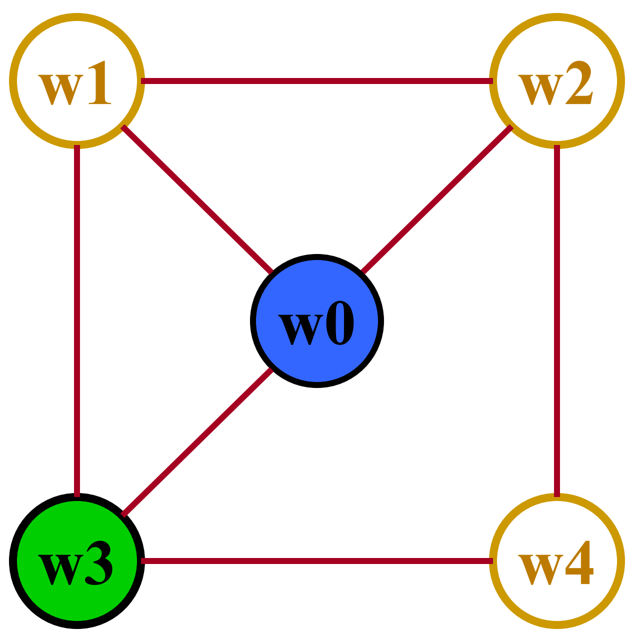
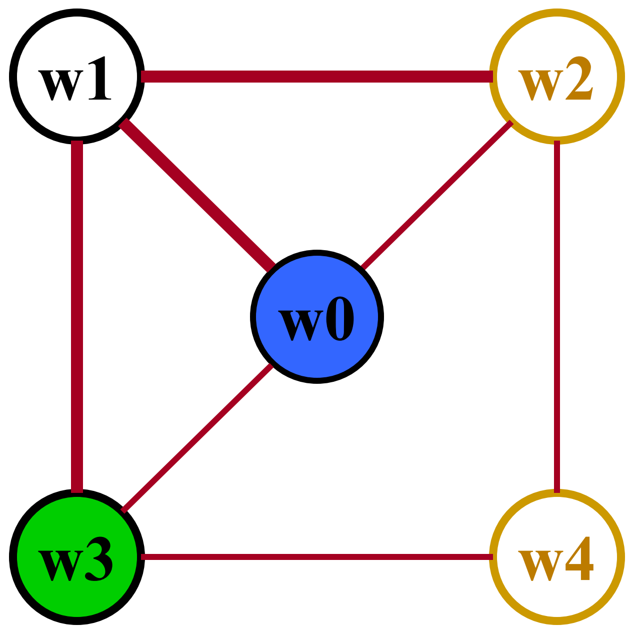
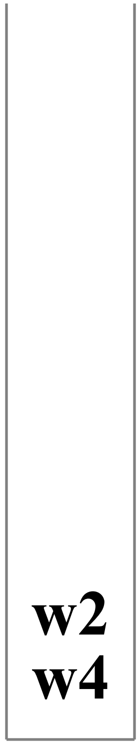


w1
w2
w4

Coloring Example

N = 3





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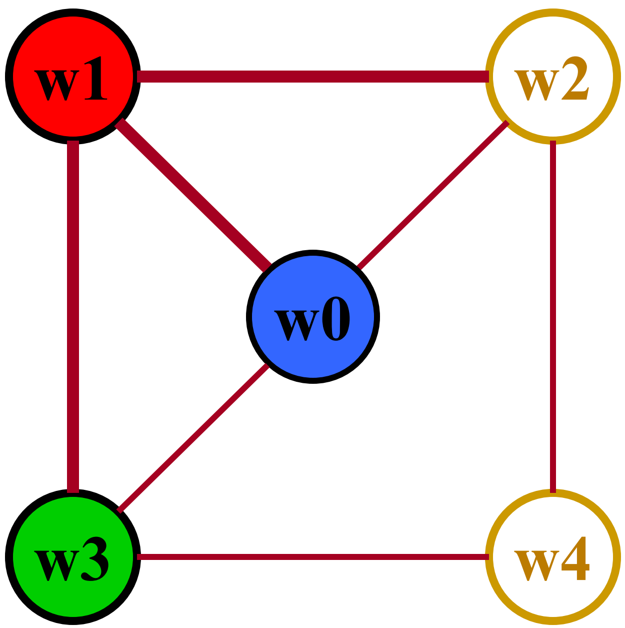

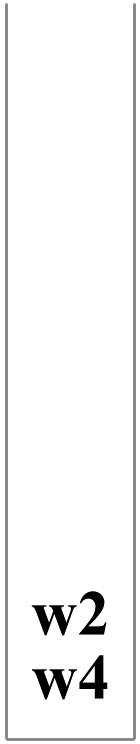

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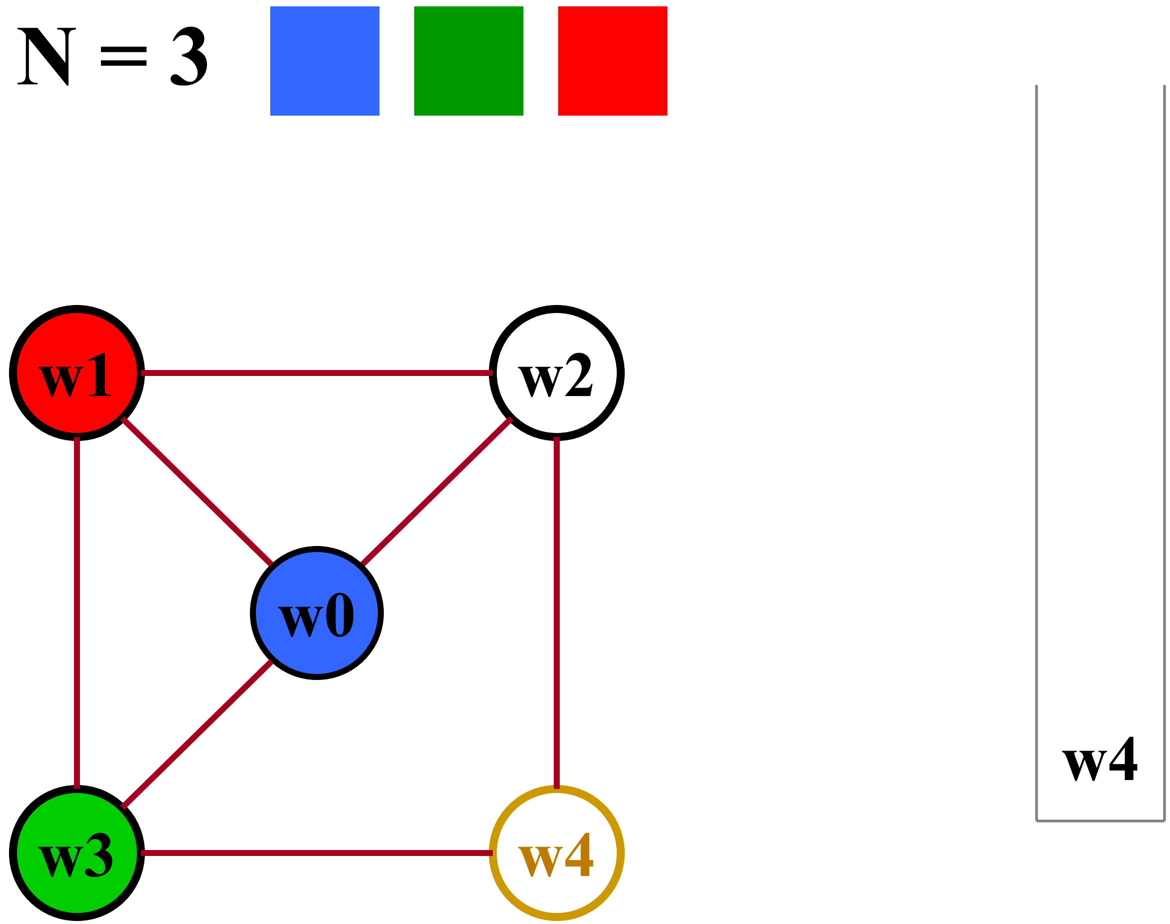

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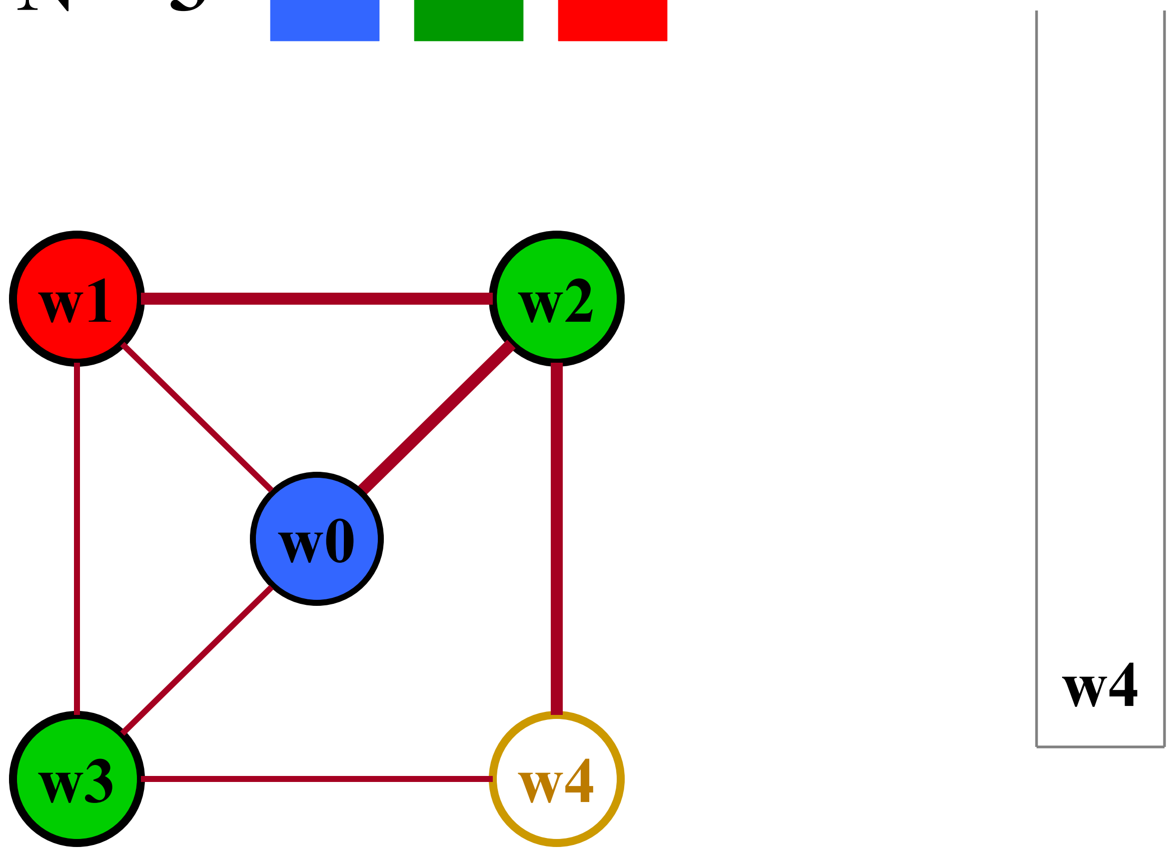

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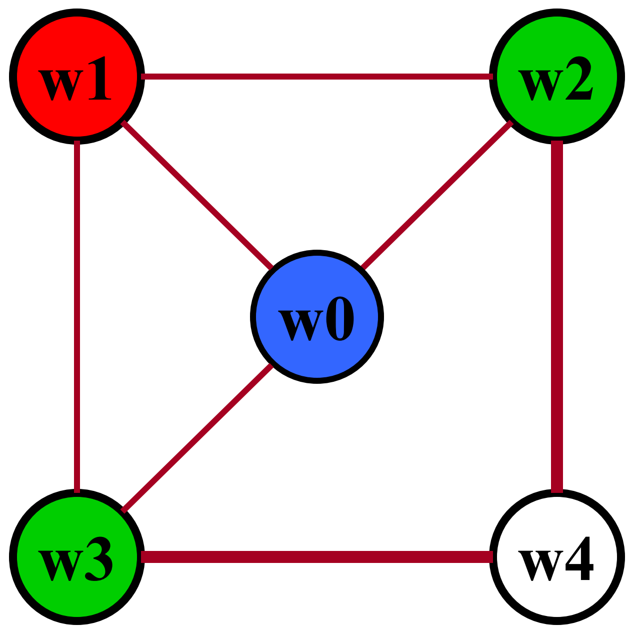

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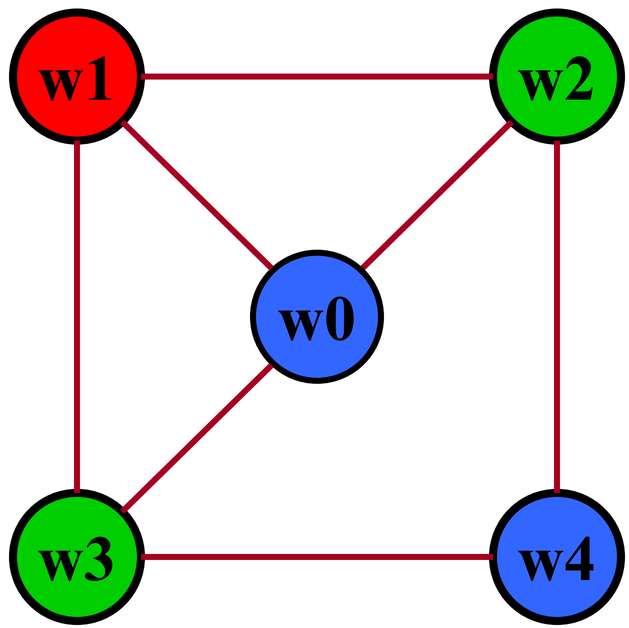
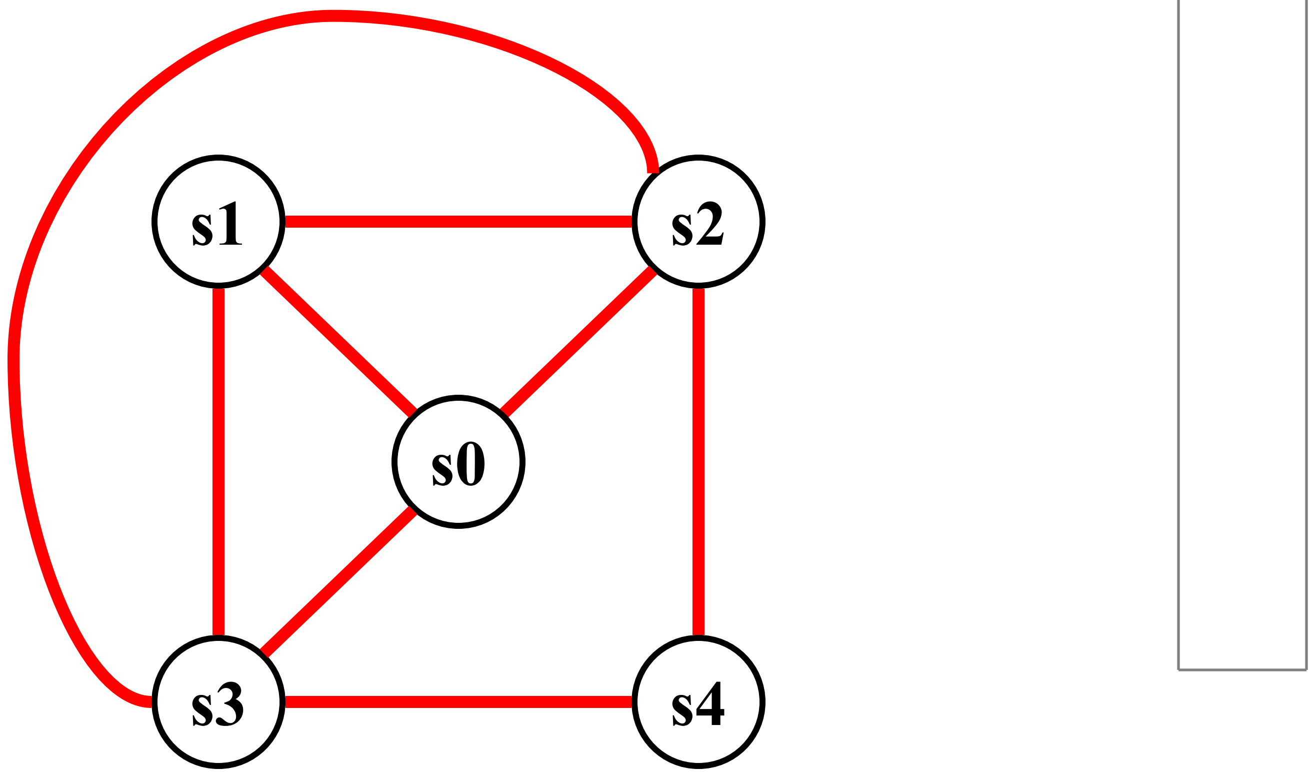


Another Coloring Example

N = 3



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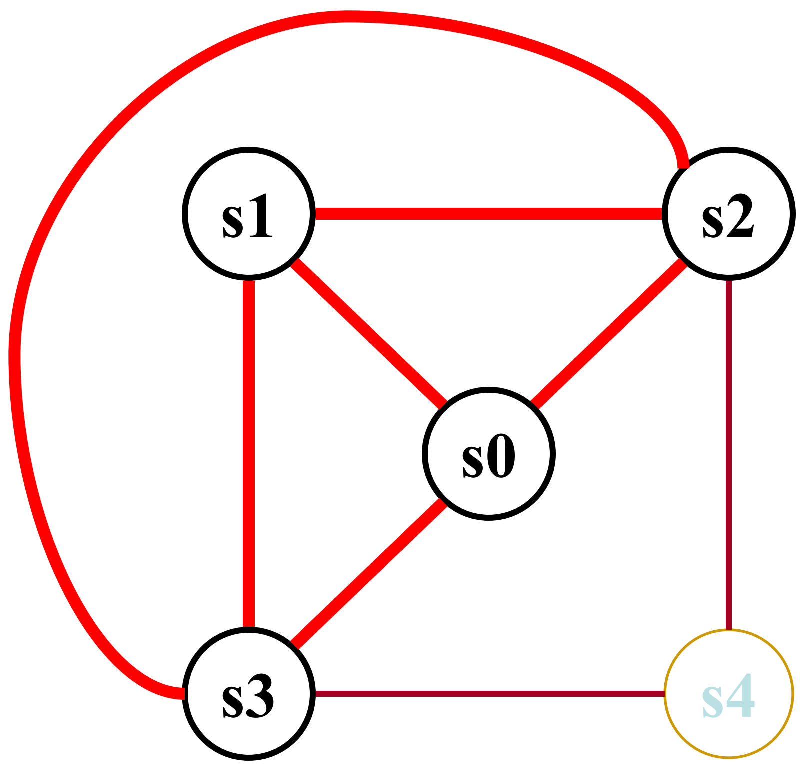

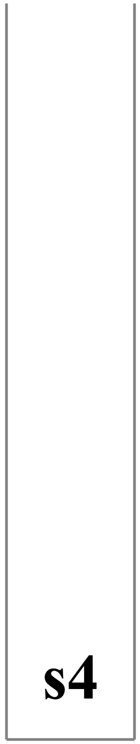

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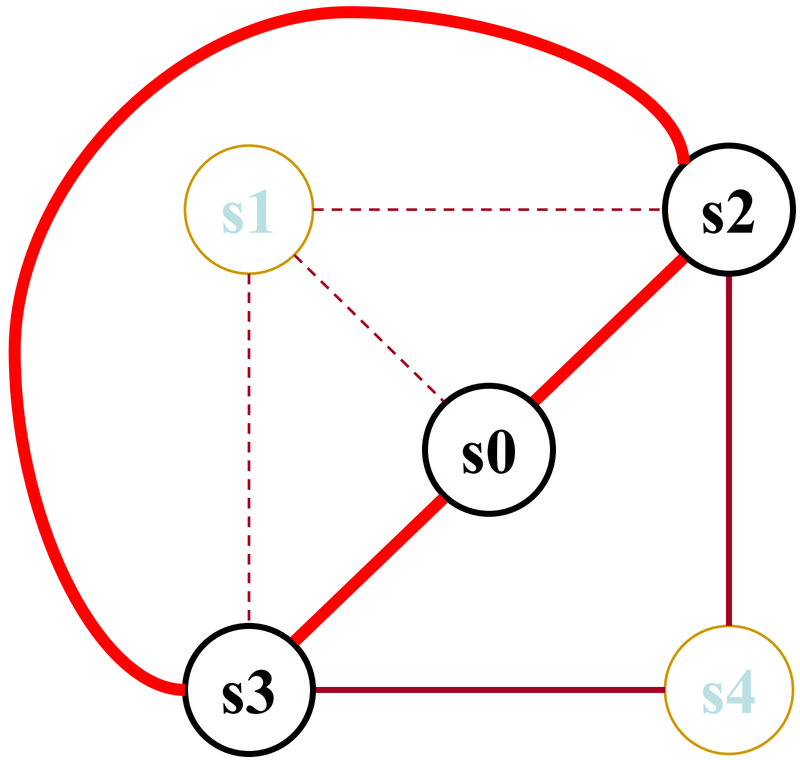

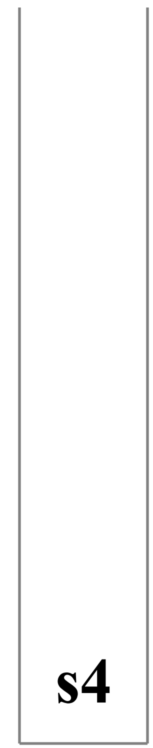

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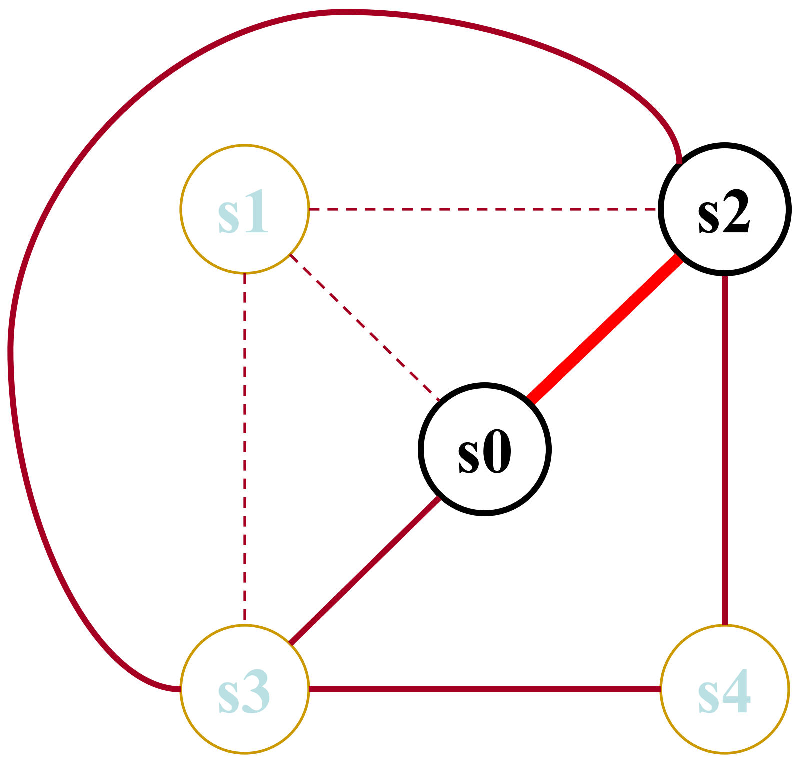

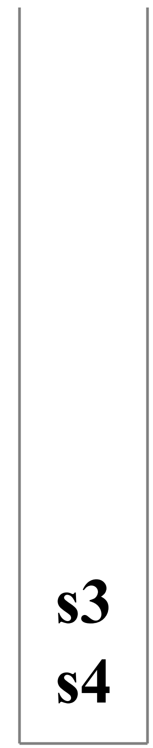

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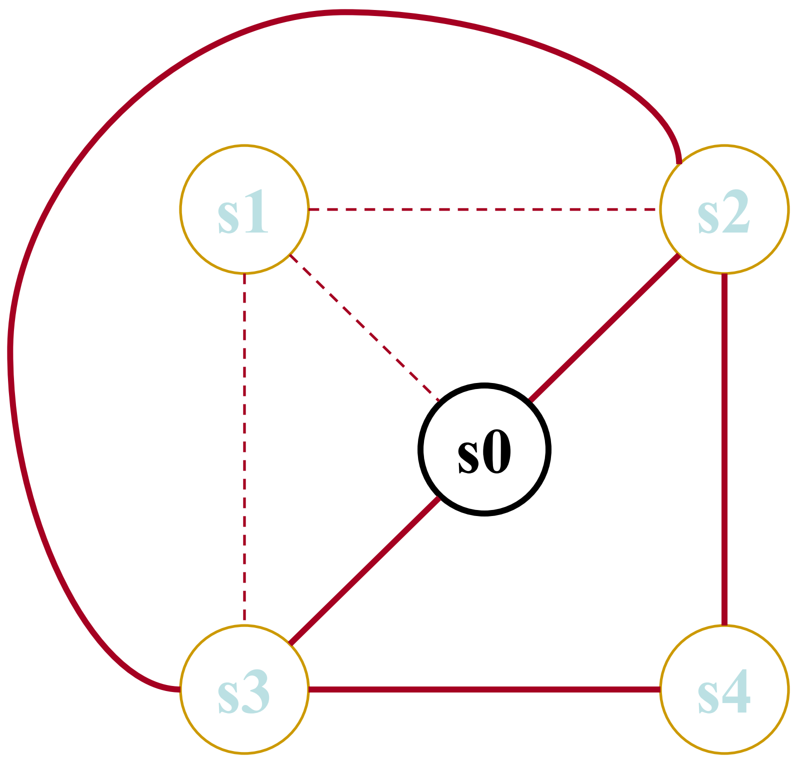
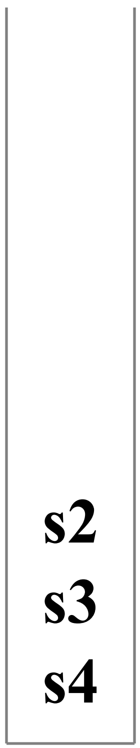
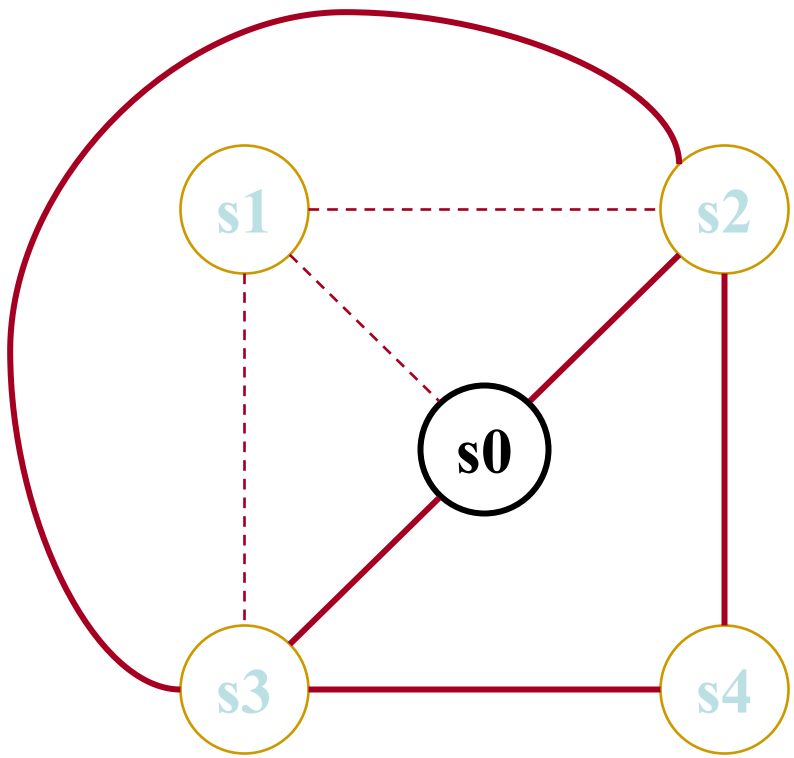
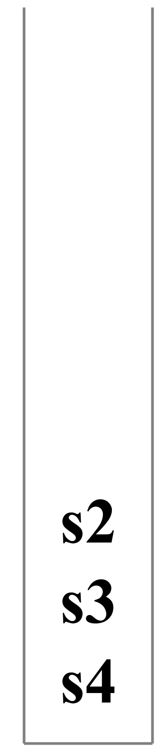




Another Coloring Example

N = 3





Another Coloring Example

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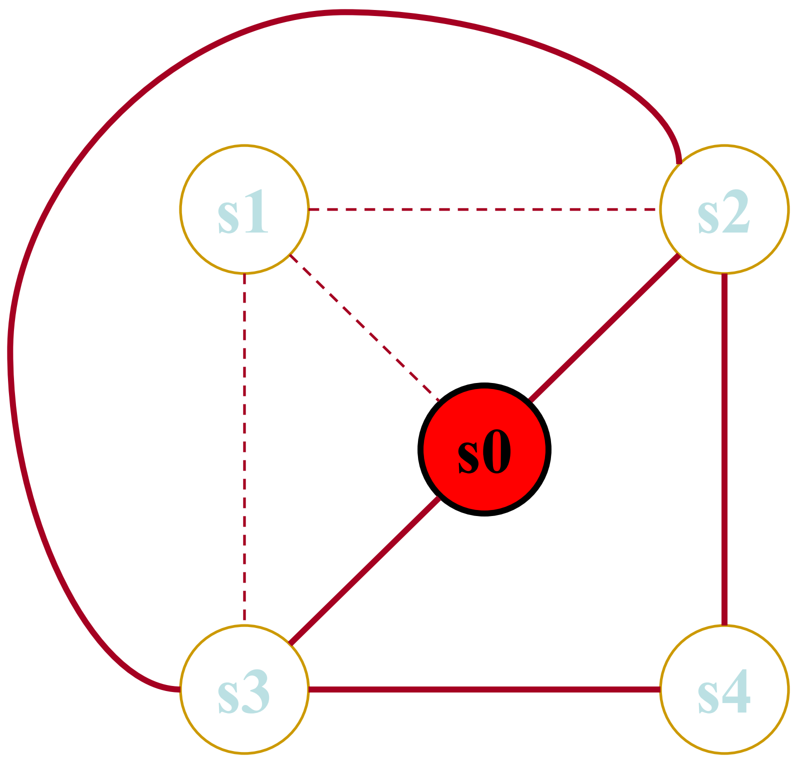
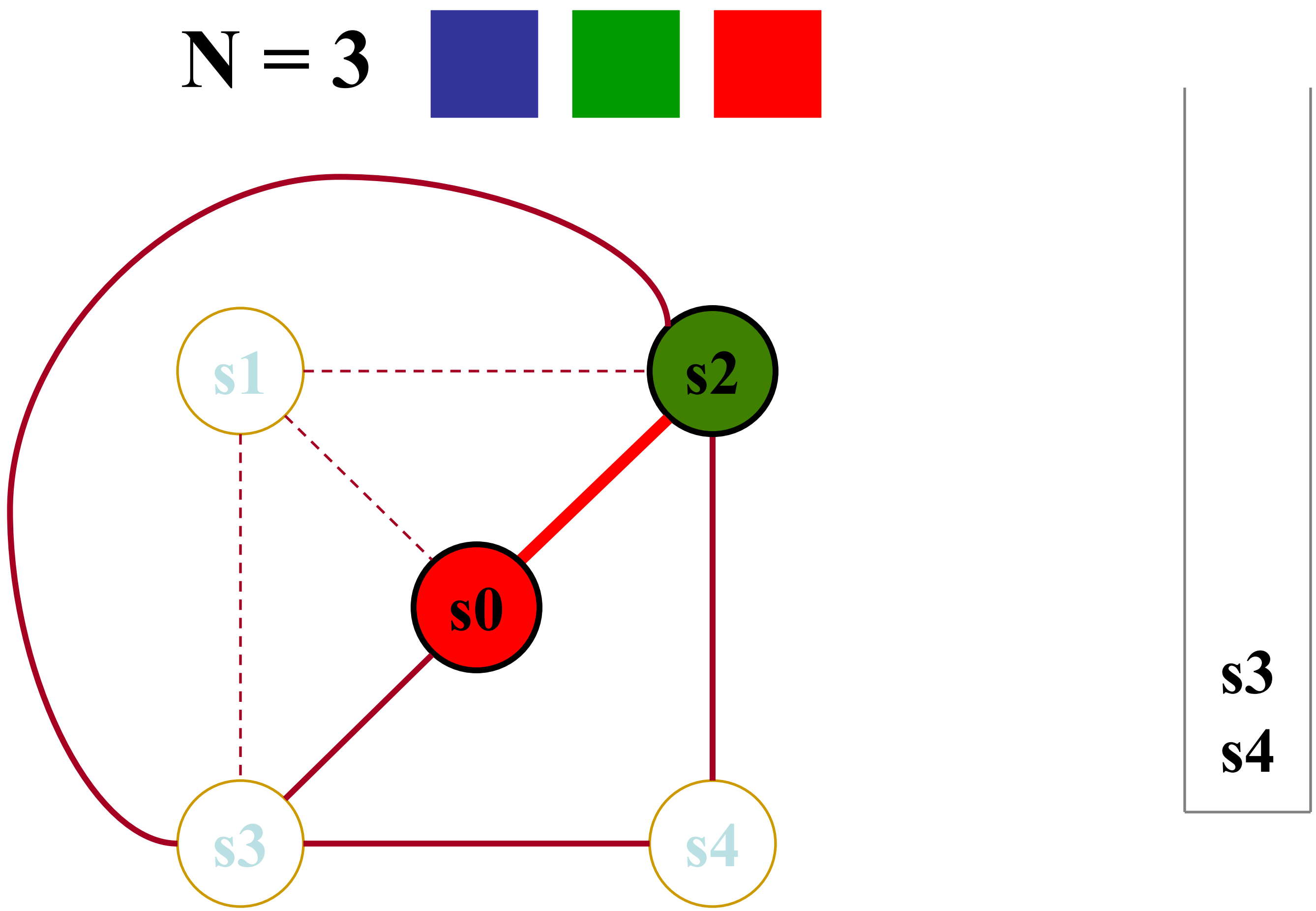


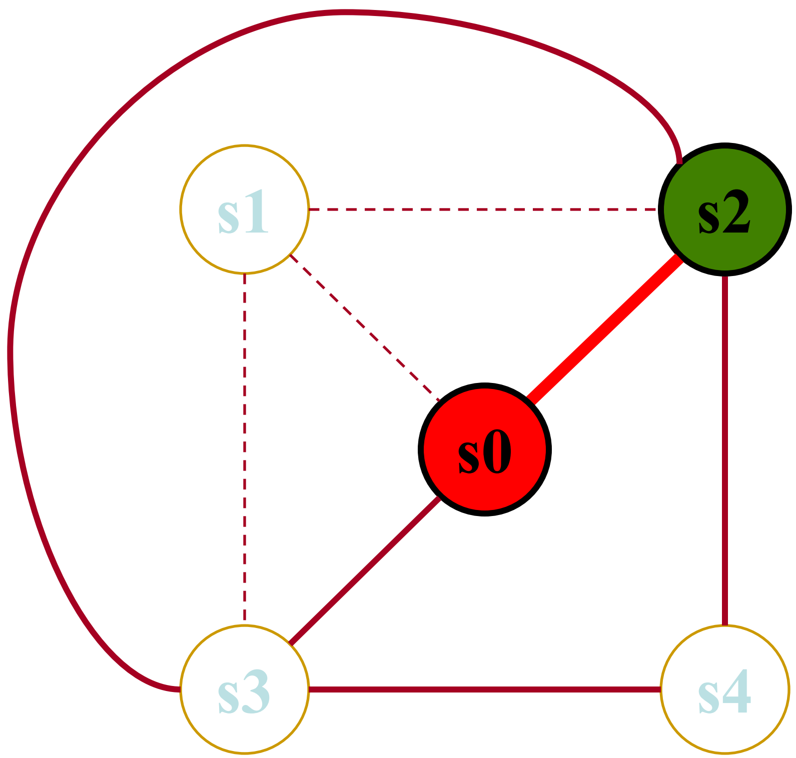
s2
s3
s4

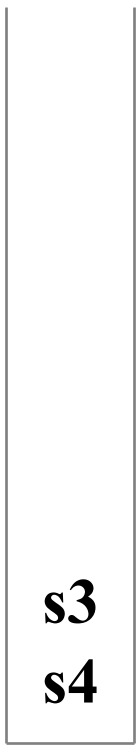
Another Coloring Example



Another Coloring Example

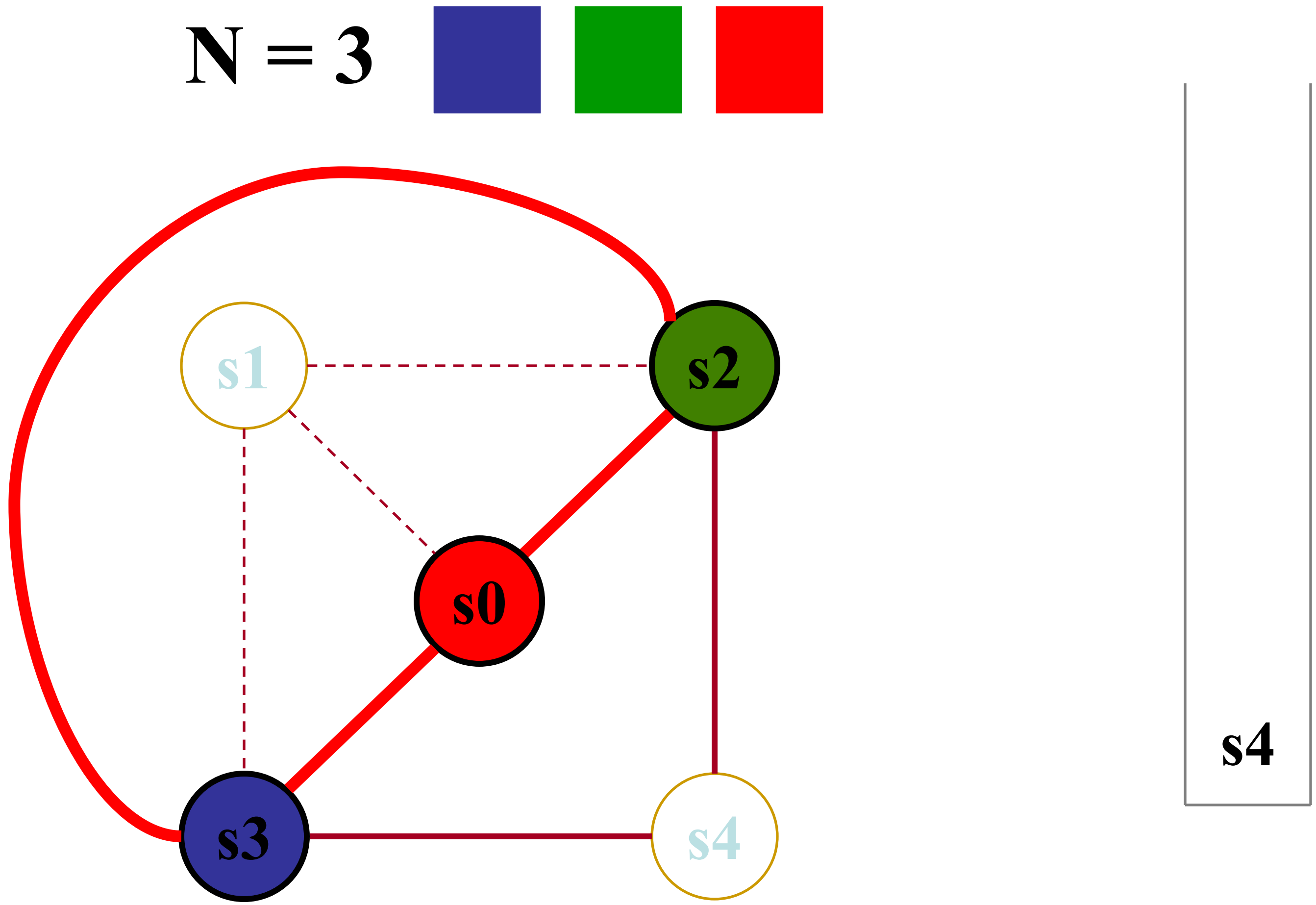
N = 3





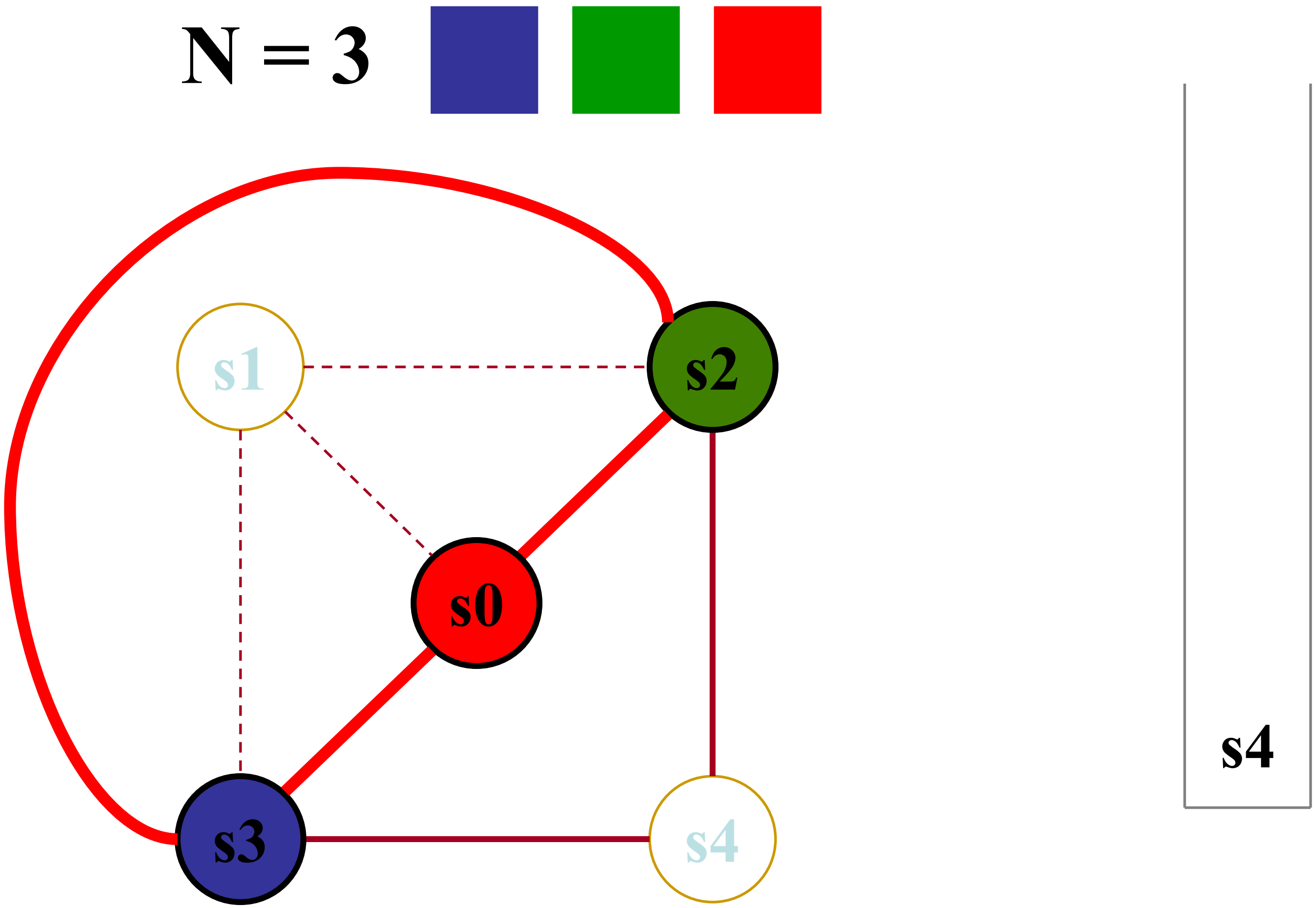
Another Coloring Example

N = 3



Another Coloring Example

N = 3



Another Coloring Example

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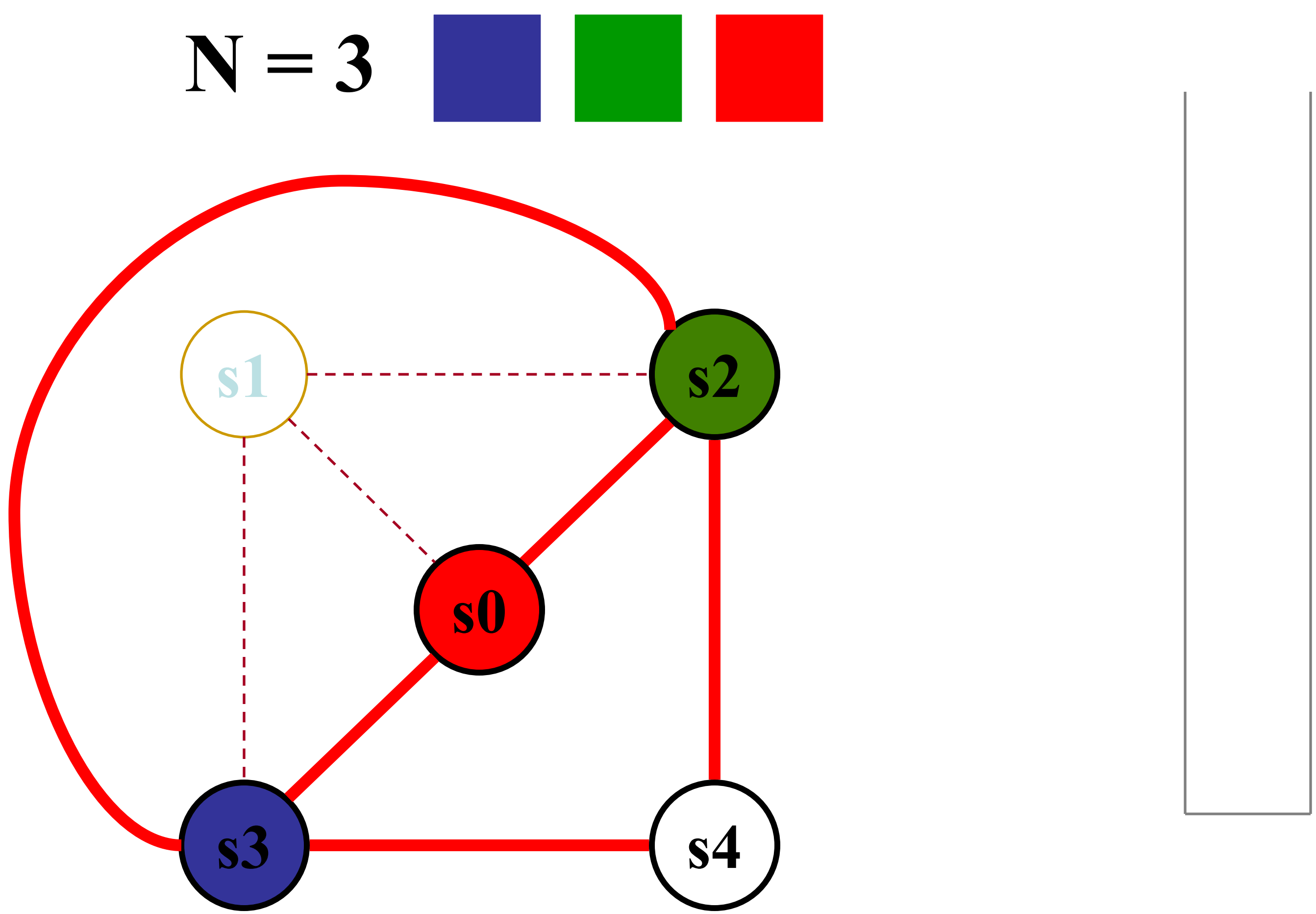
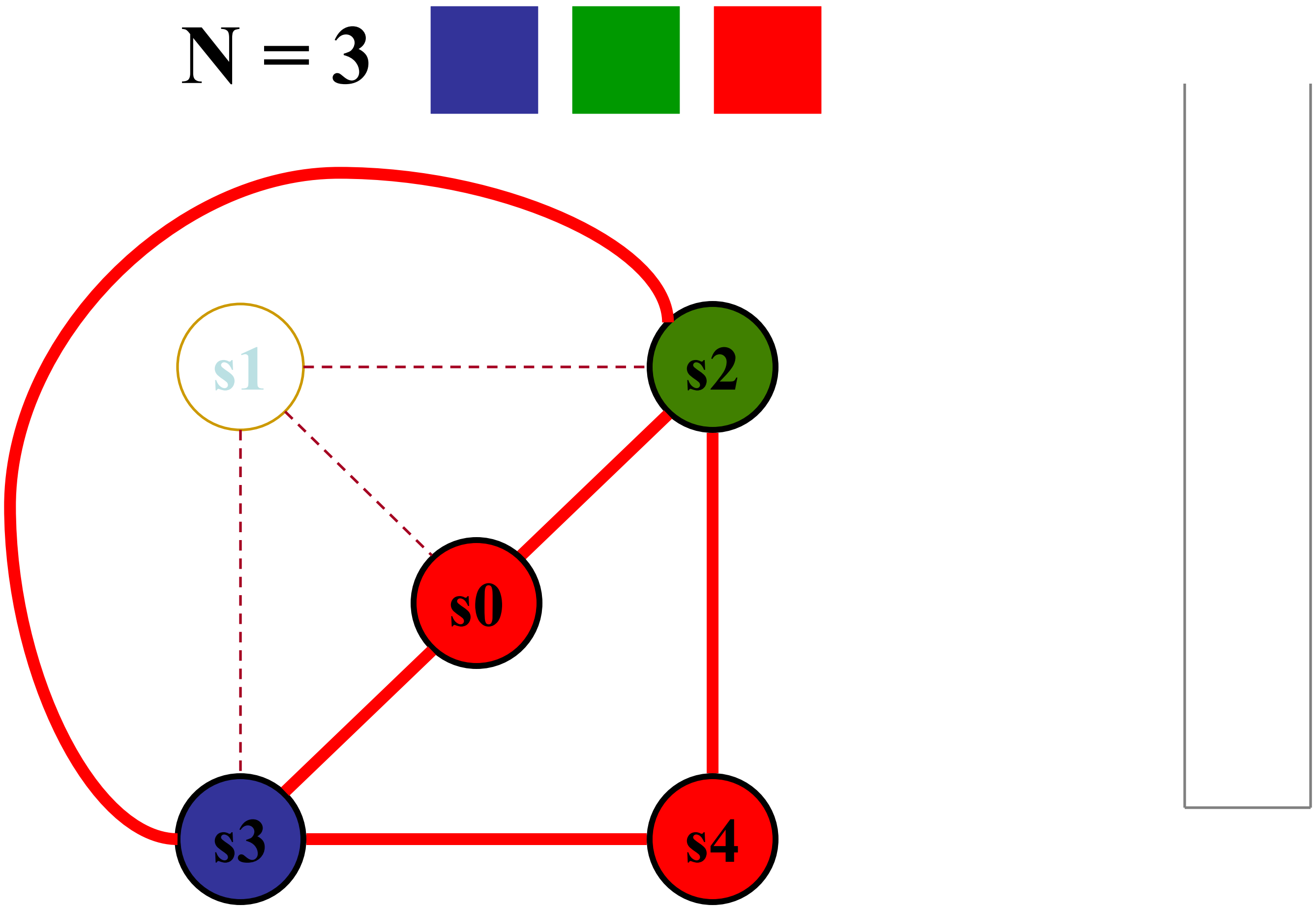


Another Coloring Example

N = 3



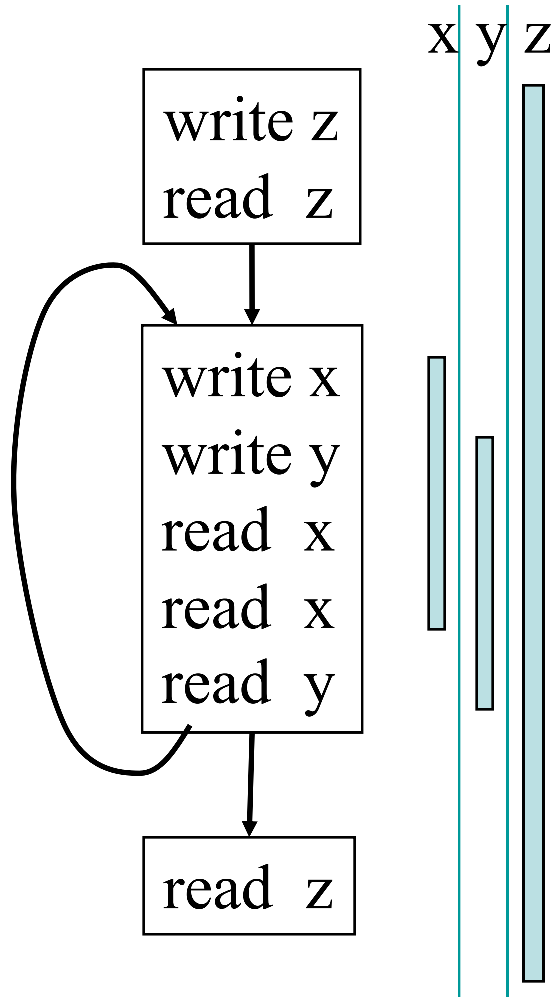
Outline

- What is Register Allocation
- A simple register Allocator
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More Optimizations

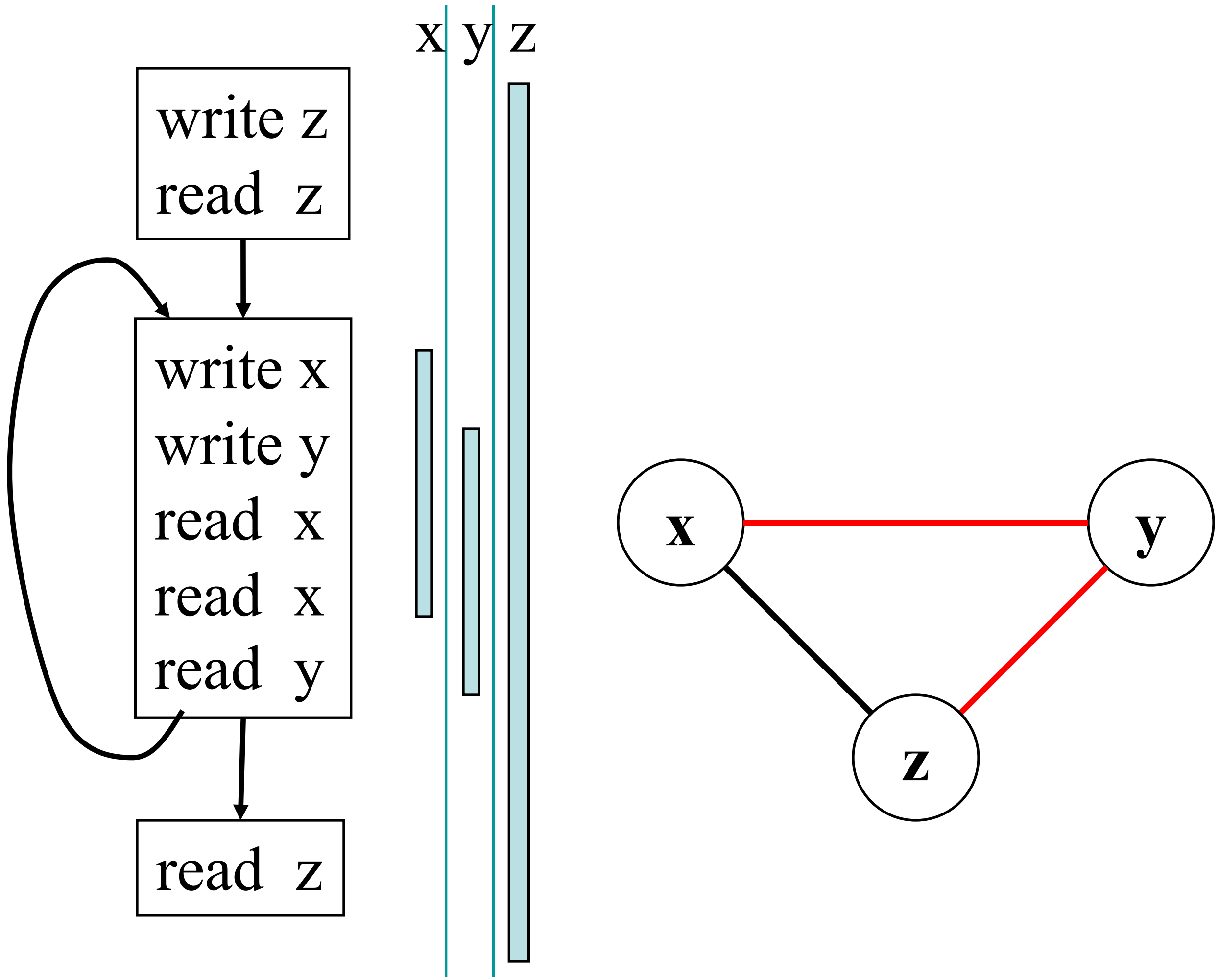
Spilling and Splitting

- When the Graph is non-N-colorable
- Select a Web to Spill
  - Find the least costly Web to Spill
  - Use and Defs of that Web are read and writes to memory
- Split the Web
  - Split a web into multiple webs so that there will be less interference in the interference graph making it N-colorable
  - Spill the value to memory and load it back at the points where the web is split

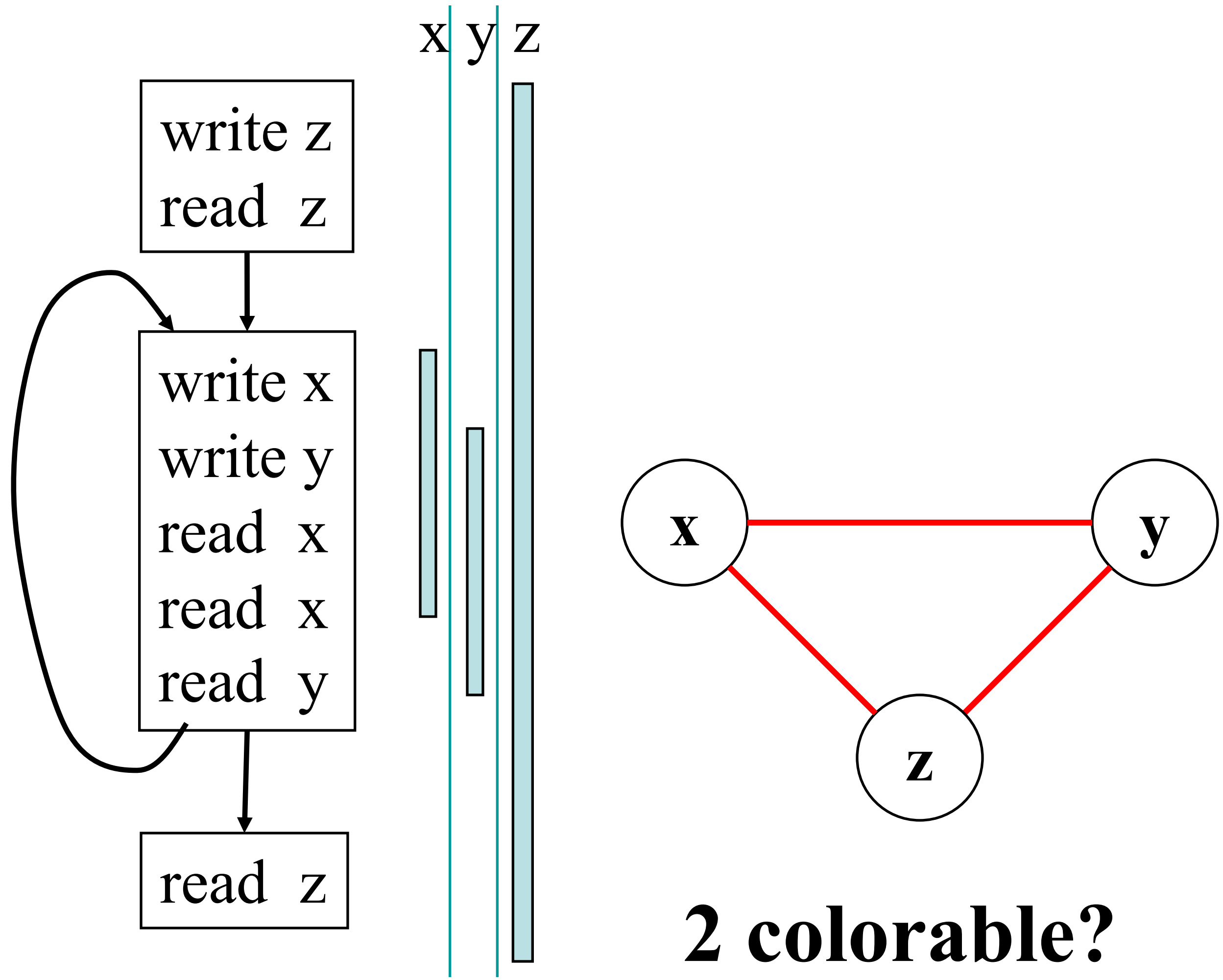
Splitting Example



Splitting Example

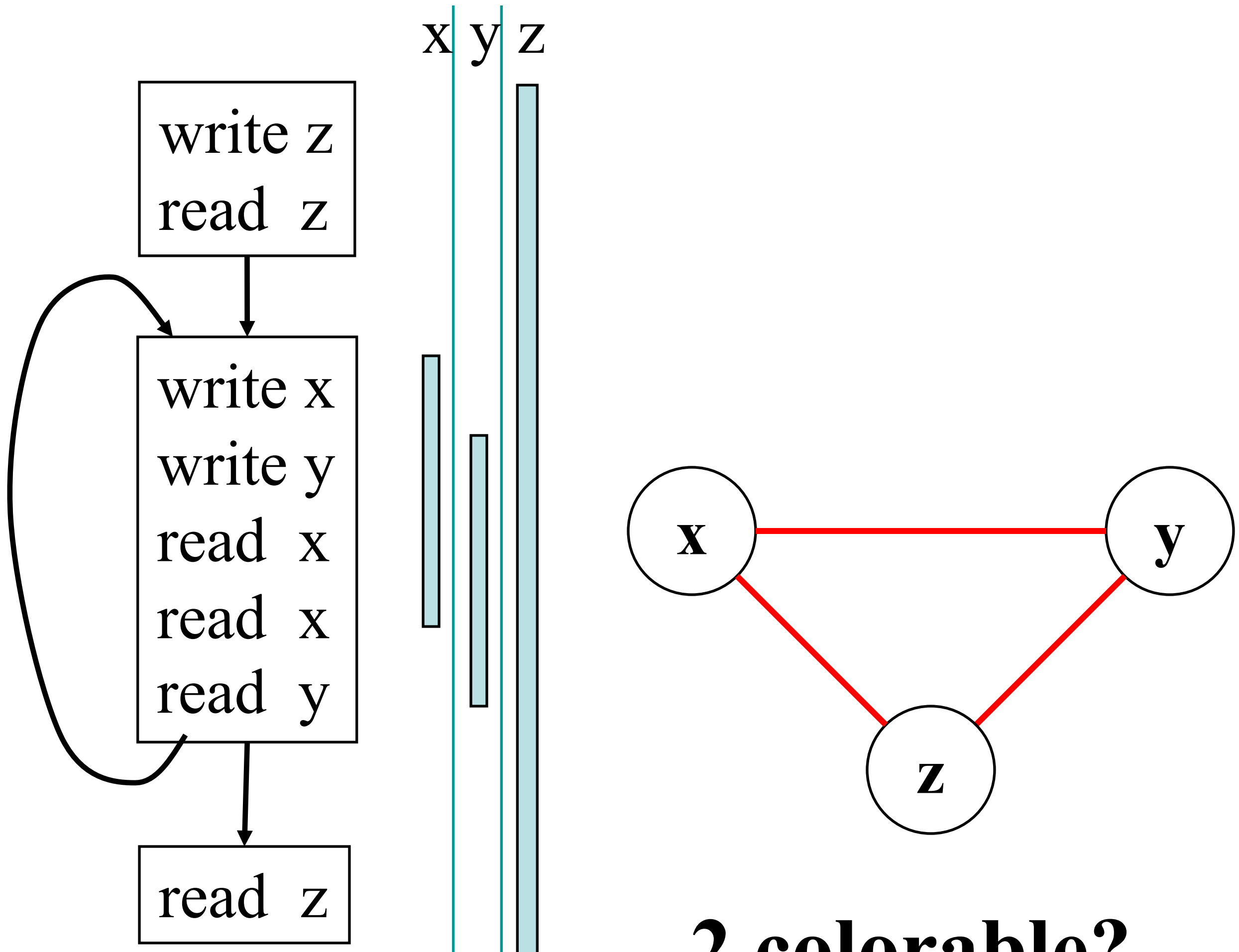


Splitting Example



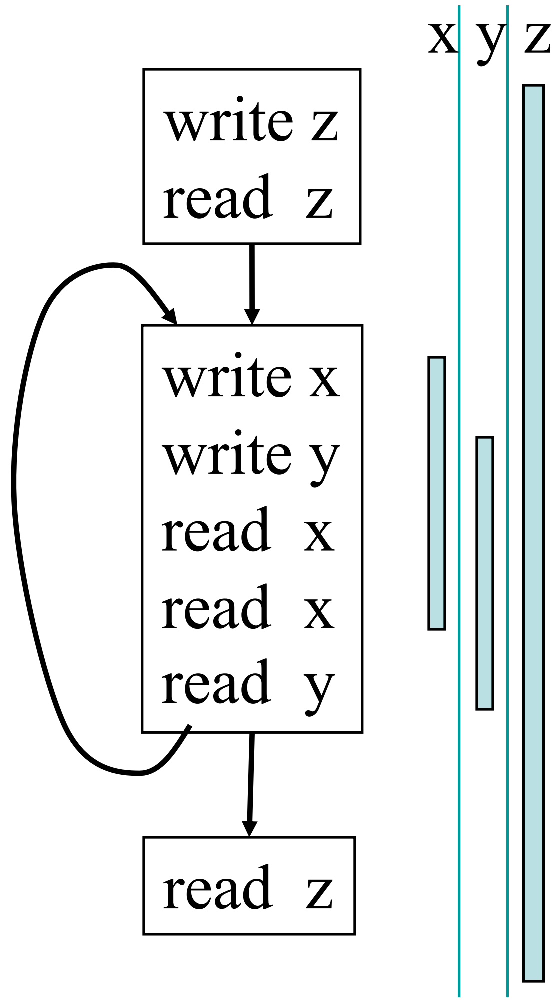
2 colorable?

Splitting Example

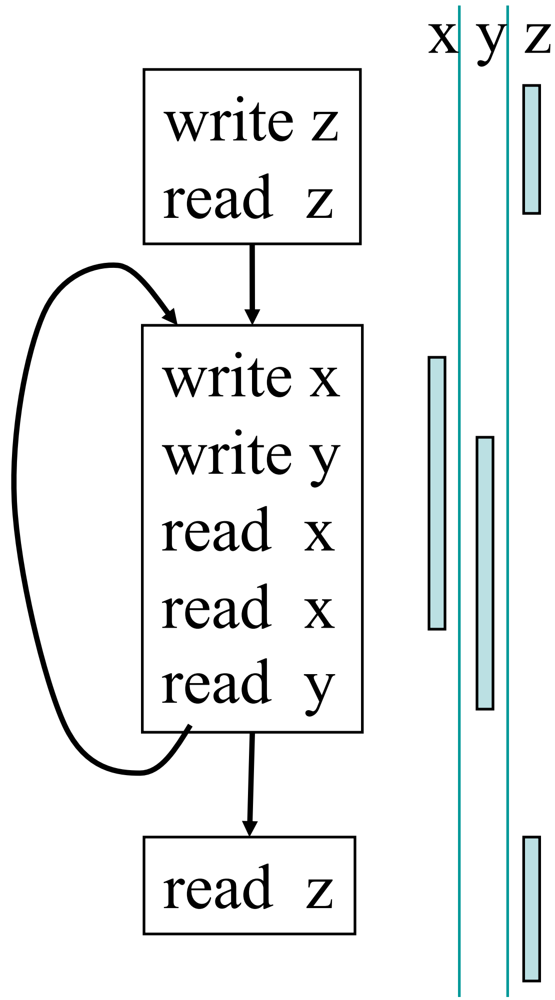


2 colorable?
NO!

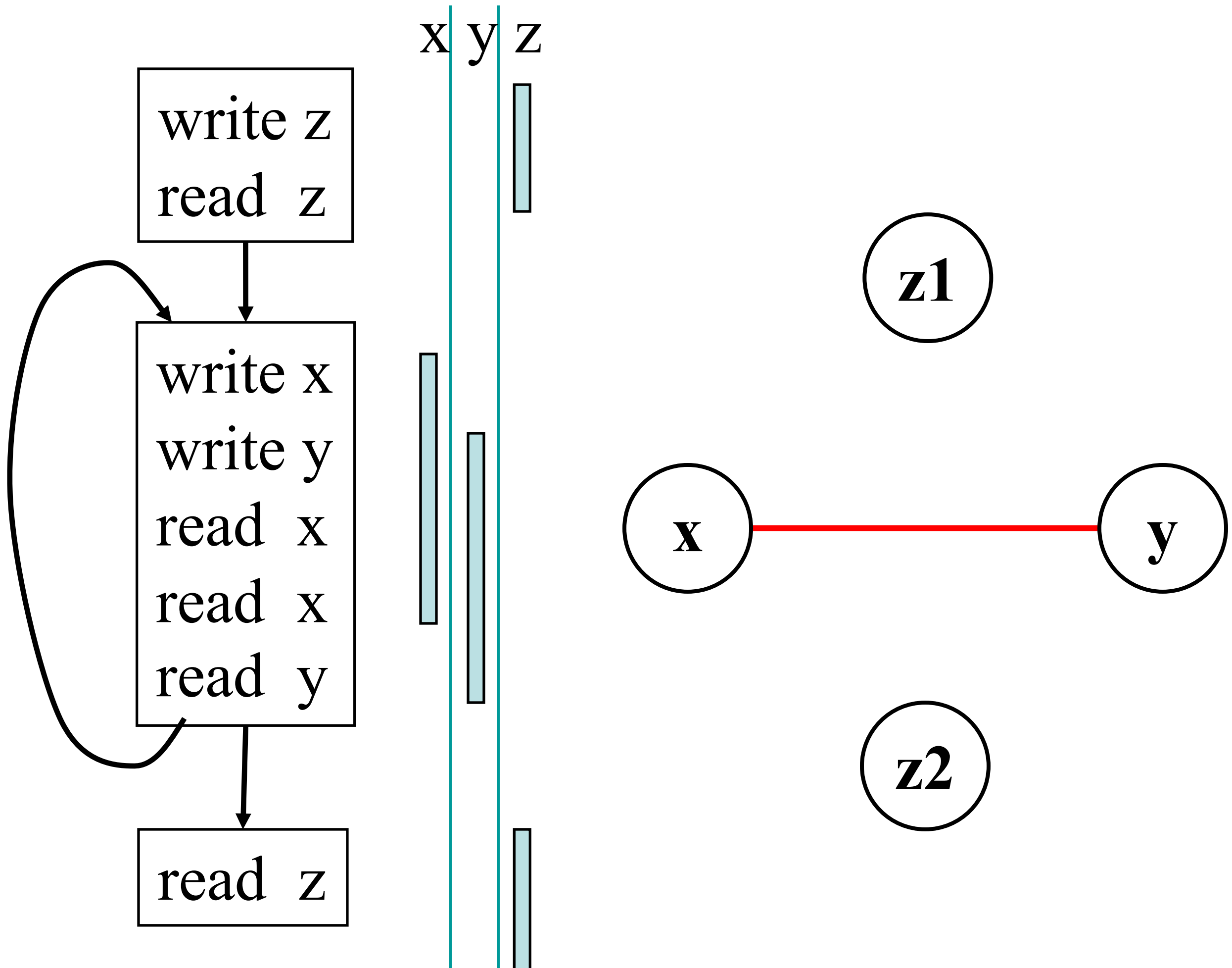
Splitting Example



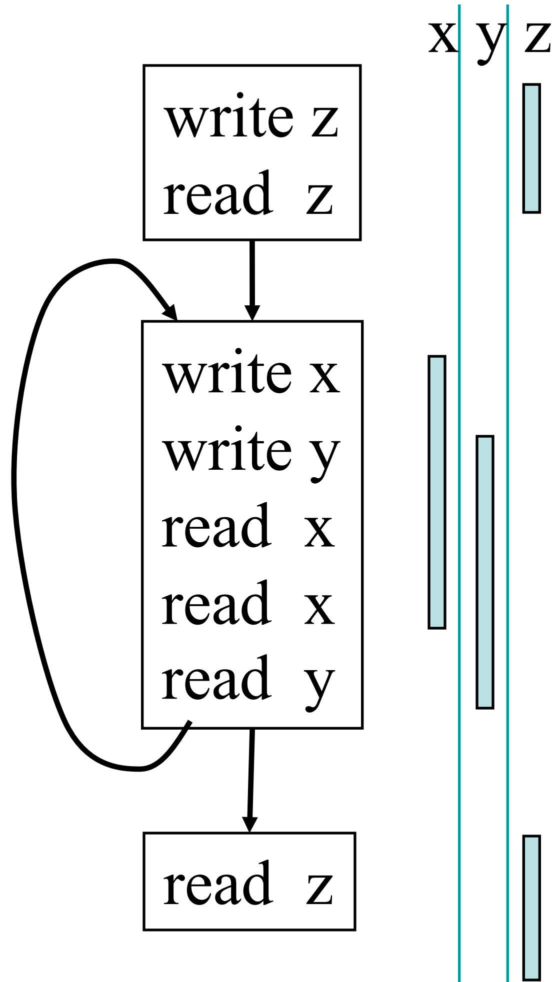
Splitting Example

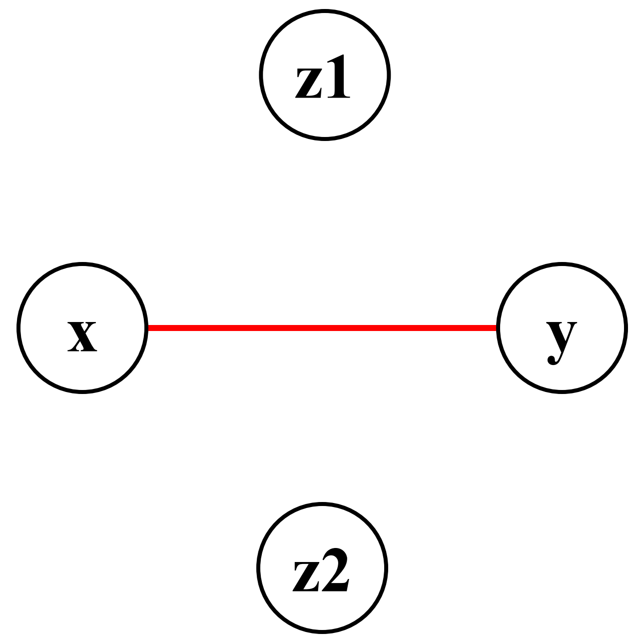


Splitting Example



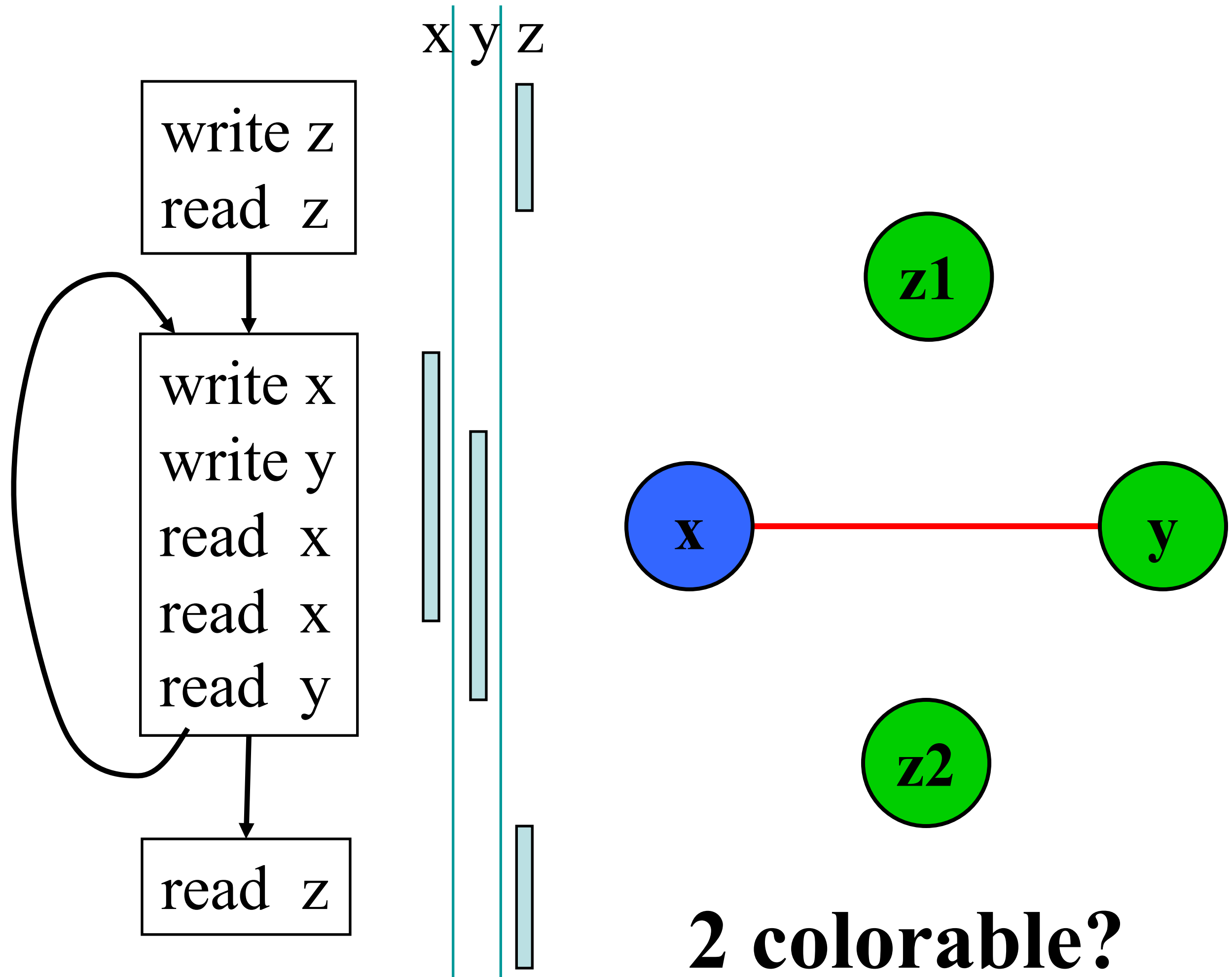
Splitting Example





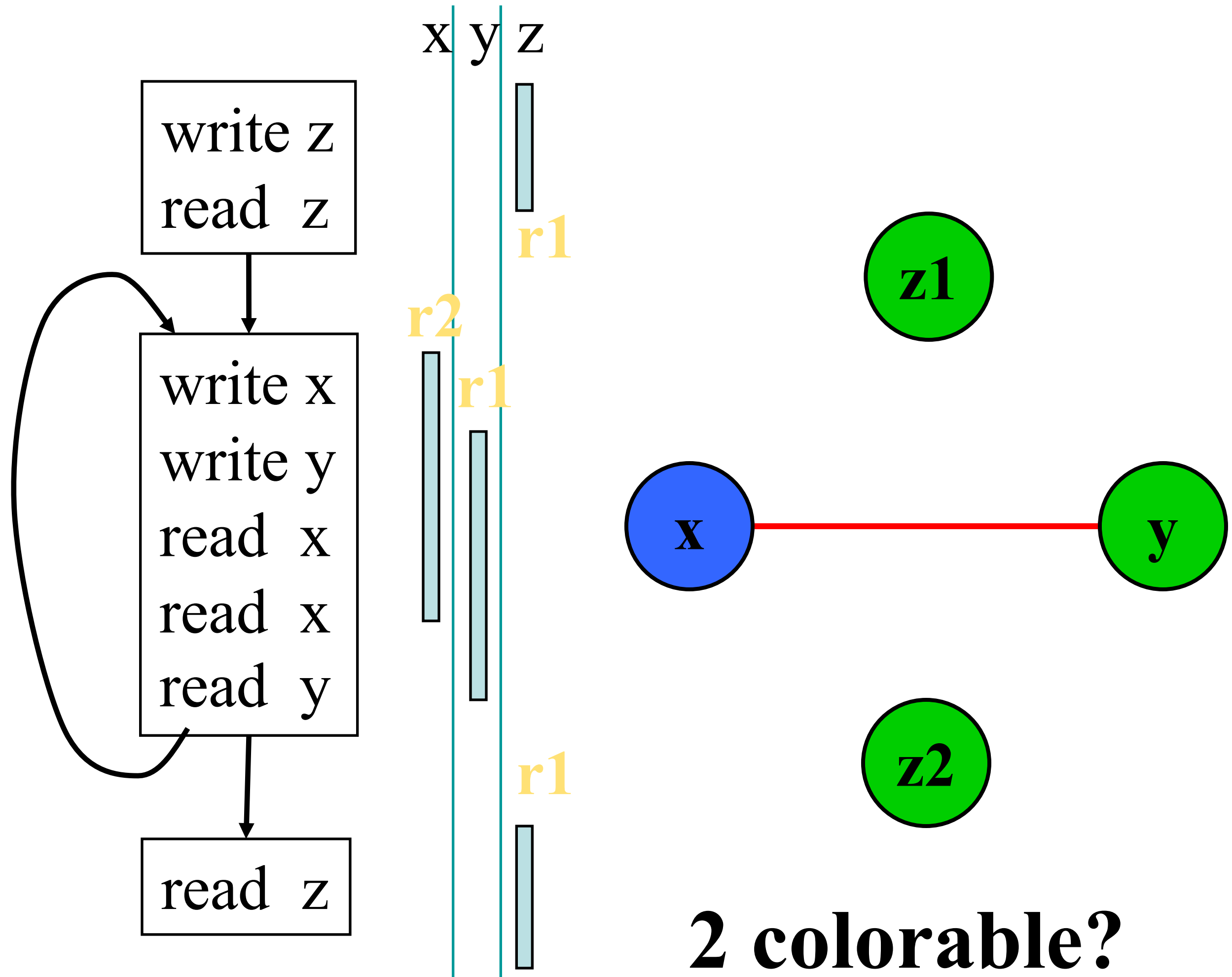
2 colorable?

Splitting Example



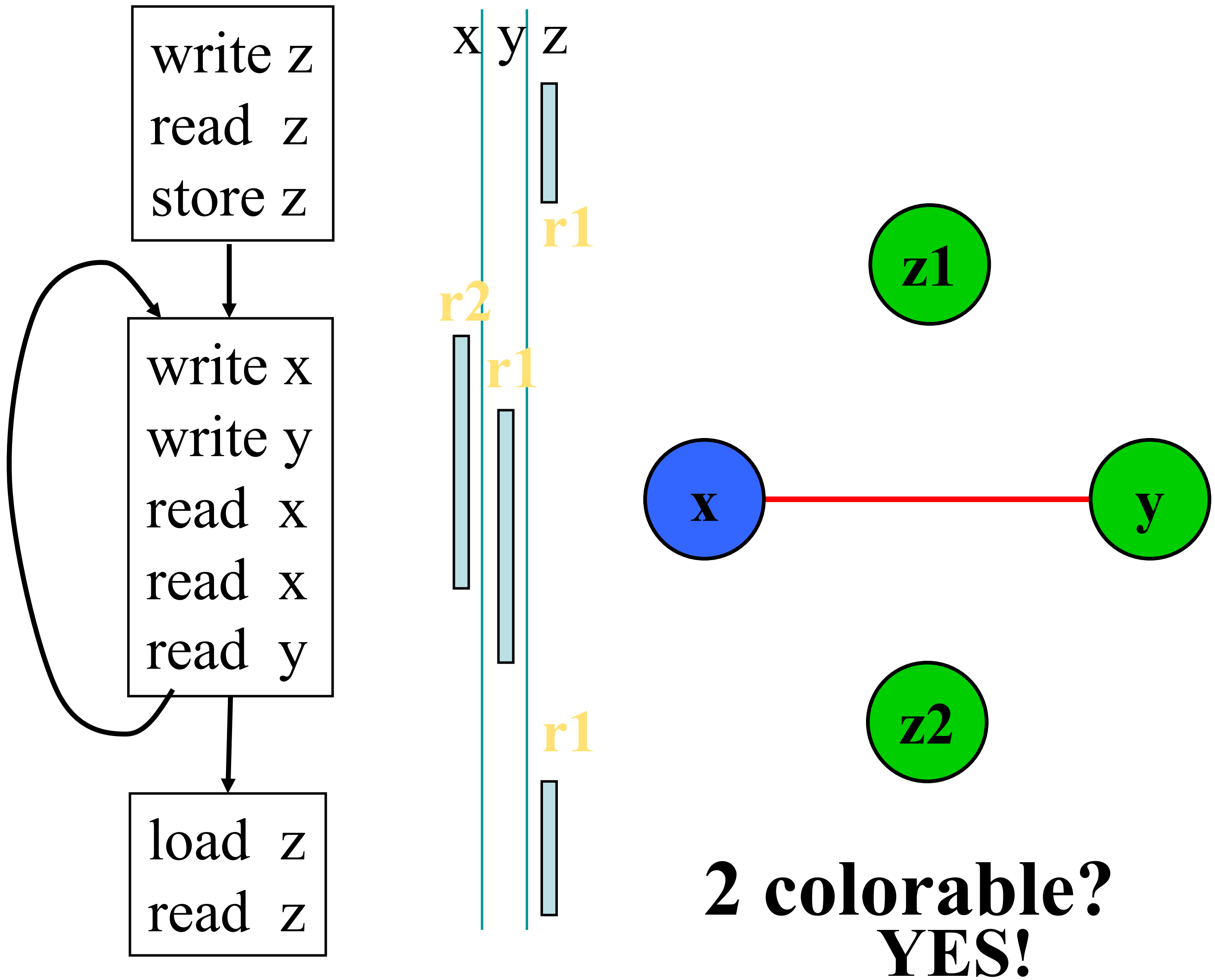
2 colorable?
YES!

Splitting Example



2 colorable?
YES!

Splitting Example



Splitting

- Identify a Program Point where the Graph is not R-colorable (point where # of webs > N)
  – Pick a web that is not used for the largest enclosing block around that point of the program
  – Split that web
  – Redo the interference graph
  – Try to re-color the graph

Cost and Benefit of Splitting

- Cost of splitting a node
  - Proportion to number of times splitted edge has to be crossed dynamically
  - Estimate by its loop nesting
- Benefit
  - Increase colorability of the nodes the splitted web interferes with
  - Can approximate by its degree in the interference graph
- Greedy heuristic
  - pick the live-range with the highest benefit-to-cost ratio to spill

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Outline

- Overview of procedure optimizations
- What is register allocation
- A simple register allocator
- Webs
- Interference Graphs
- Graph coloring
- Splitting
- More Optimizations

More Transformations

- Register Coalescing
- Register Targeting (pre-coloring)
- Pre-Splitting of Webs
- Inter-procedural Register Allocation

Register Coalescing

- Find register copy instructions $s_j = s_i$
- If s_j and s_i do not interfere, combine their webs
- Pros
  - Similar to copy propagation
  - Reduce the number of instructions
- Cons
  - May increase the degree of the combined node
  - A colorable graph may become non-colorable

Register Targeting (pre-coloring)

- Some Variables need to be in Special Registers at Specific Points in the Execution
  - first 4 arguments to a function
  - return value
- Pre-color those webs and bind them to the appropriate register
- Will eliminate unnecessary copy instructions

Pre-Splitting of the Webs

- Some Ranges have Very Large "dead" Regions
  - Large region where the variable is unused
- Break-up the Ranges
  - need to pay a small cost in spilling
  - but the graph will be very easy to color
- Can find Strategic Locations to Break-up
  - at a call site (need to spill anyway)
  - around a large loop nest (reserve registers for values used in the loop)

Inter-Procedural Register Allocation

- Saving Registers across Procedure boundaries is expensive
  - especially for programs with many small functions
- Calling convention is too general and inefficient
- Customize calling convention per function by doing inter-procedural register allocation

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Summary

- Register Allocation and Assignment
  - Very Important Transformations and Optimization
  - In General Hard Problem (NP-Complete)

- Many Approaches
  - Local Methods: Top-Down and Bottom-Up
  - Global Methods: Graph Coloring
    - Webs
    - Interference Graphs
    - Coloring
  - Other Transformations