

# **IART - Artificial Intelligence**

## **Lecture 2a: Introduction to Search**

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# Problem Solving using Search

## Presentation Structure:

- Problem Solving Methods
- Problem Formulation
- State Space
- Blind/Uninformed Search:
  - Breadth First, Depth First, Uniform Cost, Iterative Deepening, Bidirectional Search
- Intelligent/Informed Search:
  - Greedy Search, A\* Algorithm
- Practical Application Examples

# Problem Solving using Search

- **How can an agent act, setting goals and considering possible sequences of actions to achieve those goals!**
- **Problem Solving:**
  - Formulating a problem as a search problem
  - Uninformed search (search strategies)
  - Informed Search (greedy search, A\* Algorithm)
  - Iterative Improvement Algorithms
  - Search in Games (including a hostile agent!)

# Agents for Problem Solving

- **“Problem Solving Agent”**: Tries to find the sequence of actions that leads to a desirable state!
- **Problem Formulation**:
  - What are the possible actions? (what is its effect on the state of the world?)
  - What are the possible states? (how to represent them?)
  - How to evaluate states
- Search problem
  - Solution: sequence of actions
  - Final stage is execution!
- **Formulate → Search → Execution**

# Search Problems

- **Many of the problems in computer science can be defined as:**
  - A set  $S$  of STATES (possibly infinite)
  - An INITIAL state:  $s \in S$
  - A TRANSITION relationship  $T$  across this state space
  - A set of FINAL (objective) states:  $O \in S$
- **Problem can be represented by a GRAPH, where the nodes represent states and the arcs (connections) the pairs of the transition relation**
- **Problem can be solved through search finding a path between the initial state and an objective state**

# Agents for Problem Solving

- **Problem Formulation:**
  - State Representation
  - Initial (Current) State
  - Objective Test (defines the desired states)
  - Operators (Name, Preconditions, Effects, cost)
  - Solution Cost

# Simple Agents for Problem Solving

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

  state ← UPDATE-STATE(state, percept)
  if seq is empty then do
    goal ← FORMULATE-GOAL(state)
    problem ← FORMULATE-PROBLEM(state, goal)
    seq ← SEARCH(problem)
  action ← FIRST(seq)
  seq ← REST(seq)
  return action
```

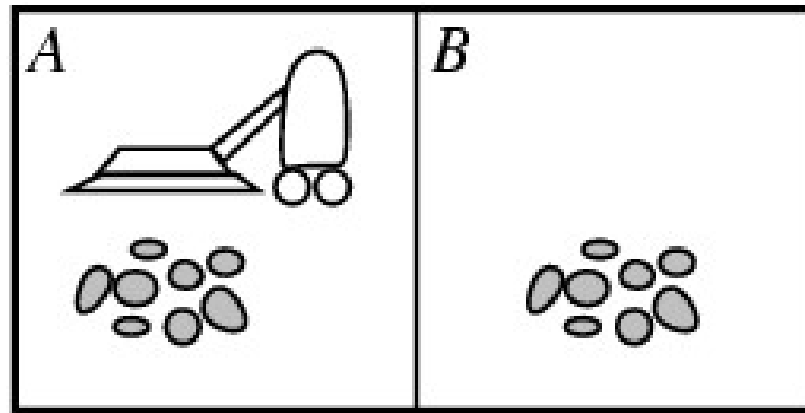
# Problem Formulation

- **What is the agent's knowledge of the state of the world and his actions?**
- **Four different types of problems:**
  - Single state problems (deterministic and accessible environment)
  - Multiple state problems (deterministic but inaccessible environment)
  - Contingency problems (non-deterministic and inaccessible environment, sensors must be used during execution, solution is a tree or policy)
  - Exploration problems (unknown state space)



# Example: Vacuum Cleaner Problem

- 2 localizations, 3 Actions (left, right, suck), 8 possible States, Objective: clear the garbage!
- Problem:



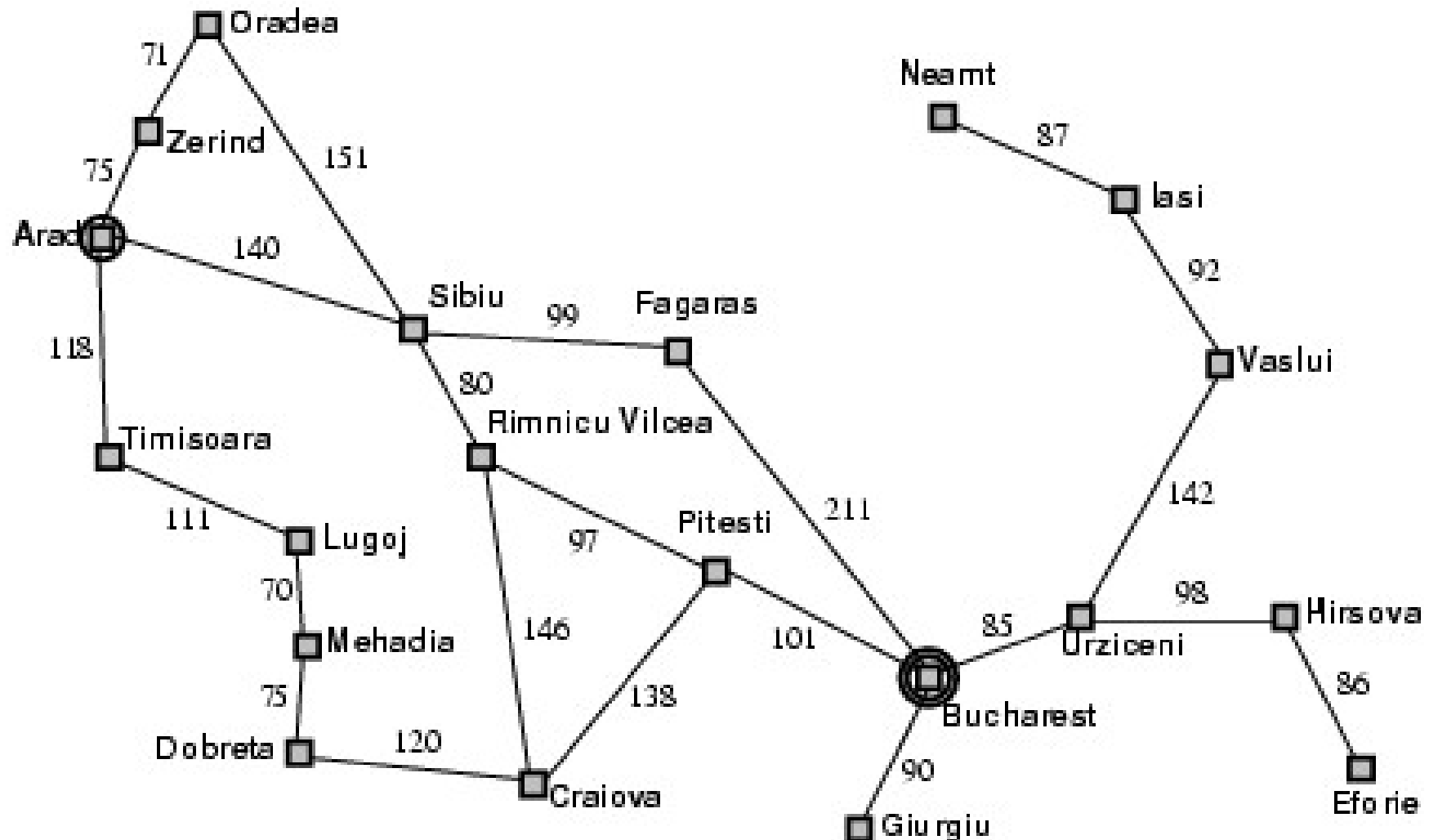
- Single state problem if... (deterministic and accessible)
- Multiple state problem if ... (deterministic but inaccessible)
- Contingency problem if ... (non-deterministic and inaccessible)
- Exploration problem if (unknown state space)

# Well Defined Problems (single state)

- **Problem:** Collection of information the agent will use to decide what to do!
- **Problem Formulation:**
  - **State Space:**
    - Initial State
    - Possible Actions Set (operators, successors role)
  - **Goal Testing**
  - **Solution Cost Function**
- **datatype** PROBLEM
  - components:** INITIAL-STATE, OPERATORS, GOAL-TEST, PATH-COST-FUNCTION
- **Solution:** Path from initial state to goal
- **Total Cost = Solution Cost + Search Cost**

# Example: Street Map or Romania

- Initial State: Arad; Objective State: Bucharest
- Operators: Arcs with respective costs



# Example: N-Puzzle Problem

- States, Operators, Objective Testing, Cost of Solution

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

# Example: N-Puzzle Problem

- **States, Operators, Objective Testing, Cost of Solution**
  - States: Specifies the position of each piece and the empty space (several representations are possible)
  - Initial state: Represented in the figure
  - Successor operators: generates valid states that result from execution. These are the four actions (move empty space left, right, up or down)
  - Objective test: Checks whether the state corresponds to the objective configuration (represented in the figure)
  - Solution cost: Each step costs 1, the cost of the solution being the number of steps to solve the problem

7	2	4
5		6
8	3	1

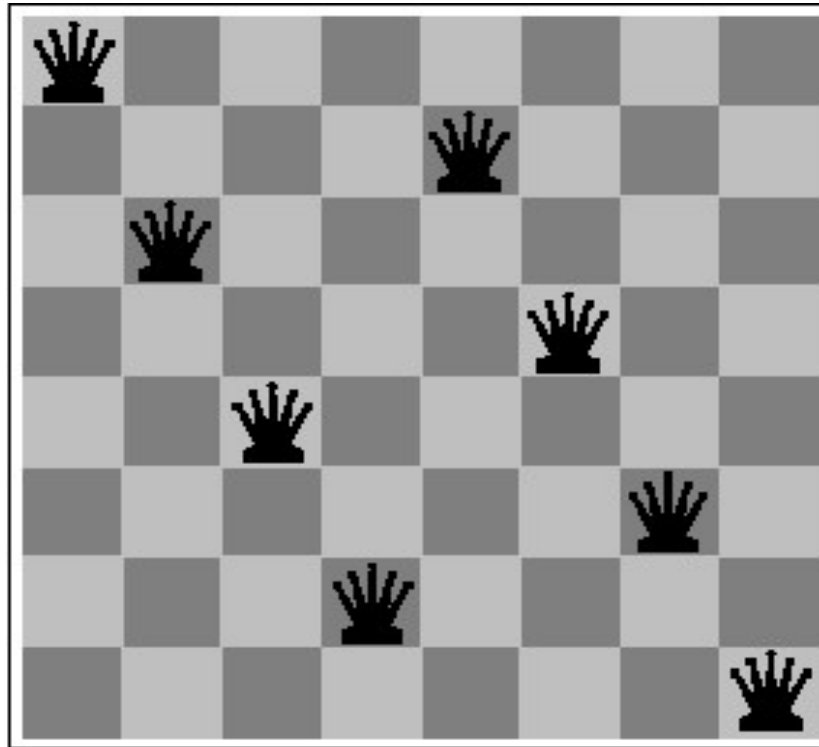
Start State

	1	2
3	4	5
6	7	8

Goal State

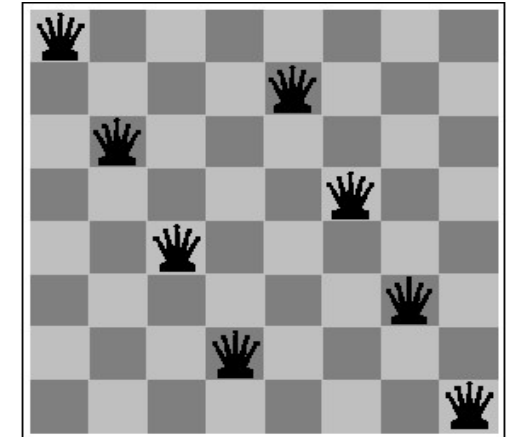
# Example: N-Queens Puzzle

- States, Operators, Objective Testing, Cost of Solution



# Example: N-Queens Puzzle

- **Objective Test:** 8 Queens on the board without any attack
- **Solution Cost:** 0 (all solutions are the same)
- **Formulation 1:**
  - Status: Any arrangement from 0 to 8 Queens on the board
  - Operator: Add a queen to any square
  - We have  $64^8$  possible sequences!
- **Formulation 2:**
  - Status: Arrangements from 0 to 8 Queens, one in each column, no attacks!
  - Operator: Add a queen to the leftmost column that is empty, without attacking any other
  - We have 2057 possible sequences!
- **Formulation 3:**
  - Status: Arrangements of 8 Queens on the board, one in each column!
  - Operator: Move attacked queen to home from the same column



# Cryptograms

- Find digits (all different), one for each letter so that the sum is correct!
- States: Puzzle with some letters replaced by numbers
- Operators: Replace all occurrences of a letter with a digit
- Objective Test: Puzzle only contains digits and the sum is correct!
- Solution Cost: 0 (all solutions are the same)

	F	O	R	T	Y
		T	E	N	
+		T	E	N	
-----					
	S	I	X	T	Y

	$C_1$	$C_2$	$C_3$	$C_4$	
	S	E	N	D	
+	M	O	R	E	
-----					
	M	O	N	E	Y



# Cryptograms

- **Cryptogram Solution:**

- Cryptogram 1: F=2, O=9, R=7, T=8, Y=6, E=5, N=0, S=3, I=1, X=4
  - Any more solutions?
- Cryptogram 2: S=9, E=5, N=6, D=7, M=1, O=0, R=8, Y=2

$$\begin{array}{r} \text{FORTY} \\ \text{+ TEN} \\ \hline \text{SIXTY} \end{array}$$

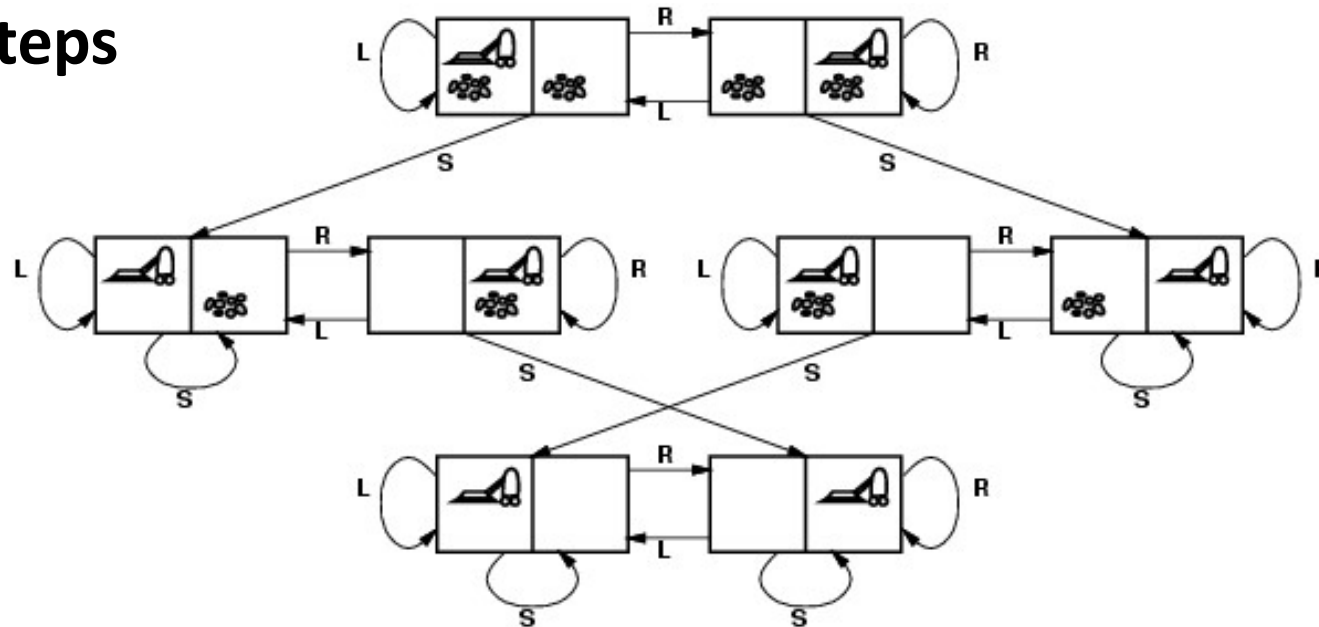
$$\begin{array}{r} 29786 \\ \text{+ } 850 \\ \hline 31486 \end{array}$$

$$\begin{array}{r} C_1 C_2 C_3 C_4 \\ \text{SEND} \\ \text{+ MORE} \\ \hline \text{MONEY} \end{array}$$

$$\begin{array}{r} 1011 \\ 9567 \\ \text{+ } 1085 \\ \hline 10652 \end{array}$$

# Example: Vacuum Cleaner Problem

- **State:** 8 states represented (defined by the position of the robot and garbage)
- **Initial state:** Any
- **Operators:** left, right, vacuum
- **Objective Test:** There is no trash in any of the squares
- **Solution Cost:** Each action costs 1 (total cost = number of solution steps)



# Example: Vacuum Cleaner without Sensors!

- **Multiple State Problem**
  - Now we have in every instant a set of possible states!
- **Problem Formulation**
  - State Set: Subset of represented states
  - Operators: left, right and suck
  - Objective Test : All states in the set cannot have garbage
  - Solution Cost: Each action costs 1

# Example of Real World Problems

- **Routes/Path Finding Problem**
  - Find the best route from one point to another (applications: google maps, computer networks, military planning, air travel)
  - Visit each point at least once in a given space (Ex: traveling salesman visit each city exactly once, find the shortest route)



# Exercise: Missionaries and Cannibals

## Missionaries and Cannibals Problem

3 missionaries and 3 cannibals are on one side of the river with a boat that only takes 2 people.

The objective is to find a way to take the 6 to the other side of the river without ever leaving more cannibals than missionaries on one of the banks during the process!



- a) Formulate this problem as a search problem, defining the representation of the state, initial state, the operators (and respective preconditions and effects), the objective test and the cost of the solution.
- b) Solve the problem through a tree search

# Exercise: Bucket Filling Problem

## Bucket Filling Problem

Two buckets, with capacities  $c_1$  (ex: 4 liters) and  $c_2$  (ex: 3 liters), respectively, are initially empty. The buckets have no intermediate markings.

The only operations you can perform are:

- empty a bucket
- fill (completely) a bucket
- pour one bucket into the other until the second is full
- pour one bucket into the other until the first is empty



The objective is to determine which operations to carry out so that the first bucket contains  $n$  liters (example: 2 liters)?

- Formulate the Problem as a search problem
- Solve the problem through a tree search

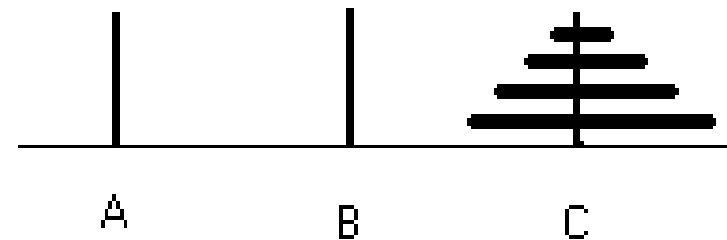
# Exercise: Hanoi Towers

## Hanoi Towers Problem

a) Formulate the problem of the Towers of Hanoi as a search problem.

Notes:

- In this version of the problem you have towers (A, B and C) and 4 disks (D1 to D4).
- Initially the disks are in tower C and the objective is to transfer them to tower A.
- In each move, the player can move a disk from one tower to another tower, if he does not place that disk on a smaller disk.



b) Suppose that the number of disks and the number of towers can be different ( $n$  disks and  $m$  towers) and formulate this generic version of the problem as a search problem.



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