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Artificial Intelligence
Lecture 4b: Dealing with Uncertainty

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The Need to Act under Uncertainty

- Partial observability
- Nondeterminism
- Adversaries

- How?
  - Keeping track of a belief state: the set of all possible world states that the agent might be in
  - Generating contingency plans for every possible eventuality

- But we should take into account that **certain states are more likely to occur than others**!

The Need to Act under Uncertainty

- Automated taxi: delivering a passenger to the airport on time
  - Plan A_{90}: leave home 90 minutes before (airport is only 5 miles away), drive at reasonable speed
  - Will plan A_{90} get us to the airport in time?
    - car doesn't break down or run out of gas
    - I don't get into an accident, and there are no accidents on the bridge
    - plane doesn't leave early
    - no meteorite hits the car
    - …
  - The plan's success cannot be inferred! Is plan A_{90} the right thing to do?
  - Is it expected to maximize the agent's **performance measure**?
  - What about plan A_{180}?

- The **rational decision** depends on the relative importance of various goals and the likelihood that, and degree to which, they will be achieved

Uncertainty and Rational Decisions

- Choosing a plan:
  - Plan A_{90}: 97% of catching the flight
  - Plan A_{180}: 99% of catching the flight
    - Perhaps not a good choice, because it probably involves an intolerable wait at the airport!
- **Preferences** over outcomes
  - Where an outcome is a completely specified state: arriving on time, waiting time at the airport, …
- **Utility theory**: represent and reason with preferences
- Combining preferences with probabilities:

Decision theory = probability theory + utility theory

  - A rational agent chooses the action that yields the *highest expected utility*, averaged over all the possible outcomes: **maximum expected utility**

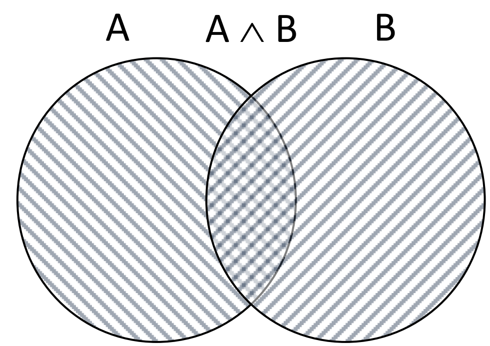
Probability Theory

- One way of dealing with uncertain knowledge: **probabilities**

- Let Ω be the set of possible worlds (the **sample space**)
- A probability model associates a probability $P(\omega)$ with each possible world $\omega \in \Omega$
  - $0 \leq P(\omega) \leq 1$ for every ω and $\sum_{\omega \in \Omega} P(\omega) = 1$

- Assign a degree of belief (between 0 and 1) to events that cannot be precisely obtained or determined
  - 0 / 1 indicates an undisputable belief that certain event is false / true
  - Probabilities between 0 and 1 correspond to intermediate degrees of belief regarding the truthfulness of the event
    - The event itself is true or false! A prob of 0.8 simply says that in 80% of the states indistinguishable from the current state we expect the event to be true

Axioms of Probability Theory



- $0 \leq P(a) \leq 1$
- $P(True) = 1$
- $P(False) = 0$

- $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

- **Mutually exclusive** events: $P(a \vee b) = P(a) + P(b)$
  - $P(a \vee \neg a) = P(a) + P(\neg a) - P(a \wedge \neg a) =$
    $P(a) + P(\neg a) - P(False) = P(a) + P(\neg a)$
  - It also follows that $P(\neg a) = 1 - P(a)$, because $P(a \vee \neg a) = 1$

Prior and Conditional Probabilities

- **Prior** (or unconditional) probabilities
  - $P(Flu) = 0.1$ may indicate, *in the absence of further information*, a probability of 10% that a person has a flu
- **Conditional** probabilities: calculated based on the presence of other interdependent events
  - $P(Flu \mid Fever) = 0.8$ is indicative that if a patient has fever, and *in the absence of further information*, the probability of having a flu is 80%

$$\boxed{P(a \mid b) = \frac{P(a \land b)}{P(b)}}$$

or $P(a \land b) = P(a \mid b)\, P(b)$
or $P(a \land b) = P(b \mid a)\, P(a)$

- With independent events: $P(a \land b) = P(a)\, P(b)$
- Two coin-tosses: $P(Heads \land Heads) = 1/2 \times 1/2 = 1/4$

In knowledge-based systems, conditional probabilities are important because usually we have only partial information on the data needed to employ certain domain knowledge

Joint Probabilities

- Conditional probabilities are defined in terms of joint events

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

- This means that in order to calculate $P(a \mid b)$, we need to know the probability that a and b occur simultaneously
- $P(Flu \mid Fever)$
  - We can build a truth table with the joint probabilities for both events:

	$Fever$	$\neg Fever$
Flu	0.04	0.06
$\neg Flu$	0.01	0.89

$$P(Flu \mid Fever) = \frac{P(Flu \wedge Fever)}{P(Fever)} = \frac{0.04}{0.04 + 0.01} = 0.80$$

- What if there are more than two variables to consider?
  - For n variables $\Rightarrow 2^n$ cells in the table!

Bayes' Theorem

- **Bayes' rule** is obtained from the equations
  - $P(a \wedge b) = P(a \mid b)\, P(b)$ and $P(a \wedge b) = P(b \mid a)\, P(a)$
- Equating both right-hand sides and dividing by $P(a)$, we obtain

$$\boldsymbol{P(b \mid a) = \frac{P(a \mid b)\, P(b)}{P(a)}}$$

  - $P(b)$: prior probability of b, that is, before discovering a
  - $P(b \mid a)$: conditional probability, that is, after discovering a

Bayes' Theorem

$$P(b \mid a) = \frac{P(a \mid b)\, P(b)}{P(a)}$$

- Why is it useful?
  - Requires 3 terms to calculate a conditional probability!
  - But in certain domains – such as in medical diagnosis – we know conditional probabilities in *causal relations* and need to derive a *diagnosis*

$$P(cause \mid effect) = \frac{P(effect \mid cause) P(cause)}{P(effect)}$$

- The doctor knows the causal $P(symptoms|disease)$
- …and wants to derive a diagnosis $P(disease|symptoms)$

Applying Bayes' Rule

- A patient has a symptom – say, a *stiff neck* (S)
- We want to determine if the symptom is due to something potentially serious – say, *meningitis* (M)
  - Doctor knows meningitis causes *stiff necks* in 70% of cases: $P(S \mid M) = 0.7$
  - The prior probability of a patient having *meningitis* is $P(M) = 1/50000$
  - The prior probability of a patient having a *stiff neck* is $P(S) = 0.01$

$$P(M \mid S) = \frac{P(S \mid M)\, P(M)}{P(S)} = \frac{0.7 \times 1/50000}{0.01} = 0.0014$$

  - Thus, only 0.14% of patients with stiff necks have meningitis
    - Even though having a *stiff neck* is common (70% of the cases) – what happens is that the prior probability of *stiff necks* is much higher than that of *meningitis*

Applying Bayes' Rule

- Why don't we know $P(M \mid S)$ right from the start?
- Diagnostic knowledge is often more fragile than causal knowledge
  - There may be no information on the probability of a person with a *stiff neck* having *meningitis*
    - $P(M \mid S)$ is diagnostic knowledge
  - But we may have a consistent notion of how many patients with *meningitis* have *stiff necks*
    - $P(S \mid M)$ is causal knowledge
- If there is a meningitis epidemic:
  - $P(M)$ will increase
  - $P(M \mid S)$ should raise proportionally to $P(M)$
  - Causal knowledge $P(S \mid M)$ will stay the same – it reflects how the disease works!

General Form of Bayes' Rule

- What if we have more than one evidence (or symptom)?
- With 2 evidence:

$$\boxed{P(M \mid S_1 \land S_2) = \frac{P(S_1 \land S_2 \mid M)\, P(M)}{P(S_1 \land S_2)}}$$

  - We need to compute $P(S_1 \land S_2) = P(S_1 \mid S_2)\, P(S_2)$

- For n evidence, we get the **general form of Bayes' Rule**:

$$P(d \mid s_1 \land \cdots \land s_n) = \frac{P(s_1 \land \cdots \land s_n \mid d)\, P(d)}{P(s_1 \land \cdots \land s_n)}$$

  - We need to compute

    $P(s_1 \land \cdots \land s_n) = P(s_1 \mid s_2 \land \cdots \land s_n)\, P(s_2 \mid s_3 \land \cdots \land s_n)\ \ldots\ P(s_n)$
  - If some of these evidence are independent of each other, i.e.,

    $P(s_i) = P(s_i \mid s_j)$, we can simplify to $P(s_i \land s_j) = P(s_i)\, P(s_j)$

Conditional Independence

- Sometimes, we can assume **conditional independence** between evidence in the presence of additional evidence E (domain knowledge) :
  - $P\left(s_i \mid s_j, E\right) = P(s_i \mid E)$
  - Car with a flat tire and faint lights: 2 independent symptoms
  - Car doesn’t start and faint lights: dependent! (both need battery to work)

- **Naïve Bayes**

$$P(d|s_1 \wedge s_2 \wedge \cdots \wedge s_n) = P(d) \prod_i P(s_i|d)$$

  - Naive because the variables are typically *not* actually conditionally independent given the cause variable
  - In practice, naive Bayes systems can work surprisingly well, even when the conditional independence assumption is not true!

Other Approaches to Model Uncertainty

- Bayesian (or Belief) Networks
- Default reasoning
- Rule-based approaches (e.g., the Certainty Factors model)
- Dempster–Shafer theory (representing ignorance)
- Fuzzy logic and fuzzy set theory (representing vagueness)