2022-Winter-Camp-Written-Test

JQ Investments

December 2021

1   Q1 [10pts]

Define a graph with n nodes and m edges as a uniform random graph, $G(n, m)$, if and only if the m edges are chosen uniformly randomly from all potential edges of a complete graph of n vertices (i.e. each one of $\binom{n}{2}$, alternatively, C_n^2, edges are chosen with the same probability).

1. What is the distribution of the degree of a vertex in $G(n, m)$? the degree of a vertex is the number of edges connected to that vertex. [**3pts**]

2. What is the expected number of triangles in $G(n, m)$? Define a triangle as a fully-connected subgraph of 3 vertices. [**7pts**]

2   Q2 [10pts]

1. A man starts from 49**. Each time he walks one unit to the left or to the right with equal probability. He stops once he has reached either** 0 **or** 100**. What is the probability that he stops at** 0**? [2pts]**

2. A man starts from 49**. Each time he walks one unit to the left with probability** p **(**$p \in (0.5, 1]$**) or to the right with probability** $1 - p$**. He still stops once he has reached either** 0 **or** 100**. Show that his random walk finishes within finite number of steps (in probability). (Hint, this is a biased random walk) [2pts]**

3. A man is in a circle with integer labels from 0 to 100, clockwise (both included). Each time he walks one unit clockwise or counterclockwise with equal probability. He stops if he has visited all labels. What is the probability that he stops at 50**? [6pts]**

3 Q3 [10pts]

Please limit your answers to 3 sentences or less and state your assumptions as well, if any. Each question below is worth 2pts.

3.1 Given a long array in RAM, why is accessing it sequentially much faster than accessing it randomly?

3.2 When your program tries to use more than the physical memory, does it crash? or does it just become slower? Why?

3.3 Do you think a quick-sort on $1e5$ elements would finish in one second? what about $1e8$ elements? what about $1e10$ elements? Why?

3.4 From a computer architecture/low level perspective, what happens when you do !++var;!, when the variable is NOT cached?

3.5 What is an environment variable? What happens when you export an env var? If you exported an env var on a server in ssh, and then logged out and re-logged in, is the env var you exported still available?

4   Q4 [10pts]

4.1   Statement

Holiday season is coming and you're preparing for a huge party. There are n dishes you'd like to order for the party, but they are all from **different** restaurants. Therefore, for each dish, you'll need to either pick it up physically or order a delivery.

The time you need to pick up dish i is p_i minutes, $\forall 1 \leq i \leq n$. The delivery time for dish i is d_i minutes, $\forall 1 \leq i \leq n$.

Delivery couriers work in parallel while you need to pick up the rest of the dishes **one by one**. Note that you can order as many deliveries as you want.

What is the minimum number of minutes to get all the dishes?

Please write actual c++ or Python code for this problem [8pts] and analyze the runtime and space complexities of your program properly [2pts].

4.2   Sample Input and Output

For example, if $n = 5$, $p = [1, 2, 3, 2, 1]$, and $d = [4, 8, 5, 6, 3]$, then you can choose to order deliveries for dish 2 and 3. This way, you'll need $max(2 + 2, max(4, 5, 3)) = 5$ minutes, which is faster than, say, ordering deliveries for dish 2, 3, and 4, which takes $max(2 + 3 + 2, max(4, 3)) = 7$ minutes.