

Periodogram

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In signal processing, a **periodogram** is an estimate of the spectral density of a signal. The term was coined by Arthur Schuster in 1898.^[1] Today, the periodogram is a component of more sophisticated methods (see spectral estimation). It is the most common tool for examining the amplitude vs frequency characteristics of FIR filters and window functions. FFT spectrum analyzers are also implemented as a time-sequence of periodograms.

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Definition

There are at least two different definitions in use today. One of them involves time-averaging,^[2] and one does not.^[3] Time-averaging is also the purview of other articles (Bartlett's method and Welch's method). So this article is not about that. The definition of interest here is that the power spectral density of a continuous function, $x(t)$, is the Fourier transform of its auto-correlation function (see Cross-correlation theorem):

$$\mathcal{F}\{x(t) * x(-t)\} = X(f) \cdot X^*(f) = |X(f)|^2.$$

Computation

For sufficiently small values of parameter \mathbf{T} , an arbitrarily-accurate approximation for $X(f)$ can be observed in the region $-\frac{1}{2T} < f < \frac{1}{2T}$ of the function:

$$X_{1/T}(f) \stackrel{\text{def}}{=} \sum_{k=-\infty}^{\infty} X(f - k/T),$$

which is precisely determined by the samples $x(nT)$ that span the non-zero duration of $x(t)$ (see Discrete-time Fourier transform).

And for sufficiently large values of parameter \mathbf{N} , $X_{1/T}(f)$ can be evaluated at an arbitrarily close frequency by a summation of the form:

$$X_{1/T}\left(\frac{k}{NT}\right) = \sum_{n=-\infty}^{\infty} \underbrace{T \cdot x(nT)}_{x[n]} \cdot e^{-i2\pi \frac{kn}{N}},$$

where k is an integer. The periodicity of $e^{-i2\pi \frac{kn}{N}}$ allows this to be written very simply in terms of a Discrete Fourier transform:

$$X_{1/T}\left(\frac{k}{NT}\right) = \underbrace{\sum_N x_N[n] \cdot e^{-i2\pi \frac{kn}{N}}}_{DFT}, \quad (\text{sum over any } n\text{-sequence of length } N),$$

where x_N is a periodic summation: $x_N[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} x[n - mN]$.

When evaluated for all integers, k , between 0 and $N-1$, the array:

$$S\left(\frac{k}{NT}\right) = \left| \sum_N x_N[n] \cdot e^{-i2\pi \frac{kn}{N}} \right|^2$$

is a *periodogram*.^[3]

Applications

The parameter **N** is known to Matlab and Octave users as NFFT.^[3] When a periodogram is used to examine the detailed characteristics of an FIR filter or window function, **N** is chosen to be several multiples of the non-zero duration of the $x[n]$ sequence, which is called *zero-padding* (see Sampling the DTFT). And when it is used to implement a filter bank, **N** is several **sub**-multiples of the non-zero duration of the $x[n]$ sequence (see Sampling the DTFT).

One of the periodogram's deficiencies is that the variance at a given frequency does not decrease as the number of samples used in the computation increases. It does not provide the averaging needed to analyze noiselike signals or even sinusoids at low signal-to-noise ratios. Window functions and filter impulse responses are noiseless, but many other signals require more sophisticated methods of spectral estimation. Two of the alternatives use periodograms as part of the process:

- The *method of averaged periodograms*,^[4] more commonly known as Welch's method,^{[5][6]} divides a long $x[n]$ sequence into multiple shorter, and possibly overlapping, subsequences. It computes a windowed periodogram of each one, and computes an array average, i.e. an array where each element is an average of the corresponding elements of all the periodograms. For stationary processes, this reduces the noise variance of each element by approximately a factor equal to the reciprocal of the number of periodograms.
- Smoothing is an averaging technique in frequency, instead of time. The smoothed periodogram is sometimes referred to as a *spectral plot*.^{[7][8]}

Periodogram-based techniques introduce small biases that are unacceptable in some applications. Other techniques that do not rely on periodograms are presented in the spectral density estimation article.

See also

- Welch's method
- Bartlett's method
- Discrete-time Fourier transform

- Least-squares spectral analysis, for computing periodograms in data that is not equally spaced

References

1. Schuster, Arthur (January 1898). "On the investigation of hidden periodicities with application to a supposed 26 day period of meteorological phenomena". *Terrestrial Magnetism*. **3** (1): 13-41. doi:10.1029/TM003i001p00013. "It is convenient to have a word for some representation of a variable quantity which shall correspond to the 'spectrum' of a luminous radiation. I propose the word *periodogram*, and define it more particularly in the following way."
2. "Periodogram—Wolfram Language Documentation".
3. "Periodogram power spectral density estimate - MATLAB periodogram".
4. Engelberg, S. (2008), *Digital Signal Processing: An Experimental Approach*, Springer, Chap. 7 p. 56
5. Welch, Peter D. (June 1967). "The use of Fast Fourier Transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms" (PDF). *IEEE Transactions on Audio and Electroacoustics*. AU-15 (2): 70-73. doi:10.1109/TAU.1967.1161901.
6. "Welch's power spectral density estimate - MATLAB pwelch".
7. Spectral Plot (<http://www.itl.nist.gov/div898/handbook/eda/section3/eda33r.htm>), from the NIST Engineering Statistics Handbook.
8. Short explanation (<http://www.itl.nist.gov/div898/software/dataplot/refman1/ch2/spectral.pdf>) of the relation between the spectral plot and the periodogram.

Further reading

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Categories: Frequency domain analysis | Fourier analysis

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