Formulae for the Helix Formed by Stacking Similar Objects

Robert L. Read *

Founder, Public Invention, an educational non-profit.

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1 Introduction

During the Public Invention Mathathon of 2018, software was created to view tetrahedra joined face-to-face in chains. It was noted by the participants that when the rules for which face to add the next tetrahedron to were periodic, the resulting chain was always a helix or a torus. A torus is a degenerate helix.

After some relfeciton, it became clear that any stack of objects of the same length joined at the same angles to each other repetitively form a helix. A specific example is a face-to-face connection of physical objects of the same length where the angles of the two joining faces relative to each other control the angle and rotation of the axis at each joint. We have not yet found this simple idea articulated elsewhere. The purpose of this paper is to prove and to provide formulae for the resulting helix.

In engineering, sometimes the term "special helix" is use for helical curves on non-cylindrical surfaces. This paper use the term "helix" only in the sense of "cylindrical helix". `https://link.springer.com/chapter/10.1007/978-94-007-2169-2_95`

2 A Warm-up: 2Dimensions

Considering the problem in two dimensions may be a valuable introduction. Suppose that we consider a polygon that that as two edges, called A and B, and that we define the length L of the polygon as the distance between the midpoints of these edges. Suppose that we are only allowed to join these polygons by aligning A of one polygon to B of another polygon, with their midpoints coincident. Let us further assume that we disallow inversions of the polygon. Let us imagine that we have a countable number of polygons P_i indexed from 0. Then what shapes can we make by chaining these polygons together?

*read.robert@gmail.com

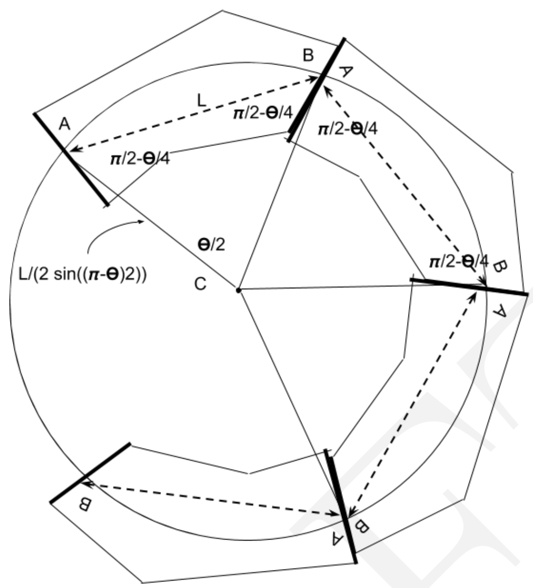

Figure 1: The rotatable prism of three objects

Each joint J_i between polygons P_i and P_{i+1} will place the axes of at the same angle, θ, since our polygons do not change shape. Let us define θ to be positive if we move anti-clockwise from P_i to P_{i+1} and negative if we move clockwise. If $\theta = 0$, the joints will be collinear.

If $\theta \neq 0$, it seems they polygon joints will always lie on a circle. A proof of this is that each polygon has associated with it an isoceles triangle A, B, C, where $\angle CBA = \angle CAB = \theta/2$, and $\angle ACB = (\pi - \theta)$. AC and BC are not necessarily aligned with an edge of the polygon. The length AB is L, and the lengths AC and BC are $(L/2)/\sin(\pi - \theta)/2$. In any chain of polygons, these triangles all meet at point C, and there all joints are on the circle centered at C with radius $\frac{L}{2\sin\frac{\theta}{2}}$.

An analogous, though far more complicated, result holds in three dimensions.

3 The Discrete Helix

In this section we consider discrete helix, or a helix evaluated only at regular points. Drawing lines between these points create 3D polygon.

Following the Wikipedia artilce `https://en.wikipedia.org/wiki/Helix`, we set up a helix parametrically.

$$
\begin{aligned}
P_x(t) &= r \sin t \\
P_y(t) &= r \cos t \\
P_z(t) &= bt
\end{aligned}
$$

Such a helix has a radius of r and slope (if $r \neq 0$) of b/r,. Note that a helix may be degenerate in two ways. If $r = 0$, these equations become a line. If $b = 0$, these equations describe a circle in the xy-plane. If $r = 0$ and $b = 0$, the figure is a point.

Such helixes are continuous, but we are investigating stacks of discrete objects. We in fact wish to derive the parameters for a continuous helix from such discrete objects which constrain discrete points, so we wish to study a helix evaluated at integral points. We call such an object a *discrete helix* or *delix (DEE-lix)*.. A discrete helix may be thought of as function that given an integer gives back a point in three space.

$$
\begin{aligned}
P_x(n) &= r \sin n\theta \\
P_y(n) &= r \cos n\theta \\
P_z(n) &= nd
\end{aligned}
$$

d is the distance along the z-axis between adjacent joints, and θ is the rotation around the z-axis between adjacent points. r is the radius of the delix. If we think of the delix as describing a polyline in 3-space, we would like to investgiate the properties of that polyline.

- L is the distance between any two adjacent points.
- c is the length of a chord formed by the projection of the segment between two points into the xy-plane.
- ϕ is the angle in the plane containing two joints which is perpendicular to xy-plane (in other words the plane parallel to the z axis containing two points.)

These quantities are related:

$$c = 2r \sin \frac{\theta}{2} \tag{1}$$

$$L^2 = c^2 + d^2 \tag{2}$$

$$\arctan \frac{c}{d} = \phi \tag{3}$$

Now we are attempting to relate these properties to properties intrinsic to the joint or interface between two segments or objects in the delix. If given an object, the length between the joint points L is easily measured.

4 The Intrinsic Properties of Stacking Objects

If we are given a physical or mathematical object that stacks, the faces may be specified by a normal vector or by the angles of the face relative to the axis. The normal vector is a more linear-algebraic approach; the face angles may be more natural to chemists and mechanical engineers. Since the two methods are interchangable, it is largely a matter of convenience. However, we seek specifically to develop a formula which allows helices to be designed, and the face-angle approach seems more suited to the carpenter grasping a hand saw or the molecular biologist designing a molecule.

As shown in figures and , We place a joint point on each face, and call these points A and B. We define the axis of the object as $\overline{AB}$. Imagine the object placed on the z axis in a right-handed coordinate system, so that A is in the negative z direction and B is at the origin. Then the cut of the B face can be described by two angles. α is the angle in the XZ plane, and β is the angle in the YZ plane. In other words, if a box or cuboid were drawn with three edges aligned with the axis and at the origin, and the vector defining the face-normal defined the diagonal of the box or cuboid, α is the angle with the z-axis of the face diagonal in the XZ plane (or the projection of the body diagonal into that plane), and β is the angle of the z-axis with the YZ-plane.

The face angles for A are denoted α_A, β_A, and likewise the independent face angles for B are α_B, β_B.

In the angle method, we start with these intrinsic properties of an object, and additionally the rule for how objects are laid face-to-face. That is, knowing the length between two joint points and a vector normal to the faces of the two joints, we almost have enough to determine the unique stacking of objects. The final piece is that we must know the *twist*. That is, when face A of a second objects is placed on face B of a first object so that they are flush (that is, their normals are in opposite directions), it remains the case that the second object can be rotated about the normals. To define the joining rule, we must attach an *up vector* to each object. Then a joining rule is "place the second object against the first, joint point conincident to joint point, and twist it so that its up vector differs by τ degrees from the up vector of the first object."

4.1 Relating Face Angles to Face Normal

Although intercomputable, we certainly want to be able to compute the coordinates or the point B_1 (the joint point of the second object), or equivalently, the axis vector of the second object.

This depends not solely on one face, but on how they combine, and in particular how they combine via the twist τ. We use ω and ρ to capture this combination.

$$\omega = \alpha_B + \alpha_A \cos\tau + \beta_A \sin\tau \tag{4}$$

$$\rho = \beta_B + \beta_A \cos\tau + \alpha_A \sin\tau \tag{5}$$

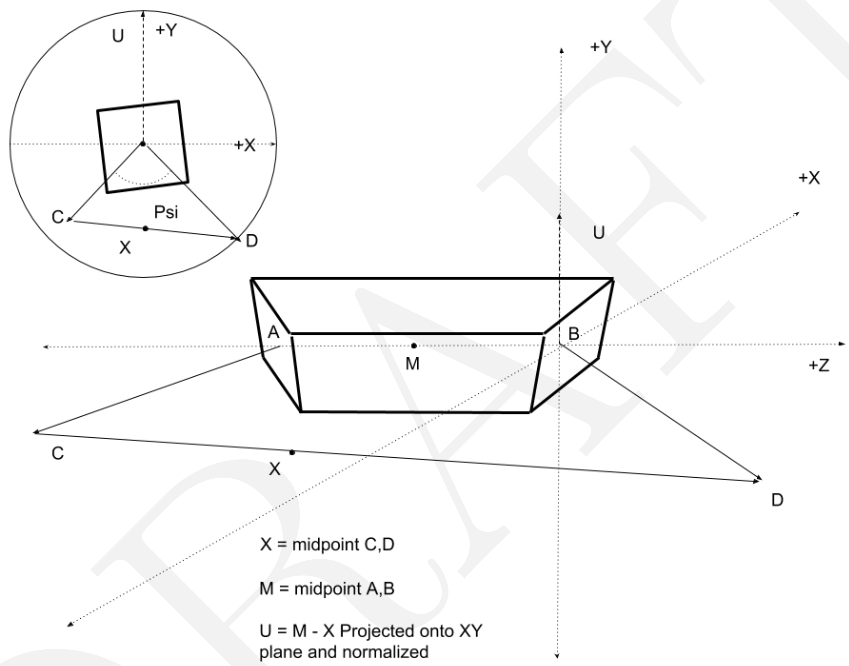

Figure 2: The rotatable prism of three objects

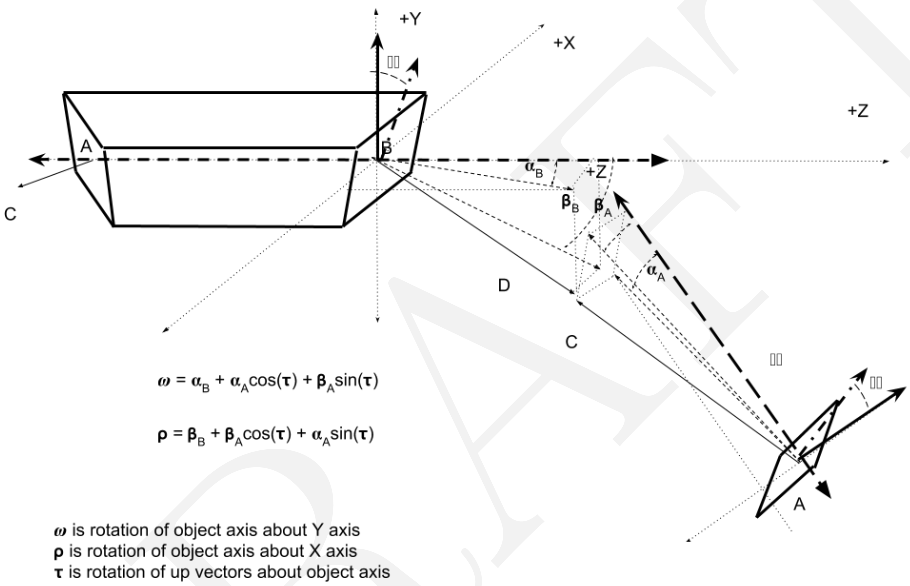

Figure 3: Joint Geometry

Given these angles and treating them as face diagonals, we can find the position of point A_1 via:

$$z = \frac{L}{\sqrt{1 + \tan\omega + \tan\rho}} \tag{6}$$

$$A[1]_x = z \tan\rho \tag{7}$$

$$A[1]_y = z \tan\omega \tag{8}$$

$$A[1]_z = z \tag{9}$$

Possibly due to my lack of imagination, these coordinates will become the basis of the formulae for r, d, and θ in the discrete helix.

The angle method requires:

- An object with two identified faces, labeled F_A and F_b. These faces are in fact planes, with normal vectors N_A and N_B. We assume that normals point out of the object rather than in. The angles in the method may be considered the angles of the face diagonals of the axis-aligned cuboid whose body digaonal is the vector normal.

- The length L of an object, measured from joint point A to joint point B.

- A joint twist τ defining the change in computed up-vector between objects, mesured at the joint face.

From the normals, we can compute an *up vector* which is intrinsic to the object. We could think of the up vector as a fine mark made on each face in the same direction (if projected onto a plane perpendicular to the axis.) When face F_A of object $N+1$ is placed against F_B of object N, object $N+1$ is twisted until the mark on F_A angles τ away from the mark on object N, measured anticlockwise looking from A to B on object N.

To define the up vector, place the object so that joint B is at the origin and the axis is aligned with the z-axis, with the point A on the negative z-axis. The projection of the face normals form a minimum angle less than or equal to π radians. Rotate the object about the z-axis so that the projections of the face normals form equal angles with the negative y-axis. Then define the direction straight up the y axis to be the up vector. If we had a physical object, we might mark the face normals with an arrow to mark the up vectors.

If one of the face normals is aligned with object axis, the up-vector is the opposite of the other face normal. (If both face normals are aligned with the axis of the object, then delix has zero radius and $d = L$, so we need do no further math.)

Mathematically, we will treat the up vector as unit-vector.

$$M = \frac{A+B}{2} \tag{10}$$

$$X = \frac{(A+F_a)+(B+F_b)}{2} \tag{11}$$

$$U = \frac{M-X}{\|M-X\|} \tag{12}$$

The up vector is not defined when the face norms point in precisely opposite directions, in which case $d = L$.

In the placement of the first object described above, it would be nice to know where the position of point B on the second object is. This can be obtained by computing the Euler angles of the face normal N_A at the origin, and then simply adding the negation of the Euler angles N_B, and using the resulting angles to compute a rotation $R(N_A, N_B)$ to be a applied to a vector of length L, and then performing the twist by $T(\tau)$. This process can be repeated at the point B_1 (though no longer at the axis) to generate four or more points. In fact the entire process of constructing the stack could be thought of a series of "rotate then translate" steps by the composition of the rotation matrixes RT applied to a vector $\overline{AB}$ and the up vector. However, it is a tiny bit of work to compute this given only the intrinsic properties of the object and joint.

Note: we have shown we can compute from 4 points. More elegant however, would be to construct a similar construction using the axis-intersecting lines without computing 4 points, perhaps from a pure geometry in some way.

TODO: Actually giving the transformation matrix here would be nice. How would we test?

TODO: Compute the vector angle between axes here. In this positioning, compute where point B of the second object will be.

Note: The twist is not the same as θ. When $\tau = 0$, $\theta = \alpha$, I think. Possible $\theta = \alpha +$ some function of τ.

Note: Possibly theta goes down as tau goes up.

Note: Considering the pitch of the discrete helix may be useful in relating variables.

Note: I now think we can think in terms of the skew lines (the joint lines), and I believe τ is directly the angle formed with the up vector, that the up-vector aims at the mid point of the axis between the intersection skew line points, and that angle bend α (or $(\pi - alpha)/2$) let's us determine the skew line as well. So we have the mid point, and we have another line based purely on τ and α, so that may allow θ to be computed more simply than the skew line intersection construction.

Todo: Try to do my 4-point solution with an Midpoint and upvector solution.

Todo: Try to prove that τ is really either the angle bisector angle or twice it.

Todo: Create a definition of the "sidedness" of delix, in terms of the number of sides in one pitch, and relate this matehmatically to the other points.

4.2 New Insight

Having arranged the objects as in our diagrams and identified the “prism” formation, it becomes clear that we can operate in the projection plane (XZ). This allows considerable simplication.

Our basic plan is to contruct the midpoint near F, M_F, which is in fact the midpoint of E and G. The line hits the axis of the helix perpendicularly. The midpoint of E adn F also hits this axis line, (and is the origin of the XZ plane.) Therefore, if we find the perpendicular distance from the the $\overline{F, M_F}$ line to the origin, we can easily compute the distance d along the axis, which will be slightly less than L.

5 Computation of Discrete Helix

In this section we attempt to derive formulae for $r, d, /theta$ from the instrinsic properties of the object.

6 Old Work

The most easily measured relationship between the segments is the angle α in the plane containing both of the segments. (If the segments are coincident and therefore parallel α will be π. If $\alpha < \pi/2$, the segment “bends backward”. The final measure that is needed is the rotation about a member between the elements attached to it. Let us call this angle ψ. That is, if you look along the axis of an object at the line between the joints, the two members form an angle between 0 and $\pi/2$.

$$
\begin{aligned}
A &= P(-1) \\
B &= P(0) \\
C &= P(1)
\end{aligned}
$$

A, B, C form an isoceles triangle that contains $\overline{AB}$ and $\overline{BC}$, and therefore the angle α is in this plane. Forming the kite $ABCO$, we see that one diagonal is r and the other diagonal is $2r \sin \theta$, where $y = r \sin \theta$ is half of this diagonal.

$$
\begin{aligned}
y &= r\sin\theta \\
z^2 &= c^2 - y^2 \\
z^2 &= c^2 - (r\sin\theta)^2 \\
q^2 + z^2 &= L^2 \\
q &= \sqrt{L^2 - z^2} \\
q &= \sqrt{L^2 - (c^2 - (r\sin\theta)^2)} \\
q &= \sqrt{L^2 + (r\sin\theta)^2 - (2r\sin\theta/2)^2} \\
\sin\alpha/2 &= q/L \\
q &= L\sin\alpha/2 \\
\alpha/2 &= \arcsin q/L \\
\alpha &= 2\arcsin\frac{\sqrt{L^2 + (r\sin\theta)^2 - (2r\sin\theta/2)^2}}{L}
\end{aligned}
$$

Note y is in the projected kite, q is in the slanted kite. Rearranging these relationships in terms of known quantites L, α, ψ, we have:

$$
\begin{aligned}
\sin\alpha/2 &= q/L \\
q &= L\sin\alpha/2 \\
z^2 &= L^2 - q^2 \\
\beta &= \frac{\pi - \theta}{2} \\
\frac{z}{c} &= \cos\beta \\
c &= \frac{z}{\cos\beta} \\
y^2 &= c^2 - z^2 \\
r &= \frac{y}{\sin\theta} \\
d^2 &= L^2 - c^2 \\
\phi &= \arctan c/d
\end{aligned}
$$

I think we can compute ψ from θ easily enough. The problem is going the other way.

6.1 An alternate approach

Rather than starting with α and ψ as inputs, we can think of a vector v that moves in the positive $\overline{Z}$ direction, $\overline{Z} = \overline{P}_1 - \overline{P}_0$, and the angle θ which Z rotates about the z-axis at each step. If a rotation matrix M represents rotation about and T represents translation along the z-axis, then $P_n = (M + T)^n P_n$.

7 Three dimensions

7.1 New Approach

We want to compute everything from properties intrinsic to the object and the joint process. These are the length, the face normal vectors, and the joint twist. These are the inputs to our process. From this we can compute several joints by: 1) Placing the object joint-to-joint aligned along their axes; 2) Computing from the face angles the rotation that makes the normal to B the negative of the normal to A, that is, the rotation that moves A into -B. 3) Apply this rotation around the joint. 4) Rotate along the axis until the up vectors match.

The "Up vectors" are computed to be in the line between the midpoints of points pointed to by the face normals, and the mid point of the axis, projected to be normal to that axis.

Using this intrinsic method, we can get as many points as we need of the stacked objects.

From these intrinsic property, the vector angle between axes of different objects is determined.

From these points, we can easily compute a "tilt and twist" approach, which defines α as the tilt and ψ as the twist. Those two parameters now become an output of this process above. Not the "twist" is measured IN THE PLANE OF FACES.

The remaining task is to find an (r, θ) helix representation that gives us what we need.

Conjecture: the twist is the same (or at least monotone in) as θ.

Proof Sketch: Imagine moving so many times around the helix that our (x, y) coordinates are similar to your starting point. The up vector for this object must point in approximately the same direction as the up axis you started with, by symmetry. If θ is anything other an integral factor of the twist, this would be impossible.

Alternative: The up vector at the midpoint of the object must aim at the axis of the helix. This must always be true. Therefore, the

TODO: We are still fundamentally seeking r, θ from these specifications. A formula for that is of the utmost importance.

Note: If the up vectors intersect the axis of the helix, we can find two points on this line using the wikipedia article: `https://en.wikipedia.org/wiki/Skew_lines` and the distance between these two points. This will allow us to find r, I think.

Using the method from skewlines to find two nearest points, we can then solve for the distance between this line and A or B and we have r.

So, the way to test this is to generate a helix. Use the helix to generate the midpoints and up vectors. Then at least check that we can back out and obtain our same radius and theta for the delix. Then the problem becomes the computation of the points and up vectors from the intrinsic properties of the block, which I believe to be possible. Which should I attack first?

Let us first prove that given 3 points and two up vectors we can recover the helix.

When we have a Mathematica procedure for that, then we can generate these inputs from the object inputs of $L, \overline{A}, \overline{B}$,and twist.

Is twist the same as theta? No, it is not—but they are related by $/phi$.

7.2 Old Approach

Consider a slender cylinder or prism with two faces F_0, F_1 cut at arbitrary angles to the axis of the cylinder. Joining two such cylinders at the axes by placing F_0 angainst F_1 produced a joint with a difference in angle between the axes of α and a rotation of the orienation of the faces about the axis of θ. Note that α and θ are a function of the vectors normal to the faces to be joined, but are not the same as them. α and θ are likely combinations of the face angles.

Theorem 1 (Stacking Helix)**.** *The joints of a sequential stack of objects of length L whose joints axis change by α and orientation rotate by θ are intersected by a helix of radius and pitch easily numerically computable from L, α, and θ.*

Proof. In Figure , let L_i be the ith instance of the length-L objects. Let α be the change in angle in the axes at a joint (the point were axes meet) measured in the plane containing the axes of both objects. (If these axes are parallel (and therefore coincident) define α to be 0. Let θ be the change in orientation relative to the axis, or, the half-angle formed by L_0 and L_2 when projected onto a plane normal to the axis of L_1. Without loss of generality, choose the measures of these angles in radians so that $0 \leq \theta < \pi/2$ and $0 \leq \alpha < \pi$. Take $\alpha < 0$ to mean that objects L_0 and L_1 bend away from each other, and $\alpha > 0$ to mean that the objects L_0 and L_1 bend toward each other.

If $\theta = 0$ and $\alpha > 0$, the stack will form a circle-like structure or radius $\frac{L}{2\sin\frac{\alpha}{2}}$, as shown in Section 2, where as if $\theta = 0$ and $\alpha < 0$, then stack will form a sawtooth-like structure.

Note that any angle α is possible, if we do not concern oursleves with the self-collision of physical objects. $\alpha > \pi/2$ means the stack "turns back on itself" to some extent.

If we arrange object L_1 so that its axis lies on the x-axis and its midpoint is at the origin, L_0 and L_1 extend from it symmetrically in the projection onto the yz-plane along the x-axis, which is always possible, and define h to be the height of the faces of L_0 and L_2

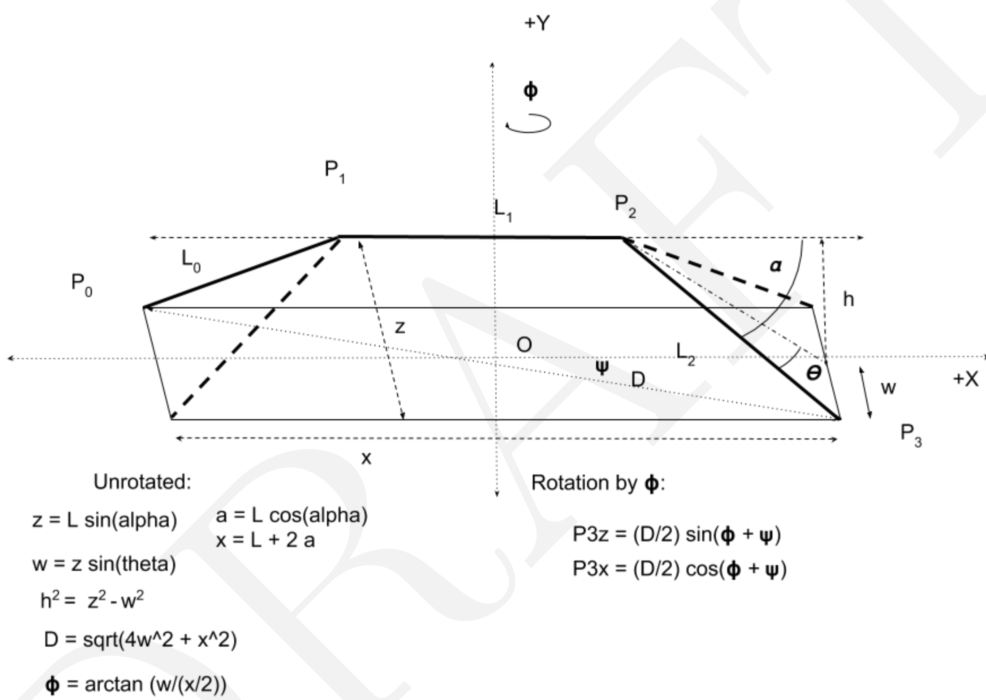

Figure 4: The rotatable prism of three objects

along the y-axis and w to be the distances between these faces in the z-dimesnion. Then we have:

$$
\begin{aligned}
z &= L \sin \alpha \\
w &= z \sin \theta \\
h &= \sqrt{z^2 - w^2}
\end{aligned}
$$

Now we seek the formula for the helix which intersects the joints. To find the radius of this helix, we conceptually place our three objects in a cylinder, with the axis of L_1 along the surface of the cylinder aligned with the axis. We can size this cylinder to include the joints at the extreme ends of L_2 and L_0 as well. However, the L_1 axis lies on the surface, but the axes of L_0 and L_2 in general do not. If there exists a helix which intersects all joints, all axes will cut through this cylinder in the same way, creating chords in the projection into the yz-plane of the same length. Name these chords $c_0, c_1, andc_2$.

Because we will need them later, we work out the geometry of this prism-like stack of three objects completely.

- The three objects are named L_0, L_1, and L_2. L_0 has joints P_0 and P_1, L_1 has joints P_1 and P_2, L_2 has joints P_2 and P_3.
- x is the total length of the prism along the x-axis.
- z is the length of the "face" of the prism, and the length of the chord before any rotation ϕ about the y-axis.
- w is half of the width of the prism in the z-dimension.
- h is the height of the prism in the y-dimension.
- D is the length of the diagonal from P_0 to P_3.
- ψ is the angle of $\overline{P_0P_3}, \arctan(w/(x/2))$.
- ϕ is the amount we will have to rotate the prism about the y-axis.

Given these defintions about our assumptions, the cord lengths are:

$$
\begin{aligned}
\psi &= \arctan(w/(x/2)) \\
D &= \sqrt{4w^2 + x^2} \\
c_1 &= L \sin \phi \\
c_2 &= \sqrt{h^2 + ((1/2)(D \sin(\psi + \phi) - L \sin \phi)^2} \\
c_0 &= c_2
\end{aligned}
$$

We can imagine turing our 3-object stack and simulataneously increasing the size of our intersecting cyliner. If we turn the stack about the y-axis by ϕ degrees and keep the cylinder intersection the two faces of L_1, then the length of the L_1 chord will gradually increase. At the same time, the L_0 and L_2 chords will change their length, possibly increasing or decrasing. When $\phi = 0$, $c_0 = z, c_1 = 0, c_2 = z$.

Equating these quantities to find when $c_2 = c_1$, we have a trigonometric equation with a single unknown, ϕ.

$$L \sin \phi = \sqrt{h^2 + ((1/2)(D \sin (\psi + \phi) - L \sin \phi)^2}$$

Although Mathematica does not appear to be able to solve this equation, it appears to be a smooth equation in the variable ϕ and we believe from the physical structure of the problem that will have only a single solution, so we can solve this numerically with a Newton-Raphson solver easily.

Thus, by rotating our stack of objects ϕ degrees around the y-axis all four faces of our three objects intersect a cylinder on its surface with equal rotational and axial distance. The axial distance between any two joints on the same object is $L \cos \phi$, and the length of the projected chord is $L \sin \phi$.

The points P_0, P_1, P_2, and P_3 now exist on a cylinder or unknown radius parallel to x-axis, and are evenly spaced along and evenly rotated about the axis of the cylinder. The joints points thus coincide with a general helix.

Let us choose our coordinate system so that the x-axis corresponds to the axis of the helix. The general equation for the helix is:

$$\begin{aligned} P_x(n) &= \kappa t \\ P_y(n) &= r \cos t \\ P_z(n) &= r \sin t \end{aligned}$$

We seek to discover r and κ based on our knowledge of P_3 and P_2. In particular, we can deduce from the axial spacing there exists some t_0 such that $P_2 = P(t_0)$ and $P_3 = P(3t_0)$. Since we know that after rotation that:

$$\begin{aligned} P_{3z} &= (D/2) \sin \phi + \psi \\ P_{3z} &= r \sin 3t_0 \\ P_{2z} &= \sin \phi \\ P_{2z} &= r \sin t_0 \end{aligned}$$

We can use symbolic computation to solve this system of 2 equations and 2 unkowns:

$$
\begin{aligned}
r \sin 3t_0 &= (D/2) \sin \phi + \psi \\
r \sin t &= \sin \phi
\end{aligned}
$$

Defining symbols:

$$
\begin{aligned}
E &= (D/2) \sin \phi + \psi \\
F &= \sin \phi
\end{aligned}
$$

Wolfram alpha solves the system:

$$
\begin{aligned}
r \sin 3t_0 &= E \\
r \sin t_0 &= F
\end{aligned}
$$

giving the result ($3F \neq E$ and $F \neq 0$), and ignoring multiples of 2π in t:

$$
\begin{aligned}
r &= \frac{2F^{\frac{3}{2}}}{\sqrt{3F - E}} \\
t_0 &= -2 \arctan x \frac{r + \sqrt{-\frac{F^2(F+E)}{E-3F}}}{F}
\end{aligned}
$$

From which, using $P_{2x} = L/2 = \kappa t_0$, we conclude:

$$
\begin{aligned}
r &= \frac{2F^{\frac{3}{2}}}{\sqrt{3F - E}} \\
\kappa &= \frac{L}{4 \arctan \frac{r+\sqrt{-\frac{F^2(F+E)}{E-3F}}}{F}}
\end{aligned}
$$

. Putting this all together we have:

$$
\begin{aligned}
a &= L\cos\alpha \\
x &= L + 2a \\
z &= L\sin\alpha \\
w &= z\sin\theta \\
h &= \sqrt{z^2 - w^2} \\
D &= \sqrt{4w^2 + x^2} \\
\phi &= \arctan\frac{z}{L} \\
E &= (D/2)\sin\phi + \psi \\
F &= \sin\phi \\
r &= \frac{2F^{\frac{3}{2}}}{\sqrt{3F - E}} \\
\kappa &= \frac{L}{4\arctan\frac{r+\sqrt{-\frac{F^2(F+E)}{E-3F}}}{F}}
\end{aligned}
$$

Using these values derived exclusively from the inputs $L, \alpha, and\theta$, we can evalute the formula for the general helix only and integral values of n to obtian a formula for precise the joint points of this any such stack.

$$
\begin{aligned}
P_x(n) &= \kappa t0(1 + 2n) \\
P_y(n) &= r\cos t0(1 + 2n) \\
P_z(n) &= r\sin t0(1 + 2n)
\end{aligned}
$$

□

8 Checking against comprehensible values

Unfortunately, the complexity of these formualae exceed the author's comprehension. However, we may check these formuale by graphing them against comprehensible examples. Obvious examples are extreme solutions, where α and θ are 0 or $\pi/2$, for example. We also have the particular non-trivial example of the Boerdick-Coxeter tetrahelix, formed by regular tetrahedra, which has been studied enough to have a known pitch.

9 Implications

One of the implcations of having a formulaic understanding of the math is that it may be possible to design helixes of any radius and pitch by designing periodic (possibly scalene) segments. Combined with slight irregularites, this means that you have a basis of design molecular helices out of “atoms” which correspond to our objects.

This would mean that if you wanted to build a brace of length exactly 3 meters with bars of exactly 1/2 meter you would be able to come as close to this as mathematically possible.

10 Applied to Periodic Regular Simplex Chains

Corollary 1. *Every regular simplex chain formed by a periodic generator has a helical structure.*

Prove or disprove that *every* periodic 3D Generator generates a figure contained within a cylinder of unbounded length but bounded diameter.

Note: Every example that we have tested exhibits this property. We believe it is a property of any repeated structure, not related to simplices. However, we do not yet know the name of this theorem or principle. We conjecture that every stack of repeated truncated prisms forms a helical aperigon, which is hinted at but not stated in the Wikipedia articlehttps://en.wikipedia.org/wiki/Skew_apeirogon.

Note: Rob believes a proof that any periodic structure fits within a cylinder is possible, and that it should be possible to give a formulaic bound on the diameter of this cylinder (under some assumptions.) The key to the proof is to use symmetry and focus on the the center of three such objects, observing that the other two must necessarily bend towards or away from each other in a way describable by two angles. A formula for the cylinder as a function of these angles would convincingly complete the proof.

Note: This paper may be critical.

https://arxiv.org/pdf/1610.00280.pdf

https://www.impan.pl/wydawnictwa/preprints_impan/p741.pdf

https://math.stackexchange.com/questions/878051/why-does-a-3d-line-of-segments-with-cons
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This reference is EXTREMELY IMPORTANT https://link.springer.com/article/10.1023/A:1015863923728 https://www.researchgate.net/publication/226976531_Helical_Structures_The_Geometry_of_Protein_Helices_and_Nanotubes This reference clearly states that any repetitive unit gives rise to a helix, and gives some species: the icosahelix, for example, and gives a 96-tet ring (not quite regular, like this work.) It gives a linear algebra way of talking about the transform. It does implies the helix is deducible from the transformation matrix, but does not appear to give any formular or method for describing it.

Need to get and read this: `https://books.google.com/books?hl=en&lr=&id=1LZlSZ7ORrQC&oi=fnd&pg=PP1&ots=0hSEwJvlUB&sig=xNG9UWv_H1OXHwaOiOBJN7TW6xA#v=onepage&q&f=false`

This may be worth reading: `https://link.springer.com/article/10.1007/PL00011063`

This is worth getting: `https://books.google.com/books?hl=en&lr=&id=FHPlDWvz1bEC&oi=fnd&pg=PP1&ots=TsOnodavEZ&sig=HO86UUVlqRVWGqY-Tv02nb7x7NA#v=onepage&q&f=false`

Some discussion of "screw transformations" `http://dergipark.gov.tr/download/article-file/56483` `https://ieeexplore.ieee.org/document/56653`

CRITICAL: `https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=56653`

This mentions Rodrigues's equation (p. 7) `https://www.cc.gatech.edu/~hic/8803-Mobile-08/slides/1_draft.pdf`

This is a valuable reference `http://www.12000.org/my_notes/screw_axis/index.htm`