Open Recreational Problems: Simplex Chains

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Founder, Public Invention, an educational non-profit.

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1 License

Contributions and suggestions from the reader are encouraged!

2 Introduction

Note: This is a work in progress as a preparation for the Public Invention Mathathon planned for Nov. 30th-Dec. 2nd 2018. Please read it as work designed to give a flavor of the problem area rather than a technical paper being prepared for publication.

The term *simplex* means the simplest regular polyhedron. In two dimensions, a simplex is an equilateral triangle. In three dimensions, a simplex is a regular tetrahedron.

Define a *simplex chain* to be a figure of many simplices adjoined face-to-face by a particular rule. The dimensionality of a face is always one less than the dimensionality of the space. In two dimensions, a simplex chain is a series of adjoined equilateral triangles joined edge-to-edge.

Let us number the simplices in a chain starting from 0. Then we can define a simplex chain via a rule that says which edge of the nth simplex to attach the $n+1$th simplex to. In two dimensions, we can label the edge of the last triangle in simplex chain as *anti-clockwise* or *clockwise*, or just *left* and *right*.

If is clear that the rules "always go left" "always go right" produced a pretty hexagon, but then starts putting the triangles right where one already is.

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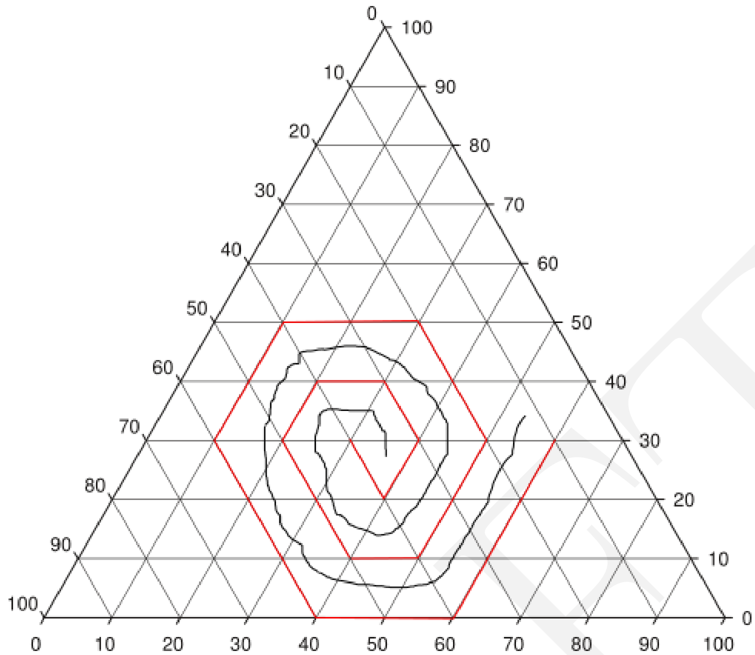

Figure 1: A Space Filling Spiral

The rule "go left if n is even, right if it is odd" is a bit more interesting; it produces a "ladder" that goes off in a straight line. That is, the whole figure is contained with in two parallel lines separated by the height of the triangle.

With the help of David Jeschke, we have written a website`https://pubinv.github.io/Mathathon-2018-Simplex-Chains/platforms/index.html` designed to allow graphic and interactive play of these concepts. Specifically it should assist in solving this exercise:

Problem 1 (TILE-THE-PLANE) *There is probably a simplex chain rule which tiles the entire plane with equilateral triangles. Can you define it?*

This website will likely evolve faster than this document!

2.1 The nature of this work

The problems posed here to make the Public Invention Mathathon interesting exist at the intersection of mathematics and computer science. The work is motivated by mechanical engineering and robotics. In particular, there is an entire class of robots, called the *tetrobot*, built entirely out of simplices which can change their member lengths. A few dozen academic papers have been published on this subject in the last 25 years[1, 2, 3, 4].

2.2 The mechanically interesting shapes

Although simplex chains are interesting in their own right purely theoretically and for recreation, they are of practical applied value as well. Mechanical and structural engineers are interested in simplex chains because when constructed out of real, physical members in the shape of the simplex chain, they may form rigid and structural strong shapes. For example, many trusses, used to hold up bridges and the roofs of buildings, are in fact simplex chains.

In two dimensions, there are a number of shapes which are interesting not just to mathematicians, but to engineers and architects. Often, these shapes are made of *trusses* which approximate geometric forms. In two dimensions the mechanically interesting shapes are:

- The line
- The triangle
- The circle
- The spiral
- The sine wave

The line is used to make columns and beams. The triangle is the basis of all trusses. Fractions of circles are arcs used to make bridges. Spirals are used to make clock springs. The sine wave is similar to the barrel vault and is used to make corrugated roofs and cardboard and baffles. Note that the triangle and the circle are closed shapes, the others are not.

General structures in three dimensions made by structural and mechanical engineers are called *space frames*. However, many space frames also follow geometric patterns. In three dimensions the mechanically interesting shapes are:

- The beam
- The triangle
- The tetrahedron
- The ring
- The helix
- The spiral

The beam, the triangle, the ring and the spiral can exist in a narrow space, like inside a wall, so they are similar to their 2-d analogs, and used for the same purposes.

The helix is used to make staircases and springs and heat exchangers.

Note that the ring and the triangle are a closed shape, the others are not. The helix does not really exist in two space, although the others do.

The only four-dimensional shape I can conceptualize is that of the beam: that is, a region close to a line embedded in 4-space.

3 On "Open Problems"

This mathathon is focused on real *unsolved open problems.* Usually when we say "open problem" in mathematics we mean "unsolved, probably has a clear objective solution, is valuable, and is not obvious." Open problems you hear about are mostly ones that have been studied a lot and remain unsolved, and are therefore probably difficult problems. However, in this mathathon, many of the open problems are probably easy—they are open and unsolved because nobody has yet cared to look very closely at them. However, they are still valuable. Hopefully the problems in this research-a-thon are "real", even if they not as "hard" as those professional researchers talk about.

We will give opinions about problems on a scale[1] of $1-10$, with 1 being quite easy and 10 being very difficult. Problems of difficulty $1, 2, 3$ are generally "easy", those of "8,9,10" hard. A problem of difficulty 10 is assumed to be publishable in a not-very-prestigious forum, though possibly easier problems will also be of interest.

4 Some Easy Problems on Generators of Regular 2D Simplex Chains

4.1 Generators

Define a *regular simplex chain generator* to be a function takes natural numbers and returns a symbol from the set $\mathbb{N} \mapsto \{L, R, S\}$, for *left*, *right*, and *stop.* The generator generates a simplex chain by defining to which edge a new triangle should be attached. That is, measured from the direction of a previous edge, should nth addition of a triangle be made by adding it to the left or right edge?

Here is an example of a generator in JavaScript:

```
function hex_generator(n) {
    return ((n < 6) ? "L" : "S");
}
```

[1] This scale is inspired by Prof. Norman J Wildberger, who has ranked famous open problem using a similar (but much more difficult!) gamut.

It produces a simple hexagon. We will assume that regular triangles have unit side length.

Although a 2D simplex chain exists in $\mathbb{R}^2$, a generator is a much simpler object. It is somewhat easier to ask questions about generators.

Problem 2 (GEN-BEAM) *(Easy 1) Can you create a generator which creates a beam?*

The following *emergent problems* were discovered during the mathathon in an attempt to solve $TILE-THE-PLANE$. They may be useful as decompositions of that problem.

Emergent Problem 1 (SIZE-N-HEXAGON) *Define a generator that makes triangle whose "hole" has n edges, starting at zero.*

Emergent Problem 2 (SIZE-N-RECURRENCE) *Create a recurrence relation that counts the number of trinagles in the SIZE-N-HEXAGON problem.*

Emergent Problem 3 (HEXAGON-COMBINATION) *Define a generator that uses SIZE-N-HEXAGON and SIZE-N-RECURRENCE to draw a completely filled hexagon of any size.*

Emergent Problem 4 (TILE-WITH-HEXES) *Define a generator that uses HEXAGON-COMBINATION to solve TILE-THE-PLANE.*

Problem 3 (GEN-PLANE) *(Easy 2) Can you create a generator which fills the plane?*

Note: at 10:00 am Central, GEN-PLANE was solved by Nathan, Sanchi, Neil, and Rob, based on rings of hexagons of increasing size.

```
(n) => {
  if (n > 10000) return "S";
  var k = n + 1;
  var h = Math.floor(Math.sqrt(k/6));
  var j = k - h * h * 6;
  if (j == 0) return "R";
  else if (j == 1) return "L";
  return (j % (2 * h + 1) % 2 == 0) ? "L" : "R";
}
```

Emergent Problem 5 (NO-CORNERS) *Prove that no 2D figure created by a generator can have a sharp point (that is, an exteranl 60-degree edge.*

Problem 4 (GEN-CIRCLE) *(Easy 3) Can you create a generator which completely covers a circle or radius r with no unnecessary triangles? In discussions with Prannoy, a mathathon participant, this problem was clarified: Suppose you took a compass and drew a razor thin circle of radius r. This circle would intersect some of the regular triangles in a grid, and not others. Is there a generator which creates only the intersecting triangles and no others.*

This discussion led me to think of the problem emergent problem

Emergent Problem 6 (PARAMETRIC-CURVE) *Supoose we have a parametric curve in 2d that gives coordinates as a function of t $(x, y)(t)$. Now if that curves intersects a triangle, we can ask: Which edge does it exit onto? Presumably we could simply choose that edge in our generator, and have a generator that can follow any parametric curve. Paradoxically, this might be easier, in a way, than some of the specific problems.*

Note: Saturday Evening at 11:23 Central, David Jeschke produce a 2D soltuion to Emergent Problem 6, thereby producing nearly perfect solutions to some of the problems below.

Problem 5 (GEN-SPIRAL) *(Medium 4) Can you create a generator which creates golden spiral?*

Note: On Sunday night at 4:30 AM CST, Sanchi produced a soltuion to a spiral which is a different form of spiral, which counts as a full solution. The value $G = 2$ creates gap of width "one triangle", the values $G = 3$ creates a gap width of "two triangles".

```
# hexagonal spiral
(n) => { if (n > 94) return "S";
const G = 3;
var k = n + 1;
var h = Math.floor(Math.sqrt(k/G));
var j = k - h * h * G;
if (j == 0) return "R";
else if (j == 1) return "L";
return (j % (2 * h + 1) % 2 == 0) ? "L" : "R"; }
```

Note: Setting G=4 creates a set of non-continuous gaps, which remind one of the vascular structure of a plant stem. In theory, one could build a "drinking straw" in this way that would have think vacuoles.

Problem 6 (GEN-SINWAVE) *(Medium 4) Can you create a generator which completely and minimal covers a sinusoidal wave?*

Problem 7 (GEN-LOGSPIRAL) *(Medium 5) Can you create a generator which defines a logarithmic spiral which does not rely on mapping first to the Cartesian plane? (variations are other forms of spirals.)*

Problem 7 raises an interesting issue. It is of course possible to create an algorithm which solves a problem in Cartesian space, and then map this back to the turnings of the generator. However, it would be more elegant to create an algorithm which never in fact uses real numbers.

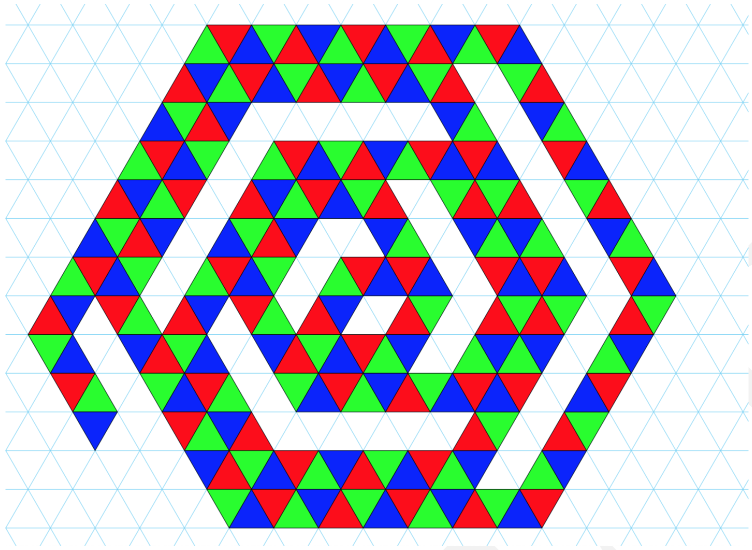

Figure 2: A Spiral with Closed Gaps

Problem 8 (GEN-ARBITRARY) *(Medium 4 R2D-10) What is the best way to define "approximating" an arbitrary curve with a simplex chain? Is it the sum of the distances of the centroid to the curve? Or some notion based on completely covering the curve and not self-colliding?*

Finally, here is a problem of some practical value to mechanical engineers:

Problem 9 (GEN-BRESENHAM) *(Medium 6) Can you create a generator draws a figure from the origin to any particular triangle as smoothly as possible as contained between the closest parallel lines as possible? (This is analogous to Bresenham's* `https://en.wikipedia.org/wiki/Bresenham%27s_line_algorithm` *line drawing algorithm with connected triangles pixels.)*

Problem 10 (GEN-PERIODICITY) *(Hard 7 R2D-08) Can we define a notional of "translational periodicity" of a regular simplex chain? That is, can we prove that a given rule for producing a simplex chain produces of series of translations of a smaller simplex chain? For example, if we have a simplex chain which approximates a sine wave or sawtooth wave, can prove that with a periodicity of k it is self-similar?*

4.2 Two dimensions

4.3 Numbering of the Regular Triangle Space

Any Regular 2D Simplex Chain, although it may be thought of as points in $\mathbb{R}^2$, may also be thought of as simply selecting a set of triangles in an abstract space, which we call the

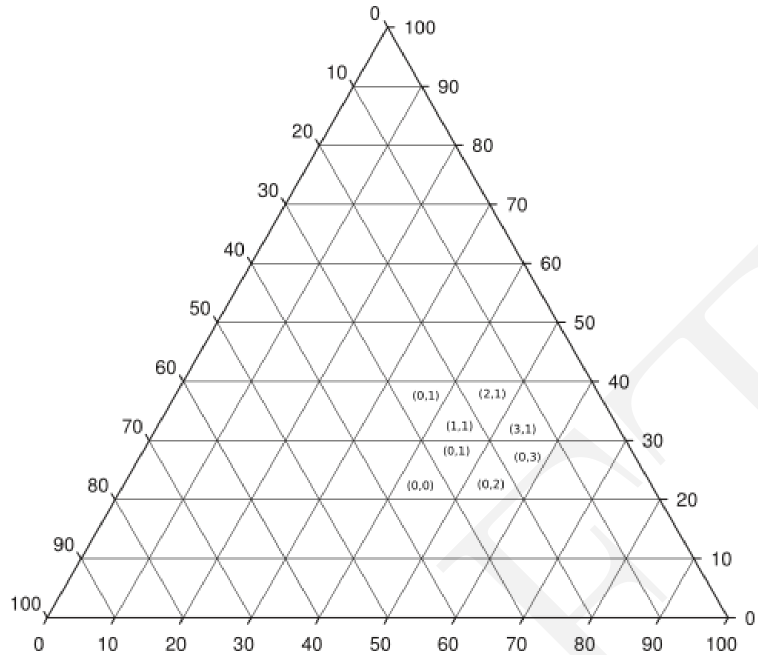

Figure 3: Triangle Space Numbering

Triangle Space, which is $\mathbb{Z}^2$. By convention, we begin numbering the Triangle Space at a equilateral triangle point upward, whose apex is at the origin. Then any triangle can be named by (x, y), where $y = 0$ on is the horizontal row of triangles below the x-axis. A triangle at position x has an apex pointing either upward or downward at position $x/2$. [2]

4.4 On the reachability of points

Although too simple for a professional mathematician in two dimensions, let us consider the problem of *reaching* certain regions or points with a simplex chain from a starting point.

Let us for the time being consider *regular* simplex chains. In the Cartesian plane, it is pretty clear that any point can *reached*, that is, intersected, by a simplex chain of finite length. Developing an algorithm to produce such a chain from the coordinates of a point is an easy problem. We could then pose questions about the shortest such chain, or the number of such chains.

Now consider the problem of reaching a point in 3D with a simplex chain formed only of regular tetrahedra.

[2] Amit proposes a different numbering scheme for regular triangles: `http://www-cs-students.stanford.edu/~amitp/game-programming/grids/`)

Problem 11 (REGULAR REACHABILITY) *(Hard 9) Given a a point in 3-space, produce a regular simplex chain of tetrahedra starting at the origin. Give the shortest simplex chain which intersects the point.*

This is a much harder problem.

It seems likely that any point can be intersected, except perhaps points very close to the starting point. Producing an algorithm to do so is more of a challenge. This is a very interesting problem to mechanical engineers, robotocists, and possibly molecular scientists.

Now if instead of merely *intersecting* a point with the body of simplex we instead seek to move the *apex* of a tetrahedra close to the point, we have a real challenge.

Problem 12 (SHORTEST REGULAR WITH d) *(Hard 9) Given a point in 3-space and a distance d, produce a shortest regular simplex chain starting at the origin whose apex is within d of the point.*

Problem 13 (ARBITRARY APPROACH) *(Hard 9) Can we come arbitrarily close to a point with arbitrary length chains?*

4.5 A diversion

An interesting, if somewhat wild idea, is to build a coordinate system around points based on reaching them with simplex chains. In this model we do not start with the Cartesian plane; rather we "name" points by the simplex chain we reaches them. What is the nature of a space of such points?

5 Relaxations of Regularity

5.1 Boundedness

If we relax the rule that all the lengths of simplex are exactly the same, then it becomes possible to create new figures which are very close to regular. In particular we can define a simplex chain, or any structure, be *x-bounded* if the ratio of the longest edge in the figure to the shortest edge is x.

We can then ask questions (in 3D) such as:

Problem 14 (SMALLEST-TORUS) *(Hard 9) What is the smallest number of tetrahedra needed to define an $b + c/b$ torus as a function of b and c?*

Or:

Problem 15 (TORUS-BOUNDEDNESS) *(Hard 9) What is the lowest x-boundedness of a figure that defines a torus?*

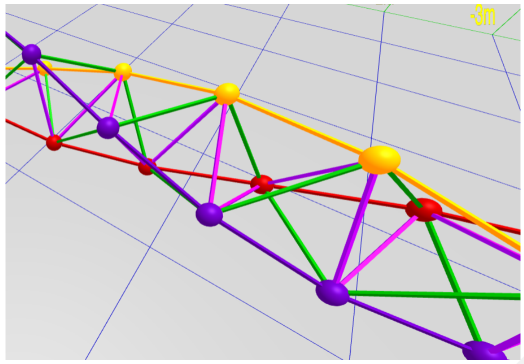

Figure 4: BC Helix Close-up (partly along axis)

A bound on this number can be given based on my previous work; however, there is almost certainly a better solution that "twists" about the torus.

One way to investigate this is simply to place points on the surface of a torus and draw edges between them so as to create an irregular tetrahelix.

5.2 Rational Regularity

Define a *k-regular tetrahelix* to be tetrahelix have at most k distinct edge lengths. Define a *k-regular rational tetrahelix* to be a k-regular tetrahelix in which all lengths are rational.

We can then ask:

Problem 16 (SMALLEST-HAS-HOLE) *What is the smallest closed k-regular/k-regular rational tetrahelix with a hole in each of these dimensions?*

Note: There exists a 3-regular rational tetrahelix that is a straight line, or a beam.

6 Three dimensions

The problem of simplex chains in three dimensions is much more interesting. In particular, there is a structure known as the tetrahelix which is analogous to a 2D ladder: it fits inside a cylinder. That is, it is a "straight", uncurved stack of tetrahedra extending infinitely.

In order to produce a tetrahelix as a simplex chain, we must have a labeling scheme for the faces of a tetrahedron. Although symmetric, the first tetrahedron has 4 faces against which we can place the second tetrahelix. Having placed the second tetrahelix, there are 3 faces against which we can place the third, if we seek not to collide with the first. These 3 are symmetric; there is no good way to distinguish them, so in a sense it doesn't matter where we place the third.

But when we come to place the fourth, the figure changes. The three tetrahelices have a clear “spine”. If we place the fourth on the face that does not touch the spine, we start to create a shape somewhat like a spindle or a top. In fact if we carry on this way we can place 6 such tetrahedral, with a 10% gap remaining. If we carry on, we just keep going around, creating a solid of revolution.

6.1 Numbering a tetrahedral chain

If we imagine a chain generator for tetrahedral chains, the fundamental decision at step is which face the next node should be attached to in order to form the next tetrahedra. This is analogous to our generators for triangle chains, which only had to make a decision {left, right, stop}, but now we must decide which face to attach to.

In order to do this we need a way to “name” or “number” faces. I propose the following scheme for (unbranched) tetrahedral chains.

First, we will number the nodes in the order in which they are added. The the starting tetreadron has nodes $0, 1, 2, 3$, and the next node will definitely attach to node 3 (the highest-numbered node). The question is, to which of the other nodes should it attach, and how should we name it?

I propsoe that we name the faces connected to a vertex numberd N high, medium, and low, as per:

1. *high* when the face has vertices $N, N-1, N-2$,
2. *medium* when the face has vertices $N, N-1, N-3$,
3. *low* when the face has vertices $N, N-2, N-3$,

In this way a *generator* could be a function from the naturals to the set H, M, L, S (high, medium, low, stop). If this is to be a generator for irregular tetrahedron, it is a function from the naturals to the 4-tuple,$[f, l_0, l_1, l_2]$, where the l_x is a positive length. In another style, $\mathbb{N} \mapsto [\{H, M, L, S\}, \mathbb{R}, \mathbb{R}, \mathbb{R}])$. The lengths l_0 goes to the lowest numbered node, l_1 to the middle-numbered node, and l_2 to the highest numbered node (which will always be N).

Problem 17 (IMPROVE 3D RULE GENERATOR) *The author has given an informal description of 3D Generator rules, can you make these definitions crisp?*

But what if we place the fourth to the left or right of the spine, and adopt this as a rule? Then we create clockwise or counterclockwise tetrahelix, respectively, and the we can go on forever, always staying inside a cylinder. It is curious that the rotation about the center of the cylinder is an irrational fraction of circle, so we will never come back to precisely the same point.

Emergent Problem 7 (PROVE HELICITY) *Prove or disprove that* every *periodic 3D Generator generates a figure contained within a cylinder of unbounded length but bounded diameter.*

Note: Every example that we have tested exhibits this property. We believe it is a property of any repeated structure, not related to simplices. However, we do not yet know the name of this theorem or principle. We conjecture that every stack of repeated truncated prisms forms a helical aperigon, which is hinted at but not stated in the Wikipedia article`https://en.wikipedia.org/wiki/Skew_apeirogon`.

These problems were discovered during the Mathathon:

Emergent Problem 8 (PARAMETRIC-CURVE-3D) *(Medium 6) Supoose we have a parametric curve in 3d that gives coordinates as a function of t $(x, y, z)(t)$. Now if that curves intersects a tetrahedron, we can ask: Which face does it exit onto? Presumably we could simply choose that face in our generator, and have a generator that can follow any parametric curve. Paradoxically, this might be easier, in a way, than some of the specific problems. Note that some curves might force self-intersection, but it might possible in this way to solve some problems with well-behaved parametric curves.*

During the Mathathon, Nathan produced a "regular beam-like conglomeration of tetrahedra" as a generator. This led me define the following emergent problem

Emergent Problem 9 (GENERATE-TETRAHELIX) *(Medium 4) Now that we have a 3D generator page, is there a simple generator that defines the famous Boerdijk-Coxeter Tetrahelix?*

Saturday mornging Nathan solved this problem. The solution is just to use either face number 0 or face 2 as a constant.

Problem 18 (IS THERE A CLOSED REGULAR FIGURE) *(Hard 9) Can we define ANY rule using only regular tetrahedra that forms a closed loop with no gaps? If not, can we prove that we cannot?*

Note: A good approximation to a torus was found in a 25 tet simplex chain:

```
(i) => { const K = 4;
 return i<25 ?
(((i % K) == K-1)
? 1 : 2 ):
 -1; }
```

Problem 19 (MAXIMUM FRACTION OF VOLUME) *(Hard 7) What is the maximum fraction of the total volume of 3-Space can we fill with regular tetrahedal chains?*

Problem 20 (RECREATIONAL RULES) *(Medium 6 R3D-01) Can we produce more interesting shapes by adopting more complex rules?*

Note: This was partially solved my Nathan by the following extraordinary generator:

```
(i) => { (i) => { const K = 22;);
return i<500 ? ((
(i % K) == -1) ||  ((i % K) == K-2)
 ? 1 : 2 ):
-1; }
```

which self-intersects, but only a little, and produces a nearly “closed” squarish cylinder. That is, it produces a slightly twisting large square cavity with no large gaps in the walls of the “container”.

An even tighter work is:

```
(i) => { const K = 11;
return i<500 ? ((
(i % K) == 7) ||  ((i % K) == 8)
 ? 1 : 2 ):
-1; }
```

To be able to discuss the generation of irregular chains of tetrahedra, we would have to amend our generator rules to include a specification of three new lengths with the addition of each new tetrahedron.

Problem 21 (FILL SPACE WITH IRREGULAR TETRAHEDRA) *(Hard 9) Produce a generator rule for irregular tetrahedra that fills all space.*

Note: Nathan suggests that we can fill a slab using our plane tiling algorithm and irregular tetrahedra with equilateral faces in the planes.

Problem 22 (IRREGULAR APEX) *(Hard 9) If we forbid self-collisions, can we define an algorithm for irregular tetrahedral simplex chains that places the apex of a simplex chain on any point? If not, can we characterize the unreachable space?*

Problem 23 (RULE FOR k-BOUNDED INSIDE TORUS) *(Hard 7) Can we define a rule that fits a k-bounded tetrahelices optimally inside a torus (as a function of the innder and out diameter of the torus)?*

Problem 24 (RULE FOR k-BOUNDED INSIDE HELIX)

(Hard 7) Can we define a rule that createsa k-bounded tetrahelix that follows a helix (that is, that stays close to a helix, as if we took a cylinder and bent it into a helical shape, like a thick wire of steel coiled into a spring? Ideally, this would be a simple algebraic expression. If no such expression is possible, show why.

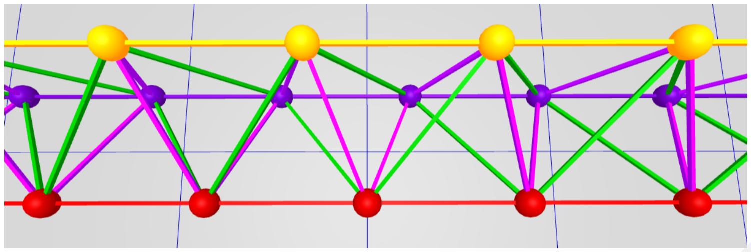

Figure 5: Equitetrabeam, an irregular tetrahelix

Note: We use the term "equitetrabeam" to refer to an irregular tetrahelix that has uncurved "rails", as in the figure belows:

In order to investigate irregular simplex chains, a software platform for tetrahedral chains similar to the 2D website we have build would be extremely useful.

Problem 25 (SOFTWARE FOR REGULAR/IRREGULAR TETRAHEDRAL CHAINS) *(Hard 8) Can we create a browser-based software toolkit that allows us to with regular or irregular tetrahedral simplex chains.*

Note: By Saturday, this problem was largely solved by Rob, Nathan and David, in that they build a generator for Regular 3D stuff. This allowed us to define a number of emergent problems:

Emergent Problem 10 (PERIODIC-REGULAR-GENERATORS) *In playing with periodic regular tenerators, we found toriodal (self-intersecting) shapes and shapes that were linear (with twists like the Boerdijk-Coxeter helix. When does a periodic generator produce linear (fitting inside a cylinder structures)? When does a periodic generator produce ring-like stuctures?*

Emergent Problem 11 (NON-CYLINDRICAL EXISTENCE) *Is there a simple generator for regular tetrahedral changes which generates a shape which cannot be containe within any cylinder? That is, is unbounded in more than one dimension.*

- (Medium 6 I3D-07) What change length is required to make a finger ring?
- (Hard 8 I3D-08) Clearly Equitetrabeams are stackable, in fact you can make a hexagon of them. Can we make a twisted tetrabeam with 3 or 6 columns that is even more efficient (in terms of the ratios of edge lengths)?
- (Medium 7 I3D-09) What is the minimum change length to make a torus?
- (Medium 6 I3D-10) What is the minimum change length of a tetrobot that can touch its toe?

- (Hard 8 I3D-11) What is the densest way to pack a tetrobot into a given volume? (Bounds acceptable)
- (Hard 8 R3D-12) Can we develop math for "regular variations" (no kinks)?
- (Hard 8 R3D-13) Can we develop math for "one kink" regular variations?
- (Hard 10 I3D-14) Can we make a torus that turns itself inside out via continuous changes of edge lengths, creating circular motion of a point around cross-section of torus? (In a sense, this would be a nano-technology motor.)[3]
- (Hard 8 I3D-15) Is there a formula for producing a helix of tetrahelices?
- (Hard 8 I3D-16) Is there a way to "knit" Tetrahelices into a slab?[4]
- (Hard 8 I3D-17)Is there a way to "wind" tetrahelices into column?
- (Hard 8 I3D-03) How fast can the tip of a tetrahelix move given a rate of change of member length? Can we define a this derivative with a closed form expression, rather than using matrix-based arbitrary Jacobians?
-

Define *point reachability* to mean place the apex of a simplex chain near a point.

- (Medium 7 R2D-03) In 2D, give an algorithm that produces the shortest regular simplex chain coming as close as possible to a point.
- (Hard 7 R3D-22) Given a limit to the number of simplices, what is the reachable space to with distance x?
- (Hard 9 I3D-23) What is the shape of reachable points for N simplices?
- (Hard 9 I3D-24) In 3D, given an algorithm that produces the shortest simplex chain that comes with x of a target point.

7 Hunting techniques

In hunting for these open problems, it would be nice to define a toolkit of techniques that we can use.

The 2D website that we have made for testing 2d-regular generators is a valuable tool. It would be nice to have such a website that supports irregular triangles as well.

[3]I might attack this problem by attempting to disprove it, and then writing code to make it happen if I could not disprove it/

[4]This is poorly defined.

Although more challenging, we could also produce software to test 3D simplex chains. If someone would do this as early in the Mathathon as possible, this would obviouosly be extremely valuable.

The author is familiar with computer science techniques. Mentioning mathematical techniques useful in addressing these problems would be very appreciated.

8 G-Brain's relation to other mathematics

In comments in a Reddit post, `https://www.reddit.com/r/math/comments/9l4qmw/a_mathathon_a_cooperative_free_virtual_hackathon/`, G-Brain point out this a simplex chain can be related to typical mathematical language:

> I would say a simplex chain is a (countable) pure simplicial d-complex in R^d such that its facet graph (vertices are facets, edges are between facets which meet in a codimension 1 face) admits a Hamiltonian path (visits each vertex exactly once, without repetition of vertices or edges).

We have not emphasized this language because it seems more user-friendly to approach some of the problems from the position of Computer Science than from pure mathematics.

9 Rules of the Mathathon

9.1 Rules

The intention of the Mathathon is to perform all the work in the light and to encourage math as a social activity. It is our hope that all members will add their written work to a GitHub repo as quickly as they can—for example, every 30 minutes or more often would not be unreasonable. To be eligible for an award on the basis of content, all content must be published in a GitHub repo whose entire contents are licensed under Create Commons By Attribution(0), or the participant must permit their work to be so published in writing. These repos must remain publicly accessible for one month after the date of the hackathon.

It is preferable to name one file in each repo "FinalSubmission.pdf", with the expectation that this will be the first file read by the judges, and that this file may be included in proceedings, if the organizers choose to create a proceedings publication.

If you don't know or can't learn how to use GitHub, you can simply email a submission, either as an image of hand-writing, as a PDF, or a plain text file or a Word file or something.

The deadline for email submissions may be early than the deadline for GitHub repo-based submissions.

9.2 Awards

Awards at present do not have monetary prizes. If a sponsor provides monetary support, my current intention is to split the award money evenly between each category below. Participants may receive more than one award.

If sufficient people volunteer, judging will be by a panel of judges.

Awards will be given for:

- Crowd favorite (to be determined by gold stars or emails)
- Best contribution by someone without a post-baccalaureate degree
- Best contribution by someone without a college degree
- Best contribution by someone without a High School degree
- Best contribution by a Senior Citizen (at least the age of 65)
- Most creative technique
- Best contribution by someone not a US Citizen
- Most worthy of publication
- Second most worthy of publication
- Best contribution by a child under 16 (may be accomplished in conjunction with a coach, so long as the child is fully engaged)
- Best contribution of a new problem statements
- Most helpful to other participants (not on same team), to be awarded based on written testimonials
- Second most helpful to other participants (not on same team), to be awarded based on written testimonials
- Most useful open repo during the mathathon
- Best mathematical presentation and writing
- Best code contribution
- Best contribution of graphic art, figure, or diagram

Awards will be announced after the hackathon, at a prompt but unspecified time, in the form of an awards file in the GitHub file. Judging will be performed by a committee of the key personnel.

10 Key Personnel and Contributors

- Robert L. Read, PhD ¡read.robert@gmail.com¿
- David Jeschke
- Stephanie Liu
- Sanchi Vaishnavi email: ¡vaishnavi.sanchi@gmail.com¿

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Is there a rule that produces “straight” 4D simplex chains?

If we imagine moving into a four dimensional space, does it make sense to imagine a simplex chain, even though this will have not solid physical reality as we know it.

The “face” of such a figure is in face a 3-dimensional shape, can “adjoining” means that two 4D simplices have faces in the same 3D volume.

Possibly we could write a computer program that would allow us to render projections of such figures. I cannot imagine such shapes, and in a way they become less interesting and less practical than 3D simplex chains. However, one question seems interesting:

Is there a rule that produces a simplex chain in 4D which would produce a “straight” figure? That is, a figure which always remained within in a fixed distance of a line embedded in 4-space?

Idea: The problem doesn’t limit the computational complexity of the chain rule producing the simplex. However, if we evaluate the simplest rules first, we could write a computer program to simply try a large number of them. A “simple” rule could be considered a function only of the number “n” modulo some small number “m”.

If you consider only simple rules, how does a 4D simplex chain behave? Does it spiral out of control, or remain within a finite region of space?

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