A Novel Passive Ferrofluid One-way (Check) Valve Based on Energy Minimization

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ABSTRACT

This work focuses on a novel passive ferrofluid check valve. The structure is free of mechanical moving parts and is based on a ferrofluid plug in a narrow channel. Check valves, also known as one-way valves, are devices that allow fluids to move in only one direction primarily to prevent backflow in the system. The valve returns to stasis (closes) under the influence of a static magnetic field when the pressure on the ferrofluid is less than the magnetic forces of the ferrofluid itself. Here, we present a simple design for a passive, normally closed ferrofluid check valve consisting of a unique channel-and-chamber geometry, a bolus of ferrofluid, and a static magnetic field. The flow is determined only by the force differential between the pressure of the fluid in the channel and the magnetic force of the ferrofluid at the intersection of the channel and the chamber

Small pumps and valves enable flow management in microfluidic systems. A novel passive ferrofluid check valve is presented. The valve consists of only a unique channel-and-chamber geometry, ferrofluid, and a stationary magnetic field. The flow is determined only by the inlet and output pressure, and the magnetic field is completely static. The prototype valve and experimental setup are explained and performance of the valves cracking and collapse pressure reported. This initial design can be used for microfluid handling and lab-on-a-chip applications.

Additionaly we present a theory of operation based on energy minimization and compare predicted performance to actual performance.

1 To Do

Fully understand the meaning of the Kelvin Force - This should probably be called a magnetophoretic force, or an inverse-magnetophoretic force. It should probably not be called the Kelvin force of the magnetic buoyancy. Perhaps we should call it energy minimization buoyancy force.

Improve Proof of 2D Manetostatic Blob Minimimization Conjecture

Understand Ferrofluid Spikes (a version of Rayleigh-Taylor or (normal-field instability) instability)

Prove that theorem that a U-tube cannot make an assymetric valve (yes it is called a U-tube) U-tubes obviously make pressure gagues, and this is not dependent on the relative areas, though the scale is.

Centrifugal force is $F = m\omega^2\rho$, where ρ is the distance from the center of the rotating frame of reference. So we can make a perfectly linear force in that way.

Get magnetohydrodynamics by Rosenweig - 250 $: https : //www.amazon.com/Ferrohydrodynamics - Dover - Books - Physics - Rosensweig/dp/0486678342ThereisaKindleEditionfor20$, I should probably buy.

ABSOLUTE MUST: Formulate and prove the perpendicular ferrofluid boundary theorem.

1.1 Writing ToDo

This may be a useful reference: `https://hal.science/hal-03416758/document`

This is a pump that uses a plug and two check valves: `https://www.researchgate.net/publication/37423698_A_ferrofluid_micropump_for_lab-on-a-chip_applications/figures?lo=1`

Write a new section on U-tube theory. Should I use a pen?

2 Outline

I. Introduction

II. Theory of magnetostatic blobs and constant and non-constant fields

  A. Cracking / Collapse
  B. Tall U-pipe on a planet (or a centrifuge)
  C. Two chambers with ferrofluid in a magnetic field

III. 2D Valve in a chamber

  A. Veronica's result
  B. Informal Theory
  C. Math
  D. Performance of 2D valves

IV. 3D case

  A. Math
  B. 3D valve perfomance

V. Scaling

  A. Math

VI. Conclusion and Future Work

  A. Scale down
  B. Improve performance
  C. Cooling, pumping

3 Introduction

Ferrofluid can be manipulated by electronically controlled fields to exert force on fluids[1, 2, 3]. This makes it possible to build pneumatic or hydraulic devices, perhaps on very small scales, such as a single chip[4, 5], to miniaturize fluid handling. This has been proposed for biomedical purposes[6] that would use water or body fluids, although this paper reports only on experiments done with air. Miniature pumps and valves could be used to make a "lab on a chip" (LOC) or even to heat or cool different chip areas.

A fundamental component of such devices is the *check* or one-way valve. Two check valves on either side of a chamber whose volume can vary creates a positive displacement pump. A perfect check valve opens or *cracks* with minimal pressure on the inlet side and sustains maximal pressure on the outlet side before *collapse*, allowing fluid to flow in only one direction. Following[7] we call the maximum pressure differential the valve can resist in the direction it is intended to check or block (from outlet to inlet) the *sustainable* or *collapse* pressure.

This article is a brief report on an initial but functioning design of a passive ferrofluid check valve (PFCV) that has no moving parts except for the ferrofluid bolus itself, which is stationary in normal operation. By passive, the authors mean a check valve that functions without changes to the magnetic field affecting the bolus, whether that field is induced by a permanent magne t or an electromagnet. That is, the flow is determined purely by the difference between the inlet port pressure and the outlet port pressure. To our knowledge, no passive ferrofluid check valve has been previously reported, despite being an active area of research and despite such a valve having significant advantages for operation and especially fabrication over valves with moving parts.

A number of papers report on ferrofluid pumps, focusing in particular on micropump and lab-on-a-chip applications[3, 8]. Many of these papers use a version of mechanical valve not based on passive ferrofluid, even though they move a ferrofluid bolus with a magnetic field. For example, a corrugated silicone micro valve[4, 9] has been reported. Other researchers use active valves, which require synchronization with the ferrofluid plug to form a pump, such as [10], which describes an active *T-Valve* with a moving ferrofluid plug, and [11] describes a complete fluid pump with valves that use active control of a ferrofluid bolus. At least two additional kinds of active valves, a *well valve* and *Y-valve*, have been described[7]. Active control is possible because the action of the plunger or bolus may be synchronized with the opening and closing of the valves. Nonetheless a passive valve would be simpler and less expensive, and would not require knowledge of the timing of the plunger.

An interesting functional micropump in which the moving ferrofluid bolus merges with a fixed ferrofluid valve and then separates on each pumping cycle has been described[5], but is not a one-way valve.

A passive ferrofluid two-way valve with tunable opening and closing pressure based on magnetic field strength[12] has been tested, but could not be passively used to make a pump.

This paper has not studied the closing pressure of the PFCV, but reports on the opening (or cracking) pressure (for flow from inlet to outlet) and sustainable (or collapse) pressure when the outlet pressure is higher than the inlet side.

Trial Name	Cracking (cmH2O)	Collapse (cmH2O)	Comment
X1	8	114	
X2	8	110	
X3	8	110	
X4	12	106	2” magnet instead of 1”
X5	8	110	System held upside down in same configuration

3.1 The 3D Valve of July 2024

On July 13, 2024, Joe Hershberger and I tested a 3D valve design that produced our best performance to date, have a *P* factor of approximately 11. This valve is depicted in Fig. 1

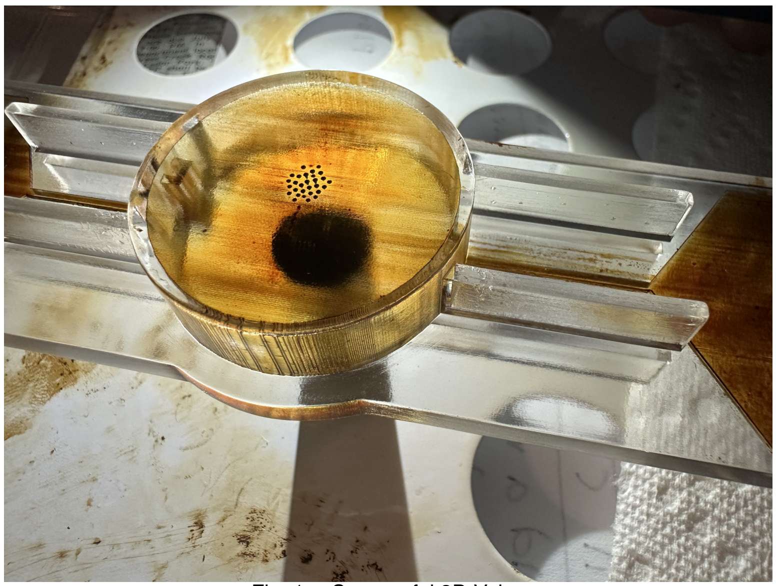

Fig. 1. Successful 3D Valve

The data produced for this valve is depicted in Table **??**

4 Passive Ferrofluid Check Valve (PFCV) Design

The PFCV depicted in Fig. 2 is a simple asymmetric volume centered in a magnetic field which holds a ferrofluid bolus in place. In the center of a radially symmetric magnetic field a narrow channel meets a larger open chamber at a right angle. The ferrofluid bolus is large enough that at rest in the field it forms a semi-circle in the open chamber. The narrow channel is longer than the radius of the bolus at rest. The broad chamber is the outlet side of the valve. The narrow chamber opens onto a recovery chamber on the inlet side of the valve. This design allows the bolus to be recovered from the recovery chamber when the pressure is equalized if the outlet pressure is raised above the collapse pressure, driving the bolus away from the magnetic field. The PFCV does not resist pressure as well as a valve of the same size made out of moving, solid parts. That

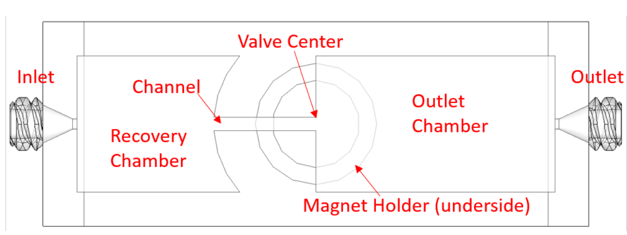

Fig. 2. The passive ferrofluid check valve components

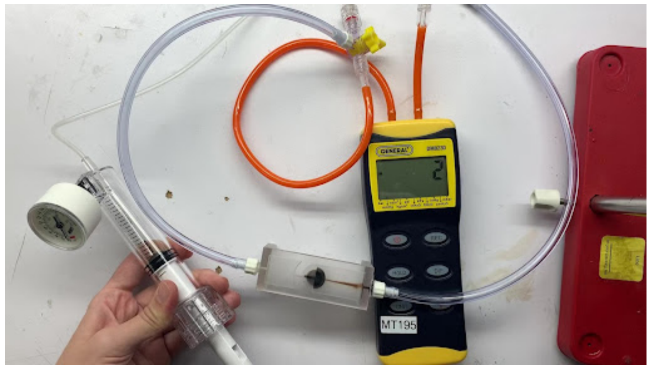

Fig. 3. Equipment set up

is, the sustainable pressure it can resist on the outlet side before failing is relatively low, and pressure required to crack it open and allow flow is relatively high. However, it may operate reliably within a range of known pressures, and thus be sufficient to build a pump-on-a-chip. Furthermore, the PFCV reported here is a preliminary design which can probably be significantly improved. The authors found the existence of the PFCV worth sharing immediately.

5 Method

The valve depicted Fig. 2 was designed using Solidworks 2016. It is freely licensed via the CERN OHL Strong Reciprocal License[13, 14]. The model consists of a 15mm long, 2mm wide channel, a large outlet chamber, a recovery chamber, two female luers, a magnet holder ring and two legs to provide room for the magnet. All volumes are 2mm high. The 3D shape of the chambers can be thought as a 2mm high extrusion of a 2D shape. Viewed from the top, one end opens up to a recovery chamber of circular profile 30mm in diameter and the other an outlet chamber with a flat wall. A magnet holder ring 12.7mm inner diameter (1/2") was created centered on the channel-chamber junction, below where the bolus is placed, to hold a permanent magnet in place at the center of the valve. When two magnets are used, the magnet on top naturally stays in the same position due to attraction to the magnet below. On the inlet side of the channel opens into a recovery chamber shaped to allow the ferrofluid to be passively drawn back into the channel by the magnetic field after a collapse of the bolus. The model was printed on a Projet MJP 2500 (3D Systems, Rock Hill, SC), using Visijet M2G-CL and VisiJet M2 SUP as material and support respectively (3D Systems, Rock Hill, SC). Support material was removed by using an EasyClean system (3D Systems, Rock Hill, SC) and Dawn dish soap (Procter & Gamble, Cincinnati, OH) to remove residuals.

As shown in Fig. 3 and in our demonstration video[15], a basixCOMPAK 30atm pressurizing syringe (Merit Medical, South Jordan, UT) is connected to the model via a two-way stopcock (Qosina, Ronkonkoma, NY), tubing (Natvar, City of Industry, CA), and male (Injectech, Fort Collins, CO) and female luer (Qosina, Ronkonkoma, NY), allowing integration of a manometer (General Tools, Secaucus, NJ) to measure pressure. A 12.7mm x 25.4mm (1/2" x 1") cylindrical neodymium magnet (Apex Magnets, Petersburg, WV) with a pull force of 14.6 kg (32.24 pounds) was placed inside the magnet channel by means of a tight fit and 0.2 mL of ferrofluid (Apex Magnets, Petersburg, WV) was injected into the model using a 3mL syringe (BH Supplies, Jackson, NJ).

To obtain values, pressure was applied through the pressurizing syringe, as demonstrated in a video [15]. Pressure was first applied from the outlet side of the model. The maximum pressure difference from the outlet side that the valve can withstand before collapsing, will be referred to as the sustainable pressure or *collapse* pressure. Pressure applied from the inlet needed to initiate flow will be referred to as the *cracking* pressure.

The cracking pressure was first measured by increasing the pressure difference on the inlet side until flow is initiated (at which point the valve is "open" and pushing the syringe plunger faster simply increases flow without increasing the pressure.) Then the collapse pressure was measured by increasing the pressure difference on the outlet side. At pressures below the sustainable pressure, the valve holds pressure well with no observable leaks of air in the short time (a few minutes) of our experiment. When the sustainable pressure is exceeded, the bolus explodes violently into the recovery chamber. When the pressure difference is equalized, the bolus may passively recover into the central position, or it may need to be actively "combed" with a magnet back into the central position.

The procedure was performed first with one magnet, named the "Single Magnet" configuration, placed below the channel-chamber connection. The "Dual Magnet" configuration was performed with the magnet in the same position as the Single Magnet case in the same position, but with a second magnet of the same kind placed vertically on top of the model, arranged to be strongly attracted to the lower magnet.

Table 1. Result pressures

Magnet configuration	Cracking Pressure kPa (mmHg)	Collapse Pressure kPa (mmHg)	Pressure Difference kPa (mmHg)	Approx. Ratio: Cracking to Collapse Pressure
Single	1.1 (8)	5.5 (41)	4.4 (33)	1:5
Dual	8.5 (64)	17.5 (131)	8.9 (67)	1:2

6 Results

The final pressures obtained demonstrate a clear difference between the inlet cracking pressure and outlet sustainable pressure, creating an effective passive check valve.

The ferrofluid had observable differences in behavior between the two configurations. After the pressure equalized following a collapse of the bolus due to exceeding the sustainable pressure, the single magnet configuration often repaired itself by drawing the fluid back into a centered bolus passively. After a collapse with two magnets, fluid further from the bolus remained stationary while the fluid closer was pulled back to the center. Following the removal of the top magnet, the stationary fluid then began to return to the bolus. This is consistent with the localization of the magnetic field between two magnets, and the weakening of the magnetic field further from the channel-chamber juncture in the dual magnet configuration.

Although the dual magnet configuration demonstrated a larger absolute pressure difference due to magnetic field strength between the cracking and the collapse pressure, the single magnet configuration granted a larger ratio of collapse pressure to cracking pressure due to the much lower cracking pressure. The authors conjecture that the low cracking pressure may have been not only to the weaker magnetic field, but the weakening at the top of the 2mm high channel, which was further away from the magnet in the single magnet configuration.

7 Theory

In constructing a theory of one-way (check) valves with no moving parts except fluid, it is perhaps worth starting with some basics, because we are unlikely to be able to make valve competitive with solid valves on their own terms.

7.1 Definitions

A check valve has an *inlet port* and *outlet port*, and is intend to allow a fluid to flow from the inlet to the outlet, and prevent a fluid from flowing from the outlet to the inlet. A *fluid chec kvalve* means one which the only moving part is a fluid, in contrast to a *sold check valve* that has, for example, a ball or a flapper which moves slightly when the valve opens. A *ferrofluid check valve* is one that uses ferrofluid as its working fluid (NOT the fluid that it valves.) As is typical of check valves, define the *cracking pressure* as the pressure at which the valve allow flow from the inlet to the outlet. A *passive check valve* is a checkvalve that opens automatically, as opposed to, for example, an electronically controlled valve that is opened by detecting the pressure in the inlet port and the outlet port (a *active checkvalve*.) This paper is about a design for an *passive ferrofluid check valve*. All valves potentially fail when the outlet port greatly exceeds the pressure on the inlet port. In a solid check valve this pressure difference may be so high that it not is stated as a performance metric of the valve. This will not be the case in the passive ferrofluid check valves discussed here. We call the pressure at which flow occurs from the outlet to the inlet as the *collapse pressure*.

The primary metric that this paper attempts to optimize is the ratio of the collapse pressure to the cracking pressure, which we call the *collapse performance* of the valve. A higher collapse performance is better.

7.2 The U-tube As A Thought Experiment

Consider as a thought experiment the venerable U-tube. When filled with water, the U-tube acts as a manometer or pressure gauge. Call the arms of the U-tube U and V. As the pressure in U becomes higher than the pressure in V, the water is pushed down in U and rises in V. The height of V above the equilibrium point in centimeters is the gage pressure of U over V expressed in $cmH2O$. In such a simple device the pressure can be literally measured as the height of a water column, allowing a slight confusion of the units of length with the units of pressure. Treated as pressure gauge, it can measure a pressure no higher than the height H of the equilibrium point of water above the bottom of the "U", since if the water in the U arm is pushed down to this point, it will continue and bubble away out the V arm, which will not cause the V arm to rise any further.

Now, let us consider the U-tube as a passive fluid check valve. It will be a miserably low-performing valve. If we consider U as the inlet point, the cracking pressure will be H. Being completely symmetric, the collapse pressure will also be H, hence the collapse performance will simple be $\frac{H}{H} = 1$. There seems to be no way practical way to build a U-tube like fluid check valve.

If we allow the fantastic idea that the U-tube reaches hundreds of time higher than the Earth's diameter above the planets surface, and that H is about the diameter of the Earth, then the gravity field will not be constant, but will be slowly decreasing with altitude. However, once again the system is completely symmmetric, so long as the area of the U arm is the same as the V arm. If, however, the area of the U arm were 100 times higher than the area of the V arm, the cracking pressure would be less than the collapse pressure. To calculate precisely how much, we would have to use calculus to integrate over the gravitational field. Qualitatively, however, it is easy to see that it would take much less pressure than H to cause the valve to crack open, since gravity would pull down harder on the water in the U arm, which would force the water in the V arm 100 times farther away from the surface of the Earth. Once the water gets very high, it is already in a weak gravitational field. The cracking pressure would be something grealy Howewver, adding pressure to V arm would cause the level of the U arm to rise only 1/100th as much, so the gravitational field would barely be affected, and the collapse pressure would be something a little less than H. By using a non-constant field, we could create a performant passive fluid check valve.

8 Other

Let us consider an ideal blob of ferrfoluid.

Assume that a blob in this section means an ideal 2-dimensional blob, approximated by a blob constrained into a very thin plane.
Assume that the B-field is purely perpendicalur to this plane, and that it completely dominates the magnetism induced in the ferrofluid itself.
Assume that all forces of viscosity and surface tension may be ignored.
Although we will not model the forces that make it so, we will assume that blob of ferrofluid is self-connected and will not split into more than one blob.
The blob is perfectly incompressible, and so has an absolutely unchanging area.
The magnetic particles are evenly distributed in the fluid no matter what the magnetic field, so that the potential energy in the fluid by being in the field depends only on the strength of the field.
We need not treat the motion, velocity, or inertia of our fluid in any way.

Under suitable assumptions, we can state two drastically simplifying assumptions.

Conjecture 1 (Magnetostatic Blob Minimization). *The magnetostaic force on an ideal 2-dimensional blob acted upon by a perpendicular magnetic field tends to minimize the potential energy in the blob and this occurs when its surfaces are in equal strength fields. Equivalently, every surface of a blob at rest with no outside forces is a line of equal field strength and all surfaces are equal.*

Proof. Suppose there existed two pixels of the blob near the boundary, such that one pixel was occupied by ferrofluid and had a lower B-field magnetism than a pixel that was not occupied by ferrofluid. The potential energy of this configuration would be higher than if the fluid moved from the occupied pixel to the unoccupied pixel. Since the fluid will minimize potential energy, all boundaries will adjust themselves to have the same field strength.

If the area is expanding the blob, it always decreases potential energy. If the area is contracting the blob, it always increasing potential energy. However, since the B-field at the two (or more) surfaces may be different, there may be a force exerted on the blob. In particular, we are interested in the force exerted on the blob counterbalancing the force exerted on by fluid pressure of non-ferrofluid at the surfaces.

Now imagine that a we have a monotone decreasing field centered on the origin which which is not necessarily circular, with the A region containg a thin tube, and the B region containing a wide open region. Imagine that the field in B is radially symmetric, but monotonically decreasing with distrance from the origin (but not necessarily the same in the region A.) A force resisting flow from the outlet to the inlet (from B to A) is equal to the field strength at a point multiplied by the surface length. In this scenario, the air pressure on the blob is a function of the length of the surface, but the magnetostatic force is also a function of length, so the length cancels out and becomes irrelevant. The collapse pressure is therefore the maximum force at any point between the rest position and the origin (this field is by construction equal on a semi-circular arc.) The collapse pressure also depends on the field strength at the point of the surface in A, but this surface has small length, and therefore contributes little to the sum of the force.

Conversely, the cracking pressure will be minimized by making the field strength in the region B as low as possible at the point when the ferrfofuid has been entirely driven into region A by air pressure from the inlet.

Conjecture 2 (Magnetostatic Bolus Minimization). *The magnetostatic force on an ideal 2-dimensional bolus acted upon by a perpendicular magnetic field is a function only of the derivative of the magnetic field intensity integrated over the changeable path.*

Proof. The potential energy of the bolus in the magnetic field is highest where the magnetic field is highest and there is no ferrofluid, and lowest when ferrofluid fills the field. Since ferrofluid is incompressible and is contrived to be in a thin plane, this can be modeled as the ferrofluid either being present (low potential energy) or not (high potential energy.) This is thus a conservative force field.

Force done by the magnetic field decreases potential energy. Force is the negative derivative of the potential energy. At an instant in time, this depends only on the boundary of the bolus. A surface patch which is not on the boundary cannot instantaneously change, and therefore cannot change the potential energy and exerts no force.

8.1 How the Valve Cracks

It is clear that a blob that has a bubble of non-ferrous fluid inside it has higher potential energy that the same blob without that bubble. Furthermore, such a bubble will feel a pressure to move away from an area of a higher magnetic strength towared an area of weaker magnetic strength. A bubble could be trapped, but if we assume that our field decreases monotonically from a central peak, a bubble will have a path to escape and will do so. In this scenario a valve will “crack” that, is, crack open allowing a fluid from the inlet to the outlet, as soon as the non-ferrous fluid crosses a central boundary.

8.2 Warmup: A Bolus in A Channel

A relatively simple problem is to consider a bolus of ferrofluid in a thin horizontal channel. Let x be the positions of the center of the bolus on the x-axis. Placing a cylindrical permanent magnet beneath it at the origin tends to lock the bolus in place, forming a plug. If a small amount of air pressure is applied to in the positive direction, the bolus will be displaced to in the positive x direction, but will find a stable point in balance with the forces exerted by the magnet in the negative x direction.

As the pressure is increased, the bolus will be displaced further in the positive x direction. It will reach a point where further displacement does not increase the force in the negative x direction, and will soon decrease it. At this point the plug will fail catastrophically, spraying messily into the channel in the x direction and letting air through.

The magnetic field in the line just above the face of the magnet can be analytically described by ellpitical integrals, but they are very difficult to work with symbolically, but easy to calculate numerically. MagPyLib is a convenient package for doing so. Figure 4 shows our setup of the problem coordinate, and Figure **??** shows the same setup in the MagPyLib simulation software. The x dimesion is the horizontal, left to right dimension. The field is polarized, which is why it is negative in the x direction, only the magnitude matters when operating on a ferrofluid bolus.

By making assumptions, we can analyze this to obtain closed form solutions which match our experience and inform more complicated cases.

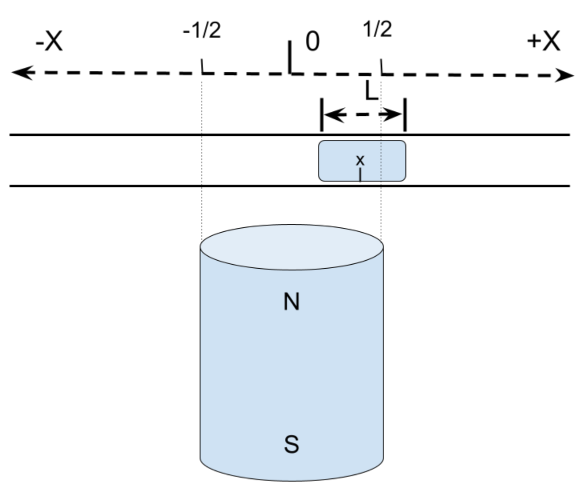

Fig. 4. Bolus Coordinate Setup

The red line represents the thin channel just slightly above the face of the magnet. MagPyLib conveniently computes the field along this line. In particular, only the x component (the East-West coordinate) of the B-field is relevant to us. The field was plotted with MagPyLib in Figure **??**.

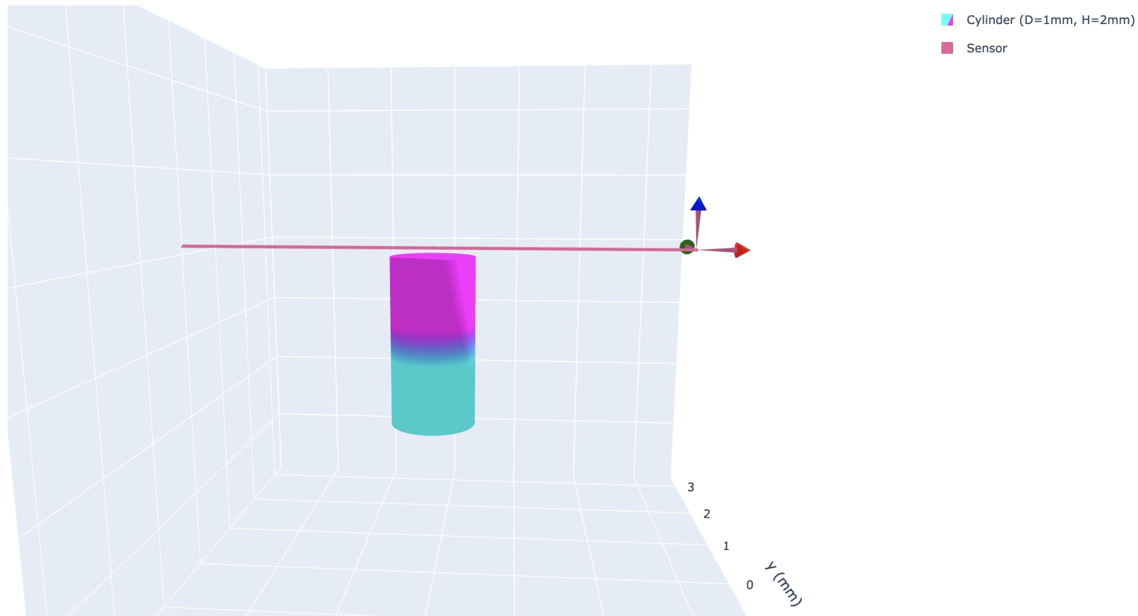

Fig. 5. Setup of the Bolus Problem in MagPyLib

8.3 Some synthetic dimensional analysis

In the work that follows, we have need to relate the ability of a B-field complete perpendicular to thin plane to exert force in the plane by virtue of the change in potential energy.

This seems to be beyond my power to define. As an approximation, we can create a fake unit, the "planar slope energy" *PSE* unit, which is a factor which coverts a change (derivative) in the perpendicular B-field intensity into Pascals (Newtons per square meter). This artificial constant folds into the nature of the ferrofluid, the geometry of the plane.

It could be experimentally determined with a manometer by placing a thin tube of known area in the planar space above a perpendicular magnetic field as depicted in the previous section. If by simulation (such as magpylib) we can know the maximum slope of the B field in the region of the thin tube, then the manometer will give us Pascals at the time of the collapse of the bolus.

We will denote this constant Ψ, in memory of Pascal.

8.4 A Thought Experiment

It is perhaps surprising that it is possible to make a passive ferrofluid check valve at all. In order to see that it is possible and to set up a mathematical framework that will be valuable later, consider the thought experiment illustrated in Fig. 6. This section could be considered an exercise, but later results will build directly on this approach.

Let x_A be the position of the interface of area A that fronts the pressures P_A. Let x_B be the (signed negative) position of the interface of the area B that faces the P_B pressure. We will treat P_A and P_B as unsigned values. Let the y axis be the across across the valve (vertical in our drawing), and let the z axis be coming out of the page toward the reader (a right-handed coordinate system.) Imagine that in the z direction the ferrofluid is sandwich between two pieces of glass so thin that it the whole system may be treated as a 2-dimensional problem.

The magnetic field intensity $B(x)$ is perpendicular to the thin plane of interest. The formula:

$$B(x) = \frac{C}{1+(\frac{x}{K})^4} \tag{1}$$

$$B'(x) = -\frac{4CK^4x^3}{(K^4+x^4)^2} \tag{2}$$

is observed by inspection with the B field computed to match the shape of the B-field computed by MagPyLib from a cylindrical magnetic in a line in the plane perpendicular to the magnet axis.

Assume that the magnetic field is strong enough to keep the bolus everywhere contiguous with itself (so that there is only one bolus), and that we can describe the configuration of the bolus only with the positions x and y (that is, the air-fluid

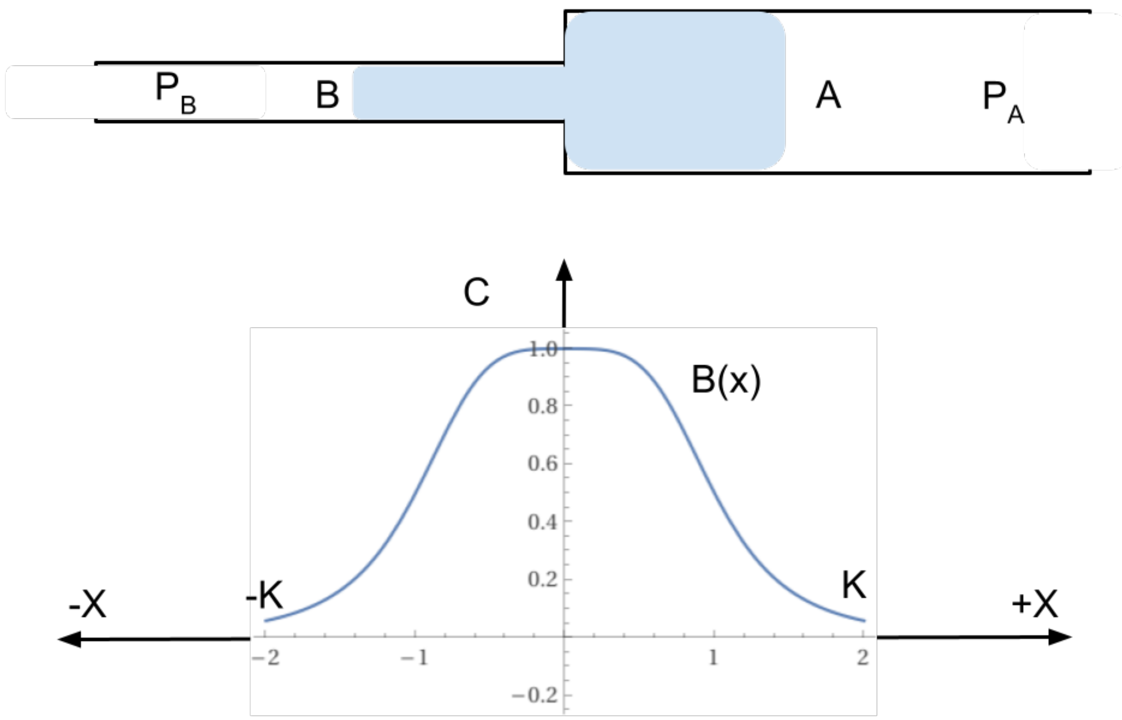

Fig. 6. Two centered on a linear B-field

interfaces in the cambers are perpendicular to the horizontal X axis) we can write the volume V as:

$$V = x_B B + x_A A \tag{3}$$
$$x_B = \frac{V - x_A A}{B} \tag{4}$$
$$x_A = \frac{V - x_B B}{A} \tag{5}$$

Thus x_B and x_A are dpendent on each other, and are in fact only one variable.

If we consider geometry qualitatively, we can imagine what will happen if the pressure is increased from the left or right, perhaps by blowing. Note that the volume of the bolus is chosen to be about a length of about K, half the length of the magnetic field in this x direction, which is $2K$. If the pressure is increased from the right, the bolus will remain intact and be forced leftward as the pressure increases. If the pressure decreases, the bolus will move back right ward untill it is centered. Potential energy in this situation depends only on the position of the bolus, so the field a conservative mechanical field.

In this model, the narrowness of the plane greatly simplifies the mechanics. In this simplified model, we can assume that the potential energy of the magnetic system can be simply modeled by the presence or absence of ferrofluid at a point in the plane. The presence of ferrofluid at a point or infinitessimal line (dx) decreases the potential energy U of the entire system in proportion to the strength of the field at that position, and that that strength is describe by our function $B(x)$ where x is the distance from the origin. We can thus describe the potential energy as an integral over surface patches da:

$$U = \int \left\{ \begin{array}{ll} B(a), & \text{if there is no ferrofluid at } a \\ 0, & \text{if there is ferrofluid at } a \end{array} \right\} da \tag{6}$$

Thus the potential energy is highest when there is no ferrofluid at all, and lowest when the entire plane is full of ferrfluid. However, we will generally treat a bolus of fixed volume and assume that our plane is thin enough that it therefore has a fixed area.

We are furthermore ignoring all effect of the ferrofluid interacting with itself in terms of the potential energy. This is an approximation; ferrofluid famously interacts with itself by producing “spikes” due to field instability. The theory we are developing is a good enough approximation to allow us to design a valve. However, we do rely on the very important property of ferrofluid that it tends to clump together into a single blob in the presence of a magentic field. Furthermore, we assume the fluid is incompressible, and therefore always has the volume V. This is true until the bolus collapses; we know that when it collapses it explodes drastically, and we do not attempt to model anything after a collapse event.

If we imagine blowing harder and harder from the right, it is clear that when the A face of the bolus reaches the origin, the bolus will explode of *collapse* to the left. This is true because by design B is shorter than A, and so the integral across that line as the same line reflected across the origin (in the A) region will be lower by B/A, and the pressure has obviously been sufficient to overcome A.

Since the absolute potential energy of our system is irrelevant, it will be easier for us to treat the absolute potential energy U as zero when there is no ferrofluid present at all and consider it to go down as ferrofluid is added. Thus a bolus at rest in a magnetic field has some potential energy which is less than zero.

Note that pushing the bolus away from its equilibrium centered on the orign takes a small amount of work. We imagine an infinite source of air at pressure, so that we need not worry about our source air pressure lessening or not being able to “do the work” of pushing the bolus away from equilibrium. However, note that doing this work causes the potential energy to increase in the formula below.

In the formuala below, we consider the region piecewise, broken into the A and B region. can reach.

$$U_B = -\int_{-\infty}^{0}\int_{-\infty}^{\infty} \Psi B(x_B)dydx \tag{7}$$

$$U_B = -\Psi\int_{-\infty}^{0} BB(x_B)dx \tag{8}$$

$$U_B = -\Psi B\int_{-\infty}^{0} B(x_B)dx \tag{9}$$

$$U_A = -\int_{0}^{\infty}\int_{-\infty}^{\infty} \Psi B(x_A)dydx \tag{10}$$

$$U_A = -\Psi\int_{0}^{\infty} AB(x_A)dx \tag{11}$$

$$U_A = -\Psi A\int_{0}^{\infty} B(x_A)dx \tag{12}$$

Qualitatively, if the bolus is pushed by pressure to the left, the potential energy U_B goes down, because there is more ferrofluid there, but the potential energy U_A goes up because there is less ferrofluid in that region. However, if A and B (the height of our space) are different, these will change at different rates. If pushed by air pressure from the right, the region B actually tugs in the same direction (to the left), but it will be resisted more strongly by the A region.

$$U = U_B + U_A \tag{13}$$

Due to Thereom 2, the forces acting on the bolus are purely a function of the two interfaces B and A. The force caused by the magnetic field depends solely on the change in potential energy at each interface, which we have contrived to make two simple lines. Let the total force in the x direction on the bolus caused by the magnetic field be F_M. Note in Eq. 22 below, the change in potential energy from the A region will increase (the derivative is negative) and the change in potential energy will decrease in the B region. The partial derivative with respect to x in the B region is negative, make its contribution to the force positive (in the x direction), which the contribution to the force in the A region is negative (in the $-x$ direction.) The force is in Newtons by virtue of the Ψ coefficient.

$$F_M = -\left(\frac{\partial U_A}{\partial x_A} + \frac{\partial U_B}{\partial x_B}\right) \tag{14}$$

$$F_M = -\Psi(AB'(x_A) + BB'(x_B)) \tag{15}$$

Note that the SI units of U is Joules, the units of $B(x)\Psi$ is $\frac{N}{m^2}$, a Joule is a Newton meter ($J = Nm$). The units of the derivative of potential energy with respect to the linear variable are thus Netwons, as we would hope. Let F_P be the force on the bolus due the pressure P_A and P_B. Let us consider both pressures to be positive. The $F_p = BP_B - AP_A$. Our overall force on the bolus at equilibrium thus becomes:

$$0 = F_P + F_M 0 = P_B B - P_A A + \Psi(AB'(x_A) + BB'(x_B)) \tag{16}$$

The Ψ factor converts the units to Newtons.

Note in this situation, the abillity of bolus to resist a pressure difference does not dependent dirctly on the strength of the magnet, but on the steepness of the slope or change in the magnetic field.

The maximum force that the magnet can exert on the bolus can be computed by simply finding the position that maximizes F_M. The force at that point times A will equal the collapse pressure.

$$F_M = -(\frac{\partial U_A}{\partial x_A} + \frac{\partial U_B}{\partial x_B}) \tag{17}$$

$$F_M = -\Psi(-A\frac{4CK^4x_A^3}{(K^4+x_A^4)^2} + -B\frac{4CK^4(\frac{V-x_AA}{B})^3}{(K^4+(\frac{V-x_AA}{B})^4)^2}) \tag{18}$$

$$F_M = \Psi 4CK^4(A\frac{x_A^3}{(K^4+x_A^4)^2} + B\frac{(\frac{V-x_AA}{B})^3}{(K^4+(\frac{V-x_AA}{B})^4)^2}) \tag{19}$$

This is sufficient complex that Wolfram Alpha can't easily deal with it. Let us use qualitative reasoning and make the assumption that as x_A decreases, as the A edge moves toward the origin, the B edge moves 3 times furthere away (if $A = 3B$) and is pushed into a region where the change in the B is very small, so that we can ignore it. Then the force on the bolus is:

$$F_M = -(\frac{\partial U_A}{\partial x_A} \tag{20}$$

$$F_M = -\Psi(-A\frac{4CK^4x_A^3}{(K^4+x_A^4)^2}) \tag{21}$$

$$F_M = \Psi 4CK^4A\frac{x_A^3}{(K^4+x_A^4)^2} \tag{22}$$

For $\Psi = C = K = 1$ and $A = 3$, Wolram Alpha tells us this has a maximum value of 3.19562 at $x_A = 0.88$, which is about what we would expect.

But I need to make this whole analysis simpler and crisper.

8.5 old

In this figure, two chambers containing ferrofluid are place in a similar B field. The chambers are in communication via a thin channel, so that that there is a single bolus of ferrofluid. Although it is quite difficult to construct a B field which is linear, let us assume for the purpose of this experiment that the B field is perfectly linear and goes to zero at point K, and is a maximized at $x = 0$, where it is C. This might actually be accomplished with two magnetic fields far from each other, but we show them on one diagram. The A chamber has a height of A and the Z chamber has a height of Z. Assume that they are encased in a very thin plane, so that we can ignore 3-dimensional effects, and so that the area of the bolus is proportional to the volume.

Assume that the magnetic field is strong enough to keep the bolus everywhere contiguous with itself (so that there is only one bolus), and that we can describe the configuration of the bolus only with the positions x and y (that is, the air-fluid interfaces in the cambers are perpendicular to the horizontal X axis) we can write the volume V as:

$$V = xA + yZ \tag{23}$$

$$y = \frac{V - xA}{Z} \tag{24}$$

$$x = (V - yZ)/A \tag{25}$$

From experiments perfomed by the authors, we know what if the air pressure in the A and Z chambers are the same, and the bolus is at rest, $x = y$. We would like to prove this configuration minimizes the potential energy.

Considering only one chamber, the potential energy is minimized, which we will call 0, when the chamber is full of ferrofluid, that is, when there is fluid between 0 and K. It is the lack of ferrofluid that has potential energy.

On the range $0 \leq q \leq K$,

$$B(q) = -qC/K + C \tag{26}$$
$$\tag{27}$$

Under our assumptions that the fluid is incompresable and homogenous, we can then write the potential energy U of each chamber as:

$$U_A = \int_x^K B(q) f_A(q) dq \tag{28}$$
$$U_Z = \int_y^K B(q) f_Z(q) dq \tag{29}$$
$$\tag{30}$$

where $f_A(x)$ is just the size of the boundary of the ferrofluid in the channel. In this thought experiment, this is a linear length, but we will soon generalize the boundary to a three dimensional surface. In this simple case, that boundary is constant, so $f_A(q) = A$, and $f_Z(q) = Z$. So,

$$U_A = A \int_x^K B(q) dq \tag{31}$$
$$U_Z = Z \int_y^K B(q) dq \tag{32}$$
$$\tag{33}$$

Performing some calculations:

$$U = A \int_x^K B(q) dq + Z \int_y^K B(q) dq \tag{34}$$
$$U = A \int_x^K \left(\frac{-qC}{K} + C\right) dq + Z \int_y^K \left(\frac{-qC}{K} + C\right) dq \tag{35}$$
$$U = A \frac{C(K-x)^2}{2K} + Z \frac{C(K-y)^2}{2K} \tag{36}$$
$$U = \frac{C}{2K} \left(A(K-x)^2 + Z(K-y)^2 \right) \tag{37}$$
$$U = \frac{C}{2K} \left(A(K-x)^2 + Z\left(K - \frac{V - xA}{Z}\right)^2 \right) \tag{38}$$
$$U = \frac{C}{2K} \left(A(K-x)^2 + \frac{(KZ - V + Ax)^2}{Z} \right) \tag{39}$$
$$\tag{40}$$

...we now have an expression for the potential energy expressed solely as a function of constants and the variable x. We can minimize this expression by taking the derivative with respect to x and setting it to zero:

$$\frac{\partial U}{\partial x} = \frac{C}{2K}\frac{(2A(x(A+Z)-V))}{Z} \tag{41}$$

$$\frac{\partial U}{\partial x} = (AC(x(A+Z)-V))/(KZ) \tag{42}$$

$$0 = (AC(x(A+Z)-V))/(KZ) \tag{43}$$

$$x = \frac{V}{A+Z} \tag{44}$$

$$\tag{45}$$

Plugging this value into Eqn. 25, we find:

$$y = \frac{V}{A+Z} \tag{46}$$

In other words, $x = y$. The two chambers, no matter their heights, will have the same level of ferrofluid when at rest. This is what we have observed in physical experiments, and it gives us confidence that our expression for the potential energy is correct.

8.6 Asymmetric Force

We would now like to answer the question: Can the two chambers with stand different pressures, or is something additional required for that to happen? In particular, it is possible that either the B-field of the shape of the chambers might have to change, so this is an interesting question.

Now let us consider the pressures that these two chambers can resist before the bolus collapses. If we increase the pressure in one chamber, the bolus will be driven away from the pressure until a stable point a found. When the pressure becomes high enough, the bolus will be drive past a point of maximum sustainable force, and the bolus will collapse. If we think of this as a valve, the valve will fail to resist the pressure.

Force times distance is work, and work increase potential energy. Therefore force exerted by the bolus can be computed as the negative of the change in potential energy. (The force exerted on the bolus will be the positive of the potential energy.) Let us assign reasonable values to A, Z, V, C, and K:

$$A = 1 \tag{47}$$

$$Z = 3 \tag{48}$$

$$K = 7 \tag{49}$$

$$V = 12 \tag{50}$$

$$C = 4 \tag{51}$$

Then our expression for potential energy becomes:

$$U = \frac{C}{2K}\left(A(K-x)^2 + \frac{(KZ-V+Ax)^2}{Z}\right) \tag{52}$$

$$U = \frac{4}{14}\left((7-x)^2 + \frac{(9+x)^2}{3}\right) \tag{53}$$

$$\frac{\partial U}{\partial x} = (AC(x(A+Z)-V))/(KZ) \tag{54}$$

$$\frac{\partial U}{\partial x} = (4(x(4)-12))/(21) \tag{55}$$

$$\frac{\partial U}{\partial x} = \frac{16}{21}(x-3) \tag{56}$$

Computing this in terms of y becomes:

$$U = \frac{C}{2K}\left(A(K-x)^2 + Z(K-y)^2\right) \tag{57}$$
$$U = \frac{4}{14}\left((-5-3y)^2 + 3(7-y)^2\right) \tag{58}$$
$$\frac{\partial U}{\partial y} = \frac{24}{7}(-1+2y) \tag{59}$$

The force exerted by the bolus, measured in the $+X$ direction, is $-\frac{\partial U}{\partial x}$. That means the force exerted by the bolus is:

$$F_A(x) = \frac{16}{21}(3+-x) \tag{60}$$
$$F_B(y) = \frac{24}{7}(1+-2y) \tag{61}$$

This means that at their strongest points (when $x = 0$ and $y = 0$), the forces that bolus exerts against pressure is different (16/7 vs. 24/7).

Thus we have proved that from simple geometry we can make an asymmetric check valve, albeit a very low performing one. QED.

8.7 Generalizing to Different Geometries

Let us now suppose that the ferrofluid bolus is three dimensional. Let us further assume that the shape of the inlet channel and the outlet channel are both variable, but depend only on the distance from the center point which we will call the origin of our manetic field. The the area of the bolus in the channels A and Z can be expressed as a function $g_A(r)$ and $g_Z(r)$.

In 3-space, it is probably not the case that we can practically construct a magnetic field which is purely radially symmetric—which depends solely on the distance from the origin. However, we may be able to approximate this. We can certainly construct a field which is generally monotonic increasing along any ray as it approaches the origin; a cylindrical magnet with the origin centered on a cylinder face produces such a field. Producing an analytic integratable function of such a field is not generally possible, since it involves elliptical integrals. However, we can either predict or constrain the shape a bolus will take in 3-space. In free space we can approximate it as a hemisphere centered on the magnet. In a constrained circular channel it will be a constanst. In either case we can represent it as a polynomial which we can tractably anti-differentiate. Let F_A and F_Z be the anti-derivatives of g_A and g_Z, so that:

$$G_A \int g_A(q)dq \tag{62}$$
$$G_Z = \int g_Z(q)dq \tag{63}$$
$$\tag{64}$$

We can then use integration by parts our on basic formulae for for potential energy:

$$U_A = \int_x^\infty B(q)g_A(q)dq \tag{65}$$
$$U_Z = \int_y^\infty B(q)g_Z(q)dq \tag{66}$$
$$\tag{67}$$

This can be integrated by parts:

$$U_A = B(\infty)G_A(\infty) - B(x)G_A(x) - \int_x^\infty B'(q)G_A(q)dq \tag{68}$$
$$U_Z = B(\infty)G_Z(\infty) - B(x)G_Z(x) - \int_x^\infty B'(q)G_Z(q)dq \tag{69}$$
$$\tag{70}$$

It is clear that $B(\infty)G_A(\infty) = 0$ for any geometry, because even a sphere feels decrease magnetic force as great distance become zero in the limit.

So:

$$U_A = -B(x)G_A(x) - \int_x^\infty B'(q)G_A(q)dq \quad (71)$$

$$U_Z = -B(x)G_Z(x) - \int_x^\infty B'(q)G_Z(q)dq \quad (72)$$

$$(73)$$

If we differentiate this generally we have:

$$\frac{\partial U_A}{\partial x} = -(B'(x)G_A(x) + B(x)g_A(x)) - \frac{\partial}{\partial x}\int_x^\infty B'(q)G_A(q)dq \quad (74)$$

$$(75)$$

relying on Wolfram alpha to differentiate the improper integral, we have:

$$\frac{\partial U_A}{\partial x} = -(B'(x)G_A(x) + B(x)g_A(x)) + B'(x)G_A(x) \quad (76)$$

$$\frac{\partial U_A}{\partial x} = -B(x)g_A(x)) \quad (77)$$

Since $g_A(x)$ is the area and force is area times pressure, the pressure that the A chamber can withstand is a function *purely* of the strength of the $B(x)$. This suggest that we define the most intense B-field that we can, likely by creating a small air gap.

8.8 Checking this math against previous calculation

Now let us check this math against the previous section. In the previous section, we computed:

$$U_A = A\int_x^K B(q)dq \quad (78)$$

$$(79)$$

we can differentitate dirrectly as:

$$\frac{\partial U_A}{\partial x} = A\frac{\partial}{\partial x}\int_x^K B(q)dq \quad (80)$$

$$\frac{\partial U_A}{\partial x} = -A(\frac{-Cx}{K} + C) \quad (81)$$

$$\frac{\partial U_A}{\partial x} = AC(\frac{x}{K} + -1) \quad (82)$$

$$(83)$$

This does not match the above because it is only one part of the valve!

For the valve B, we have:

$$U_Z = Z\int_y^K B(q)dq \tag{84}$$
$$\frac{\partial U_Z}{\partial y} = -Z(\frac{-Cy}{K} + C) \tag{85}$$
$$\frac{\partial U_Z}{\partial y} = -Z(\frac{-C(V-xA)}{ZK} + C) \tag{86}$$
$$\frac{\partial U_Z}{\partial y} = ZC(-(Ax+KZ-V)/(KZ)) \tag{87}$$
$$\frac{\partial U_Z}{\partial x} = \frac{\partial U_Z}{\partial y}\frac{\partial y}{\partial x} \tag{88}$$
$$\frac{\partial y}{\partial x} = \frac{-A}{Z} \tag{89}$$
$$\frac{\partial U_Z}{\partial x} = \frac{-A}{Z}ZC(\frac{(V-xA)}{ZK} + -1) \tag{90}$$
$$\frac{\partial U_Z}{\partial x} = -AC(\frac{(V-xA)}{ZK} + -1) \tag{91}$$
$$\tag{92}$$

$$\frac{\partial U}{\partial x} = \frac{\partial U_A}{\partial x} + \frac{\partial U_Z}{\partial x} \tag{93}$$
$$\frac{\partial U}{\partial x} = AC(\frac{x}{K} + -1) + -AC(\frac{(V-xA)}{ZK} - 1) \tag{94}$$
$$\frac{\partial U}{\partial x} = AC(x/K - (V-Ax)/(KZ)) \tag{95}$$
$$\tag{96}$$

This actually is the same expression! Yay! so probably I can do this in a very general way!

8.9 Connecting Valves in Series to Increase Collapse Pressure of a Gas

This section added July 27th, 2024.

This section may be out of place, but I wanted to record the math. We have recently had some succes with our a 3D valve model, obtaining for the first time a collapse pressure 10 times higher than the cracking pressure.

In discussion with Prof. David Trevas of Rice University he asked me if the PFCV could be made stronger by placing them in series. I suggested that that would not work. But after further thought I believe it might for gases (though not for incompressible fluids such as water.)

Unlike a flap valve, as the PFCV resists collapse pressure, its geometry changes and fluid flows toward the (nominal) inlet side. If that fluid flow decreased the volume of a chamber, the pressure would rise in proportion to the decreased volume. This would be "back pressure" against the ferrofluid blob that is in danger of collapsing, which would increase the pressure it could withstand.

Of course, if the inlet were simply a fixed chamber, the valve would have no value. Can we passively improve the collapse pressure?

If we placed two valves in series, then high pressure on the outlet side would drive some ferrofluid into the interstial space. This would increase the pressure. This would drive fluid in the second backing valve out of the interstial space, somewhat decreasing the pressure. It therefore behooves us to attempt to build an algebraic model of this situation to try to understand how much a backing valve might increase the collapse pressure, if at all.

Define:

V_F to be the fixed volume between the valves at rest.
P_C to be the valve at the inlet, which for this purpose (resising collapse gague pressure) we can consider ambient pressure.
P_B to be the pressure between the valves.

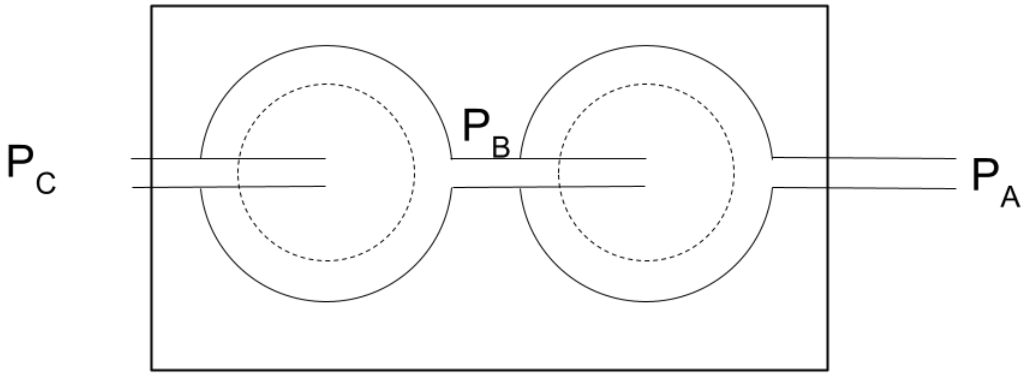

Fig. 7. Two Valves in Series

P_A to be the pressure at the (nominal) outlet, which when resiting collapse would actually be where the fluid goes in if the valve collapses.
V be the actual volume between the valves when $P_A \neq P_C$.
$D(p)$ be the function mapping the pressure of the valve into the displacement of the ferrfluid from the rest state. This is only defined when outlet pressure is higher than the input pressure, and when the gague pressure is less than the collapse pressure. $D(p)$ has been observed to a monotone function. We may approximate it as a straight line linear function, even though it probably is not. We will treat $D(p) \approx pD$, where p is a pressure, and D has units of volume divided by pressure.

From Boyle's Law, we have:

$$\frac{P_B}{P_C} = \frac{V_F}{V} \tag{97}$$

We can immediately see that this is not defined if P_C is zero. In fact in a vacuum we would not expect adding two valve to have any effect on the ability to resist collapse.

Our fundamental equation models the volume between the valves, utlizing the incompressiblity of the ferrofluid

$$V = V_F - D(P_A - P_B) + D(P_B - P_C) \tag{98}$$
$$V = V_F + 2D \cdot P_B - D \cdot P_A - D \cdot P_C \tag{99}$$
$$\tag{100}$$

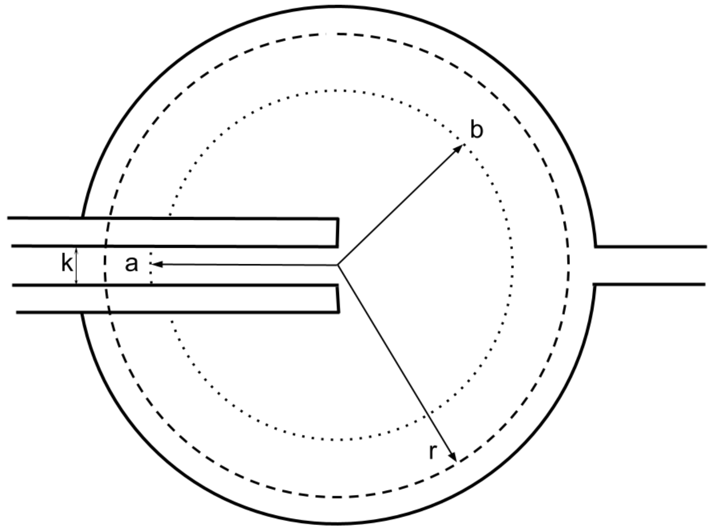

Fig. 8. Channel To Chamber Geometry

Combining Equations 97and 100, and then using Woflfam Alpha to solve for P_B we have:

$$\frac{P_B}{P_C} = \frac{V_F}{V_F + 2D \cdot P_B - D \cdot P_A - D \cdot P_C} \tag{101}$$

$$P_B = \frac{(\pm\sqrt{(-P_A D - P_C D + V)^2 + 8 P_C D V} + P_A D + P_C D - V)}{4D} \tag{102}$$

This deserves to be graphed, but we can plug in reasonable values. If we discover that P_B is relatively high (but certainly less than P_A, we believe it will push back on the bolus by that amount increasing the collapse pressure of valve complex.

We use cm H_2O as our unit of pressure milliliters as our unit of volume. We have not measured our bolus size precisely, but I believe them to approximately 1 ml. We have demosntrate collapse pressure of over 100 cm H_2O. A reasonable value for D may be 1 ml per 100 cm H_2O. Typical atmospheric pressure is 14.7 psi, which 1033.512 cm H_2O, which we will approximate as 1000 cm H_2O, making $P_C = 1000$. We set P_A to $P_C + 100 = 1100$. It is unclear how small we can make the V_F without ferrofluid from one valve being drawn to the magnet of the other when nearing collapse conditions. If we imagine that V_F is 4 ml, we have: $P_B = 25(17 + \sqrt{609}) \approx 1041.95$. This is a hopeful number! It suggests that the approach if plaing values in series will work.

8.10 The Channel-to-a-Chamber

Our goal, however, is to analyze a somewhat more complicated geometry. In particular, we are interested in a channel-leading-to-a-chamber, as depicted in Fig. 8. This geometry was discovered by intuition in attempting to build an asymmetric valve based on the observed fact that a bubble of air (or any non-magnetic material) will be pushed out of a bolus by magnetic "buoyancy" [16, 17]. A less-magnetic material "floats" toward the less intense magnetic field inside ferrofluid. This fact easily observed with a syringe by injecting air into a bolus in a field.

In that diagram, the edge of the bolus in the chamber is distance b from the origin. The edge of the bolus in the channel is a. We would ideally like to treat this as a one-dimensional problem as well. Let us assume once again that a magnetic field is placed at the center at our diagram (where the channel meets the chamber.)

The most important geometric feature of the channel-and-chamber valve design is the radius of the cylindrical magnet or magnets in it. The origin of our plane of operation will be the center of the top face of the cylindrical magnet, which is assumed to be beneath the plane, with its North face in contact with the plane.

The channel is the inlet and the chamber is the outlet.

Let us assume once again a magnetic field centered on the origin as generated by a 1/2" by 1" cylindrical magnet as defined by Equation 103. Since that field is completely radially symmetric, we can use that equation to describe the B field in the plane, where the parameter of the function is the distance from the center of the face of the magnet.

In moving to a 2-dimensional representation, we move to polar coordinates centered at the point the point where the channel and the chamber meet. Technically, the B field is a field of 3-dimensional vectors; but by working only in a plane, we treat is as a function of r, the distance from the orgin, and θ, the angle of rotation from the x-axis. However, since the B

field is completely symmetric, we can think of it as a one-dimensional function of distance from the origin $B(r)$. Essentially, we are treating the B field as the absolute value of the field we treated in the case of the 1-dimensional bolus. This can be interpreted as the strength drawing towards the origin, however, since we deduce everything from this point in terms of change in potential energy, we need not think of it as signed at all.

The authors had originally hypothesized that the valve would work with a bolus slightly larger than the radius of the cylindrical magnet. By experimentation, this was shown to be completely false. In fact, the desired affect occurs only when the bolus is smaller than the disc of the cylindrical magnet.

In this interpretation, we can simplify $B(x)$:

$$B(x) = \left\{ \begin{array}{ll} c(x), \text{ if } 0 \leq x \leq 1/2 \\ q(x), \text{ if } 1/2 < x < 3 \\ 0, & \text{if } x \geq 3 \end{array} \right\} \tag{103}$$

Since we have previously assumed (and it readily observed) that the bolus is a single blob, at least until it collapses, then we can can relate a and b via the total area of the bolus S. Let us assume that the channel of width k is narrow enough that the slight arc of the fluid inside the inlet region can be treated as a straight line.

$$S = k \cdot a + \pi b^2 \tag{104}$$

when $a \geq 0$ and $b \geq 0$, so that the areas of A and B are both positive. (Note: this ignores the small area occluded by the "walls" of the channel. I should probably assign them a width and fix that.)

It is most convenient to measure our position of the bolus, which is changing shape in this case, as the coordinate b, so we we express a in terms of b. Since we will use this algebraic relationship later, we will define it as a function A: This can be solved for a:

$$a = A(b) = \frac{S - \pi b^2}{k} \tag{105}$$

$$b_a = \frac{\sqrt{S - ak}}{\sqrt{\pi}} \tag{106}$$

We will later have need of the derivatives of these functions:

$$\frac{d}{db} A(b) = \frac{-2\pi b}{k} \tag{107}$$

$$\frac{d}{da} b_a = -\frac{k}{2\sqrt{(\pi)}\sqrt{S - ak}} \tag{108}$$

Here $A(b)$ is the value of a as a function of b, and b_a is b as a function of a.

Note that when $a = 0$, the dependence on k vanishes and:

$$b = \sqrt{\frac{S}{\pi}} \tag{109}$$

$$\tag{110}$$

Since at rest the bolus surface A will be in the same field strength as surface B, we can set:

$$B(a) = B(b) \tag{111}$$

$$B\left(\frac{S - \pi b^2}{k}\right) = B(b) \tag{112}$$

This can be solved by WolframAlpha (results not incuded here.) As expected, $b = a$, that is, both surfaces are equidistant from the center of our radially symmetric magnetic field.

8.11 A strategy for computing pressure

Our fundamental goal is to compute the collapse and cracking pressure. The collapse pressure is determined by the maximum force needed to reduce b in the outlet region past some point we will call the *collapse point.* The cracking pressure is the pressure needed to drive the ferrofluid in the inlet, represented by the variable a, to the *cracking point*, at which point a bubble forms and is pushed to the outlet.

The check valve we are trying to design will be characterized by both the collapse pressure and the cracking pressure, but in a single number the can define the performance of the valve as this ratio P, which we want to be as high as possible:

$$P = \frac{\text{Collapse Pressure}}{\text{Crack Pressure}} \tag{113}$$

If the bolus at rest is displaced, the potential energy increases, moving off a minimum. At a given point, the infinitessimal change in energy divided by the infinitessimal change in distance is equal to the force.

If we had a formula for the potential energy as a function of b, we would have a chance of computing this analytically.

In this model, the narrowness of the plane to simplifies the problem. In this simplified model, we can assume that the potential energy of the magnetic system can be simply modeled by the presence or absence of ferrofluid at a point in the plane. The presence of ferrofluid at a point or infinitessimal area (da) decreases the potential energy U of the entire system in proportion to the strength of the field at a point, and that that strength is describe by our function $B(r)$ where r is the distance from the origin (and thus the center of the face of the magnet). We can thus describe the potential energy as an integral over surface patches da:

$$U = \int \left\{ \begin{array}{ll} B(a), & \text{if there is no ferrofluid at } a \\ 0, & \text{if there is ferrofluid at } a \end{array} \right\} da \tag{114}$$

Thus the potential energy is highest when there is no ferrofluid at all, and lowest when the entire plane is full of ferrfluid. However, we will generally treat a bolus of fixed volume and assume that our plane is thin enough that it therefore has a fixed area.

We are furthermore ignoring all effect of the ferrofluid interacting with itself in terms of the potential energy. However, we rely on the very important property of ferrofluid that it tends to clump together into a single blob in the presence of a magentic field. We will, in fact, assume that our blob or bolus is perfectly circular in the chamber, and a perfect rectangle in the channel. Furthermore, we assume the fluid is incompressible, and therefore always has the area S. This is true until the bolus collapses; we know that when it collapses it explodes drastically, and we do not attempt to model anything after a collapse event.

Since the absolute potential energy of our system is irrelevant, it will be easier for us to treat the absolute potential energy U as zero and consider it to go down as ferrofluid is added. Since we only model the bolus when part of it is at the origin in our polar system, we can rewrite our potential energy as negative and measured against an absolute of zero:

$$U = -\int \left\{ \begin{array}{ll} B(a), & \text{if there is ferrofluid at } a \\ 0, & \text{if there is no ferrofluid at } a \end{array} \right\} da \tag{115}$$

Radial symmetry means only the distance from the origin is relevant, so in fact this is a one dimensional system of the coordinate b, which it is convenent to break into $U = U_N + U_R$, the channel and the chamber, respectively. We assume that the channel is narrow enough that we can model the field across the channel as if it is rectilinear. We can treat these as line integrals:

$$U_R = -\pi \int_0^b rB(r)dr \tag{116}$$

$$U_N = -k \int_0^a B(r)dr \tag{117}$$

$$U = U_N + U_R = -\pi \int_0^b rB(r)dr + -k \int_0^a B(r)dr \tag{118}$$

(This formula for U_R ignores the "walls" of the chamber.) Because pressure applied in the chamber moves the bolus into the channel, and pressue applied in the channel moves the bolus into the chamber, the potential energy changes in both. The potential energy always goes down as more fluid is added to one side (or at least never goes up), and thereofore always goes up in the chamber from which it is removed. The minimum potential energy is a point of balance. We seek a mathematical description of the change in totoal potential energy as b changes so that we can compute the force. This should allow us to choose S to be most advantageous. Eventually, this may allow us to design different magnetic fields to change the performance of the valve.

8.12 The Collapse Pressure

Let us define b_0 as the value of b when the pressure in the chamber and the channel are equal, so that the bolus is at rest. This is the point of minimum potential energy. Let $F(b)$ be the force exerted by the magnet on the bolus at postion b. Then $F(b_0) = 0$. The potential energy increases as the surrounding gases do work on it (up to the point of collapse or crack.) Within that range, the forces are conservative; the potential energy could in theory be recovered as the bolus spring back to position if the pressure on the bolus was removed. Then we have:

$$\frac{dU}{db} = F(b) \tag{119}$$

The should allow us to compute the $F(b)$ from U.

Let us consider what happens as the outlet gage pressure P is increased.

Qualitatively, we assume that at rest (ambient pressure), the bolus exerts no force. As the pressure in the chamber increases, the force (the increase in potential energy) will increase as b is decreased and the bolus is disturbed from its rest position. This force on the bolus and resiting force exerted by the bolus will increase as b decreased further from its rest postion. However, eventually this force will cease to increase and reach its maximum. At that point, the valve will collapse.

The valve is meant to withstand this pressure until a collapse point. When in balance, the force on the bolus will be zero. The from on the bolus from air pressure that is attempting to drive the bolus into the channel will be proportional to $2\pi bP$, since force is pressure times area, but in this case the area affected by the pressure is proportional to the the circumfrence of the bolus (not counting the channel, which is relatively small). Once this force has pushed much of the fluid into the channel, the inlet-side bolus boundary will be far away from the strong magnetic field, and thus may be neglected. At rest, he force on the outlet-side bolus boundary $\frac{dU}{db}$ added to the air pressure force is zero:

$$0 = 2\pi bP + -\frac{dU}{db} \tag{120}$$

$$2\pi bP = \pi \frac{d}{db}\int_0^b rB(r)dr \tag{121}$$

$$2\pi bP = \pi bB(b) \tag{122}$$

$$P = \frac{B(b)}{2} \tag{123}$$

$$\tag{124}$$

The position of the bolus b at balance is just proportional $B(b)$. So the stronger the magnetic field, the higher the collapse pressure. The strongest magnetic field as at the origin, and in fact we observe the bolus position be very near the orgin right before the collapse event.

We deduce from this that to build the best possible valve in this geometry, we should create the strongest possible magnetic field near the origin.

8.13 The Cracking Pressure

The cracking pressure P_r will be the force necessary to reach $a = 0$, because at that point, the non-magnetic fluid will form a "bubble" which will be pinched off and displaced into the chamber by the ferrofluid. This "bubble" will be forced in the direction of decreasing magnetic field strength, because it minimizes potential energy to fill the strong field with ferrofluid preferentially over non-magnetic fluid. This occurs at $a = 0$, so $A(b) = 0$, or $b = \sqrt{\frac{2S}{\pi}}$ by Equation 110. The force on the bolus inlet boundary does not change as the bolus moves, since k does not depend on a. The force on the magentic force on the outlet boundary and the inlet boundary are in fact in opposite directions. The inlet magnetic force resists the cracking air pressure, and the outlet magnetic force coooperates with it.

In considering the cracking pressure, it is more convenient to track the bolus as a function a, rather than b. Thus:

$$\frac{dU}{da} = F(a) \tag{125}$$

Then to desribe cracking, we can set the air inlet force in balance with the magnetic force.

$$P_r k = -\pi \frac{d}{da} \int_0^{b_a} rB(r)dr + k\frac{d}{da} \int_0^{a} B(r)dr \tag{126}$$

$$P_r k = -\pi b_a B(b_a) \frac{k}{2\sqrt{\pi}\sqrt{S-ak}} + kB(a) \tag{127}$$

$$P_r k = -\pi \frac{\sqrt{S-ak}}{\sqrt{\pi}} B(\frac{\sqrt{S-ak}}{\sqrt{\pi}}) \frac{k}{2\sqrt{\pi}\sqrt{S-ak}} + kB(a) \tag{128}$$

$$P_r k = -\frac{1}{1} B(\frac{\sqrt{S-ak}}{\sqrt{\pi}}) \frac{k}{2} + kB(a) \tag{129}$$

$$\tag{130}$$

We can thus create a testable hypothesis that can be verified experimentally by building an appropriate apparatus. Choosing $k = 10$ and $b_0 = 3/4$, we expect:

$$P = \frac{\text{Collapse Pressure}}{\text{Crack Pressure}} \tag{131}$$

$$P = \frac{P_c}{P_r} \tag{132}$$

$$P = \frac{\frac{F(b_m)}{\pi b_m}}{\frac{F(\sqrt{\frac{2S}{\pi}})}{k}} \tag{133}$$

8.14 Recent Thoughts

I now believe that we can set up a thought experiment of a bolus which has two widely separated faces, connected by a thin pipe. We can design the magnetic field at each end separately. On the inlet side, we can make the tube 100 times thinner (in terms of cross-sectional area) than the outlet side.

We can then set up the pressure on the faces as a function of change in potential energy. However, the factor of 100 will come into play in the area, but will also mean that a change in dx on the outlet side will produce a change of $100dx$ on the inlet side. If the $B(x)$ field magnitude varies as a function of x, this has to be taken into account. This is how the assymetry is obtained. If $B(x)$ is a constant there will be no change in force or pressure, and the system will in fact not resist any pressure at all. However, if $B(x)$ is dropping off linearly away from the staring position, the counter-balancing affect of the boundary interfaces will be different, creating the asymmetry that we observe.

In our actual geometry, the field is of course three dimensional, but that hardly matters.

8.15 Conjecture

Important note: This math strongly suggest that the valve can be made twice as performant by opening the chamber completely as shown in Figure9, so that the valve is merely a chamber in the center of the bolus. However, even though we have modeled the fluid as not reacting to itself, we would not want the ferrofluid itself to strengthen the field on the inlet side, which might occur. Creating therefore an arc of someething less than 360 degrees, such as 270 degrees, or making the walls of the thin chamber very thick, might solve this problem.

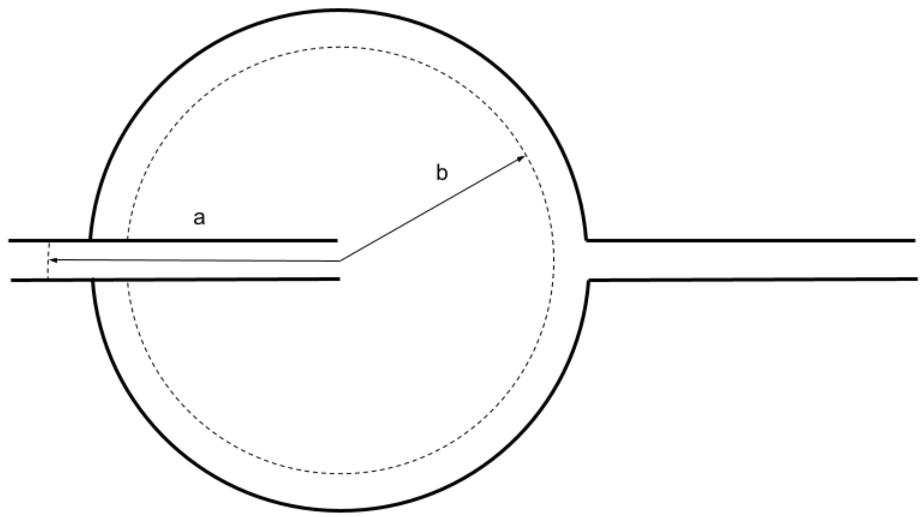

Fig. 9. 360 degree planar check valve, an improvement derived from theory

This math makes it clear that on the outlet side we want the bolus surface to be outside a strong field at r to that $rB(r)$ is maximized to maximize the collapse pressure. Let us call the value of b that maximizes $bB(b)$ the *collapse point.* We can now see that we want S to be large enough that the edge of the bolus at rest is beyond the collapse point by some comfortable margin.

However, to minimize the crack pressure, we want the bolus at rest in the chamber region inside the collapse point, because we don't want the inlet pressure to have to overcome that, even though on the inlet side the length d will be much smaller that length πb. However, by the Magnestatic Blob Minimization principle, this is impossible in a radially symmetric B field. However, we can build a magnetic field which is not radially symmetric, thereby improving the performance of the valve.

This suggest that on the inlet side we attempt to make the magnetic field, only within the channel, as gradual, linear, and long as possible. This might be possible to accomplish by simply adding a piece of ferrous metal under the channel, or by making the channel itself a steel needle. A specially designed magnet or two magnets could be used. This is somewhat depicted in Fig. 10.

8.16 Idea: a 3D version

It if we had a very powerful and very small dipole, it could in theory collect a complete sphere of ferrofluid around it. Possibly the tip of a conical magnet would nearly do this. In a spherical-like magnetic field, the same principle would apply, but the added dimensionality would be expected to further separate the collapse pressure from the crack pressue.

However, a simpler version of this idea based on accessible cylindrical bar magnets is depicted in Fig. 11. This has not yet been experimentally tested.

The advantage of having a 3D version predicted by the math is that the area of the small inlet port would be tiny compared to the area of the spherical shape. This would tend to produce a very performant valve, if the magnet field can be designed with sufficient strength. It would seem to be harder to manufacture in the intended use case of very small valves.

There are two obvious ways to manufacture such a 3D device, both of which require drilling a small hole into magnet. The first is use a small cylidrical or spherical magnet. This would be a dipole that could not be approximated as a radially symmetric field; but it might work anyway. The second would be to drill a hole through the axis of a conical magnet. Hopefully the concentration of field strength at the apex of the magnet would produce a roughly spherical field strength.

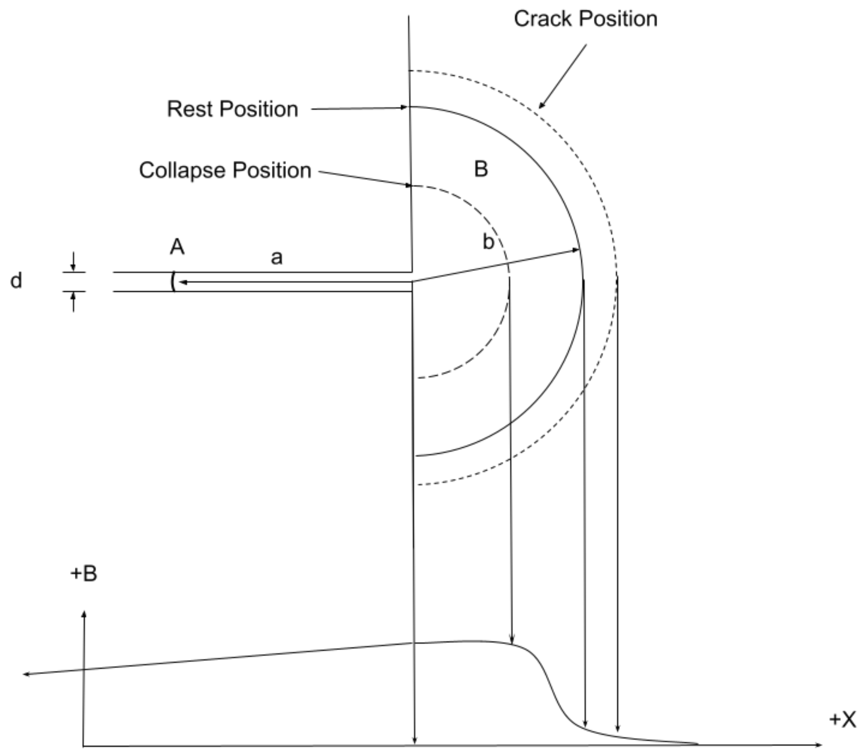

Fig. 10. (Rough) Ideal Field Profile

Possibly rather than using a sphere, a hemisphere would be more reasonable.

8.17 Idea

We can protect the valve from explosive collapse by using a separate valve that cracks before the collapse pressure, acting as a "pop-off" valve.

9 Conclusions

This paper demonstrates an apparently novel passive ferrofluid one-way valve or check valve (PFCV). This valve is completely passive in that it depends entirely on the pressure at the inlet port and the outlet port. The valve has no moving parts (except for the ferrofluid, which is almost stationary), and a remarkably simple design, consisting of nothing but a channel, an inlet chamber, and outlet chamber, and a bolus of ferrofluid in a static magnetic field.

Although no effort has been made to optimize the design, the pressure difference between the cracking pressure and the sustainable back pressure appear great enough to make an effective micropump. The performance of this one-way valve may improve with additional design effort; the authors sought to publish this result as soon as it was observed. Obvious future research possibilities are:

I. To improve the performance by varying the geometry of the passive design or shape and strength of the magnetic field.
II. Utilizing this design to make a micro-pump similar to earlier micro-pumps but with this simpler check valve design.
III. To provide an explanatory and predictive theory of operation, for example based on magnetic field strength as per [11].
IV. Studying the ability of the valve to recover after a collapse automatically when high outlet pressure is removed, which would increase robustness in some applications.

10 Possibly Venus for Publishing

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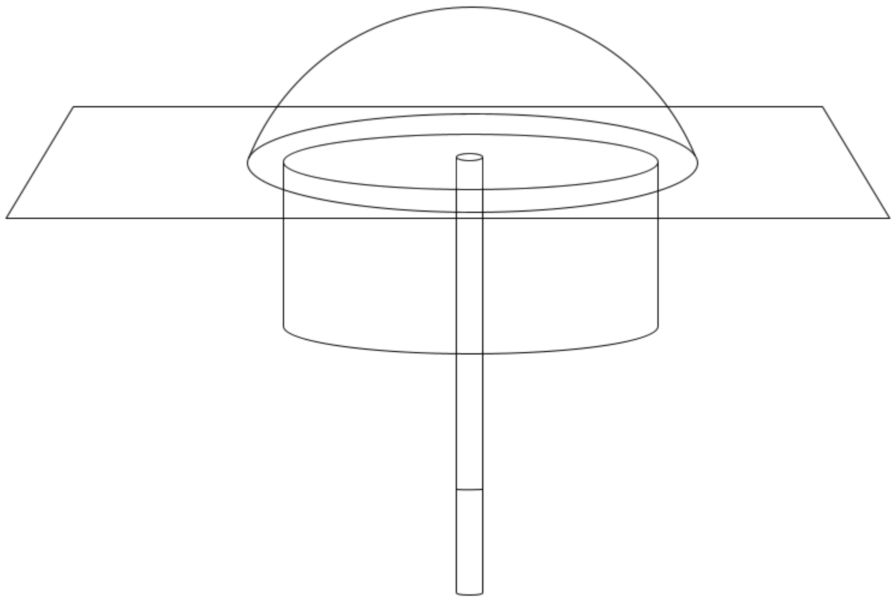

Fig. 11. Untested 3D check valve idea

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