

# A Novel Passive Ferrofluid One-way (Check) Valve Based on Energy Minimization

**Lisa Kotowski**

**Veronica Stuckey**  
Biomedical Engineer, University of Texas at Austin  
Email: stuckey002@gmail.com

**Robert L. Read**  
Founder, Public Invention  
Email: read.robert@gmail.com

## ABSTRACT

Small pumps and valves enable flow management in microfluidic systems. A novel passive ferrofluid check valve is presented. The valve consists of only a unique channel-and-chamber geometry, ferrofluid, and a stationary magnetic field. The flow is determined only by the inlet and output pressure, and the magnetic field is completely static. The prototype valve and experimental setup are explained and performance of the valves cracking and collapse pressure reported. This initial design can be used for microfluid handling and lab-on-a-chip applications.

Additionally we present a theory of operation based on energy minimization and compare predicted performance to actual performance.

## 1 INTRODUCTION

Ferrofluid can be manipulated by electronically controlled magnetic fields to exert force on fluids[? ? ? ]. This makes it possible to build pneumatic or hydraulic devices, perhaps on very small scales, such as a single chip[? ? ], to miniaturize fluid handling. This has been proposed for biomedical purposes[? ] that would use water or body fluids, although this paper reports only on experiments done with air. Miniature pumps and valves could be used to make a “lab on a chip” (LOC) or even to heat or cool different chip areas.

A fundamental component of such devices is the *check* or one-way valve. Two check valves on either side of a chamber whose volume can vary creates a positive displacement pump. A perfect check valve opens or *cracks* with minimal pressure on the inlet side and sustains maximal pressure on the outlet side before *collapse*, allowing fluid to flow in only one direction. Following[? ] we call the maximum pressure differential the valve can resist in the direction it is intended to check or block (from outlet to inlet) the *sustainable* or *collapse* pressure.

This article is a brief report on an initial but functioning design of a passive ferrofluid check valve (PFCV) that has no moving parts except for the ferrofluid bolus itself, which is stationary in normal operation. By passive, the authors mean a check valve that functions without changes to the magnetic field affecting the bolus, whether that field is induced by a permanent magnet or an electromagnet. That is, the flow is determined purely by the difference between the inlet port pressure and the outlet port pressure. To our knowledge, no passive ferrofluid check valve has been previously reported, despite being an active area of research and despite such a valve having significant advantages for operation and especially fabrication over valves with moving parts.

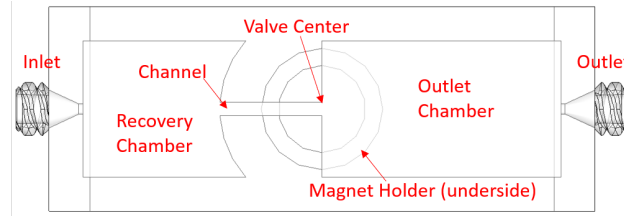


Fig. 1. The passive ferrofluid check valve components

## 2 RELATED RESEARCH

A number of papers report on ferrofluid pumps, focusing in particular on micropump and lab-on-a-chip applications[? ? ]. Many of these papers use a version of mechanical valve not based on passive ferrofluid, even though they move a ferrofluid bolus with a magnetic field. For example, a corrugated silicone micro valve[? ? ] has been reported. Other researchers use active valves, which require synchronization with the ferrofluid plug to form a pump, such as [? ], which describes an active *T-Valve* with a moving ferrofluid plug, and [? ] describes a complete fluid pump with valves that use active control of a ferrofluid bolus. At least two additional kinds of active valves, a *well valve* and *Y-valve*, have been described[? ]. Active control is possible because the action of the plunger or bolus may be synchronized with the opening and closing of the valves. Nonetheless a passive valve would be simpler and less expensive, and would not require knowledge of the timing of the plunger.

An interesting functional micropump in which the moving ferrofluid bolus merges with a fixed ferrofluid valve and then separates on each pumping cycle has been described[? ], but is not a one-way valve.

A passive ferrofluid two-way valve with tunable opening and closing pressure based on magnetic field strength[? ] has been tested, but could not be passively used to make a pump.

This paper has not studied the closing pressure of the PFCV, but reports on the opening (or cracking) pressure (for flow from inlet to outlet) and sustainable (or collapse) pressure when the outlet pressure is higher than the inlet side.

## 3 PASSIVE FERROFLUID CHECK VALVE (PFCV) DESIGN

The PFCV depicted in Fig. ?? is a simple asymmetric volume centered in a magnetic field which holds a ferrofluid bolus in place. In the center of a radially symmetric magnetic field a narrow channel meets a larger open chamber at a right angle. The ferrofluid bolus is large enough that at rest in the field it forms a semi-circle in the open chamber. The narrow channel is longer than the radius of the bolus at rest. The broad chamber is the outlet side of the valve. The narrow chamber opens onto a recovery chamber on the inlet side of the valve. This design allows the bolus to be recovered from the recovery chamber when the pressure is equalized if the outlet pressure is raised above the collapse pressure, driving the bolus away from the magnetic field. The PFCV does not resist pressure as well as a valve of the same size made out of moving, solid parts. That is, the sustainable pressure it can resist on the outlet side before failing is relatively low, and pressure required to crack it open and allow flow is relatively high. However, it may operate reliably within a range of known pressures, and thus be sufficient to build a pump-on-a-chip. Furthermore, the PFCV reported here is a preliminary design which can probably be significantly improved. The authors found the existence of the PFCV worth sharing immediately.

## 4 METHOD

The valve depicted Fig. ?? was designed using Solidworks 2016. It is freely licensed via the CERN OHL Strong Reciprocal License[? ? ]. The model consists of a 15mm long, 2mm wide channel, a large outlet chamber, a recovery chamber, two female luers, a magnet holder ring and two legs to provide room for the magnet. All volumes are 2mm high. The 3D shape of the chambers can be thought as a 2mm high extrusion of a 2D shape. Viewed from the top, one end opens up to a recovery chamber of circular profile 30mm in diameter and the other an outlet chamber with a flat wall. A magnet holder ring 12.7mm inner diameter (1/2") was created centered on the channel-chamber junction, below where the bolus is placed, to hold a permanent magnet in place at the center of the valve. When two magnets are used, the magnet on top naturally stays in the same position due to attraction to the magnet below. On the inlet

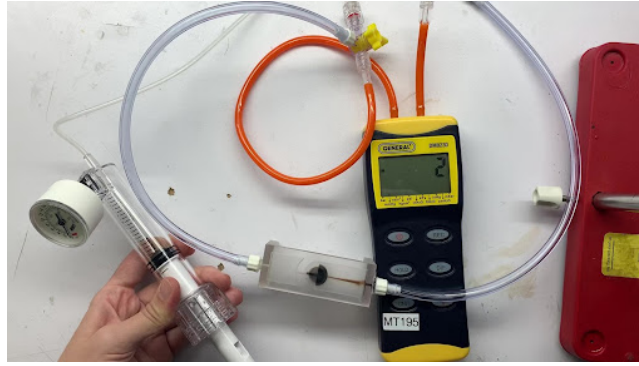


Fig. 2. Equipment set up

side of the channel opens into a recovery chamber shaped to allow the ferrofluid to be passively drawn back into the channel by the magnetic field after a collapse of the bolus. The model was printed on a ProJet MJP 2500 (3D Systems, Rock Hill, SC), using Visijet M2G-CL and VisiJet M2 SUP as material and support respectively (3D Systems, Rock Hill, SC). Support material was removed by using an EasyClean system (3D Systems, Rock Hill, SC) and Dawn dish soap (Procter & Gamble, Cincinnati, OH) to remove residuals.

As shown in Fig. ?? and in our demonstration video[? ], a basixCOMPAK 30atm pressurizing syringe (Merit Medical, South Jordan, UT) is connected to the model via a two-way stopcock (Qosina, Ronkonkoma, NY), tubing (Natvar, City of Industry, CA), and male (Injectech, Fort Collins, CO) and female luer (Qosina, Ronkonkoma, NY), allowing integration of a manometer (General Tools, Secaucus, NJ) to measure pressure. A 12.7mm x 25.4mm (1/2" x 1") cylindrical neodymium magnet (Apex Magnets, Petersburg, WV) with a pull force of 14.6 kg (32.24 pounds) was placed inside the magnet channel by means of a tight fit and 0.2 mL of ferrofluid (Apex Magnets, Petersburg, WV) was injected into the model using a 3mL syringe (BH Supplies, Jackson, NJ).

To obtain values, pressure was applied through the pressurizing syringe, as demonstrated in a video [? ]. Pressure was first applied from the outlet side of the model. The maximum pressure difference from the outlet side that the valve can withstand before collapsing, will be referred to as the sustainable pressure or *collapse* pressure. Pressure applied from the inlet needed to initiate flow will be referred to as the *cracking* pressure.

The cracking pressure was first measured by increasing the pressure difference on the inlet side until flow is initiated (at which point the valve is “open” and pushing the syringe plunger faster simply increases flow without increasing the pressure.) Then the collapse pressure was measured by increasing the pressure difference on the outlet side. At pressures below the sustainable pressure, the valve holds pressure well with no observable leaks of air in the short time (a few minutes) of our experiment. When the sustainable pressure is exceeded, the bolus explodes violently into the recovery chamber. When the pressure difference is equalized, the bolus may passively recover into the central position, or it may need to be actively “combed” with a magnet back into the central position.

The procedure was performed first with one magnet, named the “Single Magnet” configuration, placed below the channel-chamber connection. The “Dual Magnet” configuration was performed with the magnet in the same position as the Single Magnet case in the same position, but with a second magnet of the same kind placed vertically on top of the model, arranged to be strongly attracted to the lower magnet.

## 5 RESULTS

The final pressures obtained demonstrate a clear difference between the inlet cracking pressure and outlet sustainable pressure, creating an effective passive check valve.

The ferrofluid had observable differences in behavior between the two configurations. After the pressure equalized following a collapse of the bolus due to exceeding the sustainable pressure, the single magnet configuration often repaired itself by drawing the fluid back into a centered bolus passively. After a collapse with two magnets, fluid further from the bolus remained stationary while the fluid closer was pulled back to the center. Following the removal of the top magnet, the stationary fluid then began to return to the bolus. This is consistent with the localization of

Table 1. Result pressures

Magnet con-figuration	Cracking Pressure kPa (mmHg)	Collapse Pres-sure kPa (mmHg)	Pressure Differ-ence kPa (mmHg)	Approx. Ra-tio: Cracking to Collapse Pressure
Single	1.1 (8)	5.5 (41)	4.4 (33)	1:5
Dual	8.5 (64)	17.5 (131)	8.9 (67)	1:2

the magnetic field between two magnets, and the weakening of the magnetic field further from the channel-chamber juncture in the dual magnet configuration.

Although the dual magnet configuration demonstrated a larger absolute pressure difference due to magnetic field strength between the cracking and the collapse pressure, the single magnet configuration granted a larger ratio of collapse pressure to cracking pressure due to the much lower cracking pressure. The authors conjecture that the low cracking pressure may have been not only to the weaker magnetic field, but the weakening at the top of the 2mm high channel, which was further away from the magnet in the single magnet configuration.

## 6 THEORY OF OPERATION (BEST AS OF APR. 25)

Let us consider an ideal blob of ferrofluid.

Assume that a blob in this section means an ideal 2-dimensional blob, approximated by a blob constrained into a very thin plane.

Assume that the B-field is purely perpendicular to this plane, and that it completely dominates the magnetism induced in the ferrofluid itself.

Assume that all forces of viscosity and surface tension may be ignored.

Although we will not model the forces that make it so, we will assume that blob of ferrofluid is self-connected and will not split into more than one blob.

The blob is perfectly incompressible, and so has an absolutely unchanging area.

The magnetic particles are evenly distributed in the fluid no matter what the magnetic field, so that the potential energy in the fluid by being in the field depends only on the strength of the field.

We need not treat the motion, velocity, or inertia of our fluid in any way.

Under suitable assumptions, we can state two drastically simplifying assumptions.

**Conjecture 1 (Magnetostatic Blob Minimization).** *The magnetostatic force on an ideal 2-dimensional blob tends to minimize the potential energy in the blob and this occurs when its surfaces are in equal strength fields. Equivalently, every surface of a blob at rest with no outside forces is a line of equal field strength and all surfaces are equal.*

**Theorem 1 (Surface Force Computation).** *The force exerted on an ideal blob by the magnetic field is the sum over each surface of the integral of magnetic field along that surface, either pushing the surface into the blob or pulling the surface away from the blob.*

*Proof.* Force is work times distance. Within an infinitesimal distance, force is the change in potential energy. Because our fluid is uniform, the potential energy of a blob one position and a blob displaced from that position by an infinitesimal changes only by the change in the thin areas along the surfaces. The change in potential energy in these infinitesimal areas is purely a function of the their path through the B-field and the intensity of the field at that point.

If the area is expanding the blob, it always decreases potential energy. If the area is contracting the blob, it always increasing potential energy. However, since the B-field at the two (or more) surfaces may be different, there may be a force exerted on the blob. In particular, we are interested in the force exerted on the blob counterbalancing the force exerted on by fluid pressure of non-ferrofluid at the surfaces.

Now imagine that we have a monotone decreasing field centered on the origin which is not necessarily circular, with the A region containing a thin tube, and the B region containing a wide open region. Imagine that the field in B is radially symmetric, but monotonically decreasing with distance from the origin (but not necessarily the same in the region A.) A force resisting flow from the outlet to the inlet (from B to A) is equal to the field strength at a point multiplied by the surface length. In this scenario, the air pressure on the blob is a function of the length of the surface, but the magnetostatic force is also a function of length, so the length cancels out and becomes irrelevant. The collapse pressure is therefore the maximum force at any point between the rest position and the origin (this field is by construction equal on a semi-circular arc.) The collapse pressure also depends on the field strength at the point of the surface in A, but this surface has small length, and therefore contributes little to the sum of the force.

Conversely, the cracking pressure will be minimized by making the field strength in the region B as low as possible at the point when the ferrofluid has been entirely driven into region A by air pressure from the inlet.

## 6.1 How the Valve Cracks

It is clear that a blob that has a bubble of non-ferrous fluid inside it has higher potential energy than the same blob without that bubble. Furthermore, such a bubble will feel a pressure to move away from an area of a higher magnetic strength toward an area of weaker magnetic strength. A bubble could be trapped, but if we assume that our field decreases monotonically from a central peak, a bubble will have a path to escape and will do so. In this scenario a valve will “crack” that is, crack open allowing a fluid from the inlet to the outlet, as soon as the non-ferrous fluid crosses a central boundary.

## 6.2 Warmup: A Bolus in A Channel

A relatively simple problem is to consider a bolus of ferrofluid in a thin channel. Placing a cylindrical permanent magnet beneath it tends to lock the bolus in place. If a small amount of air pressure is applied to it from the West, the bolus will be displaced to the East, but will find a stable point in balance with the Westward forces exerted by the magnet. As the pressure is increased, the bolus will be displaced further Eastward. It will reach a point where further displacement to the East does not increase the Westward force, and will soon decrease it. At this point the plug will fail catastrophically, spraying Eastward into the channel and letting air through.

The magnetic field in the line just above the face of the magnet can be analytically described by elliptical integrals, but they are very difficult to work with symbolically, but easy to calculate numerically. MagPyLib is a convenient package for doing so. Figure ?? shows our setup of the problem.

By making assumptions, we can analyze this to obtain closed form solutions which match our experience and inform more complicated cases.

The red line represents the thin channel just slightly above the face of the magnet. MagPyLib conveniently computes the field along this line. In particular, only the  $x$  component (the East-West coordinate) of the B-field is relevant to us. The field was plotted with MagPyLib in Figure reffig:Bx.

By inspection, we observe that this function can be accurately approximated with a piecewise polynomial representation breaking the function at the extrema, which are in fact the edges of the magnet. The resulting representation which is quadratic to the West of the magnet, linear above the magnet, and quadratic East of the magnet is remarkably accurate, under the assumption the MagPyLib is accurate. These three approximations are shown in Figure reffig:piecewise.

The piecewise approximation  $B'$  to the field  $B$  is:

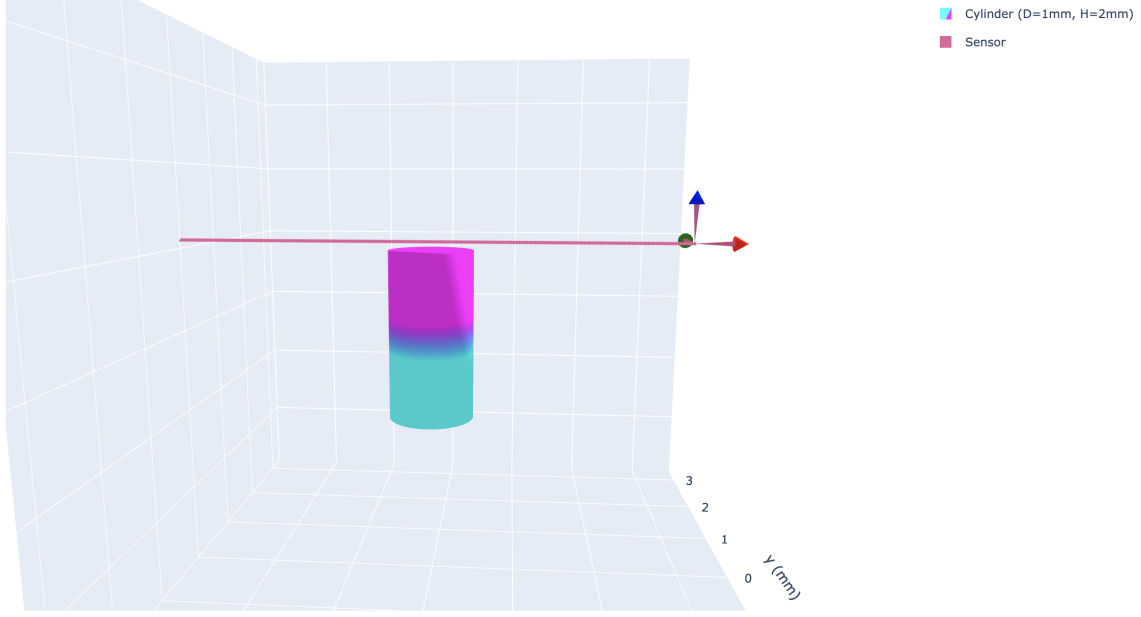


Fig. 3. Setup of the Bolus Problem

$$P(x) = -18.50670842x^6 + -230.18413653x^5 + -1174.49453555 + \quad (1)$$

$$x^4 + -3154.6263118x^3 + -4734.06101237x^2 + -3819.38958143x + -1339.63590984 \quad (2)$$

$$Q(x) = 18.50670842x^6 + -230.18413653x^5 + 1174.49453555 + \quad (3)$$

$$x^4 + -3154.6263118x^3 + 4734.06101237x^2 + -3819.38958143x + 1339.63590984 \quad (4)$$

$$B'(x) = \begin{cases} 0, & \text{if } x < -3 \\ P(x), & \text{if } -3 < x < -1/2 \\ 545x, & \text{if } -1/2 < x < 1/2 \\ Q(x), & \text{if } 1/2 < x < 3 \\ 0, & \text{if } x > 3 \end{cases} \quad (5)$$

Where  $P(x)$  and  $Q(x)$  are nearly identical 6-degree polynomials determined by NumPy's polyfit.

Setting aside issues of magnetic susceptibility of the ferrofluid used, the force is proportional to the field strength times the area. Since the area is unchanging in a simple tube, the force on a slice of ferrofluid at position  $x$  is proportional to  $B'(x)$ .

Now imagine that bolus has a length of  $L$ , and let the channel area be  $d$ , and let  $x$  be the midpoint of the bolus. Assume the magnet is a 1000 milliTesla (mT) polarized magnet. The  $x$ -axis force on the bolus can be computed as the integral of  $dx$ :

The  $x$ -axis force on a bolus in the channel centered at  $y$  generally is proportional to:

$$F(y, L) = \int_{y-L/2}^{y+L/2} B'(x) dx \quad (6)$$

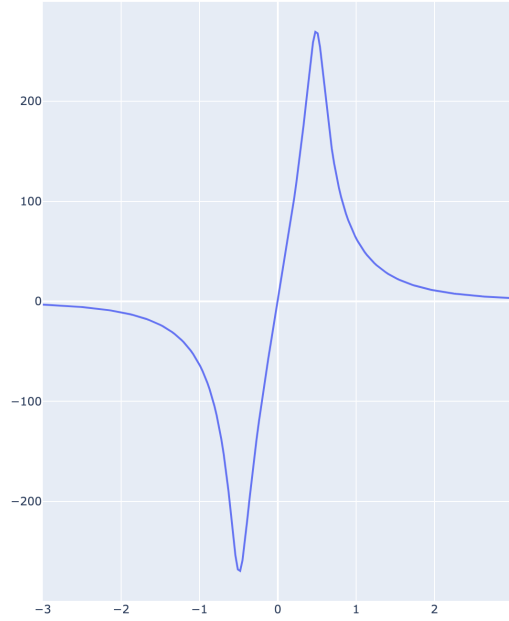


Fig. 4. Bx component of field

Let us assume that we are blowing air from the left in the positive  $x$  direction, in which case we can ignore cases when  $y < 0$ . It is clear from symmetry that  $F(0, L) = 0$  for any  $L$ .

$$F(y, L) = \int_{y-L/2}^{y+L/2} B'(x) dx \quad (7)$$

In order to compute where the bolus will be when it explodes catastrophically when blown from the negative  $x$  direction to the positive  $x$  direction, we simply need to know when the derivative of  $F(y, L)$  with respect to  $y$  becomes zero. Since  $F(y, L)$  is a piecewise function, we can find  $G(y, L)$ , integral of  $F$  such that  $d/dy G(y, L) = F(y, L)$ . Let  $C(x)$  be the integral of  $B'(x)$ .

$$C(x) = \begin{cases} 0, & \text{if } x < -3 \\ \text{anti-derivate of } P(x), & \text{if } -3 < x < -1/2 \\ \alpha x^2/2, & \text{if } -1/2 < x < 1/2 \\ \text{anti-derivate of } P(x), & \text{if } 1/2 < x < 3 \\ 0, & \text{if } x > 3 \end{cases} \quad (8)$$

Then by the fundamental theorem of calculus:

$$F(y, L) = \int_{y-L/2}^{y+L/2} B'(x) dx \quad F(y, L) = C(y + L/2) - C(y - L/2) \quad (9)$$

Call the edge of the bolus blown in the positive  $x$  direction that has the lower  $x$  value the high-side bolus and the edge with the higher  $x$  value the low-side bolus. This corresponds to the high-pressure side and the low-pressure side.

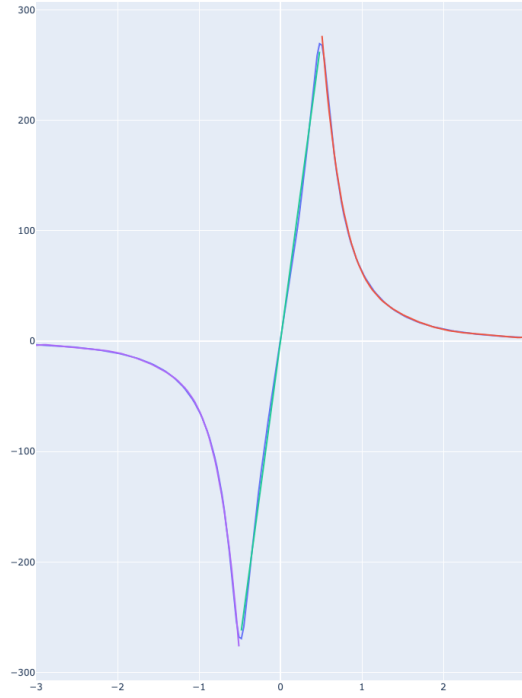


Fig. 5. Piecewise Representation of Field

It is clear the collapse point will not occur until low-side edge is past greater than  $1/2$ . It is clear it will occur before the high-side edge reaches  $1/2$ . We therefore need only consider the bolus straddling  $1/2$ . Let us assume that bolus  $L < 1$ , so that we need not consider the case that the high-side edge is the left of  $-1/2$ . Under these assumption, we have:

$$C(y + L/2) = \text{Antiderivative of } P(y+L/2) \quad (10)$$

$$C(y - L/2) = \alpha(y - L/2)^2/2 \quad (11)$$

Therefore the function  $F(y, L)$  is a simple polynomial, which we can differentiate:

$$F(y, L) = \quad (12)$$

$$F'(y, L) = \quad (13)$$

Plotting this function of  $x$ , we immediately find it matches our experience. If you put a bolus in a thin straw subject to a magnetic field and blow on it, it will be displaced a bit, increasing the force it exerts on your breath. At some point of increasing pressure, the bolus is displaced to a point that the magnetic force on it goes down, and the bolus explodes violently away from your breath, generally spattering and making a mess.

It is interesting to note that we obtain the same result by a different method. We can consider the potential energy of the bolus. Relying on the fact that the bolus always has a length of 2, the potential energy is the integral of the field



strength times the area:

$$PE(0) = \int_{-1}^1 dB(x)dx \quad (14)$$

$$PE(y) = \int_{-1+y}^{1+y} dB(x)dx \quad (15)$$

$$(16)$$

But change in potential energy is work, which is force times distance, so force is the change in potential energy divided by the distance:

$$F(y) = \frac{\Delta PE(y)}{\Delta y} \quad (17)$$

$$(18)$$

The limit of this as  $\Delta y$  goes to zero is just the derivative of  $PE(y)$ :

$$F(y) = PE'(y) \quad (19)$$

$$F(y) = \frac{dM}{1 + (3y/2)^6} \quad (20)$$

In other words, Theorem ?? gives us the same result as integrating the entire space, but gives us a shortcut of allowing us to integrate over only the edges, which in some cases can be replaced with simple algebra.

The extrema of this force  $F(x)$  occur at  $(-\frac{\sqrt[6]{2}}{3}, \frac{\sqrt[6]{2}}{3})$ , or  $(-1.0746, 1.0746)$ , which again accords with physical experience of how much a bolus can be displaced before it explodes. It must be remembered, however, that these calculations will change if the intensity of the magnetic field is modeled differently. Nonetheless, having a closed-form solution to this problem is valuable both for the intuition it provides and to allow direct design of bolus ferrofluid system.

### 6.3 The Channel-to-a-Chamber

Our goal, however, is to analyze a somewhat more complicated geometry. In particular, we are interested in a channel-leading-to-a-chamber, as depicted in Fig. ??.

We would ideally like to treat this as a one-dimensional problem as well. Let us assume once again that a magnetic field is placed at the center of our diagram (where the channel meets the chamber.) Call the region to the left A and the region to the right B, and assume the origin is at the center of the diagram.

The most important geometric feature of the channel-and-chamber valve design is the radius of the cylindrical magnet or magnets in it. It will be most convenient to take the radius of this magnet to be unity (1.0) and express all other spatial measures relative to that. The origin of our plane of operation will be the center of the top face of the cylindrical magnet, which is assumed to be beneath the plane, with its North face in contact with the plane.

Note: I believe using a magnetic force that falls off with the square of distance may not allow for a stable solution. I am not sure how to model this in a way which is realistic and convenient. Using the fourth power will probably work and may be realistic given of action is perpendicular to the magnet. I think I will try it.

In what follows, region A is the inlet region and region B is the outlet region; I should probably remove the labels A and B entirely since we are using B for the B-field.

Let us assume once again a magnetic field centered on the origin of strength

$$B(x) = \frac{1}{1 + x^2} \quad (21)$$

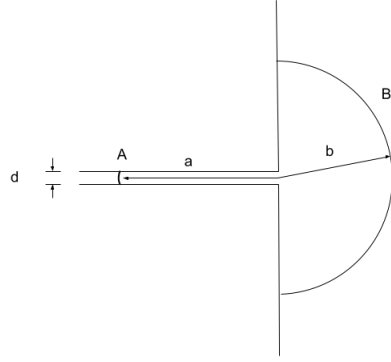


Fig. 6. Channel To Chamber Geometry

where  $x$  is the distance from the origin.

Since we have previously assumed that the bolus is a single blob, at least until it collapses, then we can relate  $a$  and  $b$  via the total area of the bolus  $S$ . Let us assume that the channel of width  $d$  is narrow enough that the slight arc of the fluid inside the inlet region can be treated as a straight line.

$$S = d \cdot a + \frac{1}{2}\pi b^2 \quad (22)$$

when  $a > 0$  and  $b > 0$ , so that the areas of  $A$  and  $B$  are both positive. It is most convenient to measure our position of the bolus, which is changing shape in this case, as the coordinate  $b$ , so we express  $a$  in terms of  $b$ . This can be solved for  $a$ :

$$a = -\frac{\pi b^2 - 2S}{2d} \quad (23)$$

Note that when  $a = 0$ , the dependence on  $d$  vanishes and:

$$b = \sqrt{\frac{2S}{\pi}} \quad (24)$$

$$b^2 = \frac{2S}{\pi} \quad (25)$$

Since at rest the bolus surface  $A$  will be in the same field strength as surface  $B$ , we can set:

$$B(a) = B(b) \quad (26)$$

$$B\left(-\frac{\pi b^2 - 2S}{2d}\right) = B(b) \quad (27)$$

This can be solved by WolframAlpha (results not included here.) As expected,  $b = -a$ , that is, both surfaces are equidistant from the center of our radially symmetric magnetic field.

#### 6.4 A strategy for computing pressure

Our fundamental goal is to compute the collapse and cracking pressure. The collapse pressure is determined by the maximum force needed to reduce  $b$  in the outlet region past some point we will call the *collapse point*. The cracking pressure is the pressure needed to drive the region  $A$ , represented by the variable  $a$ , to the *cracking point*, at which point a bubble forms and is pushed to the outlet.

The check valve we are trying to design will be characterized by both the collapse pressure and the cracking pressure, but in a single number we can define the performance of the valve as this ratio  $P$ , which we want to be as high as possible:

$$P = \frac{\text{Collapse Pressure}}{\text{Crack Pressure}} \quad (28)$$

If the bolus at rest is displaced, the potential energy increases, moving off a minimum. At a given point, the infinitesimal change in energy divided by the infinitesimal change in distance is equal to the force.

If we had a formula for the potential energy as a function of  $b$ , we would have a chance of computing this analytically.

Note: I am now concerned that the concept of the “force on the blob” may not even make sense. I may instead have to think in terms of force on one surface. The hydraulic principle suggests this; pascal’s principle is that pressure, not force is conserved in the blob.

Considering only the surfaces and using Eq. ??:

$$F_O^m(b) = \pi b B(b) \quad (29)$$

$$F_I^m(b) = dB(a) \quad (30)$$

$$F_I^m(b) = dB\left(-\frac{\pi b^2 - 2S}{2d}\right) \quad (31)$$

$$F^m(b) = F_I^m(b) + F_O^m(b) \quad (32)$$

$$F^m(b) = \frac{d}{1 + \left(-\frac{\pi b^2 - 2S}{2d}\right)^2} + \frac{\pi b}{1 + b^2} \quad (33)$$

Of course,  $F^m(b)$  is a function of  $S$  (the bolus area) and  $d$ , the channel width. For a physically realizable geometry, we might have  $d$  be about one-tenth the diameter of the bolus, and it might be convenient to have the diameter of the bolus be close to 1.

The cracking pressure will be the maximum force the magnet can exert on the blob in the range  $a < 0$ , because when  $a \geq 0$ , the air or water being valved will “bubble” out of the valve.

The force exerted by pressure is the pressure times the area, or, in our 2-dimentional model, the pressure times the length of an edge. Since our edges may be curved and our ferrofluid does not support shear forces, the force on a blob cause by a inlet or outlet pressure is the pressure times the length across the blob (as opposed to the arc length of the edge.)

We will denote the force on the blob due to magnetism by the superscript  $m$  and the force due to fluid pressure on the edges by  $p$ .

The outlet pressure force is thus:

$$F_o^p = P_o 2b \quad (34)$$

Collapse occurs when the outlet pressure force exceeds the maximum force  $F_o^m(b)$ . Both the magnetic force and the pressure force depend on the position of the edge given by  $b$ .

The collapse pressure will be the maximum force times the half-circumference in the range  $b \geq 0$ . By the time  $b$  decreases to  $b = 0$  (and probably before), the valve will explosively collapse.

Note: I now question whether this can explain the behavior of the valve at all.

It is clear that we want the bolus to be somewhat larger than the semi-circle of maximum magnetic force, so that as the outlet pressure is increased it will find this maximum point of force and hold itself against the highest outlet pressure possible without collapsing. (Nonetheless we must have a field monotonically increasing with  $b$  in order to sure inlet fluids “bubble away” from the blob to the outlet side.)

To find the collapse pressure, we want to find the  $b$  that maximizes of  $F(b)$ .

$$F(b) = \frac{d}{1 + \left(-\frac{\pi b^2 - 2S}{2d}\right)^2} + \frac{\pi b}{1 + b^2} \quad (35)$$

Recall however, the this derivation uses a suspect formula for  $B(x)$ . Possible we can design the magnetic field, for example using conical magnets, to make this valve more performant.

Important note: This math strongly suggest that the valve can be made twice as performant by opening the chamber completely, so that the valve is merely a chamber in the center of the bolus. Likewise thickening of the chamber (in the z-dimesion, violating our current 2-dimensional assuption) at the point of the strongest magent force would mean that more ferrofluid would have to be driven away at that point, increasing the potentially energy and therefore requiring more force.

However, even though we have modeled the fluid as not reacting to itself, we would not want the ferrofluid itself to strengthen the field on the inlet side, which might occur. Creating therefore an arc of something less than 360 degrees, such as 270 degrees, or making the walls of the thin chamber very thick, might solve this problem.

This math makes it clear that on the outlet side we want the bolus surface to be outside a strong field at  $r$  to that  $rB(r)$  is maximized to maximize the collapse pressure. Let us call the value of  $b$  that maximizes  $bB(b)$  the *collapse point*. We can now see that we want  $S$  to be large enough that the edge of the bolus at rest is beyond the collapse point by some comfortable margin.

However, to minimize the crack pressure, we want the bolus at rest in the region  $A$  inside the collapse point, because we don't want the inlet pressure to have to overcome that, even though on the inlet side the length  $d$  will be much smaller that length  $\pi b$ . However, by the Magnestatic Blob Minimization principle, this is impossible in a radially symmetric B field.

This suggest that on the inlet side we attempt to make the magnetic field, only within the channel, as gradual, linear, and long as possible. This might be possible to accomplish by simply adding a piece of ferrous metal under the channel, or by making the channel itself a steel needle. A specially designed magnet or two magnets could be used. This is somewhat depicted in Fig. ??.

## 6.5 Idea: An Improved Version

It is clear based on the above theory that allowing the bolus to be semi-circular is an arbitrary inefficiency. There is no reason not to allow a 360 degree bolus, as depicted in Figure ??.

## 6.6 Idea: a 3D version

It if we had a very powerful and very small dipole, it could in theory collect a complete sphere of ferrofluid around it. Possibly the tip of a conical magnet would nearly do this. In a spherical-like magnetic field, the same principle would apply, but the added dimensionality would be expected to further separate the collapse pressure from the crack pressue.

However, a simpler version of this idea based on accessible cylindrical bar magnets is depicted in Fig. ??. This has not yet been experimentally tested.

The advantage of having a 3D version predicted by the math is that the area of the small inlet port would be tiny compared to the area of the spherical shape. This would tend to produce a very performant valve, if the magnet field can be designed with sufficient strength. It would seem to be harder to manufacture in the intended use case of very small valves.

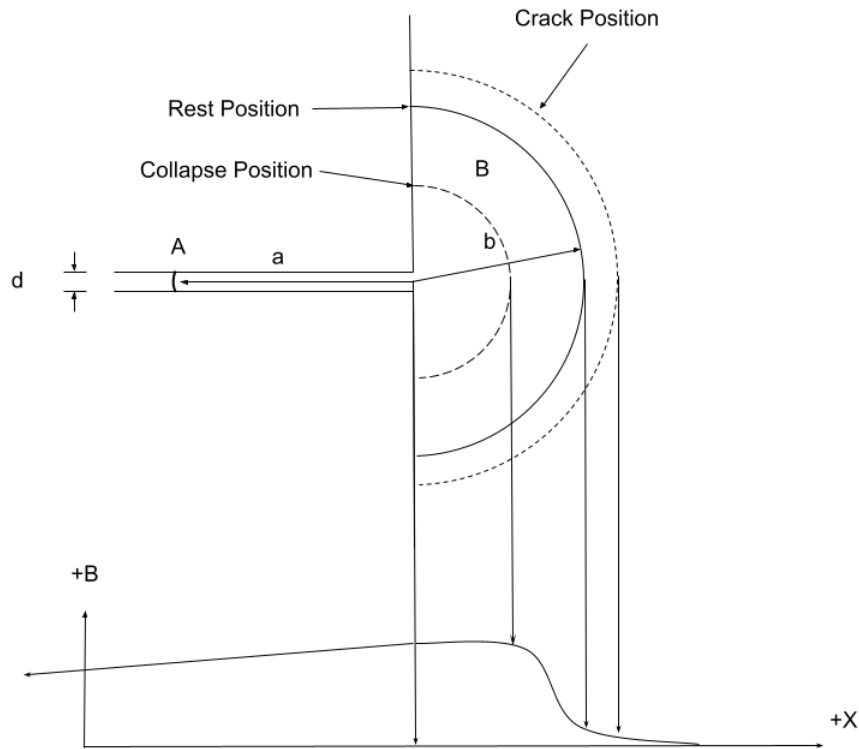


Fig. 7. (Rough) Ideal Field Profile

There are two obvious ways to manufacture such a 3D device, both of which require drilling a small hole into magnet. The first is use a small cylindrical or spherical magnet. This would be a dipole that could not be approximated as a radially symmetric field; but it might work anyway. The second would be to drill a hole through the axis of a conical magnet. Hopefully the concentration of field strength at the apex of the magnet would produce a roughly spherical field strength.

Possibly rather than using a sphere, a hemisphere would be more reasonable.

## 6.7 Idea

We can protect the valve from explosive collapse by using a separate valve that cracks before the collapse pressure, acting as a “pop-off” valve.

## 7 CONCLUSIONS

This paper demonstrates an apparently novel passive ferrofluid one-way valve or check valve (PFCV). This valve is completely passive in that it depends entirely on the pressure at the inlet port and the outlet port. The valve has no moving parts (except for the ferrofluid, which is almost stationary), and a remarkably simple design, consisting of nothing but a channel, an inlet chamber, and outlet chamber, and a bolus of ferrofluid in a static magnetic field.

Although no effort has been made to optimize the design, the pressure difference between the cracking pressure and the sustainable back pressure appear great enough to make an effective micropump. The performance of this one-way valve may improve with additional design effort; the authors sought to publish this result as soon as it was observed. Obvious future research possibilities are:

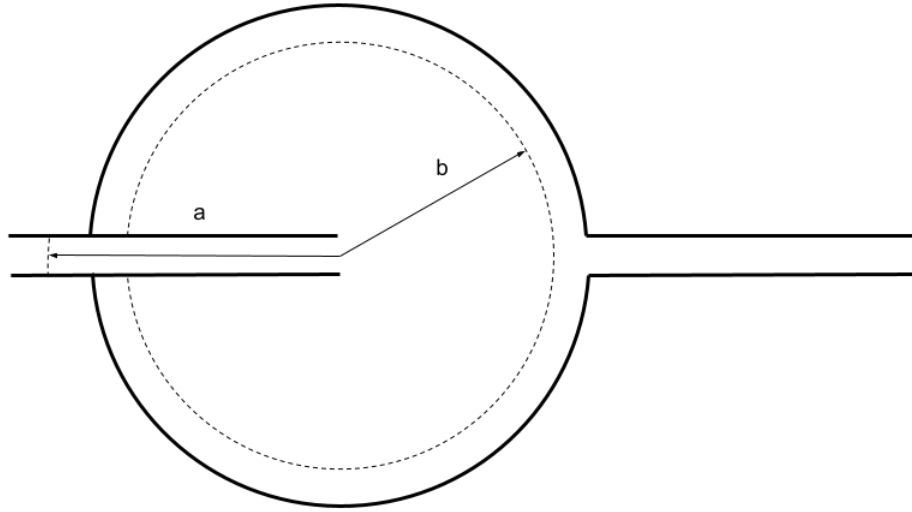


Fig. 8. 360 degree planar check valve, an improvement derived from theory

1. To improve the performance by varying the geometry of the passive design or shape and strength of the magnetic field.
2. Utilizing this design to make a micro-pump similar to earlier micro-pumps but with this simpler check valve design.
3. To provide an explanatory and predictive theory of operation, for example based on magnetic field strength as per [? ].
4. Studying the ability of the valve to recover after a collapse automatically when high outlet pressure is removed, which would increase robustness in some applications.

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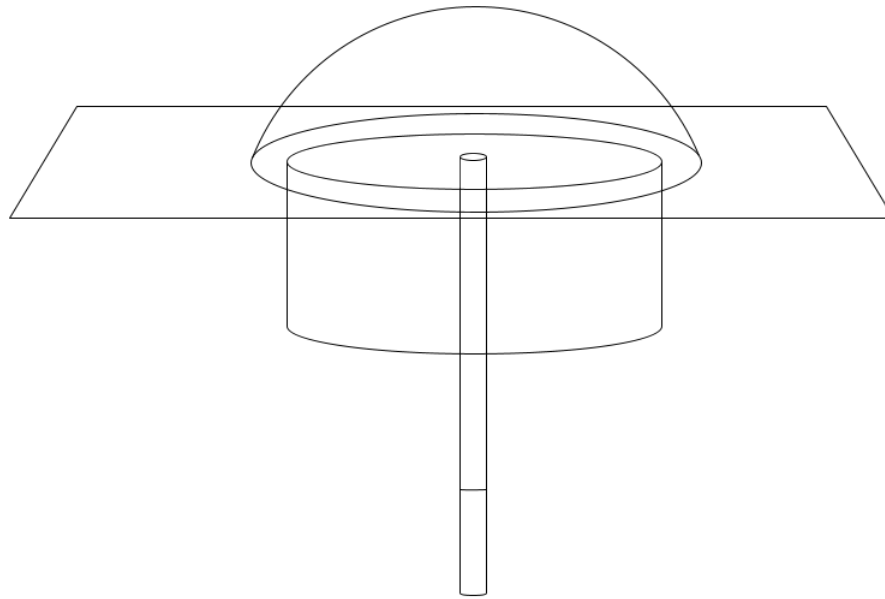


Fig. 9. Untested 3D check valve idea

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