A Novel Passive Ferrofluid One-way (Check) Valve Based on Energy Minimization

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ABSTRACT

Small pumps and valves enable flow management in microfluidic systems. A novel passive ferrofluid check valve is presented. The valve consists of only a unique channel-and-chamber geometry, ferrofluid, and a stationary magnetic field. The flow is determined only by the inlet and output pressure, and the magnetic field is completely static. The prototype valve and experimental setup are explained and performance of the valves cracking and collapse pressure reported. This initial design can be used for microfluid handling and lab-on-a-chip applications.

Additionaly we present a theory of operation based on energy minimization and compare predicted performance to actual performance.

1 INTRODUCTION

Ferrofluid can be manipulated by electronically controlled magnetic fields to exert force on fluids[1, 2, 3]. This makes it possible to build pneumatic or hydraulic devices, perhaps on very small scales, such as a single chip[4, 5], to miniaturize fluid handling. This has been proposed for biomedical purposes[6] that would use water or body fluids, although this paper reports only on experiments done with air. Miniature pumps and valves could be used to make a "lab on a chip" (LOC) or even to heat or cool different chip areas.

A fundamental component of such devices is the *check* or one-way valve. Two check valves on either side of a chamber whose volume can vary creates a positive displacement pump. A perfect check valve opens or *cracks* with minimal pressure on the inlet side and sustains maximal pressure on the outlet side before *collapse*, allowing fluid to flow in only one direction. Following[7] we call the maximum pressure differential the valve can resist in the direction it is intended to check or block (from outlet to inlet) the *sustainable* or *collapse* pressure.

This article is a brief report on an initial but functioning design of a passive ferrofluid check valve (PFCV) that has no moving parts except for the ferrofluid bolus itself, which is stationary in normal operation. By passive, the authors mean a check valve that functions without changes to the magnetic field affecting the bolus, whether that field is induced by a permanent magnet or an electromagnet. That is, the flow is determined purely by the difference between the inlet port pressure and the outlet port pressure. To our knowledge, no passive ferrofluid check valve has been previously reported, despite being an active area of research and despite such a valve having significant advantages for operation and especially fabrication over valves with moving parts.

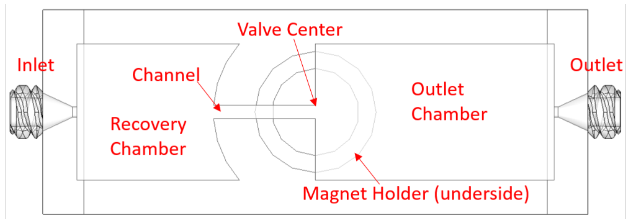

Fig. 1. The passive ferrofluid check valve components

2 RELATED RESEARCH

A number of papers report on ferrofluid pumps, focusing in particular on micropump and lab-on-a-chip applications[3, 8]. Many of these papers use a version of mechanical valve not based on passive ferrofluid, even though they move a ferrofluid bolus with a magnetic field. For example, a corrugated silicone micro valve[4, 9] has been reported. Other researchers use active valves, which require synchronization with the ferrofluid plug to form a pump, such as [10], which describes an active *T-Valve* with a moving ferrofluid plug, and [11] describes a complete fluid pump with valves that use active control of a ferrofluid bolus. At least two additional kinds of active valves, a *well valve* and *Y-valve*, have been described[7]. Active control is possible because the action of the plunger or bolus may be synchronized with the opening and closing of the valves. Nonetheless a passive valve would be simpler and less expensive, and would not require knowledge of the timing of the plunger.

An interesting functional micropump in which the moving ferrofluid bolus merges with a fixed ferrofluid valve and then separates on each pumping cycle has been described[5], but is not a one-way valve.

A passive ferrofluid two-way valve with tunable opening and closing pressure based on magnetic field strength[12] has been tested, but could not be passively used to make a pump.

This paper has not studied the closing pressure of the PFCV, but reports on the opening (or cracking) pressure (for flow from inlet to outlet) and sustainable (or collapse) pressure when the outlet pressure is higher than the inlet side.

3 PASSIVE FERROFLUID CHECK VALVE (PFCV) DESIGN

The PFCV depicted in Fig. 1 is a simple asymmetric volume centered in a magnetic field which holds a ferrofluid bolus in place. In the center of a radially symmetric magnetic field a narrow channel meets a larger open chamber at a right angle. The ferrofluid bolus is large enough that at rest in the field it forms a semi-circle in the open chamber. The narrow channel is longer than the radius of the bolus at rest. The broad chamber is the outlet side of the valve. The narrow chamber opens onto a recovery chamber on the inlet side of the valve. This design allows the bolus to be recovered from the recovery chamber when the pressure is equalized if the outlet pressure is raised above the collapse pressure, driving the bolus away from the magnetic field. The PFCV does not resist pressure as well as a valve of the same size made out of moving, solid parts. That is, the sustainable pressure it can resist on the outlet side before failing is relatively low, and pressure required to crack it open and allow flow is relatively high. However, it may operate reliably within a range of known pressures, and thus be sufficient to build a pump-on-a-chip. Furthermore, the PFCV reported here is a preliminary design which can probably be significantly improved. The authors found the existence of the PFCV worth sharing immediately.

4 METHOD

The valve depicted Fig. 1 was designed using Solidworks 2016. It is freely licensed via the CERN OHL Strong Reciprocal License[13, 14]. The model consists of a 15mm long, 2mm wide channel, a large outlet chamber, a recovery chamber, two female luers, a magnet holder ring and two legs to provide room for the magnet. All volumes are 2mm high. The 3D shape of the chambers can be thought as a 2mm high extrusion of a 2D shape. Viewed from the top, one end opens up to a recovery chamber of circular profile 30mm in diameter and the other an outlet chamber with a flat wall. A magnet holder ring 12.7mm inner diameter (1/2”) was created centered on the channel-chamber junction, below where the bolus is placed, to hold a permanent magnet in place at the center of the valve. When two magnets are used, the magnet on top naturally stays in the same position due to attraction to the magnet below. On the inlet

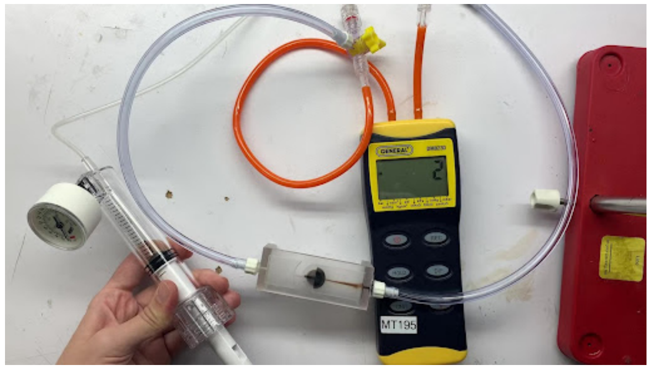

Fig. 2. Equipment set up

side of the channel opens into a recovery chamber shaped to allow the ferrofluid to be passively drawn back into the channel by the magnetic field after a collapse of the bolus. The model was printed on a Projet MJP 2500 (3D Systems, Rock Hill, SC), using Visijet M2G-CL and VisiJet M2 SUP as material and support respectively (3D Systems, Rock Hill, SC). Support material was removed by using an EasyClean system (3D Systems, Rock Hill, SC) and Dawn dish soap (Procter & Gamble, Cincinnati, OH) to remove residuals.

As shown in Fig. 2 and in our demonstration video[15], a basixCOMPAK 30atm pressurizing syringe (Merit Medical, South Jordan, UT) is connected to the model via a two-way stopcock (Qosina, Ronkonkoma, NY), tubing (Natvar, City of Industry, CA), and male (Injectech, Fort Collins, CO) and female luer (Qosina, Ronkonkoma, NY), allowing integration of a manometer (General Tools, Secaucus, NJ) to measure pressure. A 12.7mm x 25.4mm (1/2" x 1") cylindrical neodymium magnet (Apex Magnets, Petersburg, WV) with a pull force of 14.6 kg (32.24 pounds) was placed inside the magnet channel by means of a tight fit and 0.2 mL of ferrofluid (Apex Magnets, Petersburg, WV) was injected into the model using a 3mL syringe (BH Supplies, Jackson, NJ).

To obtain values, pressure was applied through the pressurizing syringe, as demonstrated in a video [15]. Pressure was first applied from the outlet side of the model. The maximum pressure difference from the outlet side that the valve can withstand before collapsing, will be referred to as the sustainable pressure or *collapse* pressure. Pressure applied from the inlet needed to initiate flow will be referred to as the *cracking* pressure.

The cracking pressure was first measured by increasing the pressure difference on the inlet side until flow is initiated (at which point the valve is “open” and pushing the syringe plunger faster simply increases flow without increasing the pressure.) Then the collapse pressure was measured by increasing the pressure difference on the outlet side. At pressures below the sustainable pressure, the valve holds pressure well with no observable leaks of air in the short time (a few minutes) of our experiment. When the sustainable pressure is exceeded, the bolus explodes violently into the recovery chamber. When the pressure difference is equalized, the bolus may passively recover into the central position, or it may need to be actively “combed” with a magnet back into the central position.

The procedure was performed first with one magnet, named the “Single Magnet” configuration, placed below the channel-chamber connection. The “Dual Magnet” configuration was performed with the magnet in the same position as the Single Magnet case in the same position, but with a second magnet of the same kind placed vertically on top of the model, arranged to be strongly attracted to the lower magnet.

5 RESULTS

The final pressures obtained demonstrate a clear difference between the inlet cracking pressure and outlet sustainable pressure, creating an effective passive check valve.

The ferrofluid had observable differences in behavior between the two configurations. After the pressure equalized following a collapse of the bolus due to exceeding the sustainable pressure, the single magnet configuration often repaired itself by drawing the fluid back into a centered bolus passively. After a collapse with two magnets, fluid further from the bolus remained stationary while the fluid closer was pulled back to the center. Following the removal of the top magnet, the stationary fluid then began to return to the bolus. This is consistent with the localization of

Table 1. Result pressures

Magnet configuration	Cracking Pressure kPa (mmHg)	Collapse Pressure kPa (mmHg)	Pressure Difference kPa (mmHg)	Approx. Ratio: Cracking to Collapse Pressure
Single	1.1 (8)	5.5 (41)	4.4 (33)	1:5
Dual	8.5 (64)	17.5 (131)	8.9 (67)	1:2

the magnetic field between two magnets, and the weakening of the magnetic field further from the channel-chamber juncture in the dual magnet configuration.

Although the dual magnet configuration demonstrated a larger absolute pressure difference due to magnetic field strength between the cracking and the collapse pressure, the single magnet configuration granted a larger ratio of collapse pressure to cracking pressure due to the much lower cracking pressure. The authors conjecture that the low cracking pressure may have been not only to the weaker magnetic field, but the weakening at the top of the 2mm high channel, which was further away from the magnet in the single magnet configuration.

6 THEORY OF OPERATION (BEST AS OF APR. 25)

Let us consider an ideal blob of ferrfoluid.

Assume that a blob in this section means an ideal 2-dimensional blob, approximated by a blob constrained into a very thin plane.
Assume that the B-field is purely perpendicalur to this plane, and that it completely dominates the magnetism induced in the ferrofluid itself.
Assume that all forces of viscosity and surface tension may be ignored.
Although we will not model the forces that make it so, we will assume that blob of ferrofluid is self-connected and will not split into more than one blob.
The blob is perfectly incompressible, and so has an absolutely unchanging area.
The magnetic particles are evenly distributed in the fluid no matter what the magnetic field, so that the potential energy in the fluid by being in the field depends only on the strength of the field.
We need not treat the motion, velocity, or inertia of our fluid in any way.

Under suitable assumptions, we can state two drastically simplifying assumptions.

Conjecture 1 (Magnetostatic Blob Minimization). *The magnetostaic force on an ideal 2-dimensional blob tends to minimize the potential energy in the blob and this occurs when its surfaces are in equal strength fields. Equivalently, every surface of a blob at rest with no outside forces is a line of equal field strength and all surfaces are equal.*

If the area is expanding the blob, it always decreases potential energy. If the area is contracting the blob, it always increasing potential energy. However, since the B-field at the two (or more) surfaces may be different, there may be a force exerted on the blob. In particular, we are interested in the force exerted on the blob counterbalancing the force exerted on by fluid pressure of non-ferrofluid at the surfaces.

Now imagine that a we have a monotone decreasing field centered on the origin which which is not necessarily circular, with the A region containg a thin tube, and the B region containing a wide open region. Imagine that the field in B is radially symmetric, but monotonically decreasing with distrance from the origin (but not necessarily the same in the region A.) A force resisting flow from the outlet to the inlet (from B to A) is equal to the field strength at a point multiplied by the surface length. In this scenario, the air pressure on the blob is a function of the length of the surface, but the magnetostatic force is also a function of length, so the length cancels out and becomes irrelevant. The collapse pressure is therefore the maximum force at any point between the rest position and the origin (this field is by construction equal on a semi-circular arc.) The collapse pressure also depends on the field strength at the point of the surface in A, but this surface has small length, and therefore contributes little to the sum of the force.

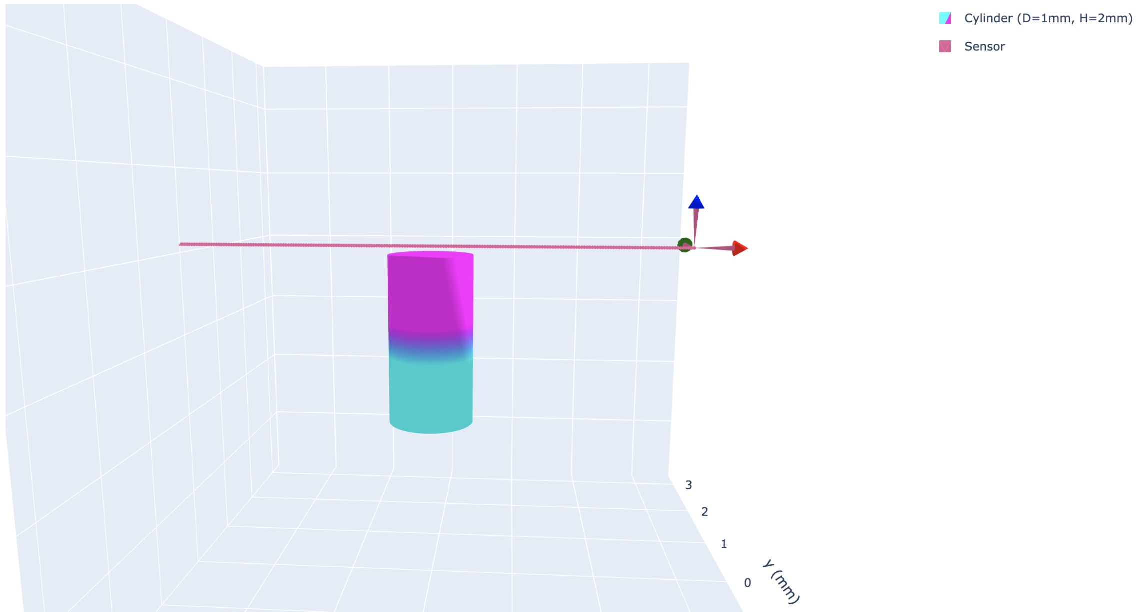

Fig. 3. Setup of the Bolus Problem

Conversely, the cracking pressure will be minimized by making the field strength in the region B as low as possible at the point when the ferrfofuid has been entirely driven into region A by air pressure from the inlet.

6.1 How the Valve Cracks

It is clear that a blob that has a bubble of non-ferrous fluid inside it has higher potential energy that the same blob without that bubble. Furthermore, such a bubble will feel a pressure to move away from an area of a higher magnetic strength towared an area of weaker magnetic strength. A bubble could be trapped, but if we assume that our field decreases monotonically from a central peak, a bubble will have a path to escape and will do so. In this scenario a valve will “crack” that, is, crack open allowing a fluid from the inlet to the outlet, as soon as the non-ferrous fluid crosses a central boundary.

6.2 Warmup: A Bolus in A Channel

A relatively simple problem is to consider a bolus of ferrofluid in thin channel. Placing a cylindrical permanent magnet beneath it tends to lock the bolus in place. If a small amount of air pressure is applied to from the West, the bolus will be displaced to the East, but will find a stable point in balance with the Westward forces exerted by the magnet. As the pressure is increased, the bolus will be displaced futher Eastward. It will reach a point where further displacement to the East does not increase the Westward force, and will soon decrease it. At this point the plug will fail catastrophically, spraying Eastward into the channel and letting air through.

The magnetic field in the line just above the face of the magnet can be analytically described by ellpitical integrals, but they are very difficult to work with symbolically, but easy to calculate numerically. MagPyLib is a convenient package for doing so. Figure 3 shows our setup of the problem.

By making assumptions, we can analyze this to obtain closed form solutions which match our experience and inform more complicated cases.

The red line represents the thin channel just slightly above the face of the magnet. MagPyLib conveniently computes the field along this line. In particular, only the x component (the East-West coordinate) of the B-field is relevant to us. The field was plotted with MagPyLib in Figure reffig:Bx. *Note: I do not understand the sign of this field. It appears to be wrong to me.*

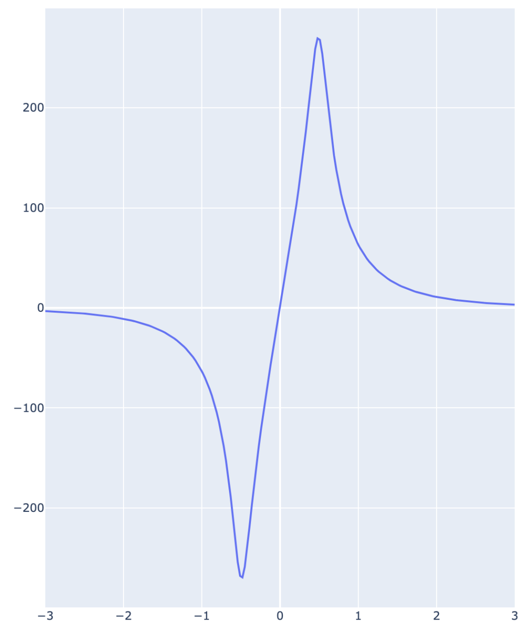

Fig. 4. Bx component of field

By inspection, we observe that this function can be accurately approximated with a piecewise polynomial representation breaking the function at the extrema, which are in fact the edges of the magnet. The resulting respresentation which is quadratic to the West of the magnet, linear above the magnet, and quadratic East of the magnet is remarkably accurate, under the assumption the MagPyLib is accurate. These three approximations are shown in Figure reffig:piecewise.

The piecewise approximation B' to the field B is:

$$p(x) = -18.5x^6 + -230.2x^5 + -1174.5x^4 + -3154.6x^3 + -4734.1x^2 + -3819.4x + -1339.6 \tag{1}$$

$$c(x) = 545.8x \tag{2}$$

$$q(x) = 18.5x^6 + -230.2x^5 + 1174.5x^4 + -3154.6x^3 + 4734.1x^2 + -3819.4x + 1339.6 \tag{3}$$

$$B(x) = \left\{ \begin{array}{ll} 0, & \text{if } x - leq - 3 \\ p(x), & \text{if } -3 < x < -1/2 \\ c(x), & \text{if } -1/2 < x < 1/2 \\ q(x), & \text{if } 1/2 < x < 3 \\ 0, & \text{if } x \geq 3 \end{array} \right\} \tag{4}$$

Note that $B(x)$ is an odd function; that is, $B(-x) = -B(x)$.

The numbers have been rounded to reflect the expected accuracy of the simulation and keep the expressions tractible. It would be preferable to find these constants analytically from the geometry and polarization of the magnet, but that is future work.

Where $P(x)$ and $Q(x)$ are nearly indentical 6-degree polynomials determined by NumPy's polyfit.

The piecewise approximations: $p(x)$, $c(x)$, and $q(x)$ are polynomials, which are easy to anti-differentiate to find the functions

P, C and Q, such that $P' = p, C' = c$, and $Q' = q$:

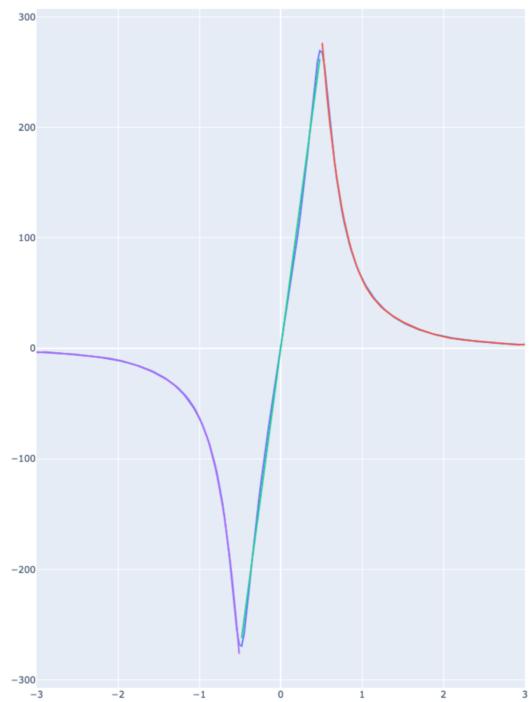

Fig. 5. Piecewise Representation of Field

$$
\begin{aligned}
P(x) &= -1339.6x - 1909.7x^2 - 1578.03x^3 - 788.65x^4 - 234.9x^5 - 38.3667x^6 - 2.64286x^7 && (5) \\
C(x) &= 272.9x^2 && (6) \\
Q(x) &= \ \ 1339.6x - 1909.7x^2 + \ \ 1578.03x^3 - 788.65x^4 + \ \ 234.9x^5 - 38.3667x^6 + \ \ 2.64286x^7 && (7) \\
& && (8)
\end{aligned}
$$

Setting aside issues of magnetic susceptibility of the ferrofluid used, the force is proportional to the field strength times the area. Since the area is unchanging in a simple tube, the force on a slice of ferrofluid at position x is proportional to $B'(x)$.

Now imagine that bolus has a length of L, and let the channel area be d, and let x be the midpoint of the bolus. Assume the magnet is a 1000 milliTesla (mT) polariazed magnet. The x-axis force on the bolus can be computed as the integral of dx:

The x-axis force on a bolus in the channel centered at y generally is proportional to:

$$F(y, L) = \int_{y-L/2}^{y-L/2} B'(x)dx \tag{9}$$

Let us assume that we are blowing air from the left in the positive x direction, in which case we can ignore cases when $y < 0$. It is clear from symmetry that $F(0, L) = 0$ for any L.

$$F(y, L) = \int_{y-L/2}^{y-L/2} B'(x)dx \tag{10}$$

In order to compute where the bolus will be when it explodes catastrophically when blown from the negative x direction to the positive x direction, we simply need to know when the derirvative of $F(y, L)$ with respect to y becomes zero. Since $F(y, L)$ is a piecewise function, we can find $G(y, L)$, integral of F such that $d/dyG(y, L) = F(y, L)$. Let $D(x)$ be the integral of $B'(x)$.

$$D(x) = \begin{Bmatrix} 0, & \text{if } x \leq -3 \\ P(x), & \text{if } -3 < x < -1/2 \\ C(x), & \text{if } -1/2 \leq x \leq 1/2 \\ Q(x), & \text{if } 1/2 < x < 3 \\ 0, & \text{if } x \geq 3 \end{Bmatrix} \tag{11}$$

Call the edge of the bolus blown in the positive x direciton that has the lower x value the high-side bolus and the edge with the higher x value the low-side bolus. This corresponds to the high-pressure side and the low-pressure side.

It is clear the collapse point will not occur until low-side edge is past greater than $1/2$. It is clear it will occur before the high-side edge reaches $1/2$. We therefore need only consider the bolus straddling $1/2$. Under these assumption, we have:

$$F(y, L) = \int_{y-L/2}^{y+L/2} B'(x)dx \tag{12}$$

$$F(y, L) = \int_{y-L/2}^{0.5} c(x)dx + \int_{0.5}^{y+L/2} q(x)dx \tag{13}$$

$$F(y, L) = C(x)\Big|_{y-L/2}^{0.5} + Q(x)\Big|_{0.5}^{y+L/2} \tag{14}$$

$$\begin{aligned} F(y, L) =& 272.90.5^2 - 272.9(y - L/2)^2 + \\ & 1339.6(y + L/2) - 1909.7(y + L/2)^2 + 1578.03(y + L/2)^3 \\ & - 788.65(y + L/2)^4 + 234.9(y + L/2)^5 - 38.3667(y + L/2)^6 + 2.64286(y + L/2)^7 \\ & - (1339.6(0.5) - 1909.7(0.5)^2 + 1578.03(0.5)^3 \\ & - 788.65(0.5)^4 + 234.9(0.5)^5 - 38.3667(0.5)^6 + \\ & 2.64286(0.5)^7) \end{aligned} \tag{15}$$

$$\begin{aligned} F(y, L) =& -278.875 - 272.9(y - L/2)^2 + 1339.6(y + L/2) - 1909.7(y + L/2)^2 + \\ & 1578.03(y + L/2)^3 - 788.65(y + L/2)^4 + 234.9(y + L/2)^5 \\ & - 38.3667(y + L/2)^6 + 2.64286(y + L/2)^7 \end{aligned} \tag{16}$$

Using Wolfram alpha and setting $L = 1$, the maximum force occurs at $y = 0.592, F \propto 142.4$, when the center of the bolus is just to the right of the break point. This accords with the graph inspection.

At $L = 2.0, y = 1.01766, F \propto 167.07$.

6.3 The Channel-to-a-Chamber

Our goal, however, is to analyze a somewhat more complicated geometry. In particular, we are interested in a channel-leading-to-a-chamber, as depicted in Fig. 6.

In that diagram, I have labelled the width of the channel d, but that is confusing when using derivatiaves, so henceforward the channel width will be represented by k. We would ideally like to treat this as a one-dimensional problem as well. Let us assume once again that a magnetic field is placed at the center at our diagram (where the channel meets the chamber.) Call the region to the left A and the region to the right B, and assume the orgin is at the center of the diagram.

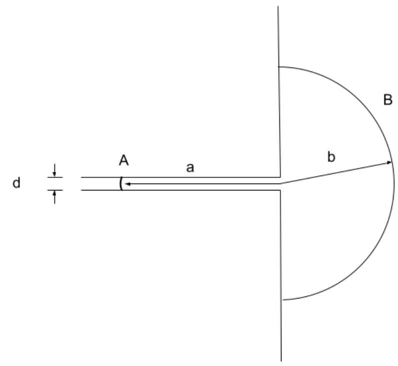

Fig. 6. Channel To Chamber Geometry

The most important geometric feature of the channel-and-chamber valve design is the radius of the cylindrical magnet or magnets in it. It will be most convenient to take the radius of this magnet to be unity (1.0) and express all other spatial measures relative to that. The origin of our plane of operation will be the center of the top face of the cylindrical magnet, which is assumed to be beneath the plane, with its North face in contact with the plane.

Note: I believe using a magnetic force that falls off with the square of distance may not allow for a stable solution. I am not sure how to model this in a way which is realistic and convenient. Using the fourth power will probably work and may be realistic given of action is perpendicular to the magnet. I think I will try it.

In what follows, region A is the inlet region and region B is the outlet region; I should probably remove the labels A and B entirely since we are using B for the B-field.

Let us assume once again a magnetic field centered on the origin as generated by a 1/2" by 1" cylindrical magnet as defined by Equation refeq:Bfield. Since that field is completely radially symmetric, we can use that equation to describe the B field in the plane, where the parameter of the function is the distance from the center of the face of the magnet.

In moving to a 2-dimensional representation, we move to polar coordinates centered at the point the point where the channel and the chamber meet. Technically, the B field is a field of 3-dimensional vectors; but by working only in a plane, we treat is as a function of r, the distance from the orgin, and θ, the angle of rotation from the x-axis. However, since the B field is completely symmetric, we can think of it as a one-dimensional function of distance from the origin $B(r)$. Essentially, we are treating the B field as the absolute value of the field we treated in the case of the 1-dimensional bolus. This can be interpreted as the strength drawing towards the origin, however, since we deduce everything from this point in terms of change in potential energy, we need not think of it as signed at all. In this interpretation, we can simplify $B(x)$:

$$B(x) = \left\{ \begin{array}{ll} c(x), & \text{if } 0 \leq x \leq 1/2 \\ q(x), & \text{if } 1/2 < x < 3 \\ 0, & \text{if } x \geq 3 \end{array} \right\} \tag{17}$$

Since we have previously assumed that the bolus is a single blob, at least until it collapses, then we can can relate a and b via the total area of the bolus S. Let us assume that the channel of width k is narrow enough that the slight arc of the fluid inside the inlet region can be treated as a straight line.

$$S = k \cdot a + \frac{1}{2}\pi b^2 \tag{18}$$

when $a \geq 0$ and $b \geq 0$, so that the areas of A and B are both positive. It is most convenient to measure our position of the bolus, which is changing shape in this case, as the coordinate b, so we we express a in terms of b. Since we will use this algebraic relationship later, we will define it as a function A: This can be solved for a:

$$a = A(b) = \frac{2S - \pi b^2}{2k} \tag{19}$$

Note that when $a = 0$, the dependence on d vanishes and:

$$\begin{align} b =& \sqrt{\frac{2S}{\pi}} \tag{20} \\ & \tag{21} \end{align}$$

Since at rest the bolus surface A will be in the same field strength as surface B, we can set:

$$\begin{align} B(a) =& B(b) \tag{22} \\ B(\frac{2S - \pi b^2}{2k}) =& B(b) \tag{23} \end{align}$$

This can be solved by WolframAlpha (results not incuded here.) As expected, $b = -a$, that is, both surfaces are equidistant from the center of our radially symmetric magnetic field.

We will late use the value of S which occurs when $a = 0.5$, $b = 0.5$, $d = 0.05$, where:

$$S = \frac{1 + 5\pi}{40} \tag{24}$$

6.4 A strategy for computing pressure

Our fundamental goal is to compute the collapse and cracking pressure. The collapse pressure is determined by the maximum force needed to reduce b in the outlet region past some point we will call the *collapse point.* The cracking pressure is the pressure needed to drive the region A, represented by the variable a, to the *cracking point*, at which point a bubble forms and is pushed to the outlet.

The check valve we are trying to design will be characterized by both the collapse pressure and the cracking pressure, but in a single number the can define the performance of the valve as this ratio P, which we want to be as high as possible:

$$P = \frac{\text{Collapse Pressure}}{\text{Crack Pressure}} \tag{25}$$

If the bolus at rest is displaced, the potential energy increases, moving off a minimum. At a given point, the infinitessimal change in energy divided by the infinitessimal change in distance is equal to the force.

If we had a formula for the potential energy as a function of b, we would have a chance of computing this analytically.

In this model, which is using a the narrowness of the plane to simplify the problem. In this simplified model, we can assume that the potential energy of the magnetic system can be simply modeled by the presence or absence of ferrofluid at a point in the plane. The presence of ferrofluid at a point or infinitessimal area (da) decreases the potential energy U of the entire system in proportion to the strength of the field at a point, and that that strength is describe by our function $B(r)$ where r is the distance from the origin (and thus the center of the face of the magnet). We can thus describe the potential energy as an integral over surface patches da:

$$U = -\int \left\{ \begin{array}{ll} B(a), & \text{if there is ferrofluid at } a \\ 0, & \text{if there is no ferrofluid at } a \end{array} \right\} da \tag{26}$$

We are furthermore ignoring all effect of the ferrofluid interacting with itself in terms of the potential energy. However, we rely on the very important property of ferrofluid that it tends to clump together into a single blob in the presence of a magentic field. We will, in fact, assume that our blob or bolus is perfectly hemispherical in the chamber, and a perfect rectangle in the channel. Furthermore, we assume the fluid is incompressible, and therefore always as the aread S. This is true until the bolus collapses; we know that when it collapses it explodes drastically, and we do not attempt to model anything after a collapse event.

Therefore, we in fact have a one dimensional system of the coordinate b, which it is convenent to break into $U = U_A + U_B$. (The channel and the chamber, respectively.) We assume that the channel A is narrow enough that we can model the field across the channel as if it is rectilinear. We can treat these as line integrals:

$$U_B = -\pi \int_0^b rB(r)dr \tag{27}$$

$$U_N = -k \int_0^a B(r)dr \tag{28}$$

$$U = U_B + U_N \tag{29}$$

This expresses our fundamental situation. Because force applied in the chamber moves the bolus into the channel, and force applied in the channel moves the bolus into the chamber, the potential energy changes in both. The potential energy always goes down as more fluid is added to one side (or at least never goes down), and thereofore always goes up in the chamber from which it is removed. The minimum potential energy is a point of balance. We seek a mathematical description of the change in totoal potential energy as b changes so that we can compute the force. This should allow us to choose S to be most advantageous. Eventually, this may allow us to design different magnetic fields to change the performance of the valve.

Let us define b_0 as the value of b when the pressure in the chamber and the channel are equal, so that the bolus is at rest. This is the point of minimum potential energy. Let $F(b)$ be the force exerted by the magnet on the bolus. Then $F(b_0) = 0$. The potential energy increases as the surrounding gases do work on it (up to the point of collapse or crack.) Within that range, the forces are conservative; the potential energy could in theory be recovered as the bolus spring back to position if the pressure on the bolus was removed. The force exerted by the magnet is positive if $b < b_0$, and negative if $b > b_0$. Then we have:

$$\frac{dU}{db} = -F(b) \tag{30}$$

The should allow us to compute the $F(b)$ from U.

But before attempting that, we simply observe that the bolus will will collapse when the force on it caused by pressure from the right (in the chamber) drives it to the point where the force resiting it is maximized, which will occur when the derivative of $F(b)$ with respect to b is zero. (The behavior is different when the pressure is from the left.) Thus we seek the values of b for which:

$$0 = \frac{dF(b)}{db} \tag{31}$$

$$0 = \frac{d^2U}{db^2} \tag{32}$$

$$0 = \frac{d^2}{db^2}U_B + \frac{d^2}{db^2}U_N \tag{33}$$

$$0 = -\frac{d}{db}(\pi\frac{d}{db}\int_0^b rB(r)dr) + -\frac{d}{db}(k\frac{d}{db}\int_0^a B(r)dr) \tag{34}$$

Let us treat subparts here, using our knowledge of the piecewise definition of $B(x)$.

$$\frac{d}{db}\int_0^b B(r)r = \frac{d}{db}\int_0^b \begin{Bmatrix} r\cdot c(r)dr, \text{ if } -1/2 < r \leq 1/2 \\ r\cdot q(r)dr, \quad \text{if } 1/2 < r < 3 \\ 0dr, \quad \text{if } r > 3 \end{Bmatrix} \tag{35}$$

$$\frac{d}{db}\int_0^b B(r)r = \begin{Bmatrix} b\cdot c(b), \text{ if } 0 \leq b \leq 1/2 \\ b\cdot q(b), \text{ if } 1/2 < b < 3 \\ 0, \quad \text{if } r > 3 \end{Bmatrix} \tag{36}$$

Having cancelled the differentiation and integration in this way, we now apply another differentiation:

$$\frac{d}{db}\begin{Bmatrix} b\cdot c(b), \text{ if } 0 \leq b \leq 1/2 \\ b\cdot q(b), \text{ if } 1/2 < b < 3 \\ 0, \quad \text{if } r > 3 \end{Bmatrix} = \begin{Bmatrix} b\cdot c'(b) + c(b), \text{ if } 0 \leq b \leq 1/2 \\ b\cdot q'(b) + c(b), \text{ if } 1/2 < b < 3 \\ 0, \quad \text{if } r > 3 \end{Bmatrix} \tag{37}$$

Now we need to consider the potential energy in the channel U_N, and substitue $A(b)$ for a:

$$\frac{d}{db}\int_0^a B(r) = \frac{d}{db}\int_0^a \begin{Bmatrix} c(x), \text{ if } 0 \leq x \leq 1/2 \\ q(x), \text{ if } 1/2 < x < 3 \\ 0, \quad \text{if } x \geq 3 \end{Bmatrix} \tag{38}$$

$$\frac{d}{db}\int_0^a B(r) = \begin{Bmatrix} c(a), \text{ if } 0 \leq a \leq 1/2 \\ q(a), \text{ if } 1/2 < a < 3 \\ 0, \quad \text{if } a \geq 3 \end{Bmatrix} \tag{39}$$

$$\frac{d}{db}\int_0^a B(r) = \begin{Bmatrix} c(A(b)), \text{ if } 0 \leq A(b) \leq 1/2 \\ q(A(b)), \text{ if } 1/2 < A(b) < 3 \\ 0, \quad \text{if } a \geq 3 \end{Bmatrix} \tag{40}$$

We can now substitute these into Equation 34:

$$
\begin{aligned}
0 =& -\frac{d}{db}(\pi\frac{d}{db}\int_0^b rB(r)dr) + -\frac{d}{db}(k\frac{d}{db}\int_0^a B(r)dr) && (41)\\
0 =& -\frac{d}{db}(\pi\begin{Bmatrix} b\cdot c'(b)+c(b), \text{ if } 0 \leq b \leq 1/2 \\ b\cdot q'(b)+q(b), \text{ if } 1/2 < b < 3 \\ 0, \qquad\qquad\qquad \text{if } r > 3 \end{Bmatrix})+ && (42)\\
& -\frac{d}{db}(k\begin{Bmatrix} c(A(b)), \text{ if } 0 \leq A(b) \leq 1/2 \\ q(A(b)), \text{ if } 1/2 < A(b) < 3 \\ 0, \qquad\qquad\quad \text{if } a \geq 3 \end{Bmatrix}) && (43)
\end{aligned}
$$

Equation 43 is a model for how we can find the extrema of force in different geometries. The number $1/2$ is of course the radius of the magnet; if we have a different geometry this equation would look different in specifics but would be generally the same if we use piecewise approximations of the B field, which seems likely to be necessary.

Unfortunately, this equation may require a careful case analysis based on the choices of k and S. Let us attempt to do this for a particular assumption. Figure 7 and Figure 4 suggest that S should be chosen so that b_0 is greater than $1/2$, and perhaps at most than 1. Let us attempt to analyze this with $b_0 = 3/4$. Let us set $k = 1/10$, which seems reasonable to manufacture. In this case,

$$S = 3/40 + (\pi 9/32) \approx 0.9585729 \tag{44}$$

We can deonte this: $A^{1/10}_{3/4}(b)$:

$$A^{1/10}_{3/4}(b) = 9.58575 - 5\pi b^2 \tag{45}$$

Now if we solve Equation 43 under the assumption that b_m is at most $1/2$, and then under the assumption that b_m is more than $1/2$, and compare these results.

Case $b_m \leq 1/2$:

In this case, $a = A^{1/10}_{3/4}(1/2) \approx 5.65876$ which is greater than 3, which allows us to simplify our base equation Equation 43 becomes:

$$
\begin{aligned}
0 =& -\frac{d}{db}(\pi b\cdot c'(b)+c(b)) + -\frac{d}{db}0 && (46)\\
0 =& \pi(\frac{d}{db}b\cdot c'(b) + \frac{d}{db}c(b)) && (47)\\
0 =& \pi\frac{d}{db}(b\cdot c'(b)) + c'(b) && (48)\\
0 =& \pi(bc''(b) + 2c'(b)) && (49)\\
0 =& \pi(0 + 2 * 545.8) && (50)\\
0 =& \pi 2 * 545.8 && (51)\\
& && (52)
\end{aligned}
$$

This is a contradiction. Possibly this means that this case cannot occur—possibly it means I have done the math wrong.

Case $b_m > 1/2$:

In this case, we know that $b_m < 3/4$. On the entire range $1/2 \leq b \leq 3/4$, $A_{3/4}^{1/10}(b) > 1/2$, which allows us to simplify our piecewise construction:

$$0 = -\frac{d}{db}\pi(b \cdot q'(b) + q(b)) + -\frac{d}{db}(kq(A(b))) \tag{53}$$

$$0 = \pi\frac{d}{db}(b \cdot q'(b) + q(b)) + \frac{1}{10}\frac{d}{db}q(A(b)) \tag{54}$$

$$0 = \pi(b \cdot q''(b) + 2q'(b)) + \frac{1}{10}\frac{d}{db}q(9.58575 - 5\pi b^2) \tag{55}$$

$$0 = \pi(b \cdot q''(b) + 2q'(b)) + \frac{1}{10} \cdot -10\pi bq'(9.58575 - 5\pi b^2) \tag{56}$$

$$0 = \pi(b \cdot q''(b) + 2q'(b)) + -\pi bq'(9.58575 - 5\pi b^2) \tag{57}$$

$$0 = bq''(b) + 2q'(b) + -bq'(9.58575 - 5\pi b^2) \tag{58}$$

$$\begin{aligned} 0 =& (555.b^5 - 4604.b^4 + 14094.b^3 - 18927.6b^2 + 9468.2b) + \\ & 2(111b^5 - 1151b^4 + 4698b^3 - 9463.8b^2 + 9468.2b - 3819.4) + \\ & - b(-1.06151 \cdot (10)^8 b^{10} + 2.53817 \cdot (10)^8 b^8 - 2.42467 \cdot (10)^8 b^6 \\ & + 1.15662 \cdot (10)^8 b^4 - 2.75489 \cdot (10)^7 b^2 + 2.62095 \cdot (10)^6) \end{aligned} \tag{59}$$

$$\begin{aligned} 0 =& 777b^5 - 6906b^4 + 23490b^3 - 37855.2b^2 + 28404.6b - 7638.8 \\ & + 1.06151 * (10)^8 b^{11} - 2.53817 * (10)^8 b^9 + 2.42467 * (10)^8 b^7 - 1.15662 * (10)^8 b^5 \\ & + 2.75489 * (10)^7 b^3 - 2.62095(10)^6 b \end{aligned} \tag{60}$$

$$\begin{aligned} 0 =& 1.06151 \cdot (10)^8 b^1 1 - 2.53817 \cdot (10)^8 b^9 + 2.42467 \cdot (10)^8 b^7 \\ & - 1.15661 \cdot (10)^8 b^5 - 6906b^4 + 2.7572410^7 b^3 - 37855.2b^2 - 2.5925510^6 b - 7638.8 \end{aligned} \tag{61}$$

According to Wolframalpha, the only real solution positive solution is: $b \approx 0.632115$. I don't trust that I haven't made a mistake, but this seems to be a plausible solution.

NOTE: Now I am out of steam. I suppose that we can do this differentiation if we choose S and k, and then we will have a polynomial in b, which we can solve to get a b value. But the piecewise nature makes it a bit complicated. I will be simplified after we choose S.

Qualitatively, we assume that at rest (ambient pressure), the bolus exerts no force. As the pressure in the chamber increases, the force (the increase in potential energy) will increase as b is decreased and the bolus is disturbed from its rest position. This force on the bolus and resiting force exerted by the bolus will increase as b decreased further from its rest postion. However, eventuall this force will cease to increase and reach its maximum. At that point, the valve will collapse.

Our strategy is thus to find the value of b b_m that maximizes the force (maximizes the change in potential energy dU). The force at b_m, $F(b_m)$ will be equal to the pressure times πb_m, and that will be the collapse pressure P_c.

$$F(b_m) = \pi b_m P_c \tag{62}$$

$$P_c = \frac{F(b_m)}{\pi b_m} \tag{63}$$

The cracking pressure P_r will be the force necessary to reach $a = 0$, because at that point, the non-magnetic fluid will form a "bubble" which will be pinched off and displaced into the chamber by the ferrofluid. This "bubble" will be forced in the direction of decreasing magnetic field strength, because it minimizes potential energy to fill the strong field with ferrofluid preferentially over non-magnetic fluid. This occurs at $a = 0$, so $A(b) = 0$, or $b = \sqrt{\frac{2S}{\pi}}$ by Equation 21.

$$P_r d = F(\sqrt{\frac{2S}{\pi}}) \tag{64}$$

6.5 Conjecture

Important note: This math strongly suggest that the valve can be made twice as performant by opening the chamber completely, so that the valve is merely a chamber in the center of the bolus. Likewise thickening of the chamber (in the z-dimesion, violating our current 2-dimensional assuption) at the point of the strongest magent force would mean that more ferrofluid would have to be driven away at that point, increasing the potentially energy and therefore requiring more force.

However, even though we have modeled the fluid as not reacting to itself, we would not want the ferrofluid itself to strengthen the field on the inlet side, which might occur. Creating therefore an arc of someething less than 360 degrees, such as 270 degrees, or making the walls of the thin chamber very thick, might solve this problem.

This math makes it clear that on the outlet side we want the bolus surface to be outside a strong field at r to that $rB(r)$ is maximized to maximize the collapse pressure. Let us call the value of b that maximizes $bB(b)$ the *collapse point.* We can now see that we want S to be large enough that the edge of the bolus at rest is beyond the collapse point by some comfortable margin.

However, to minimize the crack pressure, we want the bolus at rest in the region A inside the collapse point, because we don't want the inlet pressure to have to overcome that, even though on the inlet side the length d will be much smaller that length πb. However, by the Magnestatic Blob Minimization principle, this is impossible in a radially symmetric B field.

This suggest that on the inlet side we attempt to make the magnetic field, only within the channel, as gradual, linear, and long as possible. This might be possible to accomplish by simply adding a piece of ferrous metal under the channel, or by making the channel itself a steel needle. A specially designed magnet or two magnets could be used. This is somewhat depicted in Fig. 7.

6.6 Idea: An Improved Version

It is clear based on the above theory that allowing the bolus to be semi-circular is an arbitrary inefficiency. There is no reason not to allow a 360 degree bolus, as depicted in Figure 8.

6.7 Idea: a 3D version

It if we had a very powerful and very small dipole, it could in theory collect a complete sphere of ferrofluid around it. Possibly the tip of a conical magnet would nearly do this. In a spherical-like magnetic field, the same principle would apply, but the added dimensionality would be expected to further separate the collapse pressure from the crack pressue.

However, a simpler version of this idea based on accessible cylindrical bar magnets is depicted in Fig. 9. This has not yet been experimentally tested.

The advantage of having a 3D version predicted by the math is that the area of the small inlet port would be tiny compared to the area of the spherical shape. This would tend to produce a very performant valve, if the magnet field can be designed with sufficient strength. It would seem to be harder to manufacture in the intended use case of very small valves.

There are two obvious ways to manufacture such a 3D device, both of which require drilling a small hole into magnet. The first is use a small cylidrical or spherical magnet. This would be a dipole that could not be approximated as a radially symmetric field; but it might work anyway. The second would be to drill a hole through the axis of a conical magnet. Hopefully the concentration of field strength at the apex of the magnet would produce a roughly spherical field strength.

Possibly rather than using a sphere, a hemisphere would be more reasonable.

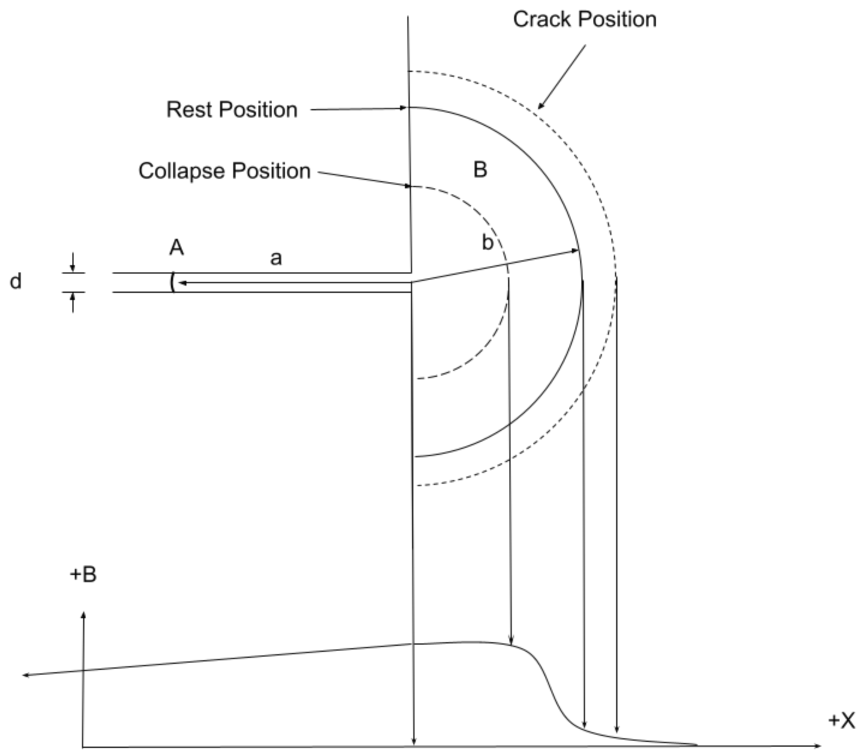

Fig. 7. (Rough) Ideal Field Profile

6.8 Idea

We can protect the valve from explosive collapse by using a separate valve that cracks before the collapse pressure, acting as a "pop-off" valve.

7 CONCLUSIONS

This paper demonstrates an apparently novel passive ferrofluid one-way valve or check valve (PFCV). This valve is completely passive in that it depends entirely on the pressure at the inlet port and the outlet port. The valve has no moving parts (except for the ferrofluid, which is almost stationary), and a remarkably simple design, consisting of nothing but a channel, an inlet chamber, and outlet chamber, and a bolus of ferrofluid in a static magnetic field.

Although no effort has been made to optimize the design, the pressure difference between the cracking pressure and the sustainable back pressure appear great enough to make an effective micropump. The performance of this one-way valve may improve with additional design effort; the authors sought to publish this result as soon as it was observed. Obvious future research possibilities are:

1. To improve the performance by varying the geometry of the passive design or shape and strength of the magnetic field.
2. Utilizing this design to make a micro-pump similar to earlier micro-pumps but with this simpler check valve design.
3. To provide an explanatory and predictive theory of operation, for example based on magnetic field strength as per [11].
4. Studying the ability of the valve to recover after a collapse automatically when high outlet pressure is removed,

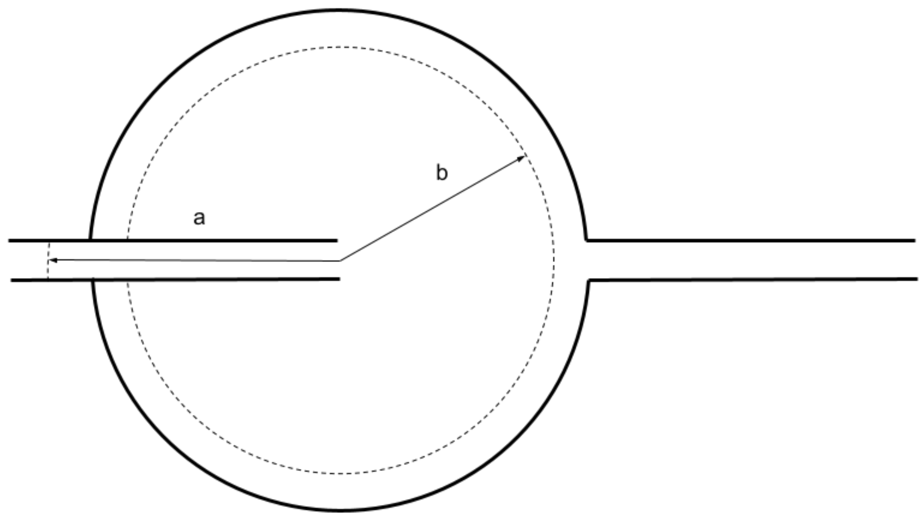

Fig. 8. 360 degree planar check valve, an improvement derived from theory

which would increase robustness in some applications.

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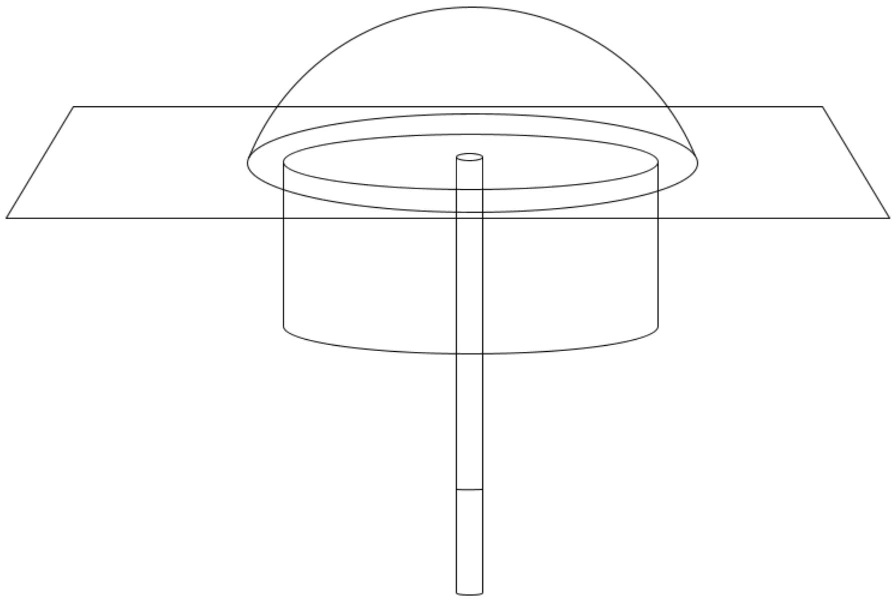

Fig. 9. Untested 3D check valve idea

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