8

PID Controllers and Modified PID Controllers

8–1 INTRODUCTION

In previous chapters, we occasionally discussed the basic PID controllers. For example, we presented electronic, hydraulic, and pneumatic PID controllers. We also designed control systems where PID controllers were involved.

It is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

Because most PID controllers are adjusted on-site, many different types of tuning rules have been proposed in the literature. Using these tuning rules, delicate and fine tuning of PID controllers can be made on-site. Also, automatic tuning methods have been developed and some of the PID controllers may possess on-line automatic tuning capabilities. Modified forms of PID control, such as I-PD control and multi-degrees-of-freedom PID control, are currently in use in industry. Many practical methods for bumpless switching (from manual operation to automatic operation) and gain scheduling are commercially available.

The usefulness of PID controls lies in their general applicability to most control systems. In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful. In the field of process control systems, it is well known that the basic and modified PID control schemes have proved their usefulness in providing satisfactory control, although in many given situations they may not provide optimal control.

In this chapter we first present the design of a PID controlled system using Ziegler and Nichols tuning rules. We next discuss a design of PID controller with the conventional frequency-response approach, followed by the computational optimization approach to design PID controllers. Then we introduce modified PID controls such as PI-D control and I-PD control. Then we introduce multi-degrees-of-freedom control systems, which can satisfy conflicting requirements that single-degree-of-freedom control systems cannot. (For the definition of multi-degrees-of-freedom control systems, see Section 8–6.)

In practical cases, there may be one requirement on the response to disturbance input and another requirement on the response to reference input. Often these two requirements conflict with each other and cannot be satisfied in the single-degree-of-freedom case. By increasing the degrees of freedom, we are able to satisfy both. In this chapter we present two-degrees-of-freedom control systems in detail.

The computational optimization approach presented in this chapter to design control systems (such as to search optimal sets of parameter values to satisfy given transient response specifications) can be used to design both single-degree-of-freedom control systems and multi-degrees-of-freedom control systems, provided a fairly precice mathematical model of the plant is known.

Outline of the Chapter. Section 8–1 has presented introductory material for the chapter. Section 8–2 deals with a design of a PID controller with Ziegler–Nichols Rules. Section 8–3 treats a design of a PID controller with the frequency-response approach. Section 8–4 presents a computational optimization approach to obtain optimal parameter values of PID controllers. Section 8–5 discusses multi-degrees-of-freedom control systems including modified PID control systems.

8–2 ZIEGLER–NICHOLS RULES FOR TUNING PID CONTROLLERS

PID Control of Plants. Figure 8–1 shows a PID control of a plant. If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical or computational approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values K_p, T_i, and T_d) based on experimental step responses or based on the value of K_p that results in marginal stability when only proportional control action is used. Ziegler–Nichols rules, which are briefly presented in the following, are useful when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical

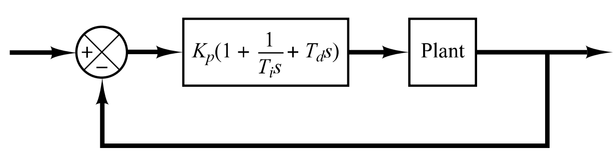

Figure 8–1 PID control of a plant.

models.) Such rules suggest a set of values of K_p, T_i, and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable. In such a case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings for K_p, T_i, and T_d in a single shot.

Ziegler–Nichols Rules for Tuning PID Controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature and from the manufacturers of such controllers.)

There are two methods called Ziegler–Nichols tuning rules: the first method and the second method. We shall give a brief presentation of these two methods.

First Method. In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 8–2. If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped, as shown in Figure 8–3. This method applies if the response to a step input exhibits an S-shaped curve. Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants, delay time L and time constant T. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure 8–3. The transfer

Figure 8–2
Unit-step response of a plant.

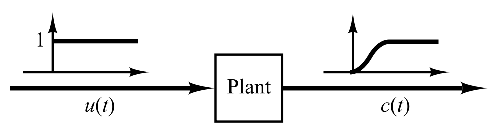

Figure 8–3
S-shaped response curve.

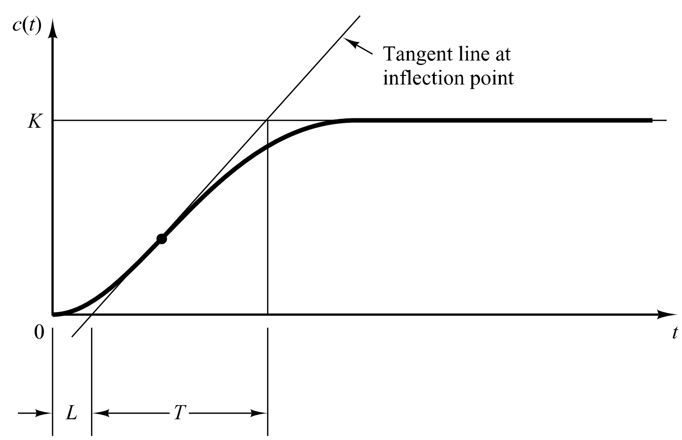

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second Method. In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 8–4), increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally

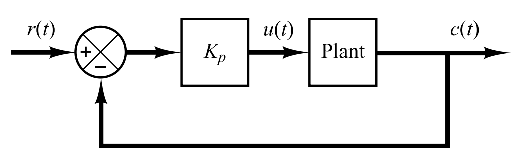

Figure 8–4 Closed-loop system with a proportional controller.

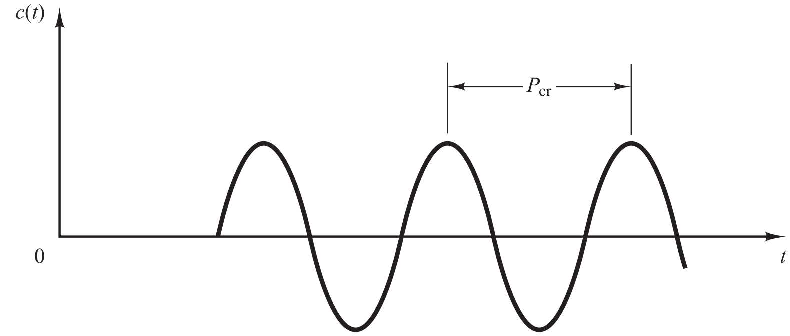

Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

determined (see Figure 8–5). Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 8–2.

Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations ω_{cr}, where $2\pi/\omega_{cr} = P_{cr}$. These values can be found from the crossing points of the root-locus branches with the $j\omega$ axis. (Obviously, if the root-locus branches do not cross the $j\omega$ axis, this method does not apply.)

Comments. Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known. Over many years, such tuning rules proved to be very useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If the plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules.)

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

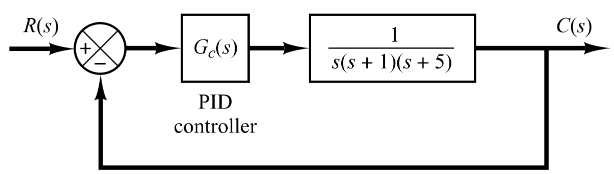

Figure 8–6 PID-controlled system.

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} (= 30), the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 8–2, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 18$$

$$T_i = 0.5P_{cr} = 1.405$$

$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right) \\ &= \frac{6.3223(s + 1.4235)^2}{s} \end{aligned}$$

The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 8–7.

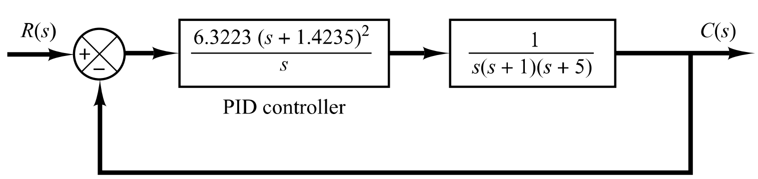

Figure 8–7
Block diagram of the system with PID controller designed by use of the Ziegler–Nichols tuning rule (second method).

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. See MATLAB Program 8–1. The resulting unit-step response curve is shown in Figure 8–8. The maximum overshoot in the unit-step response is approximately 62%. The amount of maximum overshoot is excessive. It can be reduced by fine tuning the controller parameters. Such fine tuning can be made on the computer. We find that by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$—that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\frac{(s + 0.65)^2}{s} \tag{8–1}$$

the maximum overshoot in the unit-step response can be reduced to approximately 18% (see Figure 8–9). If the proportional gain K_p is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, using the PID controller

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\frac{(s + 0.65)^2}{s} \tag{8–2}$$

MATLAB Program 8–1

```
% ---------- Unit-step response ----------

num = [6.3223  18  12.811];
den = [1  6  11.3223  18  12.811];
step(num,den)
grid
title('Unit-Step Response')
```

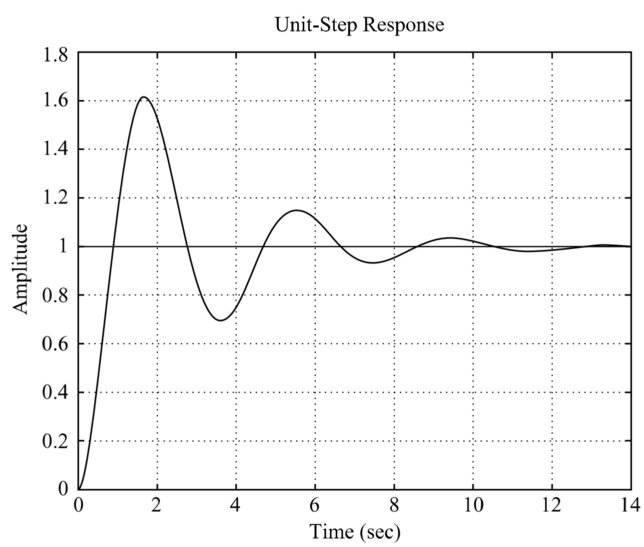

Figure 8–8 Unit-step response curve of PID-controlled system designed by use of the Ziegler–Nichols tuning rule (second method).

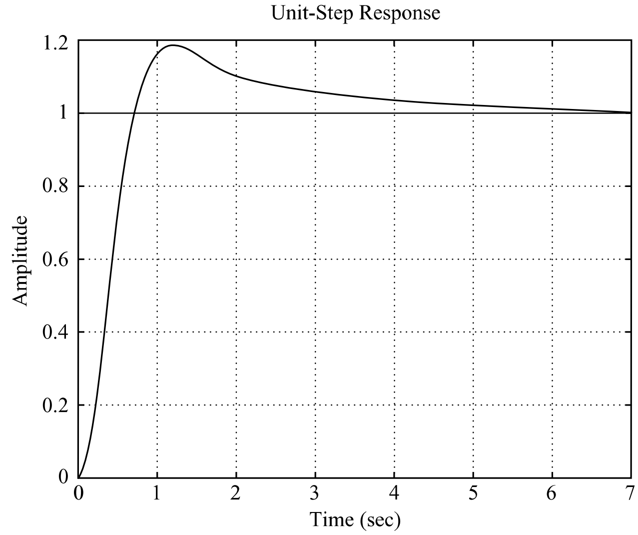

Figure 8–9
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$.

then the speed of response is increased, but the maximum overshoot is also increased to approximately 28%, as shown in Figure 8–10. Since the maximum overshoot in this case is fairly close to 25% and the response is faster than the system with $G_c(s)$ given by Equation (8–1), we may consider $G_c(s)$ as given by Equation (8–2) as acceptable. Then the tuned values of K_p, T_i, and T_d become

$$K_p = 39.42, \qquad T_i = 3.077, \qquad T_d = 0.7692$$

It is interesting to observe that these values respectively are approximately twice the values suggested by the second method of the Ziegler–Nichols tuning rule. The important thing to note here is that the Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

It is instructive to note that, for the case where the double zero is located at $s = -1.4235$, increasing the value of K_p increases the speed of response, but as far as the percentage maximum overshoot is concerned, varying gain K_p has very little effect. The reason for this may be seen from

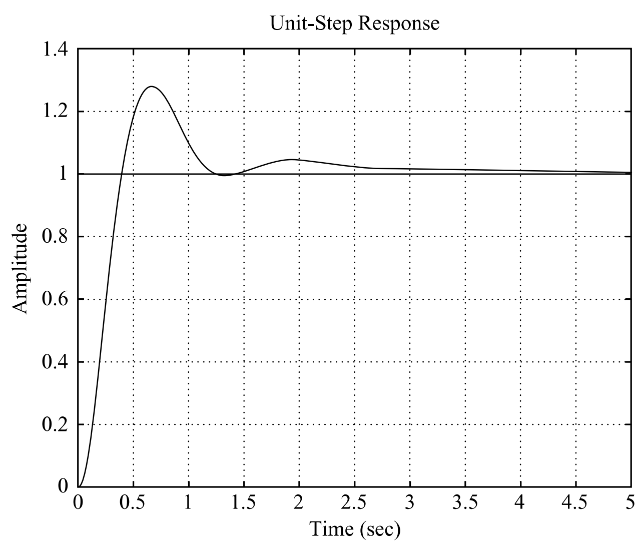

Figure 8–10
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$.

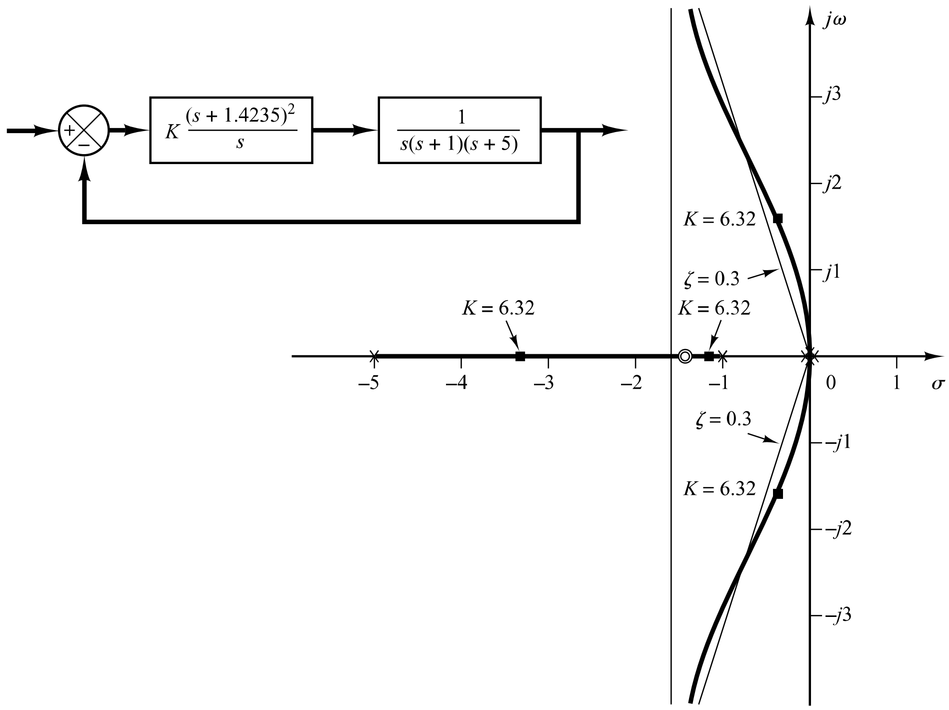

Figure 8–11
Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

the root-locus analysis. Figure 8–11 shows the root-locus diagram for the system designed by use of the second method of Ziegler–Nichols tuning rules. Since the dominant branches of root loci are along the $\zeta = 0.3$ lines for a considerable range of K, varying the value of K (from 6 to 30) will not change the damping ratio of the dominant closed-loop poles very much. However, varying the location of the double zero has a significant effect on the maximum overshoot, because the damping ratio of the dominant closed-loop poles can be changed significantly. This can also be seen from the root-locus analysis. Figure 8–12 shows the root-locus diagram for the system where the PID controller has the double zero at $s = -0.65$. Notice the change of the root-locus configuration. This change in the configuration makes it possible to change the damping ratio of the dominant closed-loop poles.

In Figure 8–12, notice that, in the case where the system has gain $K = 30.322$, the closed-loop poles at $s = -2.35 \pm j4.82$ act as dominant poles. Two additional closed-loop poles are very near the double zero at $s = -0.65$, with the result that these closed-loop poles and the double zero almost cancel each other. The dominant pair of closed-loop poles indeed determines the nature of the response. On the other hand, when the system has $K = 13.846$, the closed-loop poles at $s = -2.35 \pm j2.62$ are not quite dominant because the two other closed-loop poles near the double zero at $s = -0.65$ have considerable effect on the response. The maximum overshoot in the step response in this case (18%) is much larger than the case where the system is of second order and having only dominant closed-loop poles. (In the latter case the maximum overshoot in the step response would be approximately 6%.)

It is possible to make a third, a fourth, and still further trials to obtain a better response. But this will take a lot of computations and time. If more trials are desired, it is desirable to use the computational approach presented in Section 10–3. Problem **A–8–12** solves this problem with the computational approach with MATLAB. It finds sets of parameter values that will yield the maximum overshoot of 10% or less and the settling time of 3 sec or less. A solution to the present problem obtained in Problem **A–8–12** is that for the PID controller defined by

$$G_c(s) = K\frac{(s + a)^2}{s}$$

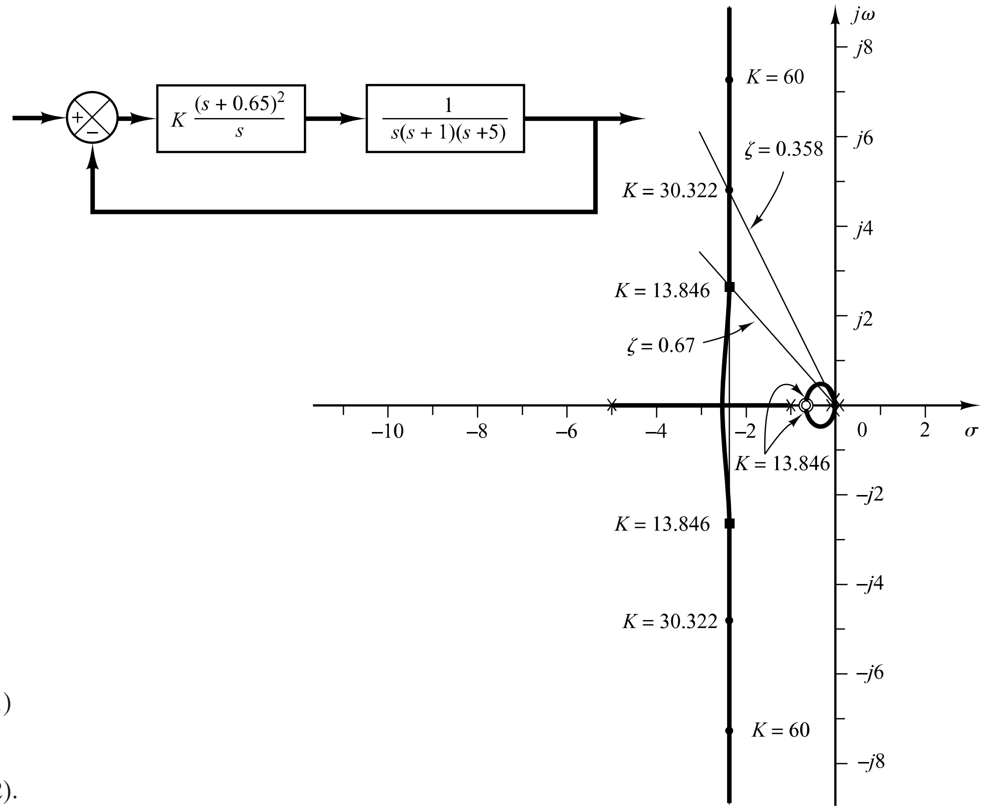

Figure 8–12
Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (8–1) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (8–2).

the values of K and a are

$$K = 29, \qquad a = 0.25$$

with the maximum overshoot equal to 9.52% and settling time equal to 1.78 sec. Another possible solution obtained there is that

$$K = 27, \qquad a = 0.2$$

with the 5.5% maximum overshoot and 2.89 sec of settling time. See Problem **A–8–12** for details.

8–3 DESIGN OF PID CONTROLLERS WITH FREQUENCY-RESPONSE APPROACH

In this section we present a design of a PID controller based on the frequency-response approach.

Consider the system shown in Figure 8–13. Using a frequency-response approach, design a PID controller such that the static velocity error constant is 4 sec^{-1}, phase margin is 50° or more, and gain margin is 10 dB or more. Obtain the unit-step and unit-ramp response curves of the PID controlled system with MATLAB.

Let us choose the PID controller to be

$$G_c(s) = \frac{K(as + 1)(bs + 1)}{s}$$