Cheating Tsiolkovsky's Equation with Highly Unrealizable Physical Approaches

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1 Introduction

Squirting a bunch of hot gases out of a rocket seems wasteful. Tsiolkovsky's equation:

$$\Delta v = v_e \ln \frac{m_0}{m_f} \qquad \text{(The Rocket Equation)}$$

is so dreary in its demand that most of a rocket be devoted to fuel that even its creator cheated it by inventing the multi-stage rocket.

If a rocket starts in free space, is it possible to build a rocket which is more efficient in power or conservative of propellant reaction mass by designing the form of the remaining propellant?

2 Conservation of Linear Momentum

Although it is promising to consider powering a rocket with a beam from a base station, we may be able to learn something interesting from considering a rocket starting in free space. After all, the base station itself or the planet it rests upon is simply a large object in free space.

A chemical rocket in free space without gravity is a system that can't change its center of mass. As the rocket fires, the system spreads out in space, with the part be call the propellant moving in one direction and the rocket moving in the other.

Since we are primarily interested in going in one direction fast, we can think of this as a one-dimensional problem. The fact that a cloud of gas spreads a little from our single axis is an unfortunate inefficiency.

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Conservation of linear moment applies to whatever form the propellant takes. If P_v is the velocity of the center of mass of the propellant (which will likely be negative), and R_V is the velocity of the rocket:

$$R_v = -\frac{m_P}{m_R} P_V \tag{CLM}$$

What would happen if instead of throwing gas molecules behind ourselves we threw solid objects? In other words, what if we used a "mass driver", without specifying how it is powered, to throw a solid propellant? Could we change Tsiolkovsky's equation in some favorable way?

3 Extruded Forms

Suppose that our rocket could extrude an object extremely quickly. If we extruded an object in the opposite direction we wish to travel, by the conservation of linear momentum our rocket will move forward.

$$R_x - P_x = \frac{st}{2} \tag{Linear Extrusion}$$

After doing a lot of math, we determine that the speed of the rocket at time t, assume an extrusion rate of s in m/s and density of extrusion of k in $kg/$ linear meter,

$$v_r = \frac{ks^2 t}{2T_m}$$

where T_m is the total inital mass of the rocket and the propellant:

$$T_m = m_R + m_P$$

or, in terms or Tsiolkovsky's symols, $m_0 = T_m$ and $m_f = m_R$.

By solving for the time it takes to use up all of the propellant and then substituting back into the form of the Tsiolkovsky equation:

$$v_r = \frac{s(m_0 - m_f)}{2m_0}$$

If we graph this against the Tsiolkovsky equation:

INSERT GRAPH HERE

we find that extruding a cylinder very quickly is not quite as good as squirting gases at the same velocity behind us. My interpretation of this is that being attached to the extruded rod during until the momenet of separation holds the rocket back, where as the acceleartion provided by each molecule of gas allows the rocket to move forward without being attached.

3.1 Develop Mathematics of Instantaneous Ejection and Relate

4 Constant Acceleration of Cylinder

Suppose that we could extrude the rod not at a constant speed, but at a constant acceleration? This is physically hard to engineer, because as the mass of the rod behind us increases, the force required to sustain a constant acceleration A increases. Nonetheless if we work out the math, we obtain:

$$R_x - P_x = \frac{At^2}{2} \tag{Constant Acceleration}$$

$$v_r = \frac{A^2 t^3 K}{m_0}$$

...where A is the acceleration we support. If we again solve for the expulsion of all of the propellant, we obtain:

$$v_r = \frac{(m_0 - m_f)^{\frac{3}{2}} \sqrt{\frac{A}{K}}}{m_0}$$

This goes up as A goes up, and goes up as we dedicated more mass to the propellant as we would expect. Note that it also goes up as K the denisty of our extrusion goes down, which is perhaps interesting: the thinnner a rod we can extrude the more efficient our system is.

NEED TO compare to Tsiolkovsky.

But of course this is unrealistic in that we cannot have infinite power in our rocket.

5 Relate to A Plume of Bouncing Machines

An alternative to producing a solid plume as a single unit is to eject discrete objects. These objects can interact by bouncing off each other or potentially accelerating objects in one direction or the other. The conservation of linear momentum remains ironclad, but we may now think of our rocket/propellant complex as a complicated machine which is working to spread itself out in space. Hopefully we are working to send as much mass behind us as quickly as possible.

Possibly our propellant could even be complete machines with their own power supply. Thuse our rocket becomes a line of mass drivers spread out in space that may drive objects with mass towards or away from the rocket.

The fact that it might be a little tricky to survice collision with or the “catching” and “rethrowing” of these objects we will ignore for the time being.

We assert that just as the Tsiolkovsky equation can be cheated by using a multi-stage rocket, it can also be cheated by bouncing objects around in a line, reusing reaction mass.

This can potentially result in final propellant shape more efficient than a plume of gas flying through and slowly spreading through space.

Even better, if one line of the mass drivers is anchored on a planet, we can imagine a system in which the rocket is powered from the planet, where presumably power is cheap, via the transfer or momentum to the rocket via objects driven to and fro from the rocket to the planet.

6 TODO

Fill out the math in a reasonable way. Fill in graphs. Do dimensional analysis.