

# Cheating Tsiolkovsky's Equation with Highly Unrealizable Physical Approaches

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## 1 Introduction

Squirting a bunch of hot gases out of a rocket seems wasteful. Tsiolkovsky's equation:

$$\Delta v = v_e \ln \frac{m_0}{m_f} \quad (\text{The Rocket Equation})$$

is so dreary in its demand that most of a rocket be devoted to fuel that even its creator cheated it by inventing the multi-stage rocket.

If a rocket starts in free space, is it possible to build a rocket which is more efficient in power or conservative of propellant reaction mass by designing the form of the remaining propellant?

## 2 Conservation of Linear Momentum

Although it is promising to consider powering a rocket with a beam from a base station, we may be able to learn something interesting from considering a rocket starting in free space. After all, the base station itself or the planet it rests upon is simply a large object in free space.

A chemical rocket in free space without gravity is a system that can't change its center of mass. As the rocket fires, the system spreads out in space, with the part we call the propellant moving in one direction and the rocket moving in the other.

Since we are primarily interested in going in one direction fast, we can think of this as a one-dimensional problem. The fact that a cloud of gas spreads a little from our single axis is an unfortunate inefficiency.

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Conservation of linear moment applies to whatever form the propellant takes. If  $P_v$  is the velocity of the center of mass of the propellant (which will likely be negative), and  $R_v$  is the velocity of the rocket:

$$R_v = -\frac{m_P}{m_R}P_v \quad (\text{CLM})$$

What would happen if instead of throwing gas molecules behind ourselves we threw solid objects? In other words, what if we used a “mass driver”, without specifying how it is powered, to throw a solid propellant? Could we change Tsiolkovsky’s equation in some favorable way?

### 3 Extruded Forms

Suppose that our rocket could extrude an object extremely quickly. If we extruded an object in the opposite direction we wish to travel, by the conservation of linear momentum our rocket will move forward.

$$R_x - P_x = \frac{st}{2} \quad (\text{Linear Extrusion})$$

After doing a lot of math, we determine that the speed of the rocket at time  $t$ , assume an extrusion rate of  $s$  in  $m/s$  and density of extrusion of  $k$  in  $kg/\text{linear meter}$ ,

$$v_r = \frac{ks^2t}{2T_m}$$

where  $T_m$  is the total initial mass of the rocket and the propellant:

$$T_m = m_R + m_P$$

or, in terms of Tsiolkovsky’s symbols,  $m_0 = T_m$  and  $m_f = m_R$ .

By solving for the time it takes to use up all of the propellant and then substituting back into the form of the Tsiolkovsky equation:

$$v_r = \frac{s(m_0 - m_f)}{2m_0}$$

If we graph this against the Tsiolkovsky equation:

INSERT GRAPH HERE

we find that extruding a cylinder very quickly is not quite as good as squirting gases at the same velocity behind us. My interpretation of this is that being attached to the extruded rod during until the moment of separation holds the rocket back, whereas the acceleration provided by each molecule of gas allows the rocket to move forward without being attached.

### 3.1 Develop Mathematics of Instantaneous Ejection and Relate

## 4 Constant Acceleration of Cylinder

Suppose that we could extrude the rod not at a constant speed, but at a constant acceleration? This is physically hard to engineer, because as the mass of the rod behind us increases, the force required to sustain a constant acceleration  $A$  increases. Nonetheless if we work out the math, we obtain:

$$R_x - P_x = \frac{At^2}{2} \quad (\text{Constant Acceleration})$$

$$v_r = \frac{A^2 t^3 K}{m_0}$$

...where  $A$  is the acceleration we support. If we again solve for the expulsion of all of the propellant, we obtain:

$$v_r = \frac{(m_0 - m_f)^{\frac{3}{2}} \sqrt{\frac{A}{K}}}{m_0}$$

This goes up as  $A$  goes up, and goes up as we dedicated more mass to the propellant as we would expect. Note that it also goes up as  $K$  the density of our extrusion goes down, which is perhaps interesting: the thinner a rod we can extrude the more efficient our system is.

NEED TO compare to Tsiolkovsky.

But of course this is unrealistic in that we cannot have infinite power in our rocket.

## 5 Relate to A Plume of Bouncing Machines

## 6 TODO

Fill out the math in a reasonable way. Fill in graphs. Do dimensional analysis.