Cheating Tsiolkovsky's Equation with Highly Unrealizable Physical Approaches

Robert L. Read *

Founder, Public Invention, an educational non-profit.

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1 Introduction

Squirting a bunch of hot gases out of a rocket seems wasteful. Tsiolkovsky's equation:

$$\Delta v = v_e \ln \frac{m_0}{m_f} \qquad \text{(The Rocket Equation)}$$

is so dreary in its demand that most of a rocket be devoted to fuel that even its creator cheated it by inventing the multi-stage rocket.

If a rocket starts in free space, is it possible to build a rocket which is more efficient in power or conservative of propellant reaction mass by designing the form of the remaining propellant?

2 Conservation of Linear Momentum

Although it is promising to consider powering a rocket with a beam from a base station, we may be able to learn something interesting from considering a rocket starting in free space. After all, the base station itself or the planet it rests upon is simply a large object in free space.

A chemical rocket in free space without gravity is a system that can't change its center of mass. As the rocket fires, the system spreads out in space, with the part be call the propellant moving in one direction and the rocket moving in the other.

Since we are primarily interested in going in one direction fast, we can think of this as a one-dimensional problem. The fact that a cloud of gas spreads a little from our single axis is an unfortunate inefficiency.

*read.robert@gmail.com

We choose coordinates so that the directions we wish to go is the positive x axis. The origin is at the center of mass of our starting machine, which includes the rocket and its propellant. Conservation of linear moment applies to whatever form the propellant takes. If $\dot{P}_x$ is the velocity of the center of mass of the propellant (which will likely be negative), and $\dot{R}_x$ is the velocity of the rocket:

$$\dot{R}_x \cdot M_R = -\dot{P}_x \cdot M_P \tag{CLM}$$

What would happen if instead of throwing gas molecules behind ourselves we threw solid objects? In other words, what if we used a "mass driver", without specifying how it is powered, to throw a solid propellant? Could we change Tsiolkovsky's equation in some favorable way?

3 Extruded Forms

Suppose that our rocket could extrude an object extremely quickly. If we extruded an object in the opposite direction we wish to travel, by the conservation of linear momentum our rocket will move forward.

$$R_x - P_x = \frac{st}{2} \tag{Linear Extrusion}$$

After doing a lot of math, we determine that the speed of the rocket at time t, assume an extrusion rate of s in m/s and density of extrusion of k in $kg/$ linear meter,

$$\dot{R}_x = \frac{ks^2 t}{2m_0}$$

where m_0 is the total inital mass of the rocket and the propellant:

$$m_0 = m_R + m_P$$

or, in terms or Tsiolkovsky's symols, $m_0 = T_m$ and $m_f = m_R$.

By solving for the time it takes to use up all of the propellant and then substituting back into the form of the Tsiolkovsky equation:

$$v_r = \frac{s(m_0 - m_f)}{2m_0}$$

If we graph this against the Tsiolkovsky equation:

INSERT GRAPH HERE

we find that extruding a cylinder very quickly is not quite as good as squirting gases at the same velocity behind us. My interpretation of this is that being attached to the extruded rod during until the momenet of separation holds the rocket back, where as the acceleartion provided by each molecule of gas allows the rocket to move forward without being attached.

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3.1 Develop Mathematics of Instantaneous Ejection and Relate

4 Constant Acceleration of Cylinder

Suppose that we could extrude the rod not at a constant speed, but at a constant acceleration? This is physically hard to engineer, because as the mass of the rod behind us increases, the force required to sustain a constant acceleration A increases. Nonetheless if we work out the math, we obtain:

$$R_x - P_x = \frac{At^2}{2} \tag{Constant Acceleration}$$

can be differentiated:

$$\dot{R}_x - \dot{P}_x = A \cdot t \tag{Velocities under C.A.}$$

and the mass of the propellant will be:

$$M_P = (R_x - P_x) \cdot K \cdot 2$$

or

$$M_P = At^2 \cdot K$$

Rearranging and using (CLM), we obtain:

$$\dot{R}_x(1 + \frac{M_R}{M_P}) = A \cdot t$$
$$=$$
$$\dot{R}_x(\frac{M_P + M_R}{M_P}) = A \cdot t$$
$$=$$
$$\dot{R}_x = A \cdot t \cdot (\frac{M_P}{M_P + M_R})$$
$$=$$
$$\dot{R}_x = A \cdot t \cdot (\frac{M_P}{m_0})$$
$$=$$
$$\dot{R}_x = A \cdot t \cdot (\frac{At^2 \cdot K}{m_0})$$
$$=$$
$$\dot{R}_x = \frac{A^2 t^3 K}{m_0}$$

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...where A is the acceleration we support. If we again solve for the expulsion of all of the propellant, we obtain:

$$v_r = \frac{(m_0 - m_f)^{\frac{3}{2}}\sqrt{\frac{A}{K}}}{m_0}$$

This goes up as A goes up, and goes up as we dedicated more mass to the propellant as we would expect. Note that it also goes up as K the denisty of our extrusion goes down, which is perhaps interesting: the thinnner a rod we can extrude the more efficient our system is.

NEED TO compare to Tsiolkovsky.

But of course this is unrealistic in that we cannot have infinite power in our rocket. My friend Mr. John Gibbons has criticized this equation as simply obscuring an unrelistic way to obtain a higher exhaust velocity.

5 On the mathematics of constant force extrusion

I have attempted to develop the mathematics not of constant acceleration, but of constant force. Generally, this would require that the amount of extruded material per meter is rapidly decreasing, as each addition adds to the total mass.

I was not successful in describing this. Possibly this is simple failure on my part. Possibly it is because the differential equations which result have not solutions. It is possible that this is worth returning to, but I believe it is less valuable than the idea of a spatially distributed mass transfer machine.

6 Relate to A Plume of Bouncing Machines

An alternative to producing a solid plume as a single unit is to eject discrete objects. These objects can interact by bouncing off each other or potentially accelerating objects in one direction or the other. The conservation of linear momentum remains ironclad, but we may now think of our rocket/propellant complex as a complicated machine which is working to spread itself out in space. Hopefully we are working to send as much mass behind us as quickly as possible.

Possibly our propellant could even be complete machines with their own power supply. Thuse our rocket becomes a line of mass drivers spread out in space that may drive objects with mass towards or away from the rocket.

The fact that it might be a little tricky to survice collision with or the “catching” and “rethrowing” of these objects we will ignore for the time being.

We assert that just as the Tsiolkovsky equation can be cheated by using a multi-stage rocket, it can also be cheated by bouncing objects around in a line, reusing reaction mass.

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This can potentially result in final propellant shape more efficient than a plume of gas flying through and slowly spreading through space.

Even better, if one line of the mass drivers is anchored on a planet, we can imagine a system in which the rocket is powered from the planet, where presumably power is cheap, via the transfer or momentum to the rocket via objects driven to and fro from the rocket to the planet.

7 The Simulation Showing That Bouncing Leads to Higher Speed

A simulation which animates the principle of projectiles interacting with elastic collions has been created http://pubinv.github.io/projpop/RocketSim.html which is perhaps illustrative.

8 A Recursively Created Rocket Base

Let us imagine attempting to send a rocket very fast and very far away. Let us assume that we begin on a large and airless planetoid. On this planetoid, mass and energy are free. We furthermore have a mass driver, and the mass driver can hurl projectiles of some limited mass at some velocity into space that has a strict limit V_{md}. Imagine that this mass-drive is in otherways perfect. It can hurl an object with perfect precision as to any momentum which does not exceed its limited total velocity and its limited total kinetic energy.

Furthermore, let us assume that we have extremely good robots and additive, or 3D printing, technology, and that we can hurl such machines into space using our mass driver.

Then we can imagine the following situation. We launch a rocket, and shoot into it projectiles, which it catches and reuses as reaction mass to increase its velocity. It seems clear that we can raise the velocity of this rocket to the velocity limit of our mass driver, given enough time.

But then we are stuck: we cannot reach the rocket with our mass driver.

So, we begin building a new rocket base in space, between the planetoid and the rocket. This base also has the ability to construct a mass driver with the same velocity limit. When this base has attained something similar to velocity of the rocket, then it may shoot projectiles at the rocket and indeed reach it, adding momentum to the rocket. In principle, we can drive the rocket to twice the limit of our mass driver velocity.

However, there is no reason we do not create a third base, and in general any number of bases. Looking at the entire system, we have creating a very long machine which has very long distances between its parts, which transfer momentum and energy by the transfer of projectiles. This machine is not only large, but is enlarging itself at the velocity of the rocket. It is “dilating” at the rate of $N \cdot V_{md}$.

Until we reach some massive target, we cannot make our machine contract. However, by firing mass in the direction of the rocket journey but not being caught by the rocket, we can begin contracting the part of the machine which is not the reaction mass used for contraction.

It is important to note that the projectiles send to the rocket may be used to further the rocket on its journey, or it may be used to as reaction mass for some other purpose, such decelerating to match the velocity of the target or simply to return to the home base. One of way of thinking of this is that the rocket can only be driven by the bases to a velocity of $N \cdot V_{md}$, but that the rocket at that point is full of reaction mass. It has not exhuasted its own reaction mass, but rather is "full". If we can build a mass rocket base, we can certainly fuel the rocket. If the rocket has its own power supply, it can use this reaction mass for its own purposes.

We might even imagine a rocket that is attempting to decelerate receiving a mass packet from the last base at slow velocity. This mass pushes it along the track away from the home base, but if the rocket has enough power, it can fire that mass away from the base at a much higher speed than it received it, thus accelerating itself toward the home base. Conceptually, the rocket is climbing back along a machine which continues to dilate.

9 TODO

Fill out the math in a reasonable way. Fill in graphs. Do dimensional analysis.

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