Continued...

Three Sphere Math

We seek to compute the normal of the tangent plane[1, 2]. Observe that this plane is tangent to all three cones. Observe that the AB cone intersects the A sphere in a circle on the surface of the A perpendicular to and centered on the X axis.
A vector of length r_A that begins pointing in the X direction and is rotated counterclockwise about the Z-axis by $\pi/2 - \alpha$.
However, we must rotate this vector about the X axis by an unknown amount θ in order to bring capture the tilt which is not purely a rotation about the Z axis. Since this angle is computed in the ZY plane, we compute a projection of the point V into that plane, forming a triangle in the ZY plane. Call this point I_z, the intersection of the apex line with the Z-axis.

$$\phi = \arcsin \frac{r_c}{I_z} \tag{1}$$
$$\theta = \frac{\pi}{2} - (\frac{\pi}{2} - \phi) \tag{2}$$
$$\theta = \phi \tag{3}$$
$$I_z = U_x(\frac{V_z}{(U_x - V_x)}) \tag{4}$$

Diagram

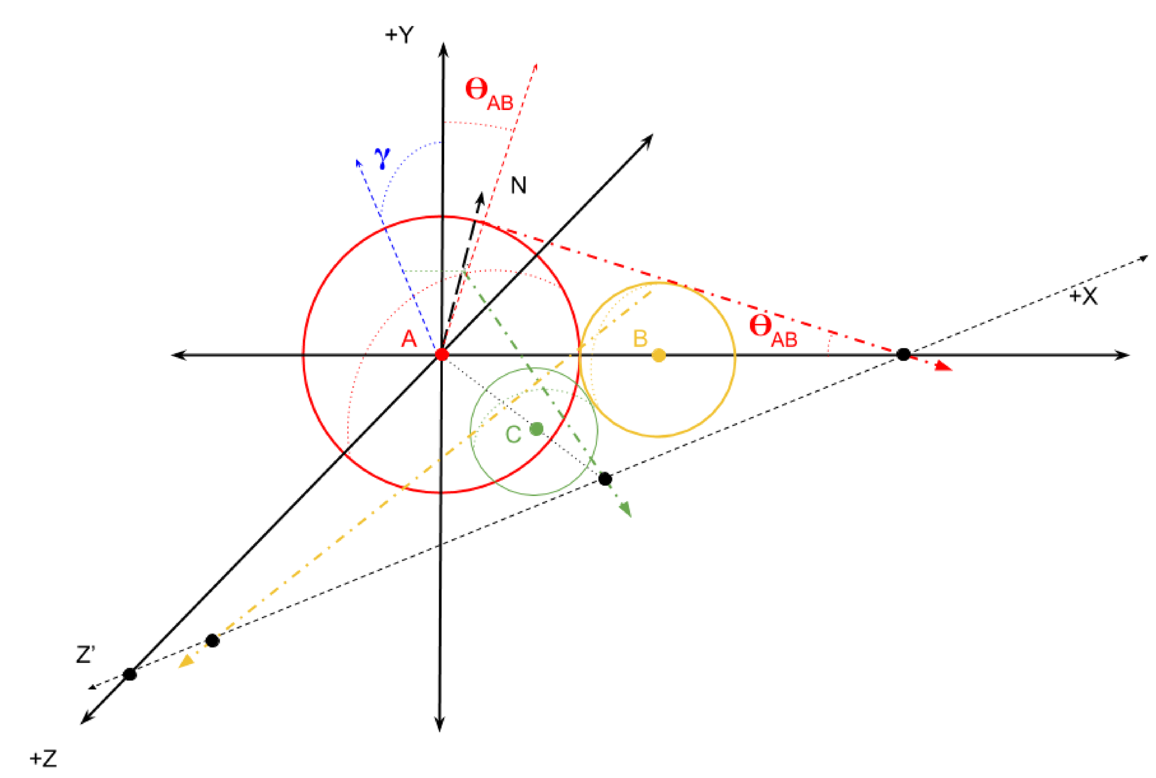


Fig. 1: Rotation Math

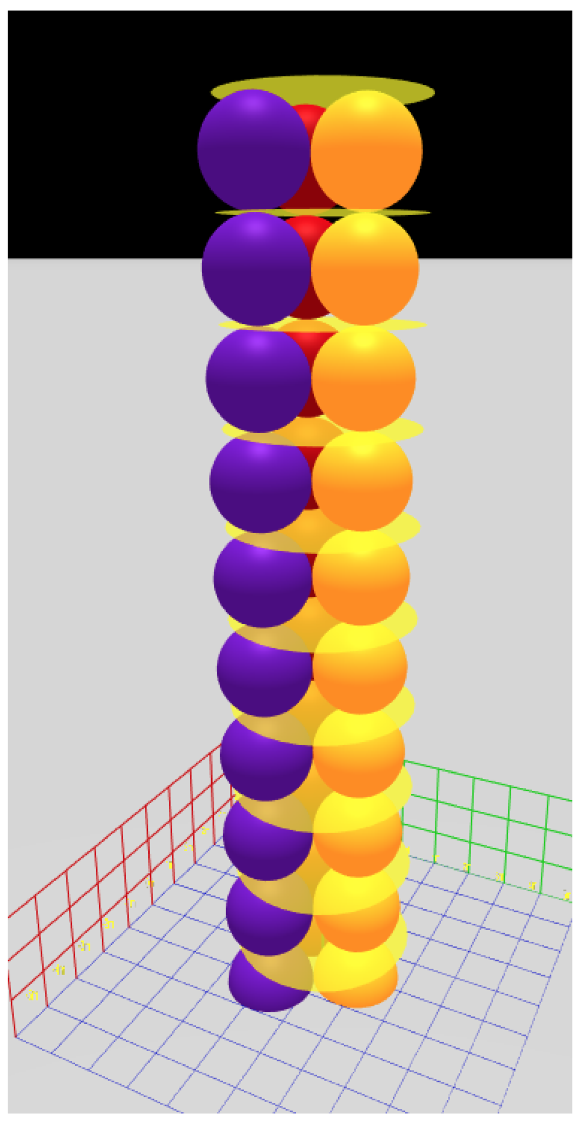
Rotations

Compute these by considering the three cones that envelope the three spheres taken in a pair-wise fashion. Let θ_{xy} be the half-angle of the aperture of the cone that envelopes sphere x and sphere y.
Compute the apices of the three cones, naming the apex of the cone that envelopes sphere X and sphere Y $\overrightarrow{XY_{apex}}$. It is interesting to note that the three apices lie on a line, which is where the normal plane intersects the XZ-plane where the center of all the spheres lie.
Let Z' be the z value of the intersection of this line with z-axis.
Let the normal of the plane tangent to the three spheres be generated by a (negative) rotation θ_{AB} of the $[0, 1, 0]$ vector about the Z-axis followed by a rotation γ about the X-axis.

Rotations

$$\overrightarrow{AB} = \overrightarrow{A} - \overrightarrow{B} \tag{5}$$
$$\overrightarrow{BC} = \overrightarrow{B} - \overrightarrow{C} \tag{6}$$
$$\overrightarrow{CA} = \overrightarrow{C} - \overrightarrow{A} \tag{7}$$
$$\theta_{ab} = \arcsin \frac{r_a - r_b}{r_a + r_b} \tag{8}$$
$$\theta_{bc} = \arcsin \frac{r_b - r_c}{r_b + r_c} \tag{9}$$
$$\theta_{ca} = \arcsin \frac{r_a - r_c}{r_a + r_c} \tag{10}$$
$$\overrightarrow{AB_{apex}} = \overrightarrow{A} + \hat{\overrightarrow{AB}}\frac{r_a}{\sin\theta_{ab}} \tag{11}$$
$$\overrightarrow{BC_{apex}} = \overrightarrow{B} + \hat{\overrightarrow{BC}}\frac{r_b}{\sin\theta_{bc}} \tag{12}$$
$$\overrightarrow{CA_{apex}} = \overrightarrow{C} + \hat{\overrightarrow{CA}}\frac{r_c}{\sin\theta_{ca}} \tag{13}$$
$$\psi = \arccos \hat{\overrightarrow{CA}}_z \text{(angle of CA against the z axis)} \tag{14}$$
$$L = \|A - CA_{apex}\| \tag{15}$$
$$Z' = L \cos\psi \frac{AB_{apex}[x]}{AB_{apex}[x] - CA_{apex}[x]} \tag{16}$$
$$\gamma = \frac{\pi}{2} - \arccos \frac{r_a}{Z'}; \tag{17}$$

Tentacl3



Future Work: Inflatable Stewart Platform

By constructing soft mechanism that serves the same positioning perfomed by a mechanical Stewart Platform[3], we might build machines scalable up or down that that are gentle enough to be used for medical purposes.
By solving the problem[1] with closed-form expressions as shown here, we allow the possibility of computing the derivative of the tilt with respect to change of the radii, a function of the pressure in the spheres.

Future Work: A Soft Tentacle

By stacking such mechanisms, we propose to make a soft *tentacle*. By composing the derivative of tilt with respect to many such platforms, we may construct a Jacobian which allows positioning of the tentacle and even motion planning.
Such a tentacle could be used as an endoscope or arthroscope. Because potentially scalable down to minute sizes, arterial catheterization may be possible.

Future Work: Joule Heating Phase Change for Inflation

Although inflatable spheres could be controlled by pneumatics, it would be more elegant to build a sphere that inflates not by air tubes, but with a simple two-wire electrical connection.
Gas changes pressure when heated, but the change in pressure is proportional to the absolute temperature. Doubling this temperature is relatively impractical.
Water or alcohol can be vaporized at low temperatures. We hope to design a way to add simple heating wires inside an inflatable sphere in order to accomplish a phase change, and therefore a drastic pressure change, with a simple application of voltage.

References

References

[1] Payne, J. *Practical Solid Geometry*, 4 ed. Murby's "Science and Art Departemnt" Series of Text-Books. Thomsas Murby, 32 Bouverie Street, Fleet Street, E.C.;, 1 1881. Available free as an electronic book, see Problem CXLVIII (148), Page 195.

[2] Read, R. L., and Cadena, M. Plane Tanget to 3 Spheres, 2019. [Online; accessed 13-November-2019].

[3] Wikipedia contributors. Stewart platform — Wikipedia, the free encyclopedia, 2019. [Online; accessed 9-October-2019].