A Consideration of Inflatable Circles

Robert L. Read [*]email: read.robert@gmail.com
Megan Cadena [†]email: megancad@gmail.com

September 25, 2019

1 Introduction

This is a study of the basic math of inflatable spheres as a tool for soft robotics. We begin with a study in two dimension to simplify the problem. Our final goal is to analyze three dimensional soft robots composed of inflatable spheres.

2 Problem I: Circles in Fixed Postions

A very simplified version of the problem is to imagine that a two-dimensional plane. Instead of spheres, we will assume we have circles of changable radius. This is in fact realistic of a soft robot constrained to a plane.

Eventually we hope to have circles pressing against each other, or tangent or "kissing". However, the problem is a bit simpler if we assume we have two circles, each of which is constrained to have its center on the a vertical line (see Figure 1.) We place the circle C_1 with radius r_1 on the $x = -1$ line, and assume that it rests on a shelf or plane on the x-axis. Assume the C_2 circle whose radius if r_2 is on the $x = 1$ line.

Let A the intersection of the tangent line supported by the inflatable circles with the x-axis. Call the distance of A on the x-axis x. Let ψ be the angle formed by the circle centers with the x axis (measured counterclockwise).

If the radii are less than 1 so that no issues of intersection of the circles arise, we have:

[*]read.robert@gmail.com
[†]megancad@gmail.com

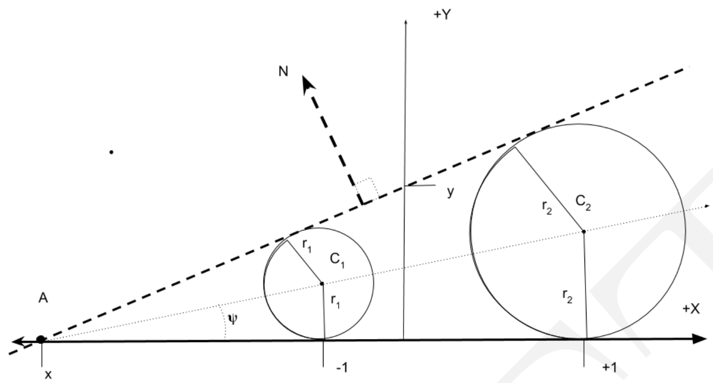

Figure 1: Problem I: Fixed Circles Centers

$$
\begin{aligned}
\tan\psi = \frac{r_1}{x-1} = \frac{r_2}{x+1} && (1)\\
r_2(x-1) = r_1(x+1) && (2)\\
r_2x - r_2 = r_1x + r_1 && (3)\\
r_2x - r_1x = r_1 + r_2 && (4)\\
x(r_2 - r_1) = r_1 + r_2 && (5)\\
x = \frac{r_1 + r_2}{r_2 - r_1} && (6)\\
\tan\psi = \frac{r_1}{x-1} && (7)\\
\psi = \arctan\frac{r_1}{x-1} && (8)\\
N = \begin{bmatrix} -\sin\psi \\ \cos\psi \end{bmatrix} && (9)\\
&& (10)
\end{aligned}
$$

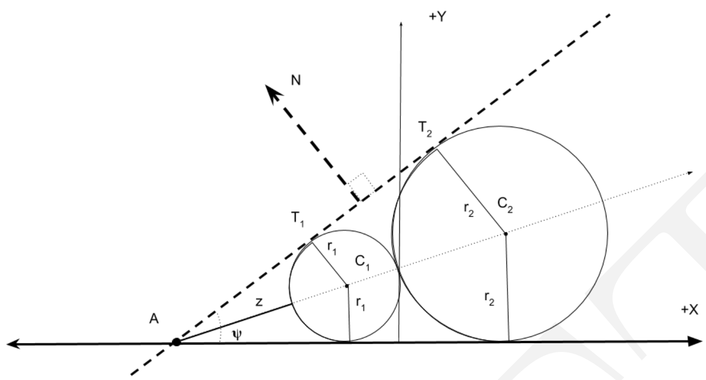

Figure 2: Problem II: Tangent Circles

3 Problem II: Tangent Circles

$$\frac{r_2}{||(C2-A)||} = \frac{r_1}{||(C_1-A)||} \tag{11}$$

$$||C2-A|| = r_2 + 2r_1 + z \tag{12}$$

$$||C1-A|| = r_1 + z \tag{13}$$

$$\frac{r_2}{r_2+2r_1+z} = \frac{r_1}{r_1+z} \tag{14}$$

$$z = -\frac{2r_1^2}{r_1-r_2} \text{ and } r_1 \neq r_2 \text{ and } r_1 r_2 (r_1+r_2) \neq 0 \tag{15}$$

$$\sin\psi = \frac{r_1}{z+r_1} \tag{16}$$

$$\theta = \frac{2\arcsin r_1}{z+r_1} \tag{17}$$

$$Nx = -\sin\pi/2+\theta \tag{18}$$

$$Ny = -\cos\pi/2+\theta \tag{19}$$

$$\tag{20}$$

4 Problem: Three Touching Circles

Our goal is to be able to determine the orientation of plane resting on top of three spheres of different radii which are touching each other. Because there is

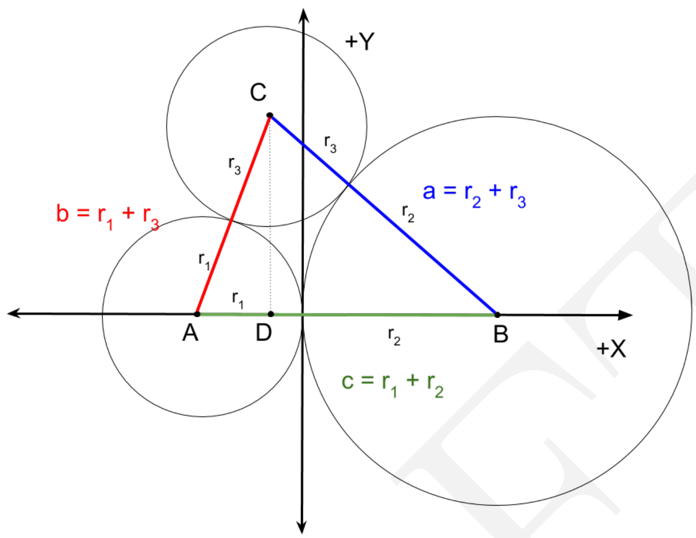

Figure 3: Three Touching Circles

a plane through any three points and we have three spheres, we can construct the plane through the center of these points. The projection of the edges of the spheres onto this plane form three touching circles. Knowing the position of these circles is a valuable prelude to solving the three dimensional problem.

To solve this problem most conveniently, we place the first circle on the negative x-axis, and the second circle on the positive x-axis, with the circles intersecting at the origin. The third circle is place in the positive y direction. It's center will not be on the y-axis itself unless the radii of the first two circles are equal.

We seek a formula for the coordinates of the third circle in terms of three input radii r_1, r_2, r_3.

Because the distance between adjacent circles is the sum of their radii, define:

$$a = r_2 + r_3 \tag{21}$$
$$b = r_1 + r_3 \tag{22}$$
$$c = r_1 + r_2 \tag{23}$$

Then we can use the cosine law to compute the angle $\angle ABC = \theta$:

$$\theta = \arccos \frac{a^2 + b^2 - c^2}{2bc} \tag{24}$$
$$\theta = \arccos \frac{(r_2 + r_3)^2 + (r_1 + r_3)^2 - (r_1 + r_2)^2}{2(r_2 + r_3)(r_1 + r_3)} \tag{25}$$
$$\tag{26}$$

It is clear that once θ has been calculated:

$$C_y = a \sin \theta \tag{27}$$

Allowing us to form a right triangle $\triangle ACD$ and use the Pythagorean theorem:

$$b^2 = y^2 + (r_1 - C_x)^2 \tag{28}$$

$$b^2 - y^2 = (r_1 - C_x)^2 \tag{29}$$

$$\sqrt{b^2 - y^2} = r_1 - C_x \tag{30}$$

$$C_x = r_1 - \sqrt{b^2 - y^2} \tag{31}$$