Three Inflatable Spheres as a Theoretical Basis for a Soft Design for a Stewart Platform and Soft Tentacles

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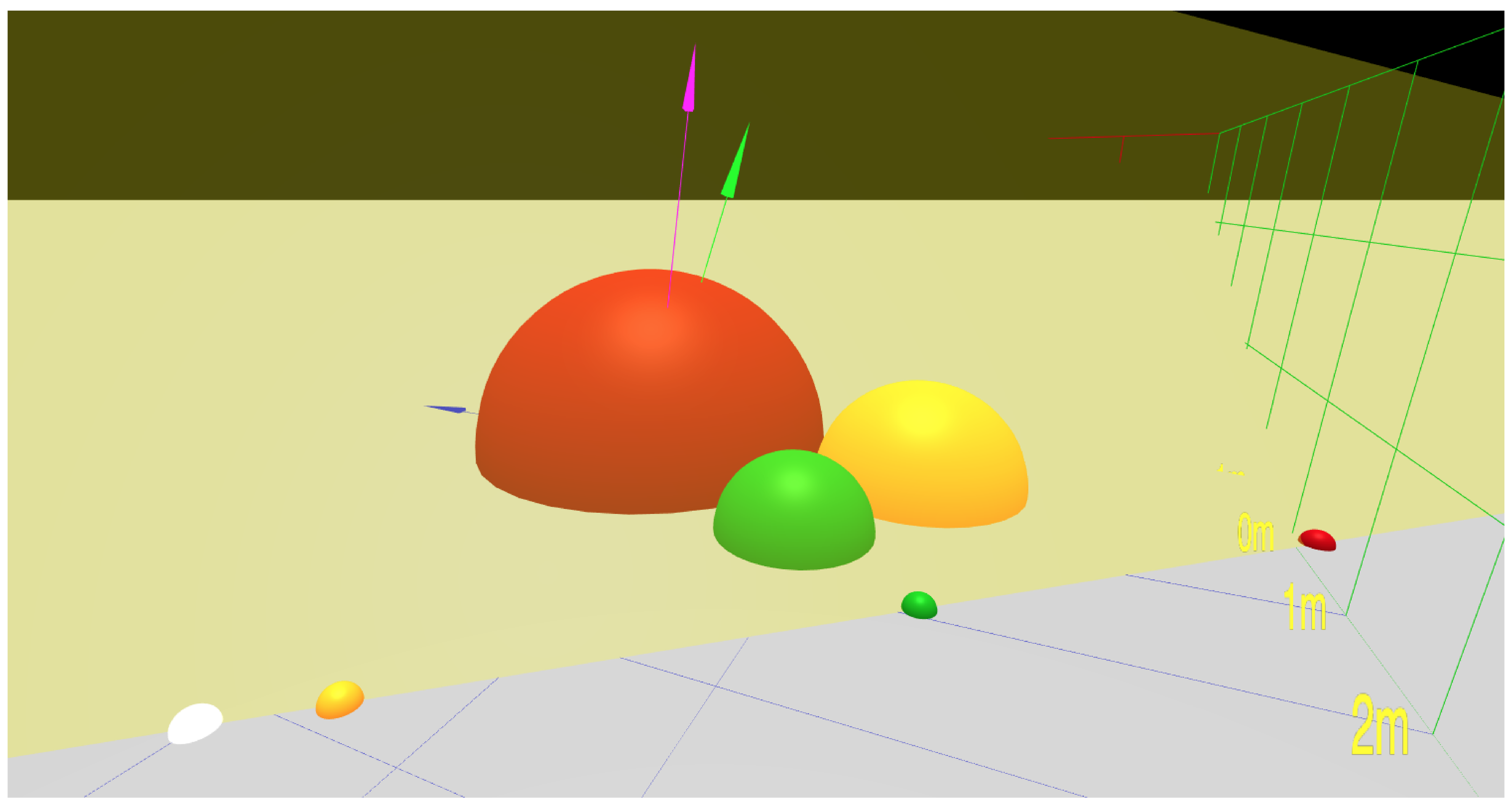
Public Invention

Abstract

In 1881, the problem of computing the plane tangent to three spheres was set as an exercise in a textbook, *Practical Solid Geometry*[1]. We have solved this problem in JavaScript, producing an interactive, browser-based web page that dynamically solves the problem[2]. All of the code is released under the GNU Public License. We propose constructing a "soft" Stewart platform using this theory which allows us to control the inclination of two planes with a very simple mechanism powered only by phase change of a liquid to a gas or by pneumatics. Future work includes computing the Jacobian of a "tentacle" formed by stringing together many such Stewart Platforms, thus providing a theoretical model for a simple, inexpensive, "soft" tentacle. Such a tentacle can be scaled down nicely for *in vivo* medical applications such as endoscopy and arterial catheterization.

Screenshot of Online Calculator: `https://pubinv.github.io/softrobotmath/`[2]



Mechanical Stewart Platform

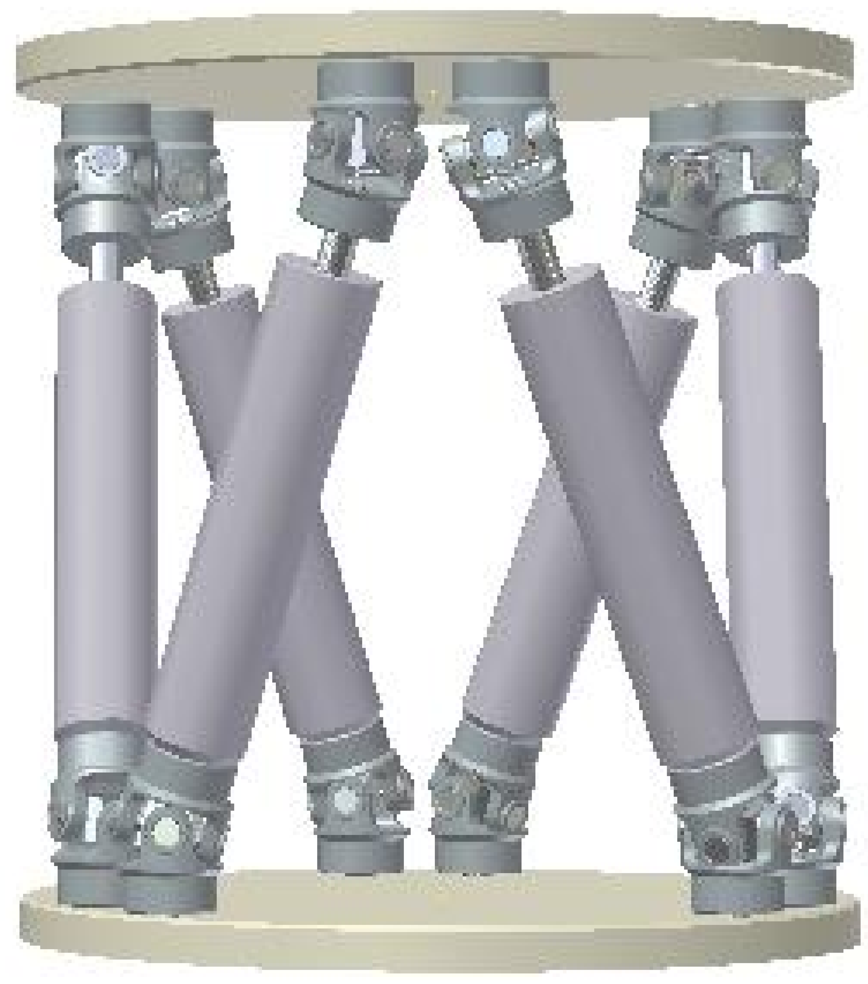
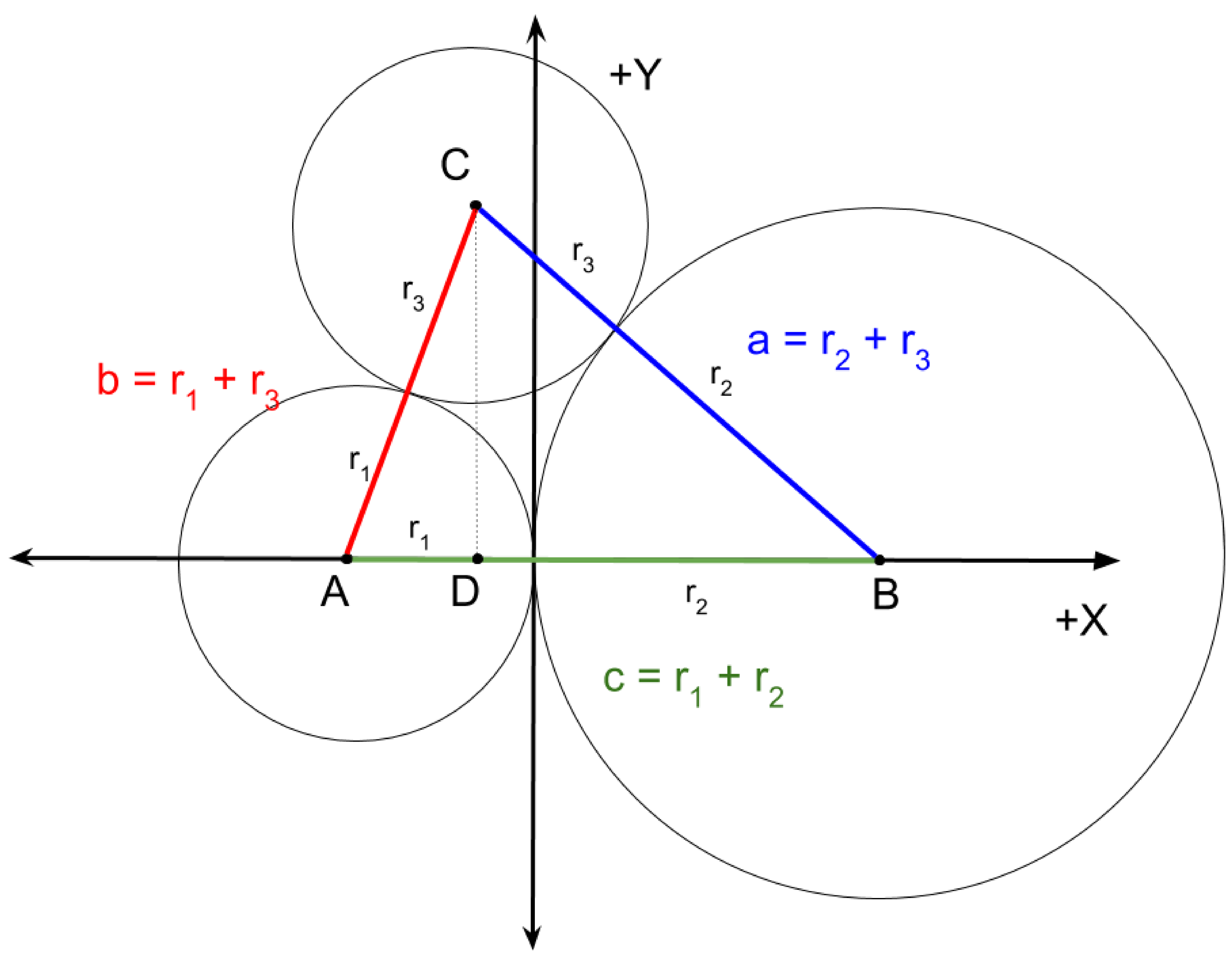


Fig. 1: Stewart Platform

Three Touching Circles



Problem: Three Touching Circles

Our goal is to be able to determine the orientation of plane resting on top of three spheres of different radii which are touching each other. Because there is a plane through any three points and we have three spheres, we can construct the plane through the center of these points. The projection of the edges of the spheres onto this plane form three touching circles.

Knowing the position of these circles is a valuable prelude to solving the three dimensional problem.
We seek a formula for the coordinates of the third circle in terms of three input radii r_1, r_2, r_3.
Because the distance between adjacent circles is the sum of their radii, define:

$$a = r_2 + r_3 \quad (1)$$
$$b = r_1 + r_3 \quad (2)$$
$$c = r_1 + r_2 \quad (3)$$

Then we can use the cosine law to compute the angle $\angle ABC = \theta$:

$$\theta = \arccos \frac{a^2 + b^2 - c^2}{2bc} \quad (4)$$
$$\theta = \arccos \frac{(r_2 + r_3)^2 + (r_1 + r_3)^2 - (r_1 + r_2)^2}{2(r_1 + r_3)(r_1 + r_2)} \quad (5)$$
$$(6)$$

It is clear that once θ has been calculated:

$$C_y = a \sin \theta \quad (7)$$

Allowing us to form a right triangle $\triangle ACD$ and use the Pythagorean theorem:

$$b^2 = C_y^2 + (r_1 - C_x)^2 \quad (8)$$
$$b^2 - C_y^2 = (r_1 - C_x)^2 \quad (9)$$
$$\sqrt{b^2 - C_y^2} = r_1 - C_x \quad (10)$$
$$C_x = r_1 - \sqrt{b^2 - C_y^2} \quad (11)$$