Rebuttal: JMR-17-1184 (Research Paper), A Continuum of Optimal Tetrahelices between the Boerdijk-Coxeter Helix and a Planar-faced Tetrahelix

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1 Thanks

The author wishes to thank the reviewers for their careful and attentive work, that has significantly improved the paper. Each comment is responded to below, on a point-by-point basis.

2 Response to Reviewer #1

2.1 Overview

Slightly irregular edge lengths consisting the tetrahelix are introduced to develop one exact revolution of the tetrahelix structure using the integer number of the tetrahedron in this paper. Therefore, the efficient modeling of the tetrahelix using tetrahedral is proposed, and this research can be applied to the robot or architecture field.

2.2 Remarks

In this paper, the optimality criterion is proposed to minimize the minimax which means the difference between longest and shortest edge. For optimization, 7 conditions were considered. However, these 7 conditions are just based on the authors experience, but is not fully supported by academic background such as kinematic measure to make any comparison between optimal design and non-optimal design.

To parameterize the tetrahelix with respect to the rail angle, the relationship between one-hop edge, two-hop edge, and rail edge is proposed.

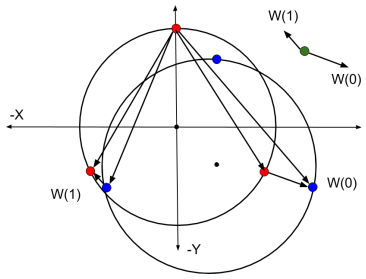

Figure 1: Wobble Vectors from Non-Coincident Axes

The inradius of the tetrahelix was also calculated to consider the insertion of material inside the tetrahelix.

The equitetrabeam is introduced as a special case that the rail angle is zero. It is proposed that transformation from tetrahelix to equitetrabeam is possible by changing the minimax ratio about 16%.

Overall, this paper is acceptable, but requires some revision. The following includes some comments.

2.3 Modification

- The sentences of this paper need to be compact. Please read several times and then clean the sentences. "without loss of generality" → "generally" The term "without loss of generality" is being used specifically as mathematical jargon[1] to represent an assumption allowed in a proof because of symmetry or an ability to make a choice that does not change the validity of the proof. However, because this was unclear to the reviewer, in order to clarify this, I have added the word "assume" to the first use of the phrase "without loss of generality" and the phrase "By symmetry" to the second use.

- The explanation for figure 8 can be erased by drawing the general case for figure 8. Yes, you are correct, thank you, I have made the diagram (see fig. 1) more general and removed the disclaiming text.

- Instead of sentence starting to we , the passive sentence is recommended. I believe editorial opinion is somewhat divided[2] on the use of the word "we" when it is used to mean "the author and the reader together" as I have used it. Most editors do not prefer passive sentences in general. Nonetheless, in deference to the reviewer, I have searched the paper for each use of the word "we" and removed it in most, but not all cases.

- The following sentence is obscure. You'd better suggest a way to measure optimality using some kinematic indices :

> We have not yet proved that a two-class tetrahelix is optimal, but it suffices to show that there exists such a better tetrahelix to show that different radii imply a suboptimal tetrahelix.

The reviewer has generally noted that I may not have motivated my definition of "optimality" strongly enough. To improve this, I have made some changes: ... displays a continuum of tetrahelices optimal **in the sense of being as regular as possible, which is the main result of this paper.**

Additionally, in the section on Optimal Tetrahelices, I have inserted the following sentence which helps to explain the definition of optimality: **By maintaining all lengths as close to the same "regular" length as possible, such as the mean length of the actuator, one retains the greatest possible freedom of motion for the robot.**

This specific sentence has been clarified to show that "not yet proved" means "not yet reached the proof in this paper of that fact that" by changing the sentence to refer to the next theorem:

> We have not yet *proved theorem 3 which asserts* that a two-class tetrahelix is optimal, but it suffices to show that there exists such a better tetrahelix to show that different radii imply a suboptimal tetrahelix.

I unfortunately am not sure how to relate this notion of optimality to kinematic indexes, but have attempted elsewhere to clarify why this definition of optimality is valuable for Tetrobot-style robots.

- It is recommended to change: "conincident" → "coincident" Corrected–an embarrassing mistake in my spelling correction software.
- In page 3, after the eq. (4) In this formula, "integral values of n" → In this formula, "integer values of n" Corrected.
- In eq. (23), need to be changed as

$$\frac{\text{two-hop}(r)}{\text{one-hop}(r)} = \frac{\sqrt{\frac{4}{9} + r^2(g_\rho + j_\rho)}}{\sqrt{\frac{1}{9} + r^2(f_\rho + k_\rho)}}. \tag{1}$$

→

$$\frac{\text{two-hop}(r)}{\text{one-hop}(r)} = \frac{\sqrt{\frac{4}{9} + r^2(g_\rho + k_\rho)}}{\sqrt{\frac{1}{9} + r^2(f_\rho + j_\rho)}}. \tag{2}$$

This has been corrected, thanks for the careful catch of an embarrassing mistake.

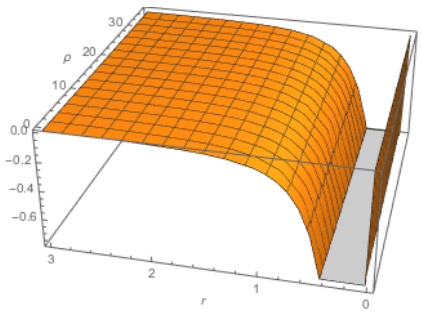

Figure 2: Partial Derivative against ρ and r: $\frac{\partial \frac{\text{two-hop}(r)}{\text{one-hop}(r)}}{\partial \rho, r}$

- - In page 8, the graph could not be observed through this following address, https://github.com/PubInv/tetrahelix/blob/master/tetrahelix.nb . For user not using the Mathematica, it is recommended to show the tendency of the graph of eq. (23) by setting rail angle as 0, 15, and 30(deg).

  In order to make this clearer and avoid forcing the user to follow a link, I have included the graphic inline in the document with the following change to the relevant sentence:

  > By inspection of the graph of the derivative of this ratio rendered in fig. 2 observe that the partial derivative of this with respect to radius r is always negative, for any $\rho \leq \rho_{bc}$.

- - In eq. (27) and (28), it needs to be changed as 9/2 instead of 9. Corrected–thank you!

- - Figure 12 interferes to understanding the related contents. It is recommended to change figure 12 referring the following figure,

  The reviewer has provided an excellent simplifying diagram. I have created a diagram similar (see fig. 3) to theirs but retaining the numbering features, which are useful in the first place the figure is referenced, and in explaining the derivation of the inradius. I thank the reviewer for this improvement.

- - It would be great to add the snapshot and explanation showing the application of this conclusion

  In order to more clearly tie this to the actual research robot for which it was developed, I have re-referenced the photograph which occurs early in the paper and modified the text:

  > A machine, such as the Tetrobot or a variable-geometry truss depicted in fig. 5, that can change the length of some of its members by 16% can thus twist and untwist itself. A completely

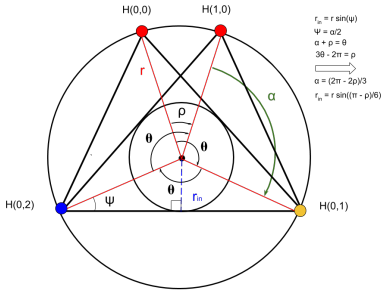

Figure 3: General One-hop Projection Diagram

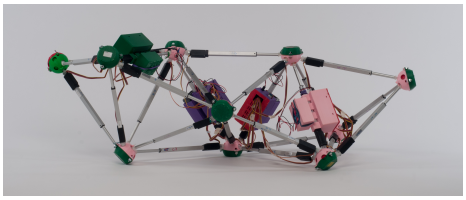

Figure 4: 7-Tet Tetrobot in relaxed, or BC helix configuration

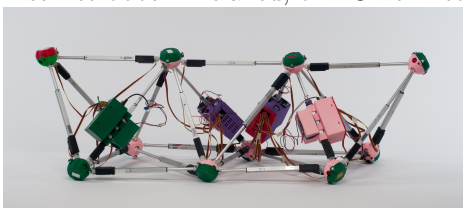

Figure 5: The Equitetrabeam: Fully Untwisted 7-Tet Tetrobot in Hexapod Configuration

regular Tetrobot can untwist itself to create a planar face on the ground for locomotion via standard gaits.

3 Response to Reviewer #2

Title: A Continuum of Optimal Tetrahelices between the Boerdijk-Coxeter Helix and a Planar-faced Tetrahelix

This paper proposed a method to transform the BC helix to Planar-faced tetrahelix by minimizing the difference between the longest and shortest edges, under some constraints. The author tried to explain his finding logically, and the background of the research is explained well. Also, the author shared the materials for this research at github. I have some comments for this paper as follows:

1. I can see the organization of this paper is as follows: Intro (2pages) A designers formulation for the BC helix(2 pages) Optimal tetrahelices are triple helix(3 pages) Parameterizing tetrahelices via rail angle(2 pages) The inradius(0.5 pages) The Equitetrabeam (0.5 pages) An Untwisted continuum (0.5 pages) Conclusion

2. I think the author tried to explain all the findings but I think the organization of the paper should be revised to be emphasized carefully. Even for the researchers in this field, I am not convinced the paper is clearly written to emphasize the contribution well. Some sections can be combined, and the organization should be changed to emphasize the major contribution of this paper. I think this is the most significant point to improve the readability of the paper.

   In order to address this, I have added the sentence below at the end of the introduction.

   The remainder of this paper:

   (a) provides a new formulation for the BC helix that is more natural for engineers,

   (b) defines a concept of minimum maximum edge-length optimality that maintains the structure as close to regularity as possible,

   (c) proves that all optimal tetrahelices are in fact triple helices,

   (d) provides formulae for constructing optimal tetrahelices of different twists or for optimally transforming a robot from a twisted to a planar configuration in a way that maintains optimum regularity,

   (e) gives a formula for the inradius of an optimal tetrahelix,

   (f) describes a completely planar tetrahlix (the *equitetrabeam*) useful as a structural beam or for applying standard robot gaits to a tetrahedral robot, and

(g) discusses the continuum of optimal tetrahelices between the BC Helix and equitetrabeam.

3. I think the explaining optimization part should be revised. For a certain optimization problem, I think mathematics definition or algorithmic explanation can be helpful to understand the problem quickly. I think the author should refer problem definition standards from other optimization papers to revise this. In order to clarify the "Optimal Tetrahelices are Triple Helices" section as requested, I have provided a mathematical description of the optimization, and a more mathematical description of the definition of the tetrahelix.

4. In my opinion, the problem definition and proof of theorems looks being done based on personal experiences. I recommend using mathematical derivation or expression if possible. I believe the theorems are all true (and verified by computer experimentation) and that the proofs are rigorous. However, in deference to the reviewer I have significantly rewritten parts of them more formally in several places. Unfortunately, it is beyond my ability to make them completely formal in the space allowed in this paper. I hope readers will find them convincing at the available length.

5. I think the demonstration by experiment can be helpful to understand the idea. Video attachment is recommended. I have included a public link to a video[3] in the citations, and will upload the same video to the journal if appropriate.

   All math developed here is available in JavaScript and demonstrated by an interactive design website `https://pubinv.github.io/tetrahelix/`[4] **and a video[3], from which figs. 1 and 2 and later figures in this paper are taken.**

Minors:

- I am very confusing to the term "An untwisted continuum" and "equitetrabeam." I have attempted to clarify this by adding the phrase: We call this the *equitetrabeam*, **a portmanteau of "equilateral tetrahedral beam"**. Some other terminologies should be checked for Conincident should be coincident This has been corrected, thank you.

- Title is little bit not easy to be understood. A continuum of optimal tetrahelices can be changed to some other expression. I have changed the title to: "Transforming Optimal Tetrahelices between the Boerdijk-Coxeter Helix and a Planar-faced Tetrahelix"

References

[1] Without Loss of Generality, 2017. Without loss of generality — Wikipedia, the free encyclopedia. [Online; accessed 5-March-2018].

[2] Joshi, Y., 2014. https://www.editage.com/insights/is-it-acceptable-to-use-first-person-pronouns-in-scientific-writing, January. [Online; accessed 8-March-2018].

[3] Read, R. L., 2017. Utwisting the Tetrahelix. https://www.youtube.com/watch?v=yGc-G4YFTxo, May.

[4] Read, R. L., 2017. Untwisting the tetrahelix website. https://pubinv.github.io/tetrahelix/.