Gluss = Slug + Truss

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Abstract

An innovative approach to modular robotics that is capable of both locomotion and exerting and withstanding structural forces is described. Extending the TETROBOT[1, 2, 3, 4] concept of a variable-geometry truss which is composed only of joints and linear actuators with a new 3D-printable embodiment of a spherical joint[5] produces a completely modular, mechanically strong, tentacle-like machine capable of independent locomotion. We call this truss that can ooze like a slug a *gluss*. The *turret joint* is shown to theoretically support a maximum ratio of maximum actuator length to minimum actuator length of $\varphi \equiv \frac{1+\sqrt{5}}{2} \approx 1.618....$ The simplest glussbot capable of crawling and turning, the 3TetGlussBot, is constructed of inexpensive open-source modules comprising an Arduino Mega, a custom PCB, servo controller chips, and a Bluetooth module. A 3-module robot, the 5TetGlussBot, that uses spherical magnetic joints is described which oozes at 27 cm/min (11 in/min).

1 Introduction

The best introduction to this paper is to watch the 2:10 minute video of the walking robot, or *glussbot*, which this paper describes:

`https://www.youtube.com/watch?v=zl0AEfxyVMw`.

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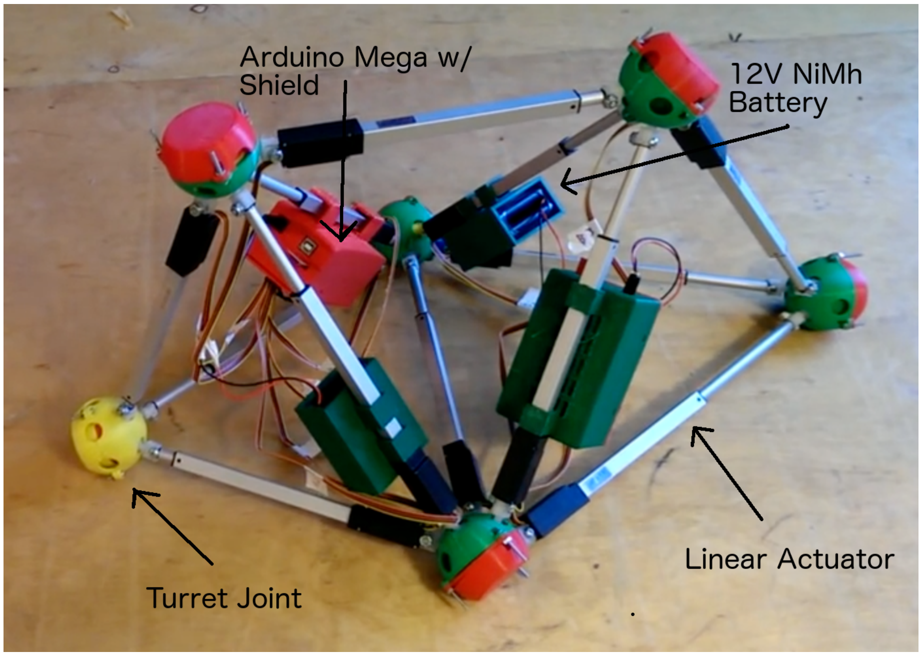

Figure 1: 3TetGlussBot Components

Between 1996 and 2002 years ago, Arthur C. Sanderson and his colleagues published a series of papers[1, 2, 3] on modular robots. The "TETROBOT" was a variable-geometry truss, in which motion was accomplished by the change in length of linear actuators, connected in a modular geometry based on the tetrahedron and octahedron. Such a system requires a special joint which allows the actuators to remain aimed at the center of joints while supporting a certain amount of rotation about this center.

This paper builds upon that work by introducing 3D-printable embodiments of a recently invented spherical joint[5], and gives some results related to the underlying geometry and math, as well as providing references to all of the open-source materials needed to duplicate and expand on this work. This is an open-source embodiment of the TETROBOT with physically smaller actuators which is more accessible to the hobbyist or researcher with a limited budget. The development of 3D printers, Bluetooth, microprocessors such as the Arduino, and inexpensive commercial actuators has made this possible. A very simple robot having only three tetrahedra is shown to be capable of locomotion.

1.1 Motivation

Imagine a strong, light, metamorphic material that can exert or resist force. You can command it to form any shape, limited only by its flexibility. Using that coformability you can get it to crawl across very rough terrain. You can command it to form into a bridge or to lift and move heavy objects, or to perform all the functions of a crane, a forklift, a backhoe and/or bulldozer.

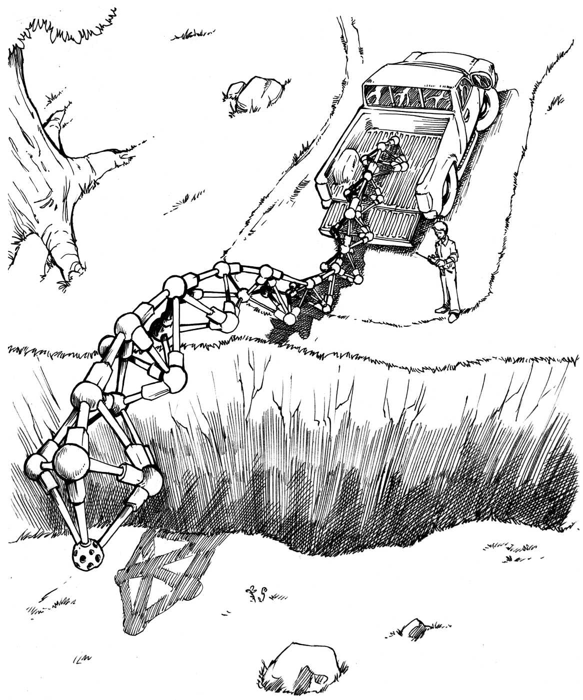

Figure 2: Concept art of GlussBot Spanning a Chasm

It is a truss that crawls like a slug—a *gluss*. If you need more of it, you buy it by the kilo and when it arrives you order it to crawl to your existing gluss. You easily join it there, creating larger, stronger, combined mass.

The advantages of snakebots have been widely recognized [6]. In general, these have been constructed with angular joints. In this paper we propose a different, truss-like approach to providing similar mobility that uses only linear actuators and spherical joints that eliminates non-axial forces so that only compressive and tensile forces act on the actuators. This potentially combines the advantages of forceful machines with snake-like mobility.

Other geometries, such as moving planes, are possible with the same material.

Additional videos at the YouTube channel, Public Invention, reachable from the above link, further motivate the *gluss* concept.

1.2 Concept: Gluss = Slug + Truss

Imagine a metamorphic or polymorphic machine that forcefully assumes a variety of shapes. It moves like a mollusc or amoeba, oozing into position as commanded. It is technically a "machine" because it can exert force reliably, but it may be thought of as a material, because unlike most robots its components are not differentiated.

Although someday an actual chemical substance may do this, today it can be constructed from commercial components and 3D-printable parts. This paper introduces the *gluss* approach to building metamorphic dynamic robots and static machines.[1]

Massively scalable robots have often been proposed. Our particular approach is to use linear actuators, which are rod-like machines that can make themselves shorter or longer. These are tied together using a relatively new joint [5] which allows, for example, as many as 12, but more realistically 4 or 6, members to be joined together sturdily at a single point. A 3-D printed embodiment presented here, called the *turret joint*, allows the change of angle required for the gluss to ooze about. Some gluss consists of some actuators joined together with some turret joints and whatever batteries and control microelectronics are needed. In the first crawling robot discussed here, the *3TetGlussBot*, there are two controllers and two batteries in addition to the 12 actuators and the 6 multi-member turret joints, but 3TetGlussBot is only a small amount of gluss compared to what we hope to build. Additionally, we present *5TetGlussBot*, an obvious extension of the 3Tet geometry implemented with magnetic joints.

[1] "Gluss" is a portmanteau of "Slug" and "Truss" because we are attempting to build a truss, or space frame, that is capable of moving like a slug or octopus. The word *gluss* should be used as a substantive noun in English, much like the word *clay* is used. The use of *glusses*, the plural of *gluss*, should be rare and refer to different kinds of metamorphic material, such as the expression "four clays" suggests four distinct types of clay without specifying how many kilograms of each one means.

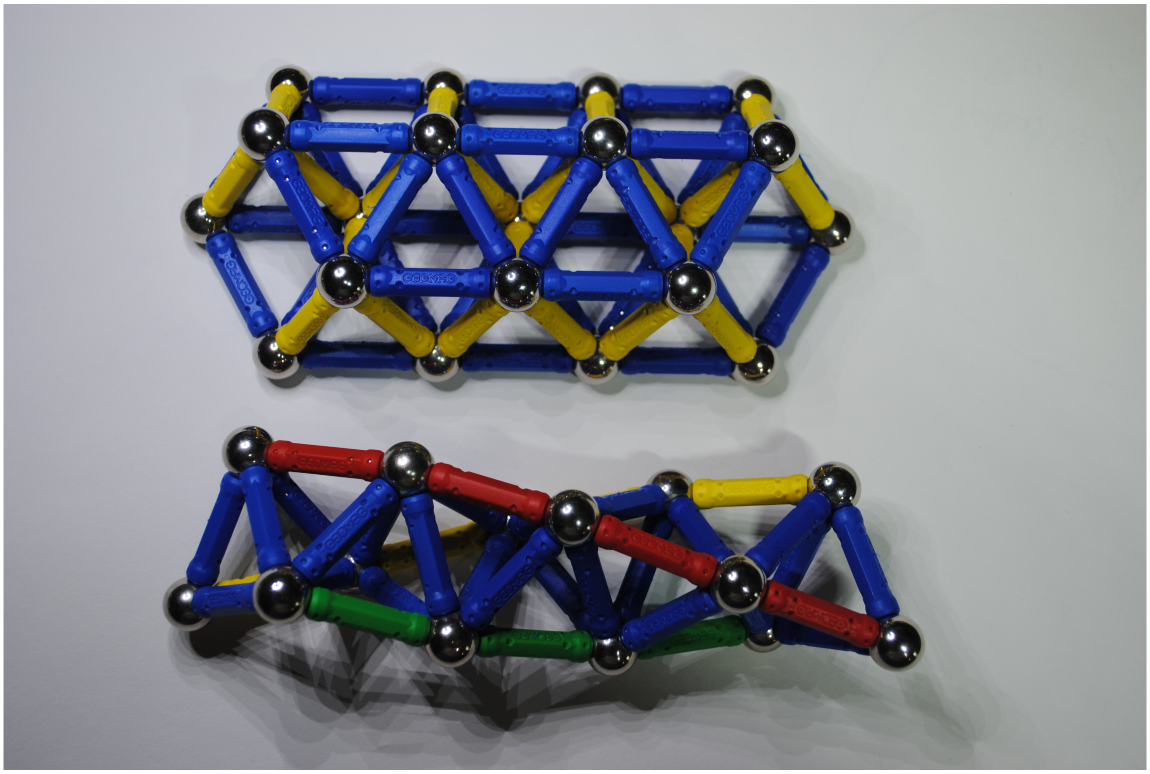

Figure 3: The Octet Truss (above) and Tetrahelix (below)

Although unlimited geometries are possible, two geometries (See Figure 3) have the advantage of being regular, thus allowing us to seamlessly connect any number of actuators into any amount of gluss with some hope of effective software control.

2 The *Turret Joint*

2.1 The Need

The way to make something large, light, and strong is to make it inherently rigid by building it out of triangles. In a single plane, this is called a *truss* [7], and more generally is called a *space frame*. Space frames made completely from triangles tend to be rigid even if the joints that connect members allow motion, such as a pin joint or a ball-and-socket joint. This is an advantage because non-axial strain (that is, a slight change in the angular geometry of the frame) cannot cause the joint to fail, as it can with a welded joint.

But we seek a space frame that can change its shape dramatically. Imagine a radio tower in which each girder has been replaced with an actuator that can get longer or shorter. Such a tower could bend its top down to the ground, or even tie itself into a knot. To accomplish this, the joints must support significant but limited range of angular motion.

The spherical joint invented by Song, Kwon and Kim [5] is such a joint, the essence of which is rendered in their patent drawing, Figure 4. We name this joint the *Turret Joint*.

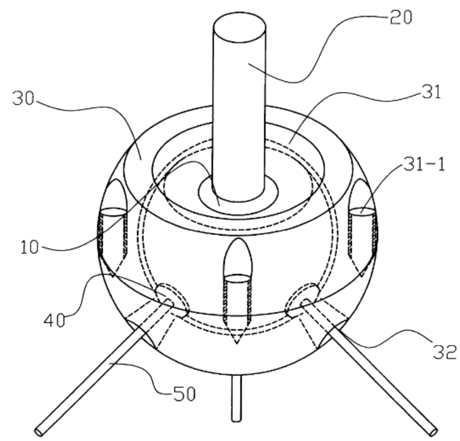

Figure 4: Song, Kwon, Kim, patent image.

When properly configured to support regular nets of actuators, it allows the gluss to be a moving space frame. It happens that the specific actuators we use are geared such that when no power is applied, they strongly resist outside forces that would change their length, essentially becoming rigid members. The resulting gluss can move into position and then be powered off to be a temporarily static space frame.

One could also use this joint with members which are not actuators. For example, we first constructed the joint with carbon fiber rods. In essence it is then a construction kit with continuously variable member lengths liberated from using a finite set of angles.

2.2 Geometry

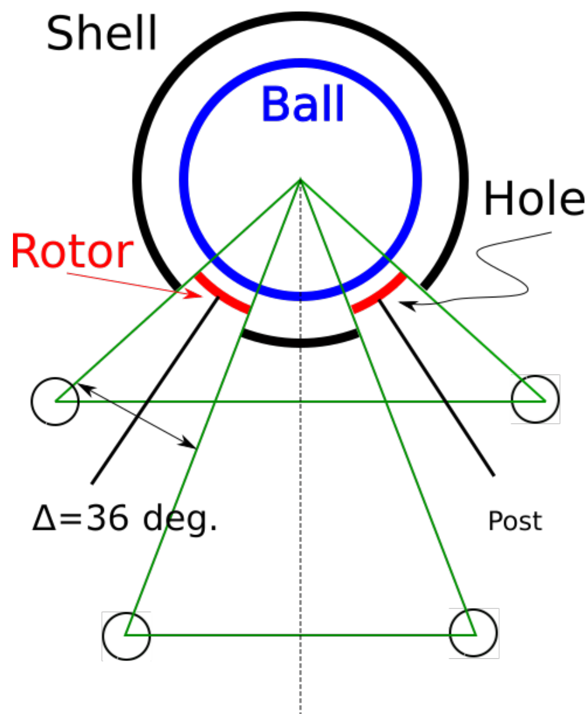

Figure 5: Turret Joint Planar Geometry

How versatile can we make the turret joint?

In particular, since we are attempting to build gluss which is regular in its use of actuators, we may ask: What is the maximum range of motion in our actuators which we can usefully employ in our gluss? To work independent of scale, we use the symbol Q to denote the ratio of the actuator length at its longest to its length at its shortest. The particular Actuonix$^{\text{TM}}$ actuators we use have a Q of 1.5. But is that the maximum Q that we could utilize? Or is it already too high?

One way to approach this problem is to consider a single triangle formed by joints and actuators. The joint must support the most acute triangle that can be formed with the three actuators and the most obtuse triangle that can be formed with the actuators.

In fact it is a surprising result that we prove in Appendix A that the maximum Q which can be utilized by an ideal turret joint happens to be the famous golden ratio, $\varphi \equiv \frac{1+\sqrt{5}}{2} \approx 1.618...$, and the maximum deviation for any one member coming into the joint is 36°. Thus in Figure 5 the triangles drawn are in fact a Golden Triangle and a Golden Gnomon. A real-world joint, which will support less variation because the "post" must have a certain thickness and the "rotor" must have a lip slightly larger than the hole in order to remain locked in place. Furthermore, the joint adds a certain necessary thickness, the minimum length from joint-center to joint-center will be somewhat greater than from actuator tip to actuator tip.

However, the theoretical ideal result is a valuable approach to physical computation and makes a Q for a physical actuator of 1.5 seem quite appropriate.

2.3 Embodiment

Although the joint could be machined or formed in some other way, 3D Printers have made the construction of the Turret Joint far easier. We have designed a complete set of components needed to 3D print the joint and the rotors to attach to the linear actuators. These models are created with OpenSCAD, a functional parametric modeling program.

Our experience has been that the common plastics PLA and ABS are adequate for the Turret Joint, but have found that nylon, which is far tougher and less prone to cracking, is superior for the rotors which bolt directly to the actuators.

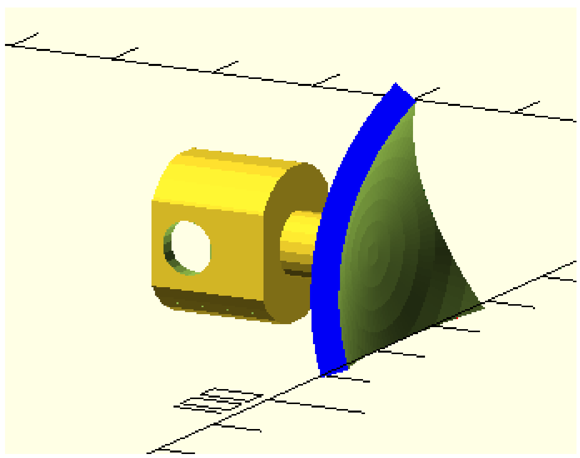

Figure 6: Triangular Rotor Model.

We have innovated the design of the rotor by using a triangular section of a sphere as the rotor rather than a circular section, as shown in Figure 6. Assuming that each actuator is free to rotate about its axis as well as revolve about the center of the ball joint, this shape does not limit motion even in the most pinched configurations. The triangular rotor provides greater extent of contact, which presumably makes the joint motion smoother and less likely to bind.

Figure 7 shows most of the parts. The nylon triangular rotor is white and rests upon the red ball. The green part is a Tetrahelix lock, and the yellow parts are the locks for the Octet Truss geometry.

Figure 7: 3D Printed Parts

In Section 3, one of our actuators and also a carbon rod with 3D-printed tubular mounts are shown in Figure 10. The carbon rod and mounts can be used as a construction system to build a static structure. By cutting the rods to any length (with $Q \leq \varphi$), you can build a static version of any geometry that the actuators are capable of dynamically achieving.

2.4 Specific Geometries

Although the possible ways to configure actuators and joints is limitless, the simplest thing is to use regular, repeatable geometries. The two most obvious are the Boerdijk-Coxeter helix (more easily called the *tetrahelix*) `https://en.wikipedia.org/wiki/Boerdijk%E2%80%93Coxeter_helix` and the *Octet Truss* [8]. It is instructive to compare Figure 8 with the photo of the same object made with the Geomag$^{\text{TM}}$ toy shown in Figure 3.

Roughly speaking, the Tetrahelix is a good way to make a long shaft or tentacle, and the octet truss is a good way to make a planar shape. The purpose of a tentacle is to curl, and the we have no word for a stingray-like plane that rolls itself up into a cylinder or cone or forms a barrel vault. Buckminster Fuller discusses both the tetrahelix and octet truss [9] in terms of static structures and geometries.

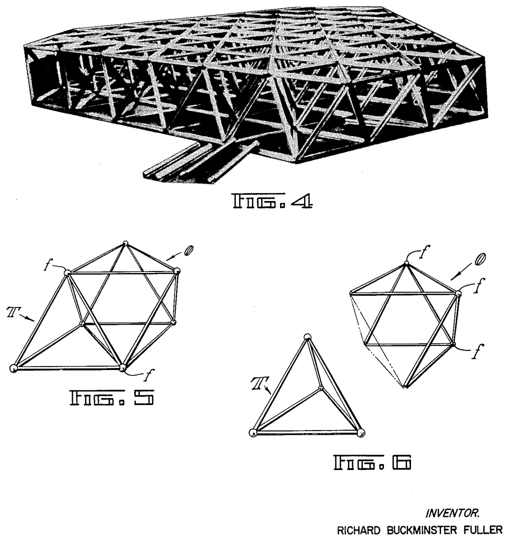

Figure 8: The Octet Truss, selected from Buckminster Fuller's patent.

The Octet Truss reflects the geometry called the *cuboctahedron*.

Because the Turret Joint does not support infinite revolution of all members, the joint, or the "lock" or "stator" in particular, must be adjusted to the regular geometry that one is implementing. Figure 9 indicates this difference. The image is from TurrentJoint.scad: `https://github.com/PubInv/turret-joint/blob/master/Models/TurretJoint.scad`, an open-source file that can be used to 3D print all of the turret joint components. It shows a lock part for the Tetrahelix geometry on the left, and the more expansive Octet Truss geometry on the right. Observe that some of the holes are semicircular, half in this part, and half in the "cap" parts not shown here.

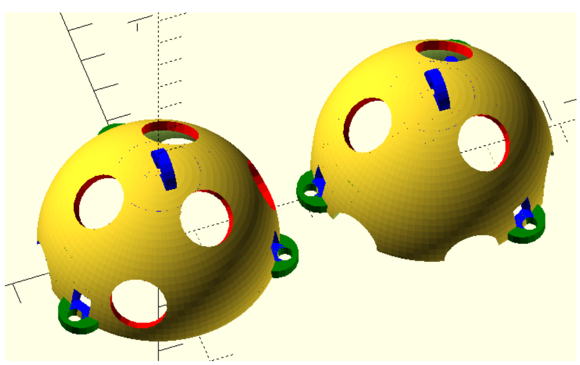

Figure 9: Lock Comparison

We have additionally experimented with magnetic joints similar to those use in the Geomag™ toy. These joints use 1/2” diameter by 1” long neodymium magnets restrained within small cages such that the circular face may contact a 2” diameter hollow steel ball. This provides a pull force greater than the 50 Newton force applied by our linear actuators.

Although this joint functions well and easily achieves the intention of making the robot quick to assemble and disassemble for travel, it is easily broken by non-axial forces exerted from outside the robot. Since climbing over obstacles and stairs will necessarily involve side forces on the actuators, we intend to move away from the magnetic joints.

Furthermore, it would be very expensive to scale upward (and similarly becomes less expensive when scaling downward.) We believe the magnetic joint could be a good engineering solution for glussbots smaller than the current general scale of 50 Newton, 400mm-average-length actuators.

3 Linear Actuators

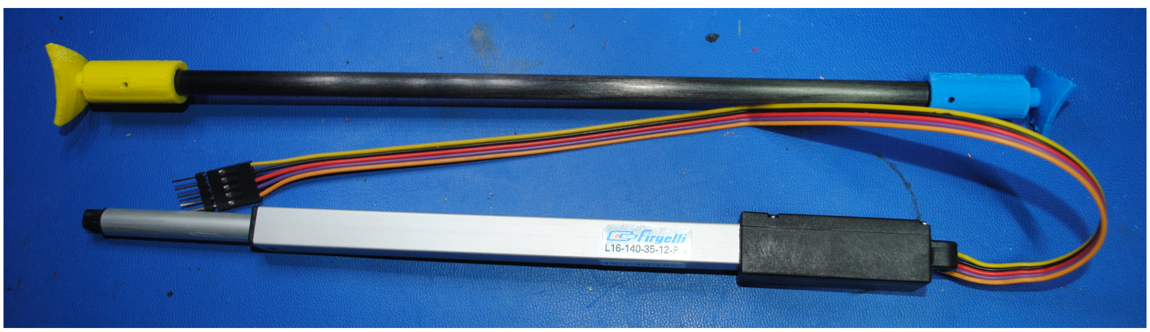

Figure 10: Static Rod and Actuator

We use the term Linear Actuator to refer to any device that is capable of changing its length. When used in gluss it might be called a *glussion*, a single particle of gluss. Our current gluss uses a commercial linear actuator (the Firgelli/Actuonix L-16-140-35-12-P, `http://www.firgelli.com/L16_Linear_Actuators_p/l16-p.htm`)[2] that costs about $80 and exerts about 50 Newtons of force. They have the tremendous advantage of providing positional feedback. Turret joints that are 3D printed in plastic have been strong enough for this level of force.

However, in principle there is no reason we could not use the enormous hydraulic cylinders used by backhoes, bulldozers, and other earth moving equipment. These would of course require much stronger joints, for example joints in cast in aluminum.

[2] At the time of this writing, Firgelli Technologies Inc. has changed its name to Actuonix Motion Devices.

We could also attempt to make smaller actuators, to make a pocket-sized gluss. This is probably a greater technical challenge because it is relatively difficult to make small electric motors.

The current actuators use a rotary motor with a lead screw. This has the advantage of great strength relative to their size, and of retaining position forcefully with relatively little backlash. It has the disadvantage of being relatively slow. It would be interesting to build a gluss out of linear motors, for example tubular linear synchronous motors. These would be much faster than the lead-screw type actuators, resulting in a much faster gluss, and would require dynamic, rather than purely kinematic control.

4 Control and Motion

4.1 Architecture of the 3TetGlussBot and 5TetGlussBot

We have constructed what we believe to be the simplest possible Tetrahelix-based glussbot that is capable of locomotion without relying on inertia. It comprises:

- 12 actuators,
- 6 turret joints,
- 2 12V battery packs, and
- 2 controller units, each of which comprises:
  - 1 Arduino Mega microcontroller, and
  - 1 custom Arduino Mega shield hosting 6 1-amp motor channels and a Bluetooth module,

Using these components, an NTetGlussBot can be constructed for any number of tetrahedra N

- $(N+1) \cdot 3$ actuators,
- $N+3$ turret joints,
- $\lceil \frac{N+1}{2} \rceil$ 12V battery packs, and
- $\lceil \frac{N+1}{2} \rceil$ controller units,

for a cost of about $400 per tetrahedra.

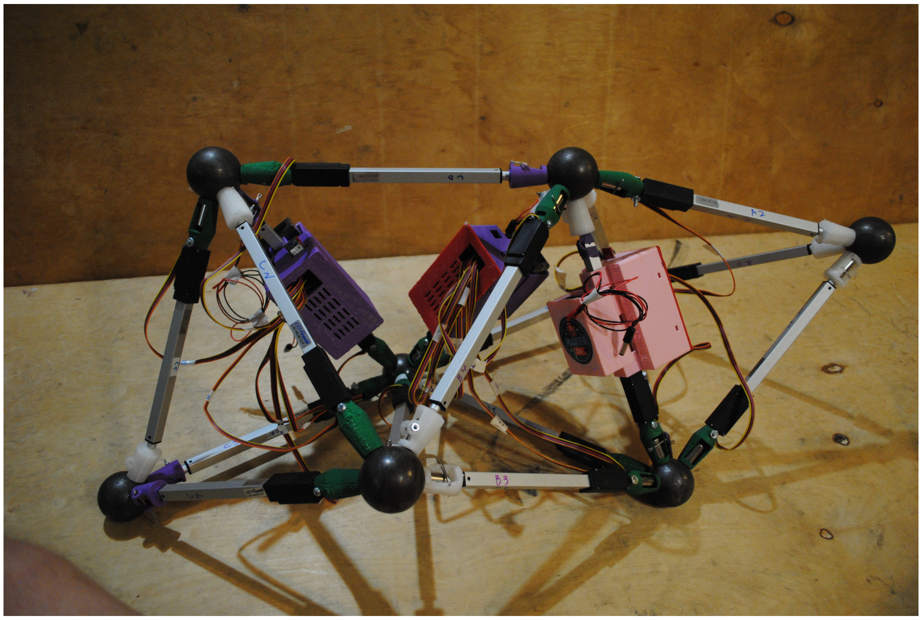

Figure 11: 5TetGlussBot Components

Each controller module electrical system can support up to 6 actuators, so two are required for the 3TetGlussBot. The Bluetooth modules open serial connections controlled by a computer. The Arduino Mega is programmed to support commands addressed to each or all of the actuators, and to convert electrical actuator feedback into digital position information. The computer and the Arduino control programs communicate via S-Expressions implemented with our own module. This is very analogous to sending JavaScript Object Notation (JSON) back and forth.

Commands to and feedback from the controller modules are managed by an Emacs E-Lisp program. This program organizes motion into “poses”. A series of poses make a “dance”. In order to dance, the program must synchronize the completion of poses, because there is no guarantee how long it will take to achieve a pose. The time to reach a position in theory depends on the voltage level provided by the battery and the force resisting the motion. In practice, when lightly loaded by moving only itself, the time is relatively predictable. It takes about 3 seconds for the 3TetGlussBot to go from its most compact state to its largest, most expanded state.

The current software has been written in anticipation of a much larger number of controllers and much more gluss. Nonetheless as mentioned in Section 6 the current software is specific to either the 3-tetrahedron geometry or the 5-tet geometry.

4.2 Open Source Realizations

We have provided everything necessary to construct gluss in the form of FLOSS. Providing an "Instructable"-style how-to guide is beyond the scope of this article, but a semi-skilled electronics technician can use the following resources to construct their own GlussBot.

Turret Joint The Turret Joint is implemented via OpenSCAD files. The file itself is here: `https://github.com/PubInv/turret-joint/blob/master/Models/TurretJoint.scad`. The model may be observed without even installing OpenSCAD, and even 3D printed, from the same file using the customizer feature of Thingiverse: `http://www.thingiverse.com/thing:1043716`

Robot Controller Shield An Arduino Mega Shield may be ordered directly from OshPark and then hand-soldered with through-hole components, where it is titled the *3x2 Motor Controller MegaShield, v0.2*. The `https://oshpark.com/shared_projects/fijpozoB` shield may be useful to anyone who wants to control up to six DC motors (up to 1 amp each) at the same time, via Bluetooth. Alternatively, you may use and modify the Creative Commons-licensed Eagle Soft CAD files, which can be found here: `https://github.com/PubInv/gluss/tree/master/GlussPCBv0.1`

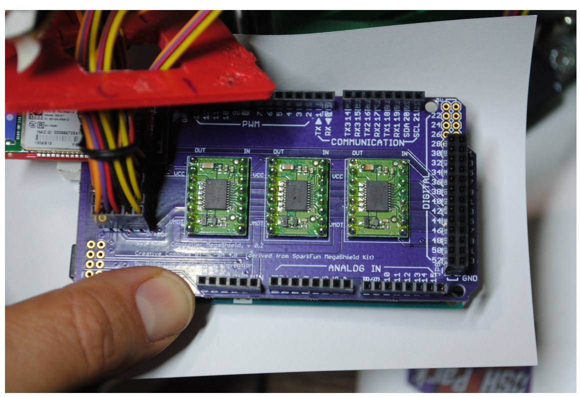

Figure 12: Arduino Mega Shield board

Mountable Enclosures Files to 3D Print the mountable enclosures for the battery pack and the Arduino Mega with the Robot Controller Shield are: `https://github.com/PubInv/gluss/tree/master/MegaAndShieldEnclosure`

Control Software An Emacs LISP program which controls the 3TetGlussBot via a command-line interface: `https://github.com/PubInv/gluss/blob/master/emacs-ctl.el`

An Arduino Sketch which implements a device driver for the GlussBot Controller Shield is required as well:

`https://github.com/PubInv/gluss/blob/master/GlussPCBv0.1/GlussPCBv0.1.ino`. This sketch needs the Arduino module S-Expr.

S-Expr The `https://github.com/PubInv/S-Expr` Arduino S-Expr project and its test project `https://github.com/PubInv/Arduino-S-Expr-Test` may be useful to anyone who wants to control an Arduino from Lisp or prefer S-Expressions to the closely related JSON format.

4.3 Speed Performance

The 3TetGlussBot is capable of "walking" and turning. Some might prefer the term "crawl" to "walk" in this case. It can move forward awkwardly at a rate of about five inches per minute. It can turn 30 degrees in about 60 seconds. We believe the 3TetGlussBot is the simplest amount of gluss that is capable of locomotion.

The present mode of walking avoids dragging the pseudopods by leaning to one side or front or back and then lifting the pseudopod, moving it forward (without ground contact), and placing it down again. Such a gait may be able to handle rugged terrain better than a gait that drags. Furthermore, the 3TetGlussBot gait assumes and has no anisotropy of friction against the ground, as is typical of snakebots. It is also non-inertial, in that it simply assumes that the actuators are powerful enough to move to any extent commanded of them. No attempt to model force has thus been made in the control of this gait. Since the gait was constructed by hand, it does not represent a modeling of the idea of leaning and or unweighting a given joint.

The 5TetGlussBot is significantly faster, moving about 19 centimeters (7.5 inches) per minute in the "broadwalk" mode and about approximately 28 centimeters (11 inches) per minutes in a walk in the "thin" direction using a "slide" in its gait.

The current programmed gaits are not optimal. We are developing a browser-based simulation using the Cannon.js physics engine that should allow for development of more efficient gaits without having to actually have a robot.

5 Related Research

Between 1996 and 2002 years ago, Arthur C. Sanderson and his colleagues published a series of papers[1, 2, 3] on modular robots. The "TETROBOT" was a variable-geometry truss, in which motion was accomplished but the change in length of linear actuators, connected in a modular geometry based on the tetrahedron and octahedron. A quadrupedal robot was constructed completely out of the tetrahedral/octahedral geometry. The TETROBOT robots successfully walked and even rolled. The TETROBOT hardware was significantly heavier and more powerful that then hardware used here. The glussbots have so far demonstrated no greater functionality, although we have demonstrated that very simple robots consisting of only 3 tetrahedra can locomote.

The technology presented in this article has drastically lowered the cost, thus making the glussbot/TETROBOT concept accessible to hobbyists and researchers on a limited budget.

The TETROBOT used a joint called the CMS joint. Although possibly superior in not allowing an extra degree or rotational freedom, it would be a challenge to use the CMS joint with the Actuonix actuators because the pushrod must fully retract, or the length of the pushrod would be have to be extended with an attachment. Sanderson's students used actuators that extended from the middle, avoiding this problem. If the Gluss Project ever develops its own actuators, it should explore using this joint.

If you read the introduction of the brilliant book by Shigoe Hirose[10] substituting "even simpler soft squiggly thing that might not be as cylindrical as a snake" for the word "snake", you will have an excellent motivation for the gluss concept. More generally, much of the work developed for snakebot locomotion[6] is directly reusable, in the sense that a long enough tetrahelix can model a snake, and further inspires the idea of using simpler models mapped into a gluss model to perform complex movements.

Buckminster Fuller also promoted *tensegrity*, and some research on Tensegrity Robots has been done, the work of Paul, Valero-Cuevas, and Lipson[11] being a good starting point. This work has developed into a serious effort[12] by NASA to explore tensegrity robots for extraterrestrial exploration.

Tensegrities are closely related to the gluss concept, more researched, and potentially more performant. In fact a gluss could be considered a special case of a tensegrity, using vanishingly short cables and, in the terminology of [11], *strut-collocated actuation*. It has been reasonable to produce a static gait for the 3TetGlussBot and 5TetGlussBot because its behavior is not very dynamic: it is so slow and strong that velocity is irrelevant at the current scale. Reported tensegrity robots have focused on dynamic, "hopping" and rolling gaits.

It is possible that gluss is easier to work with for an actual human being on the ground. Although of course both systems will use computer control systems, one can imagine a large robot crawling into place imperfectly, and some workperson making a manual adjustment: "Actuator #37, get shorter!" This is intellectually more difficult for a tensegrity, wherein changing a cable length has less predictable impact on the tensegrity geometry. However, many of the future steps outlined in Section 6 apply to both gluss and tensegrity robots.

This paper presents the gluss as a "machine", rather than a "mechanism". That is, it motivates gluss by asserting it can exert and resist force, yet currently treats gluss positioning as a purely kinematic, rather than dynamic problem. The actuators currently in use are geared such that they are so slow and powerful that the behavior is not really dynamic. If static analysis of a resulting geometry is needed, for example to ask if structure used as a bridge will bear a load, a finite element approach[?] will be adequate.

If one chooses to attempt to exert a high enough force or to move more quickly, classic robot control theory which models forces and velocities based on Lagrangian mechanics will be required.

6 Future Steps

We have organized these future research areas from most difficult to least difficult to offer a range of problems accessible to the researcher and hobbyist.

Remove Scale: (Pure math, physics, computer science) At present the 3- and 5-TetGlussBot is programmed as a specific geometry. Any program written for it would have zero value for a robot of the same general shape but built with considerably more actuators. Ideally, we would be able to program gluss completely independent of the scale of implementation. We would treat it as a true metamorphic material, rather than as a net of actuators arranged in a particular configuration. This can be considered a problem of pure mathematics, touching on wavelet theory, for example. It blends into computer science and finally robotics.

Math of Ideal Systems: (Geometry) What configurations are possible of ideal and real-world gluss? For example, what is the maximum radius of curvature of an octet truss employing joints and actuators of specific properties? This is primarily in the realm of geometry, and perhaps computational geometry.

Throw a Ball: (Mechatronics) Develop the dynamic control needed to throw a ball, thereby moving from kinematics to dynamics.

Cheap Actuator: (Physics, Electrical Engineering) The gluss concept heightens the need for very inexpensive linear actuators. The current actuators at $80 make constructing a 100-actuator glussbot expensive. It should be possible to build a linear actuator for 1/10th of this cost, although it may require giving up some advantages. If we imagine a design spectrum of actuators, creating a new and less expensive point in the spectrum of actuators designs would greatly expand the possibilities of gluss.

Applications: (Philosophy, Design, Entrepreneurship) An exciting application would help motivate the gluss concept. This does not require engineering, but requires careful thought.

Static Force Performance: (Structural Analysis) It would be be nice to answer the such questions as:

- If a 5-tetrahedron tetrahelix were bolted to a frame and extended horizontally, how much weight can it support at the free end?

- If a gluss bot crawled under a car and sought to jack up the car enough for a tire to be changed, how forceful would each actuator have to be?
- If a large glussbot crawled across a chasm, could a truck or a human being safely move across it without danger of collapse?

We have not attempted to analyze gluss in terms of force performance. We believe this would be analogous to analysis of static space frames using the dynamic configuration at a point. Finite element analysis is a standard approach to this.

Quick Joint: (Mechanical Engineering) If all of the turret joints in a 3TetGlussBot are disassembled so that the linear actuators can be stacked together for efficient transport, it takes more than an hour to bolt them back together. A joint that could be disassembled and assembled more quickly would be convenient. We have found that it is possible to build a magnetic joint similar (but larger) than those used in the Geomag™ toy. However, it does not scale up in size and is weak against external non-axial forces. We would like to investigate the turret joint using bolt-free "bayonet mounts" which can be easily snapped into place.

Construction System: (Mechanical Engineering) If we imagine the members connecting the joints are not actuators but simply rods that can be cut to any length, we have a construction system for making space frames. Exploring the possibilities of such a construction system to make organic and interesting shapes freed of rectilinear limits would be worthwhile.

Build Simulator: (Computer Science) A 3TetGlussBot can be constructed for about $1300. Nonetheless this is beyond the means of many hobbyist. A simulator would allow one to research motion without monetary cost. The open-source, browser deliverable physics engine "Cannon.js" provides all the basic machinery needed to build such a simulator.

7 Contact and Getting Involved

The Gluss Project `http://pubinv.github.io/gluss/` is a free-libre, open-source research, hardware, and software project that welcomes volunteers. It is our goal to organize projects for the benefit of all humanity without seeking profit or intellectual property. To assist, contact <read.robert@gmail.com>.

A Turret Joint Geometric Limitations

A.1 Geometric Preparation

Defining all angles against a center line between two rotor holes in a plane, let:

$A \equiv$ Min Actuator Length. Without loss of generality, assume this is 1.0.
$Z \equiv$ Max Actuator Length
$Q \equiv \frac{Z}{A}$, The ratio of the actuator lengths, noting that $Q \geq 1$. Since $A = 1$, $Q \equiv Z$.
$G \equiv$ Greatest Angle Reachable by Center of Rotor
$L \equiv$ Least Angle Reachable by Center of Rotor
$P \equiv$ Angle from the Post to the edge of the Rotor
$\theta \equiv$ Angle of Inmost Edge of the Rotor Hole
$\psi \equiv$ Angle of Outmost Edge of the Rotor Hole

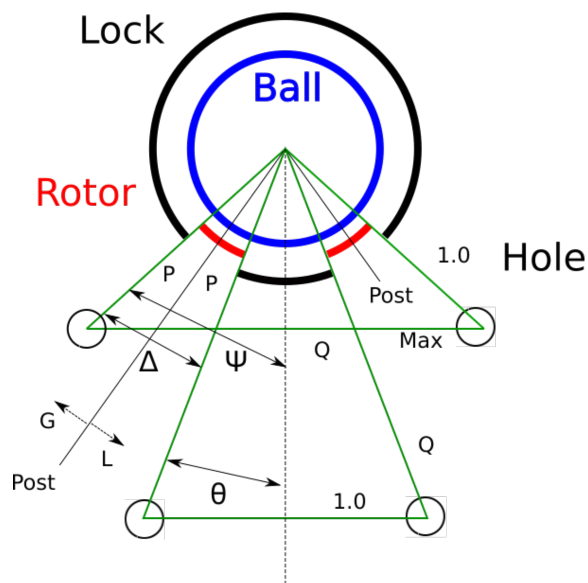

Figure 13: Turret Joint Geometry Constraints

What θ, ψ maximizes Q and what is Q?

We will define the angle of the hole to be:

$$\Delta \equiv \psi - \theta \qquad \text{(Delta Definition)}$$

From the diagram:

We then place engineering constraints upon these variables to represent physical conditions which define the limits of the joint. We name these constraints *meet*, *bump*, *capture*.

The *meet* condition means that post is actually within the hole.

The *bump* condition means that the rotors don't bump into each other in their extreme position.

The *capture* condition means that the the rotor can't fall out of the hole.

In both cases, we have have the capture constraint:

$$\Delta \geq G - L \tag{capture}$$

To realize the most acute position, we have constraints:

$$\theta \leq L \tag{acute meet}$$

$$\frac{\Delta}{2} \leq \theta \tag{acute bump}$$

To realize the least acute position, we have constraints:

$$\psi \leq G \tag{obtuse meet}$$

It is perhaps obvious that the rotors cannot bump in the most obtuse position, but we observe that:

$$360° - 2 \cdot \psi \geq \frac{G - L}{2} \tag{obtuse bump}$$

is invariably true because $2 \cdot \psi < 180°$ because psi is a half angle of a triangle, and likewise $\frac{G-L}{2} < 180°$.

A.2 Maximum usable $\mathbf{Q} = \varphi$

In the previous subsection we set up geometric conditions generally so that we could consider them from both a Classical Trigonometry and from Rational Trigonometry. Although we used the world "angle", we did not rely additive properties of angles except in the definition of Delta and the "obtuse bump" condition. This will allow us to perform an analysis in Rational Trigonometry in the next section.

When we attempt to set G and L to be as wide apart as possible by asserting $G \equiv \psi$ and $L \equiv \theta$, we must ensure that all of thse constaints are true. The two meet conditions become true by equality. Likewise the capture condition is trivially true by substitution. The obtuse bump condition can never be false. However, the acute bump conditions remains:

$$\frac{\Delta}{2} \leq \theta \quad \equiv \quad \frac{G - L}{2} \leq L \quad \equiv \quad G \leq 3 \cdot L \tag{acute bump}$$

$$G \equiv \arcsin \frac{Q}{2}$$

$$L \equiv \arcsin \frac{1}{Q \cdot 2}$$

Returning to our definitions of G and L,

$$\arcsin \frac{Q}{2} \leq 3 \cdot \arcsin \frac{1}{Q \cdot 2}$$

So our question becomes what is the maximum Q that satisfies this inequality. Noting from the graph of the arcsin function in the relevant ranges where $Q \geq 1$ that G is a monotonically increasing function and L is a monotonically decreasing function of Q (since Q is in the denominator), the maximum Q will be the solution to:

$$\arcsin \frac{Q}{2} = 3 \cdot \arcsin \frac{1}{Q \cdot 2} \tag{original}$$

Upon investigating this with a spreadsheet we noted that Q suspiciously approached the famous number 1.618..., the golden ratio, φ. We investigated further with the help of Wolfram Alpha Pro (https://www.wolframalpha.com). Wolfram Alpha gave us a closed-form solution to *original* of $\frac{1+\sqrt{5}}{2}$ that we recognized as φ, but oddly would not show us the set of transformations. After verifying that $Q = \varphi$ was a solution in that way, we used the special property of φ that $\varphi = \frac{1}{\varphi} + 1$, to rewrite our constaint as:

$$\arcsin \frac{Q}{2} = 3 \cdot \arcsin \frac{Q-1}{2} \tag{assuming $Q = \varphi$}$$

which led Wolfram Alpha Pro to indeed provide a long and complex chain of trigonometric transformations to show that $Q \equiv \frac{1+\sqrt{5}}{2} \equiv \varphi$ is actually an algebraic solution to this equation.

Thus the surprising result is obtained that, for a turret joint where the problem of the rotors physically bumping is not solved by some other means, the highest Q that we can take advantage of is φ, and that it would be perfectly correct in Figure 13 to label the slenderest triangle as a Golden Triangle and the obtuse triangle as a Golden Gnomon. Furthermore, $\theta = 18°$, and $\psi = 54°$, and $\Delta = 36°$.

This is a valuable ideal to strive for, but a physically realizable joint may not support such a high Q, because any naive physically realizable turret joint is likely to have a rotor diameter with a lip significantly larger than the hole, and the post will have an actual thickness that must be considered in computing G and L. Nonetheless this analysis is a starting point for analyzing more realistic joints, even if such joints will have to be dealt with numerically rather than having an elegant analytic solution.

A.3 Rational Trigonometry

A magician may pull a rabbit out of a hat, but a mathematician should avoid it. The Classical Trigonometry result in the last section feels like pulling a rabbit out of a hat because it relies on results beyond our ability of human calculation. Norman J. Wildberger has a presented a simpler re-formulation of trigonometry called Rational Trigonometry[**?**] that proposes to avoid such rabbits.

This section assumes the reader is familiar with Rational Trigonometry.

In our diagram and our previous formulation, we can almost treat the angles of that diagrams as spreads. However, since spreads are non-linear, we cannot use *Delta Definition*, or *acute bump* in their formulation from that section.

Now if we consider *theta*, G and L as spreads, we have the definition of the spread as the quadrance of the opposite leg over the quadrance of the hypotenuse:

$$\theta \quad \equiv \quad \frac{(1/2)^2}{Q^2} \quad \equiv \quad \frac{1}{4 \cdot Q^2} \tag{rational-theta}$$

$$\psi \quad \equiv \quad (\frac{Q}{2})^2 \quad \equiv \quad \frac{Q^2}{4} \tag{rational-psi}$$

Rather we assert to avoid the problem of the rotors bumping in the case of the most acute triangle we assert:

$$P \leq \theta \tag{rational acute bump}$$

Our goal is to find the maximum Q satisfying all constraints. It is clear that θ goes down as Q goes up, and that ψ goes up as Q goes up. P goes up as Q goes up (this remains to be rigorously shown).

Therefore Q is maximized when $P = \theta$.

We can thus use Theorem 56 (Three equal spreads) [**?**] on θ:

$$\psi \equiv \theta(3 - 4 \cdot \theta)^2$$

together with *rational-psi* via substitution to obtain a series of purely algebraic manipulations:

$$\frac{Q^2}{4} = \theta(3 - 4 \cdot \theta)^2$$

then by substituting *rational-theta*:

$$\frac{Q^2}{4} = \frac{1}{4 \cdot Q^2}(3 - 4 \cdot \frac{1}{4 \cdot Q^2})^2$$

performing elementary algebraic simplification we obtain:

$$Q^4 = (3 - \frac{1}{Q^2})^2$$

Perform the substitution $x = Q^2$:

$$x^2 = (3 - \frac{1}{x})^2$$

Take the square root of both sides and converting to common fraction:

$$x = \pm\frac{(3 \cdot x - 1)}{x}$$

Cross multiplying and simplifying:

$$x^2 - 3 \cdot x = -1$$

Completing the square by adding $\frac{9}{4}$ to both sides:

$$x^2 - 3 \cdot x + \frac{9}{4} = \frac{5}{4}$$

Writing left hand side as a square:

$$(x - \frac{3}{2})^2 = \frac{5}{4}$$

Take the square root of both sides:

$$\pm(x - \frac{3}{2}) = \pm\frac{\sqrt{5}}{2}$$

Adding $\frac{3}{2}$ to both sides:

$$x = 1 + \frac{1}{2} + \frac{\sqrt{5}}{2}$$

Being previously alerted to the existing of φ in our solution, we recognize this as $1 + \varphi = \varphi^2$.

$$Q = \sqrt{x} = \sqrt{1 + \varphi}$$

Substituting back into $x = Q^2$,

$$Q = \varphi$$

Note that we have taken square roots twice in this operation, and therefore must check that the negative solutions are not also valid solutions. Wolfram Alpha Pro combined with the fact that only $Q > 1.0$ allows us to conclude this is the only valid solution.

We have thus obtained the same algebraic result as the trigonometrical more understandably and with less recourse to computer aids.