

圆的内接正多边形逼近法改进

由于 $X_n - > X_{n+1}$ 需要四次矩阵乘法，考虑用递推计算。记 P_{xi} 为两点之间x坐标的差值：

$$P_{xi} = X_{i+1} - X_i = (\cos a + \sin a - 1)X_i$$

$$P_{xi+1} = X_{i+2} - X_{i+1} = (\cos a + \sin a)(\cos a + \sin a - 1)X_i = (\cos a + \sin a)P_{xi}$$

同理可得 $P_{yi+1} = (-\sin a + \cos a)P_{yi}$ 。

因此求得初始 P_x, P_y 后，每次求新的 X_i, Y_i 只需要两次乘法和两次加法求出新的 P_{xi+1}, P_{yi+1} ，再根据递推用两次加法求出下一个 X_{i+1}, Y_{i+1} ，总共用到 $2n$ 次乘法和 $4n$ 次加法即可求出整个圆。比原始求法 $4n$ 次乘法要有显著的加速。

中点画椭圆

由于椭圆x, y轴对称，只要求第一象限然后对称到四个象限即可

首先，求得斜率：

$$\frac{2xdx}{r_x^2} + \frac{2ydy}{r_y^2} = 0$$

$$\frac{dx}{dy} = -\frac{2r_y^2x}{2r_x^2y}, \text{ 临界点在 } 2r_y^2x = 2r_x^2y, \text{ 此时切换主导部分}$$

主要目标是取中点，根据中点在椭圆的内外来判断下一个点取哪个

对于主导部分是x（x变的比y快）：

记 $P_k = f(x_k + 1, y_k - \frac{1}{2}) = r_y^2(x_k + 1)^2 - r_x^2(y_k - \frac{1}{2})^2 - r_x^2r_y^2$ ，则有两个式子。假如中点在内部，则下一个点的判别式如下：

$$P_{k+1} = r_y^2(x_k + 2)^2 + r_x^2(y_k - 0.5)^2 - r_x^2r_y^2 = P_k + r_y^2(2x_k + 3) \\ \delta = r_y^2(2x_k + 3)$$

假如中点在外部，则下一个点的判别式如下：

$$P_{k+1} = r_y^2(x_k + 2)^2 + r_x^2(y_k - 1.5)^2 - r_x^2r_y^2 = P_k + r_y^2(2x_k + 4) + r_x^2(-2y_k + 2) \\ \delta = r_y^2(2x_k + 4) + r_x^2(-2y_k + 2)$$

下半部分（y变化比x快）同理，记：

$$P_k = f(x_k + \frac{1}{2}, y_k - 1) = r_y^2(x_k + \frac{1}{2})^2 - r_x^2(y_k - 1)^2 - r_x^2r_y^2$$

假设中点在内部，则

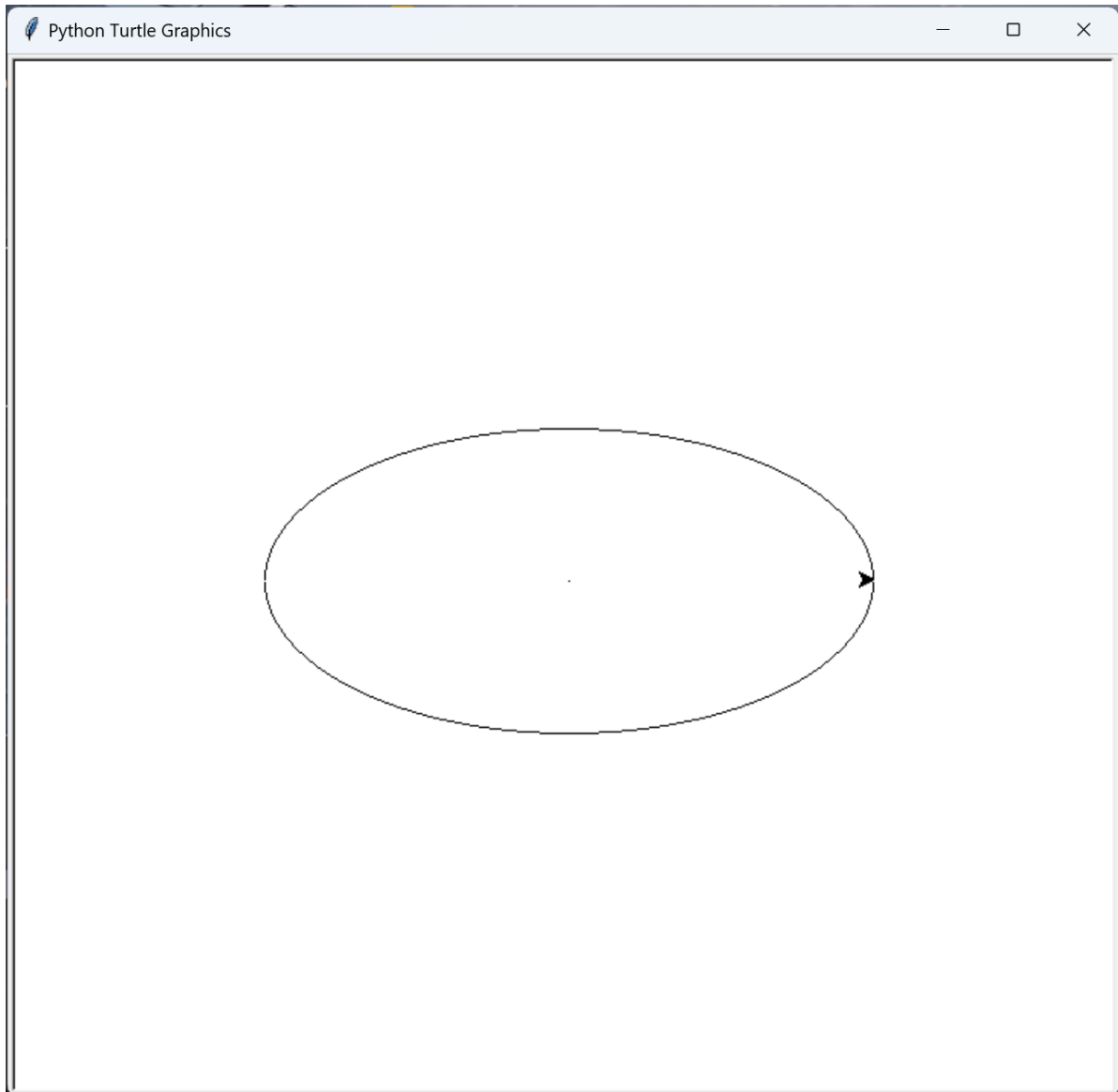
$$P_{k+1} - P_k = r_x^2(2y_k + 3)$$

假设在外部，则

$$P_{k+1} - P_k = r_x^2(2y_k + 3) + r_y^2(-2x_k + 2)$$

对于起始点 $(0, r_y)$ ，第一个值为 $r_x^2 + r_y^2(-r_x + 0.25)$

[这里](#) 是我的python实现，使用了turtle库。结果如下：



第四题

首先边的数据结构为 $(x, \Delta x, y_{\max})$, 排序方法是先排 x , x 相同排 Δx , 以此类推

0:

1:->(6,-2,3)->(6,0.5,5)

2:

3:->(2,0,6)

4:

5:->(8,-1,7)

6:->(2,4,7)

7:

接下来进行AET,从 $y=1$ 开始,到指定高度自动进边, $=>$ 代表有边要被踢出

1. AET:(6,-2,3),(6,0.5,5)
2. AET:(4,-2,3),(6.5,0.5,5)

3. AET:(2,-2,3),(7,0.5,5),(2,0,3)=>(7,0.5,5),(2,0,6)
4. AET:(7.5,0.5,5),(2,0,6)
5. AET:(8,0,5),(2,0,6),(8,-1,7)=>(2,0,6),(8,-1,7)
6. AET:(2,0,6),(7,-1,7),(2,4,7)=>(7,-1,7),(2,4,7)
7. AET:(6,-1,7),(6,4,7)=> \emptyset

第五题

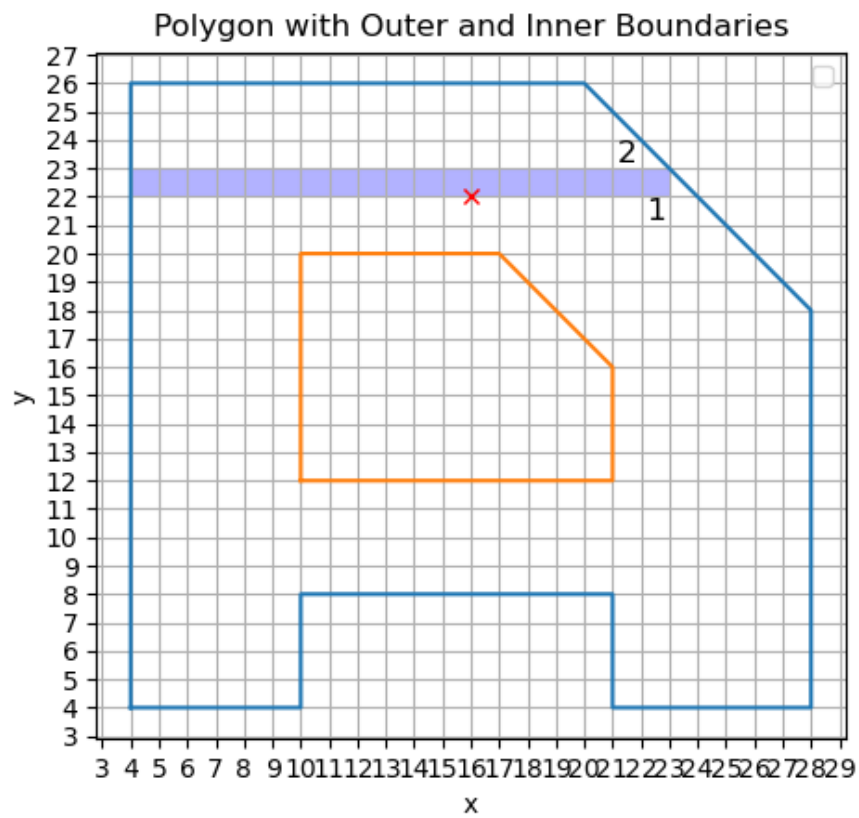
最右边是栈顶，最左边是栈底，每次画栈顶被弹出的一点，以此类推

1. stack=[(3,3)]
2. stack=[(4, 3), (2, 3), (3, 4), (3, 2)]
3. stack=[(4, 3), (2, 3), (3, 4), (4, 2), (2, 2)]
4. stack=[(4, 3), (2, 3), (3, 4), (4, 2)]
5. stack=[(4, 3), (2, 3), (3, 4), (5, 2)]
6. stack=[(4, 3), (2, 3), (3, 4), (6, 2), (5, 3)]
7. stack=[(4, 3), (2, 3), (3, 4), (6, 2), (6, 3), (5, 4)]
8. stack=[(4, 3), (2, 3), (3, 4), (6, 2), (6, 3), (4, 4)]
9. stack=[(4, 3), (2, 3), (3, 4), (6, 2), (6, 3)]
10. stack=[(4, 3), (2, 3), (3, 4), (6, 2)]
11. stack=[(4, 3), (2, 3), (3, 4)]
12. stack=[(4, 3), (2, 3), (2, 4)]
13. stack=[(4, 3), (2, 3)]
14. stack=[(4, 3)]
15. stack=[]

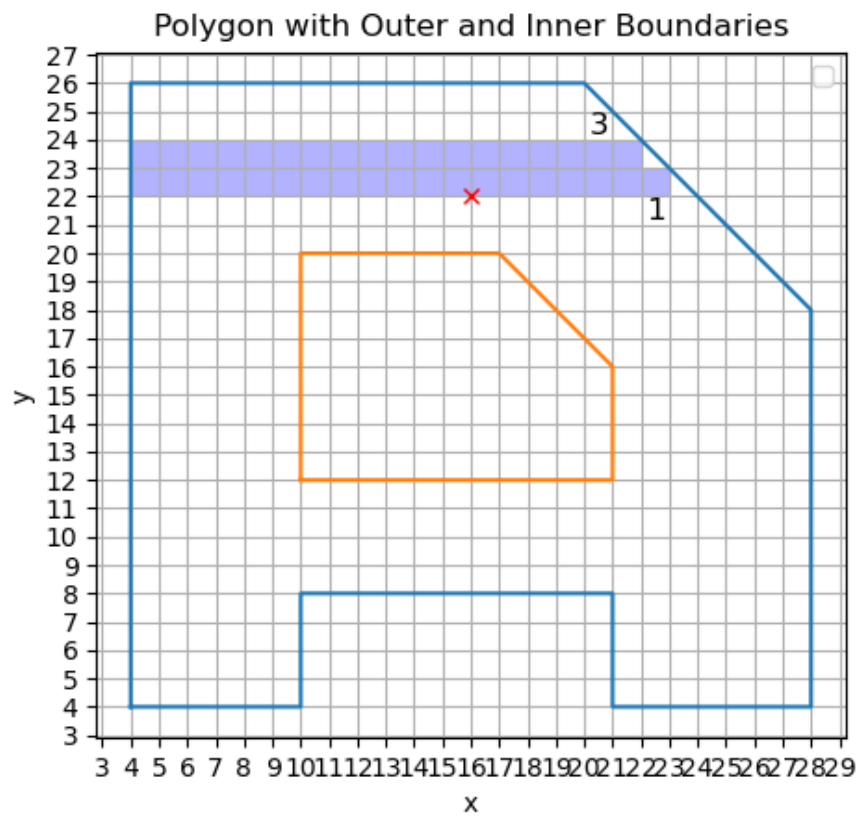
第六题

过程如下：

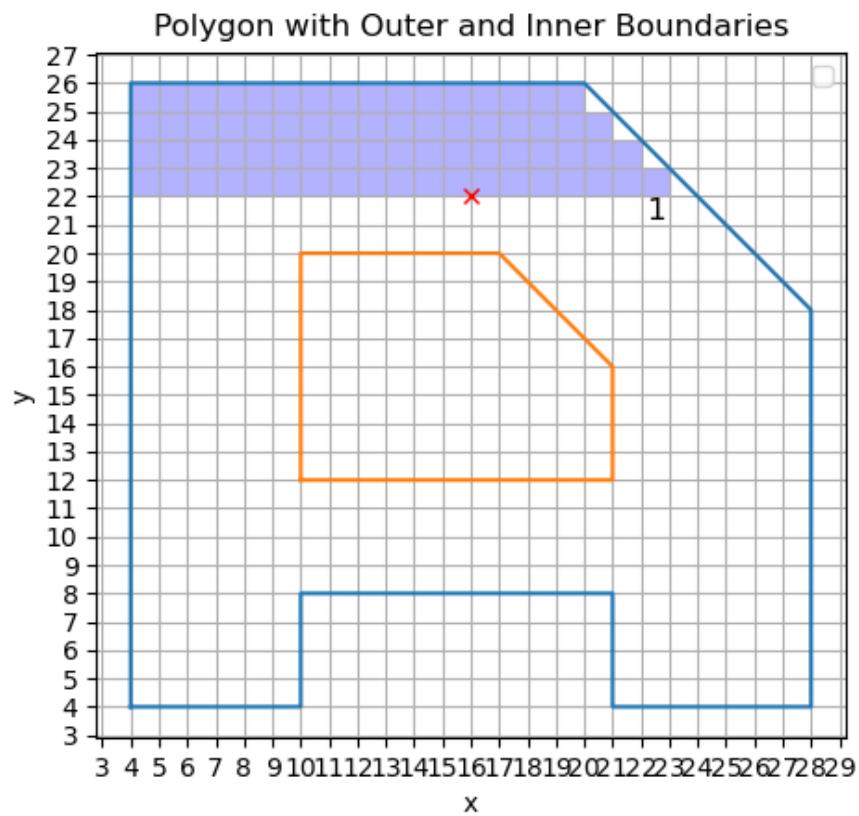
stack is [1, 2]



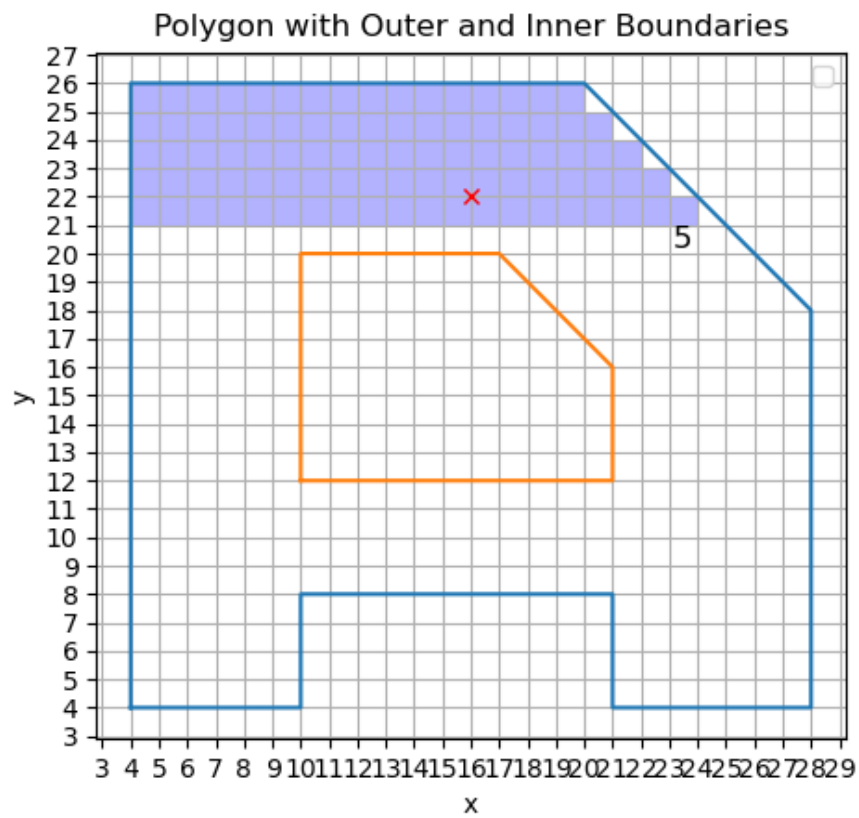
stack is [1, 3]



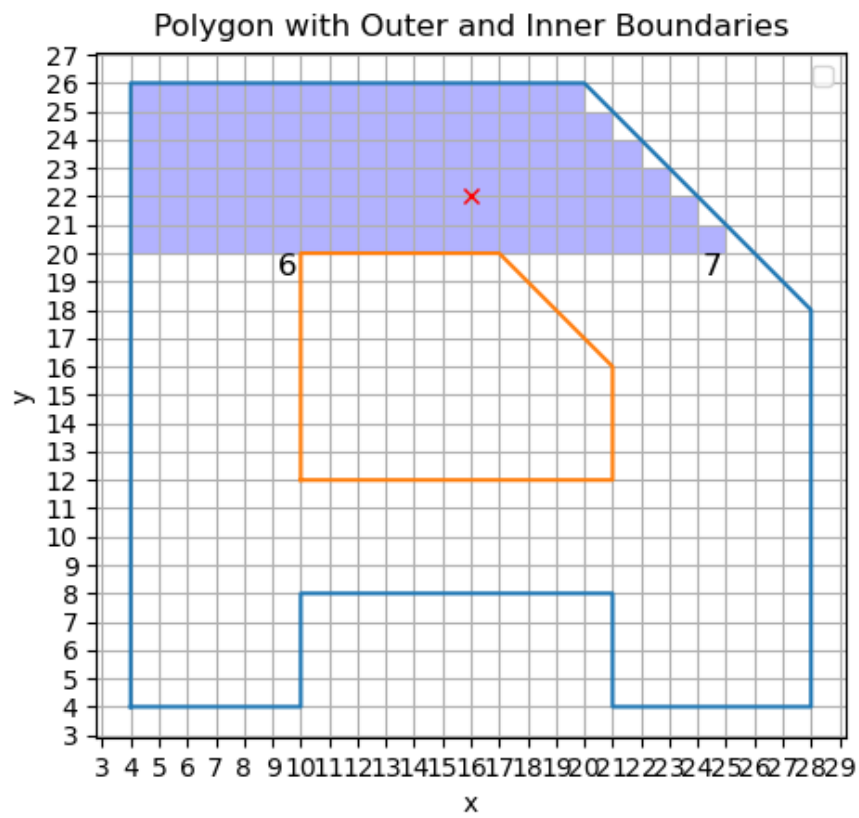
stack is [1]



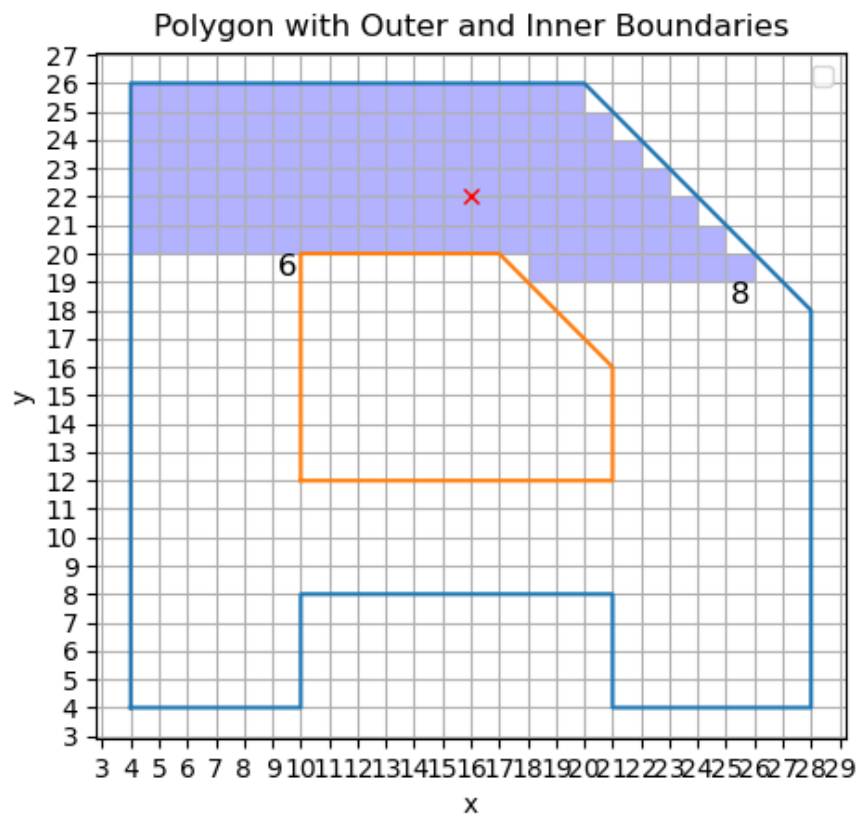
stack is [5]



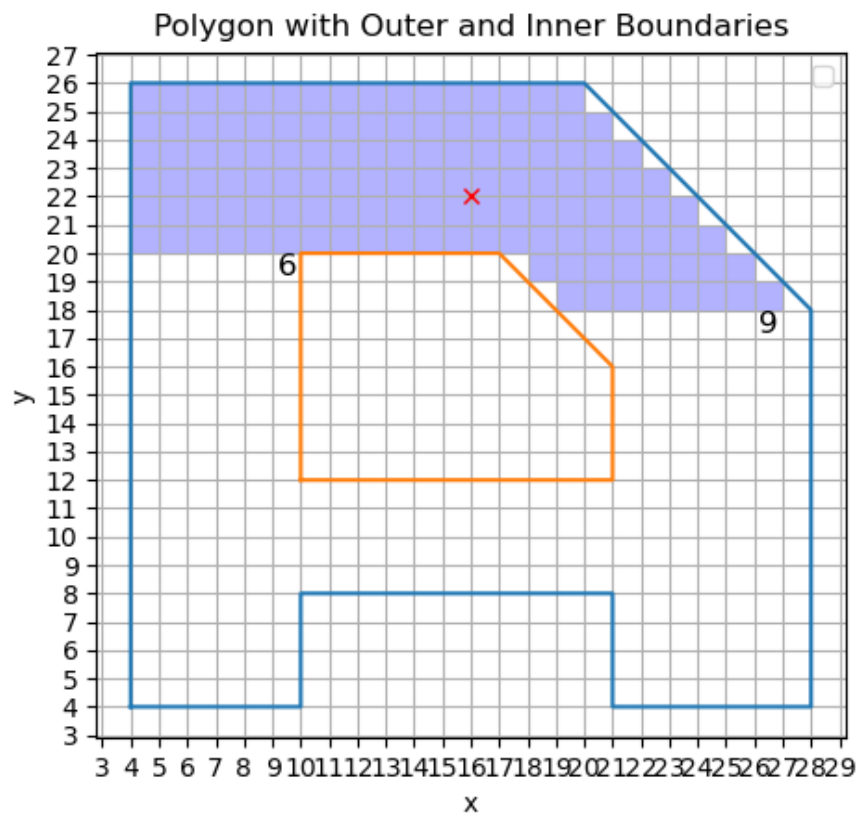
stack is [6, 7]



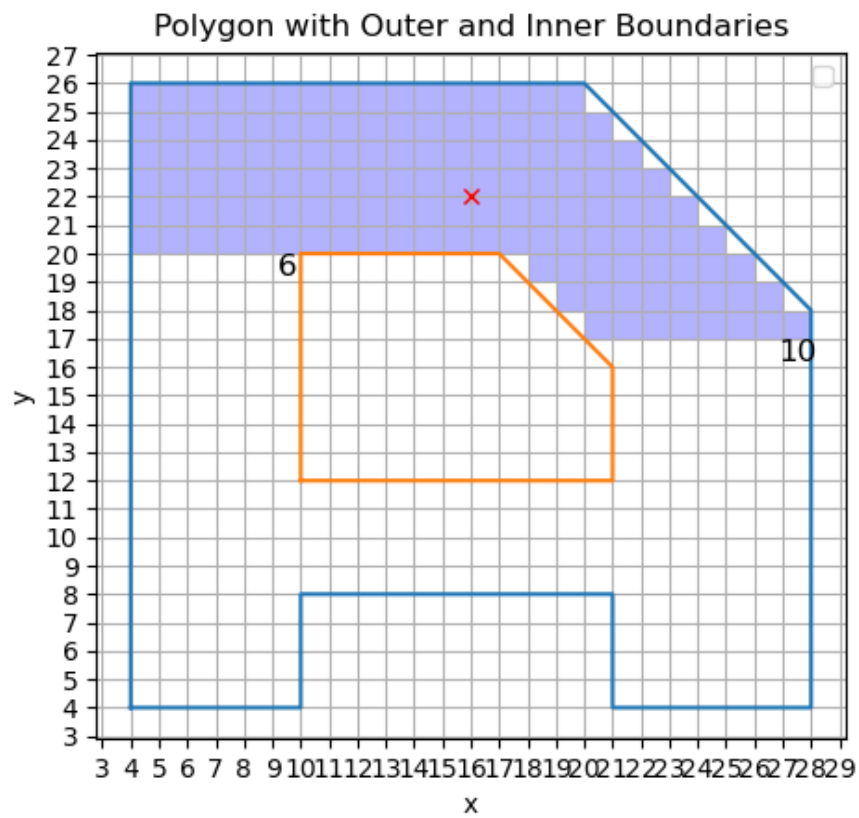
stack is [6, 8]



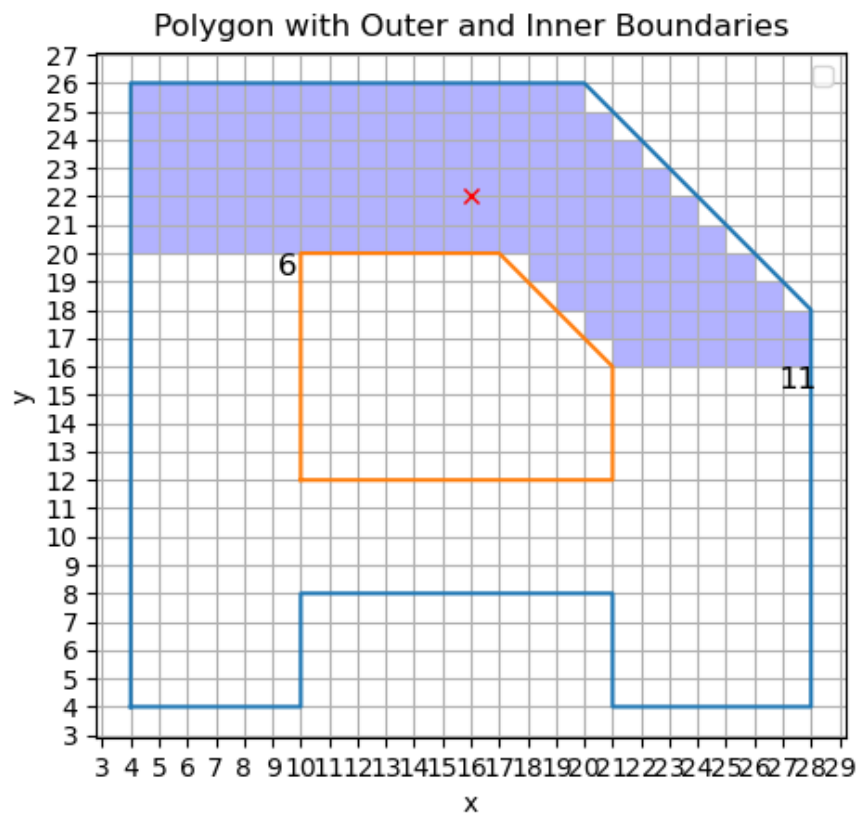
stack is [6, 9]



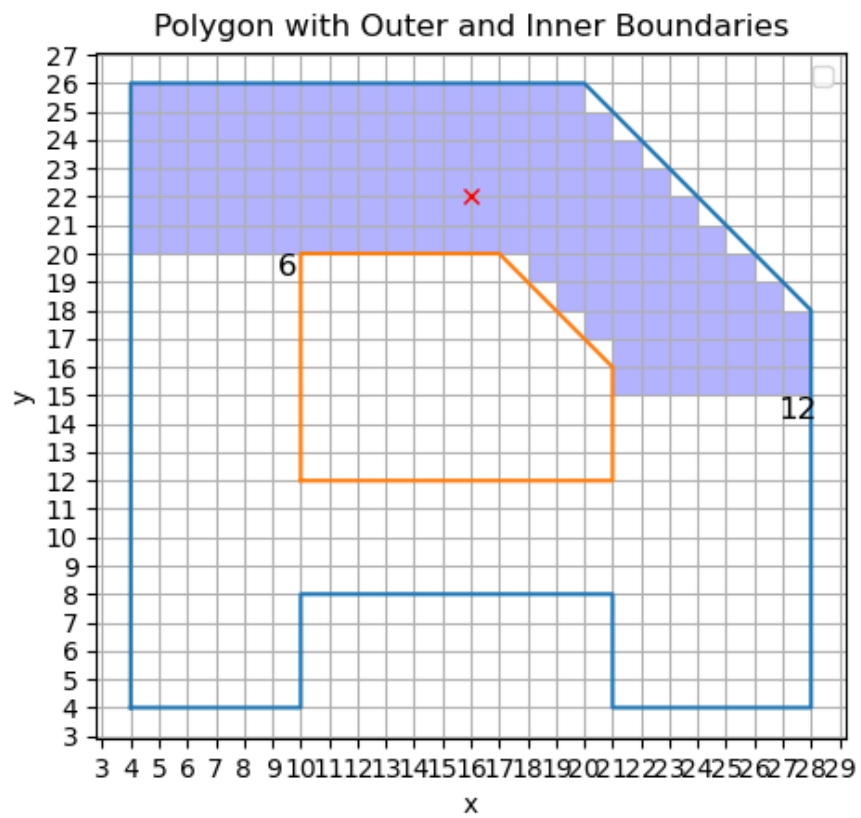
stack is [6, 10]



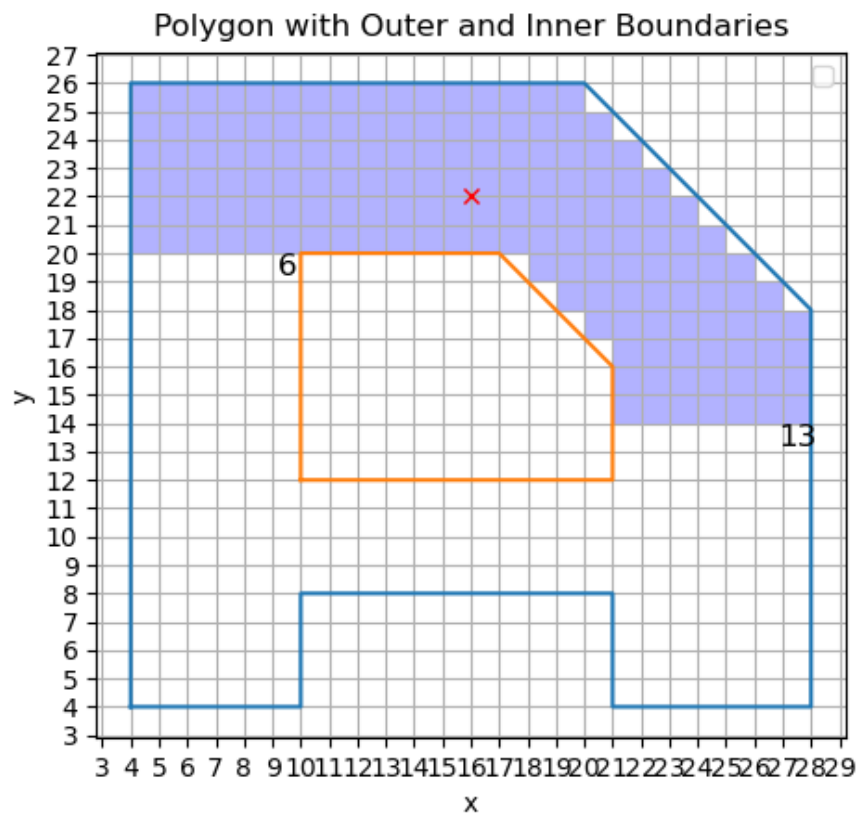
stack is [6, 11]



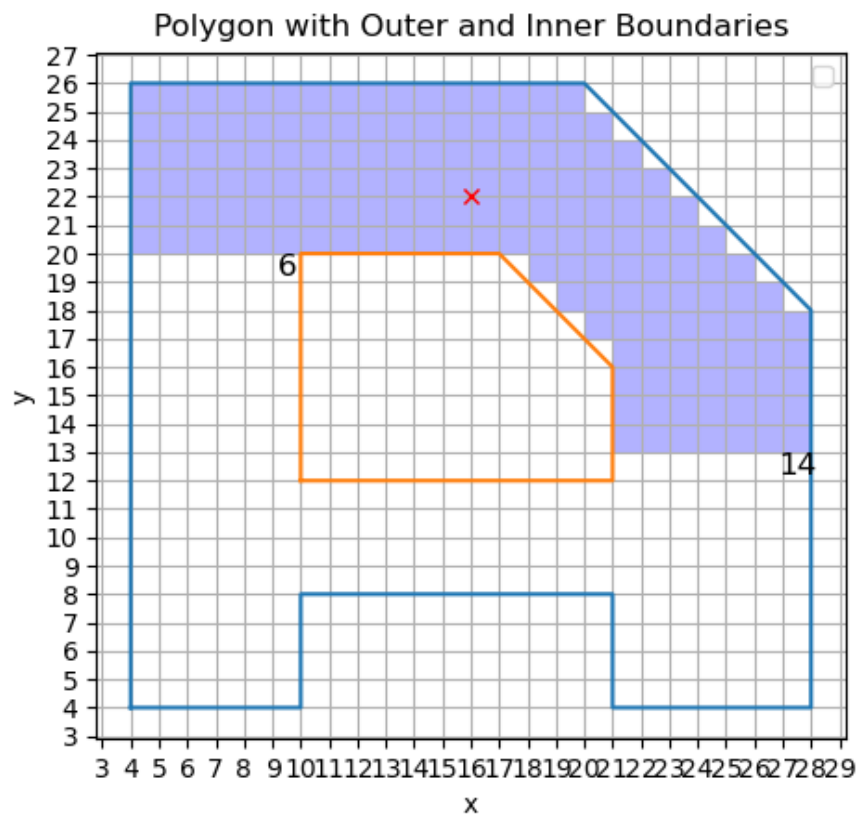
stack is [6, 12]



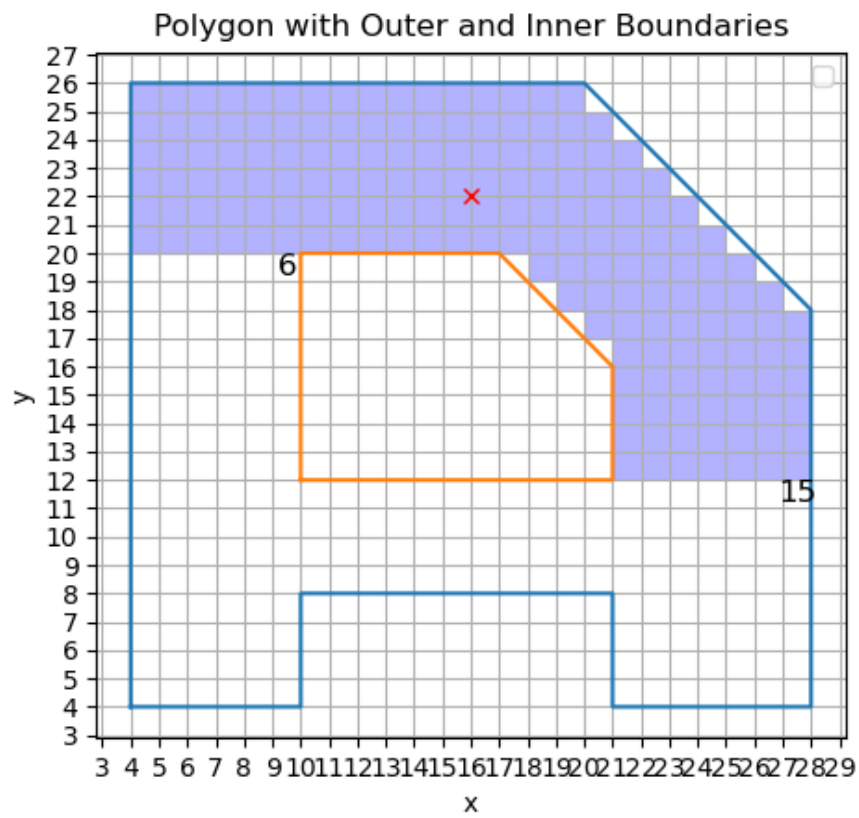
stack is [6, 13]



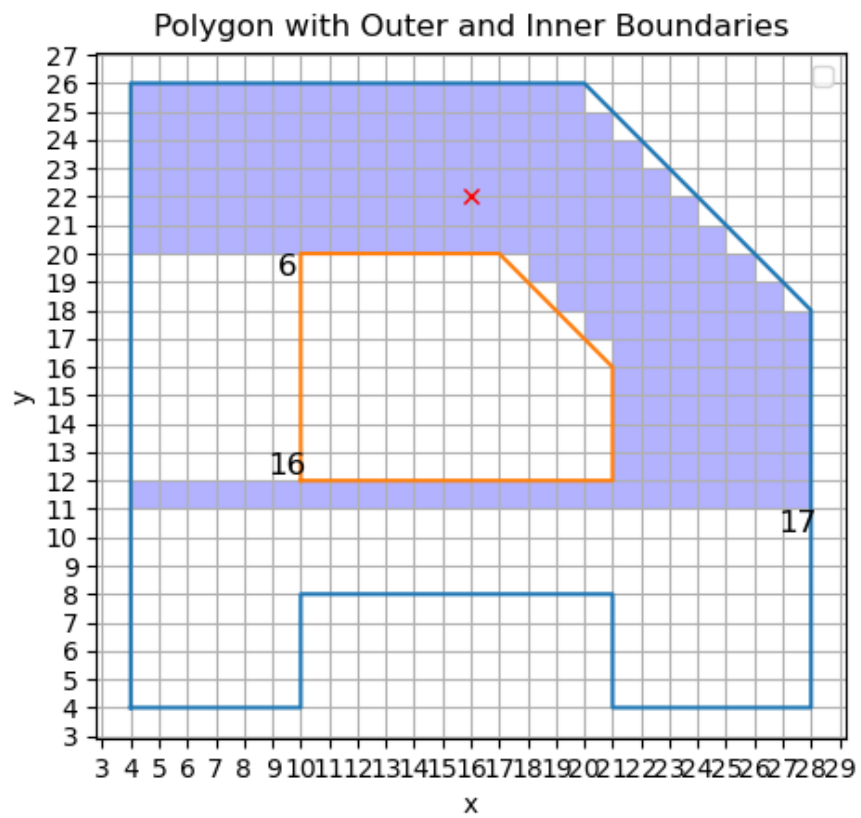
stack is [6, 14]



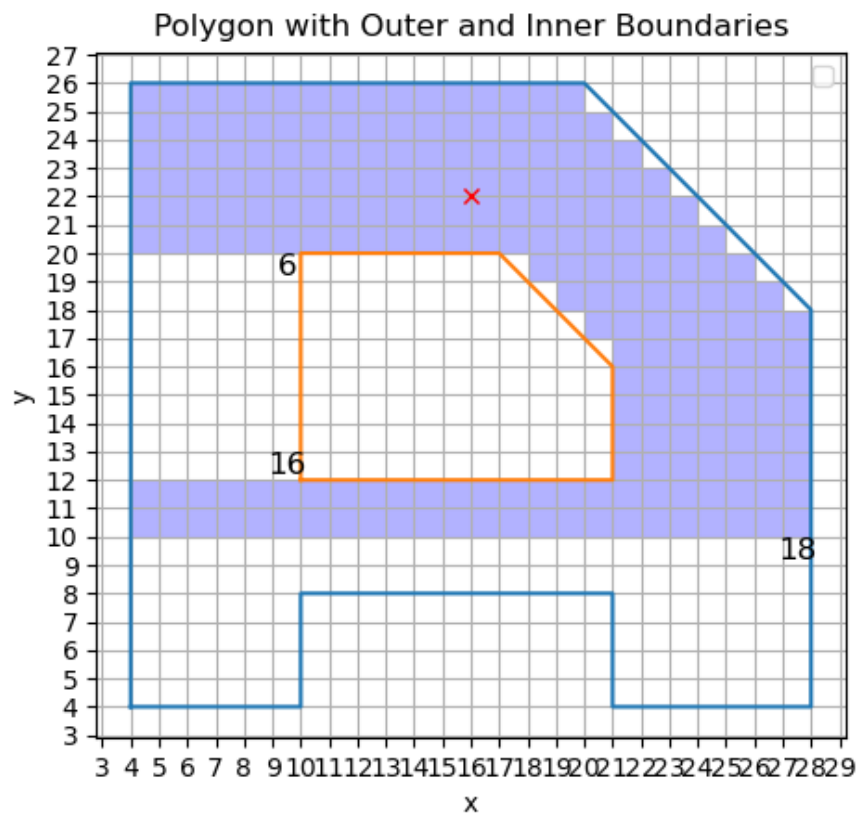
stack is [6, 15]



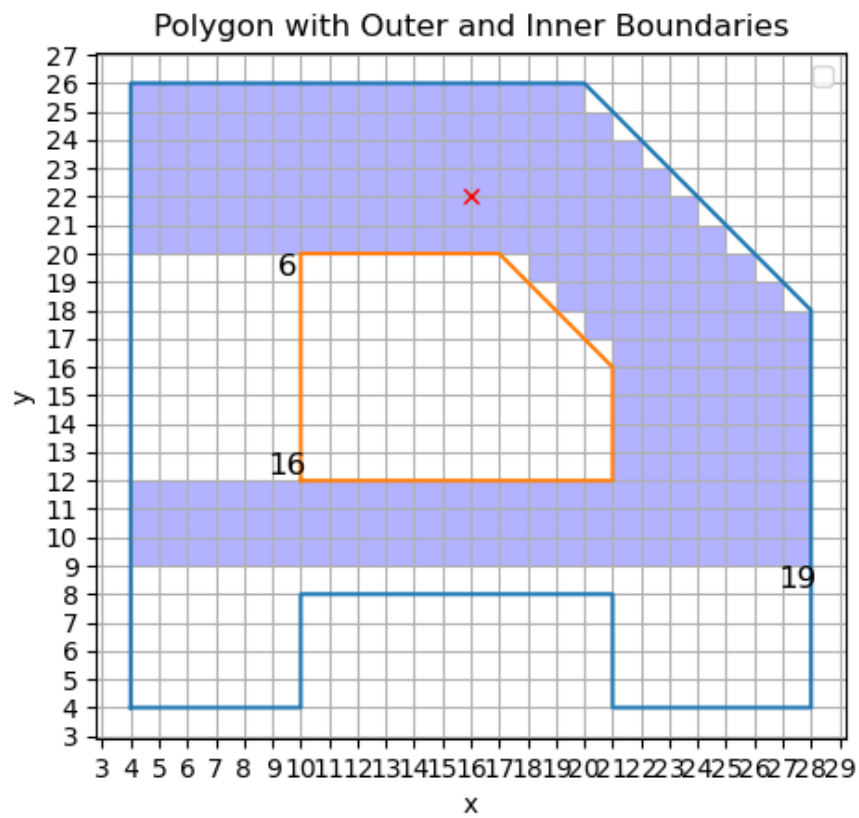
stack is [6, 16, 17]



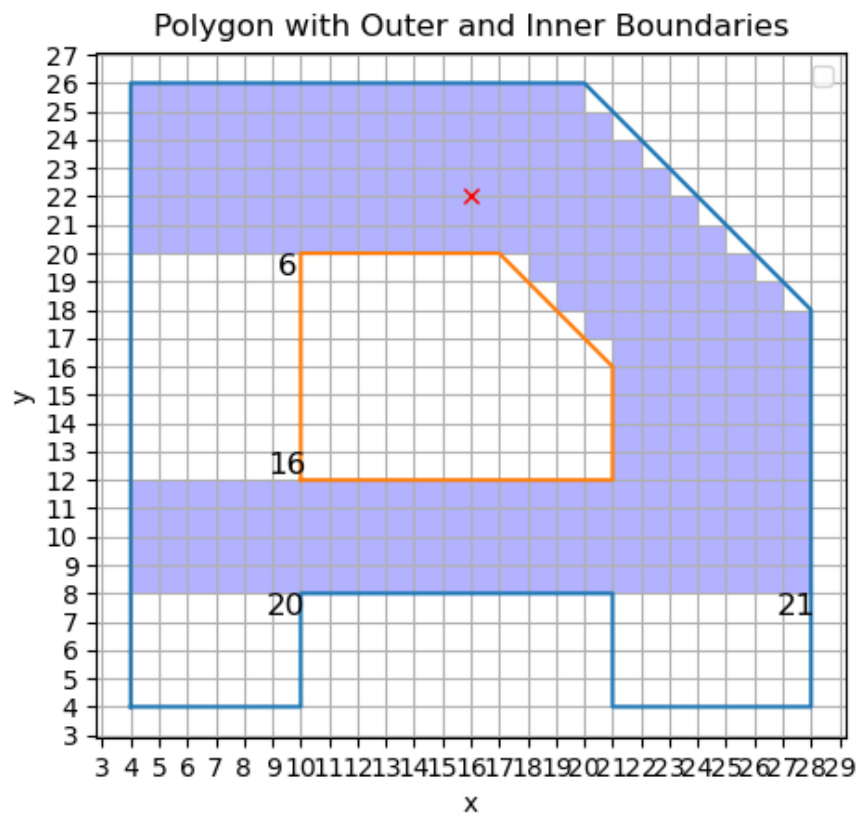
stack is [6, 16, 18]



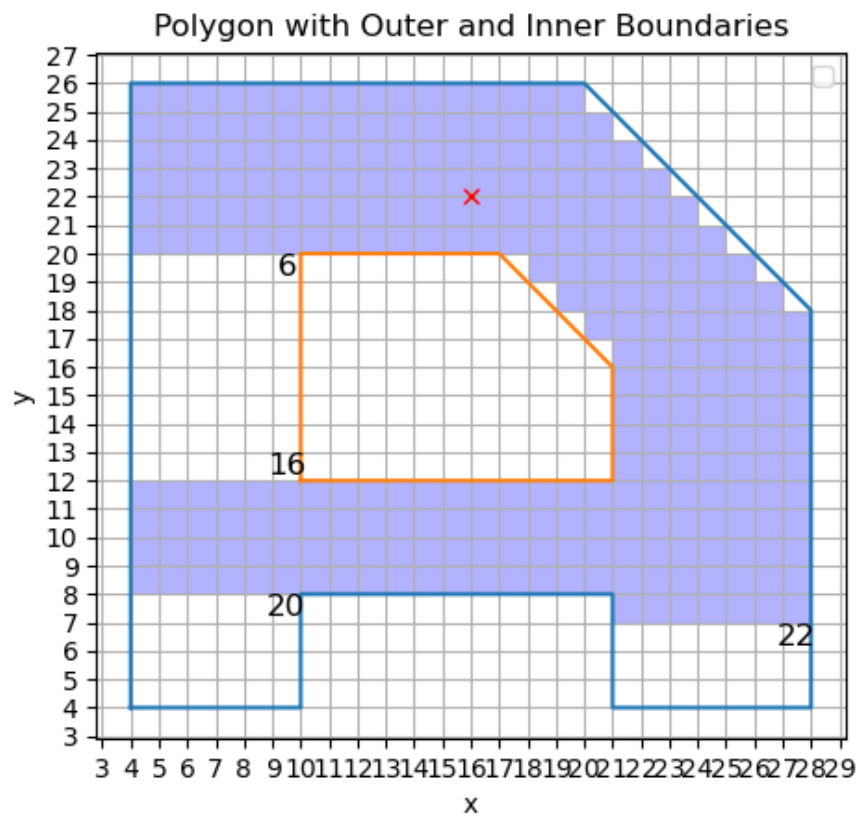
stack is [6, 16, 19]



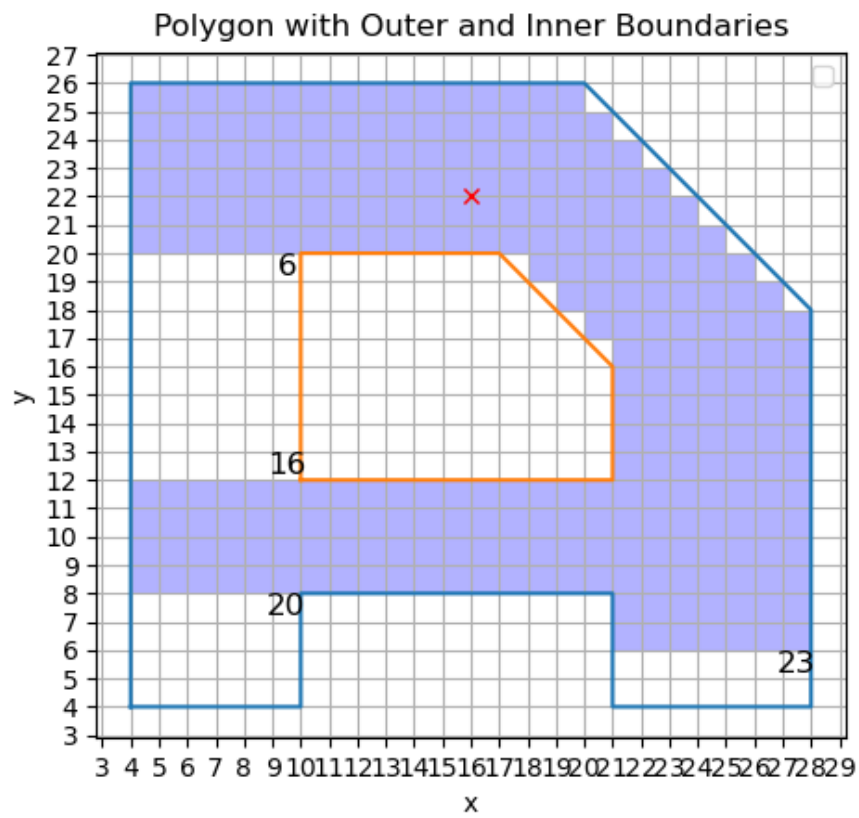
stack is [6, 16, 20, 21]



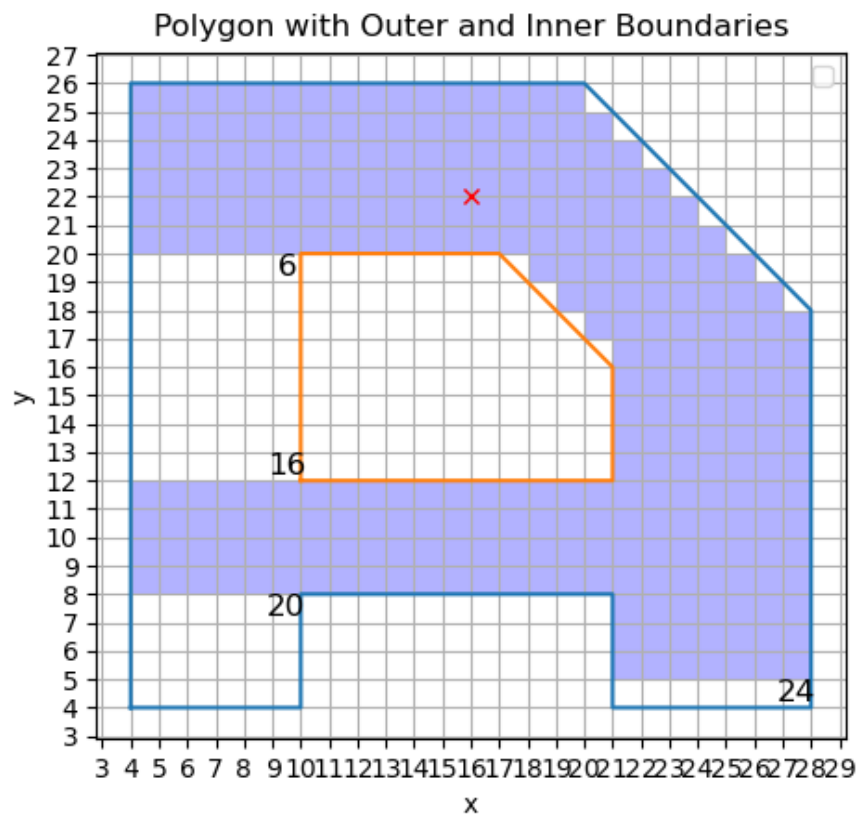
stack is [6, 16, 20, 22]



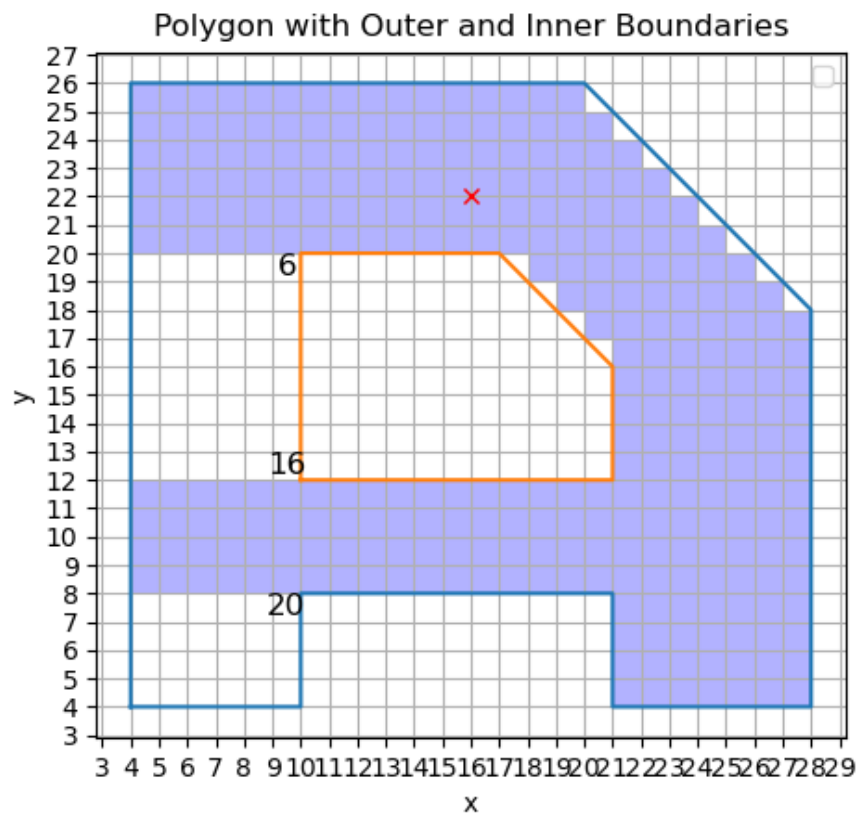
stack is [6, 16, 20, 23]



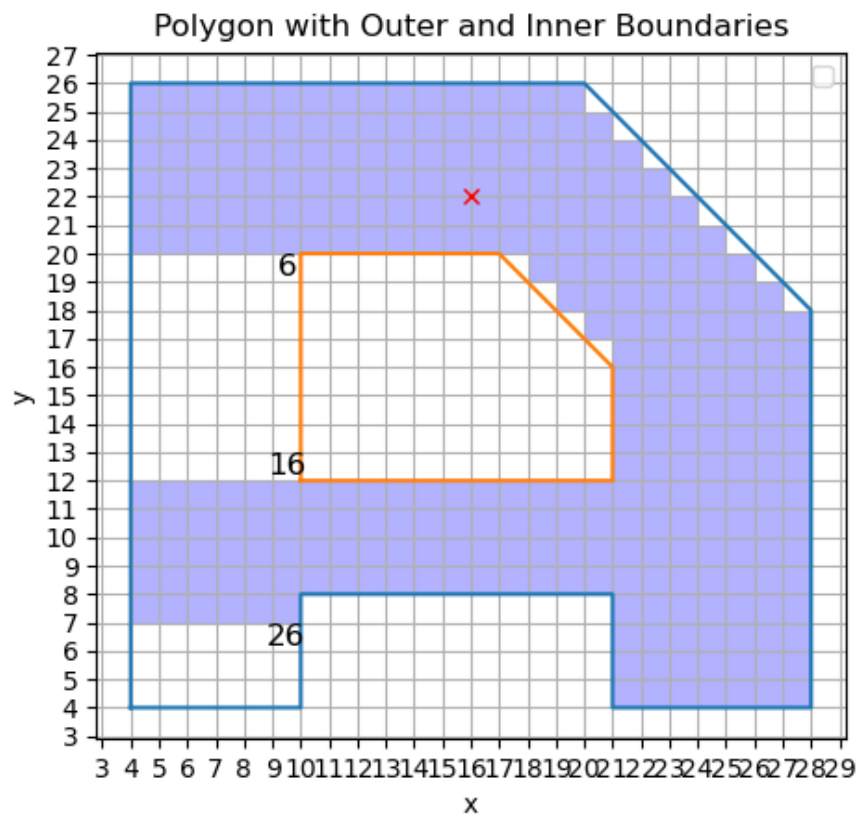
stack is [6, 16, 20, 24]



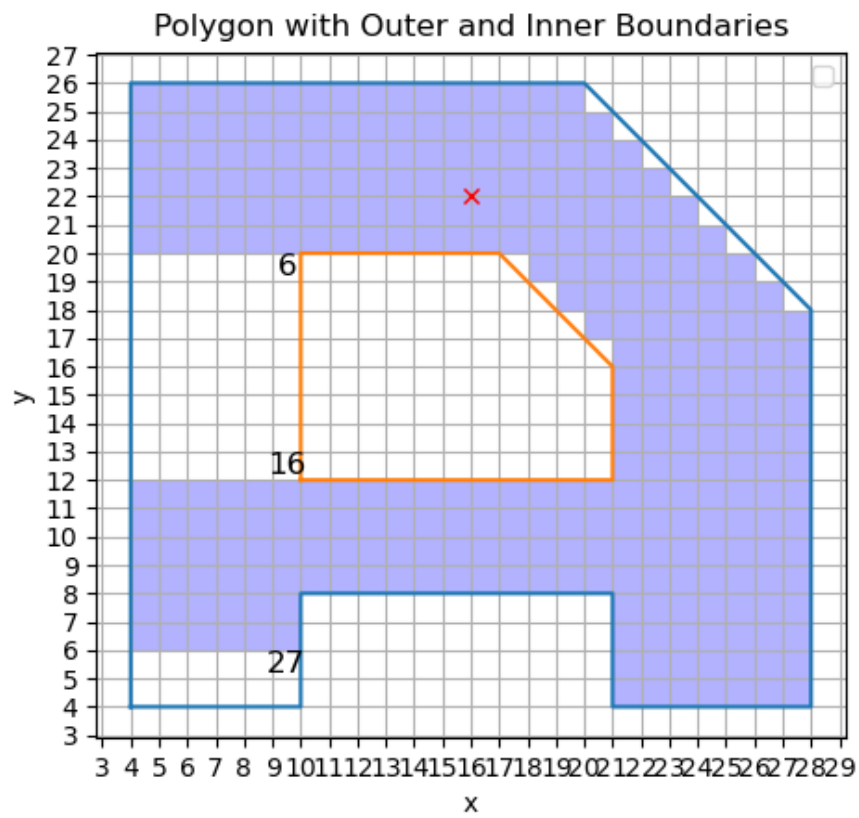
stack is [6, 16, 25]



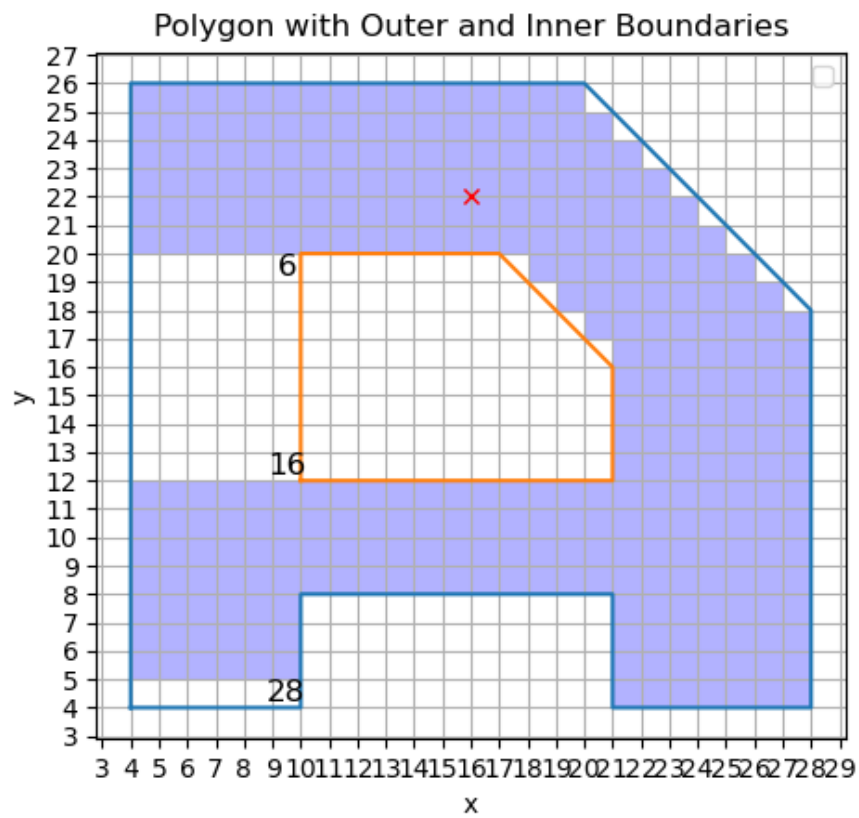
stack is [6, 16, 26]



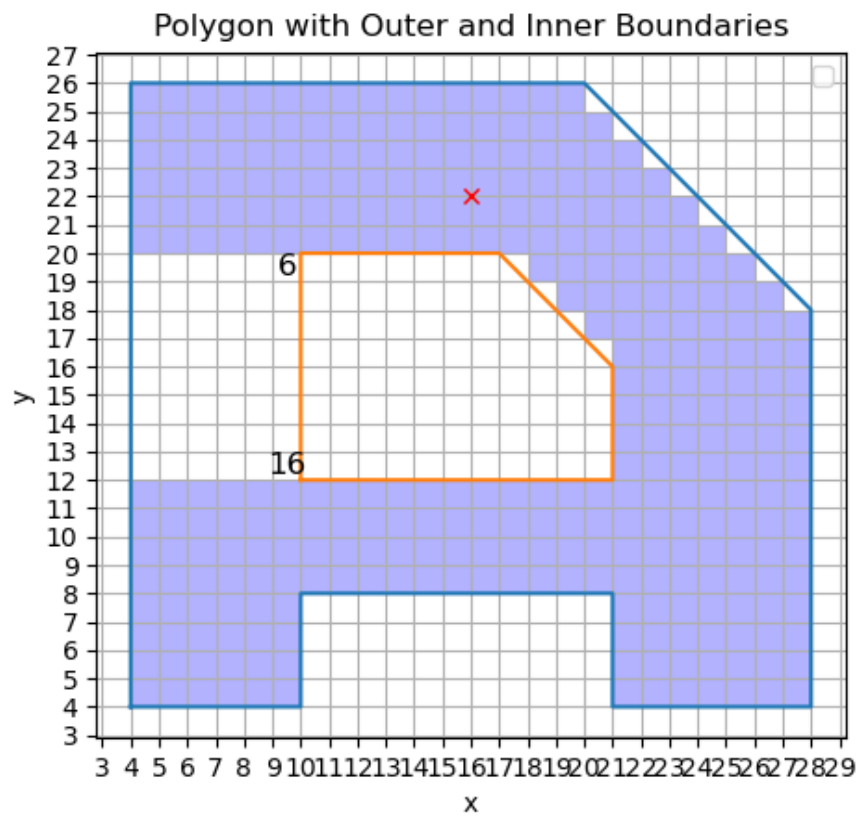
stack is [6, 16, 27]



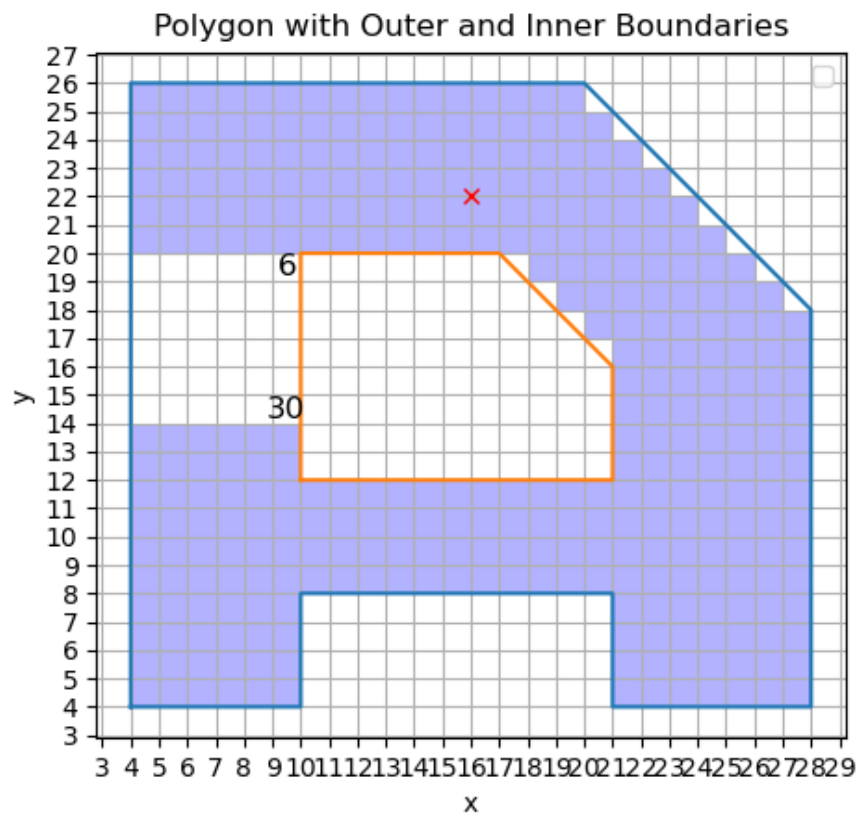
stack is [6, 16, 28]



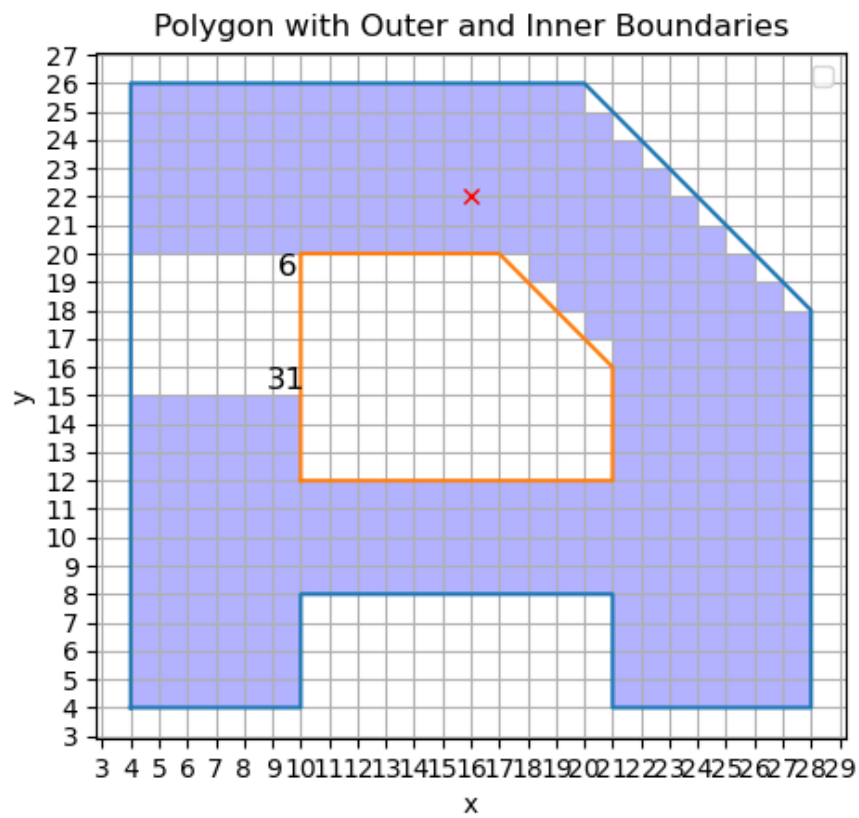
stack is [6, 16]



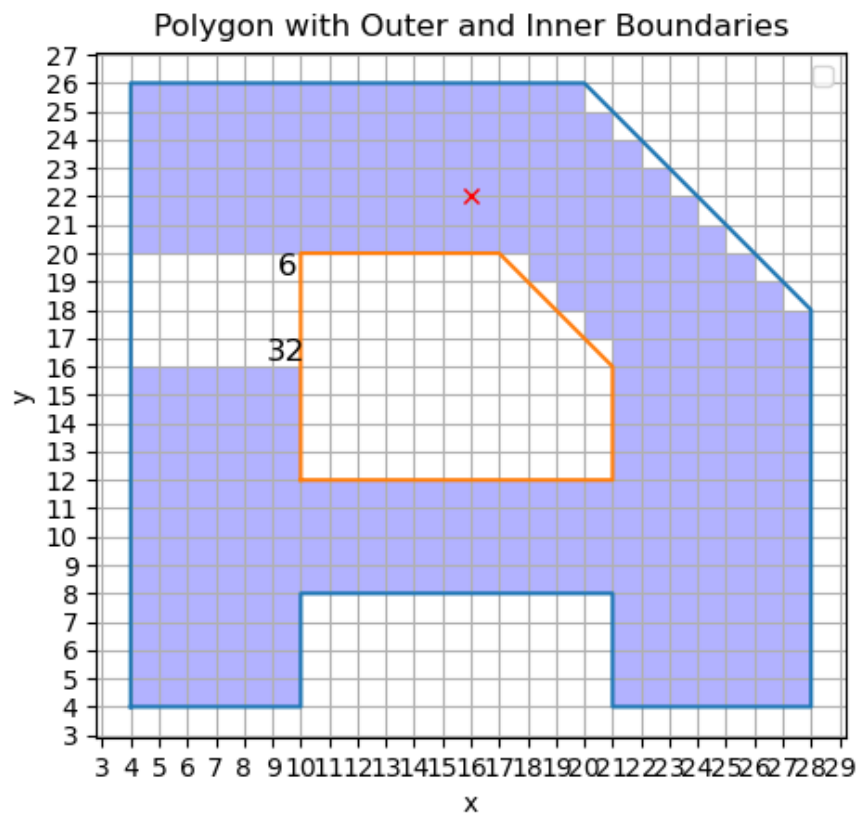
stack is [6, 30]



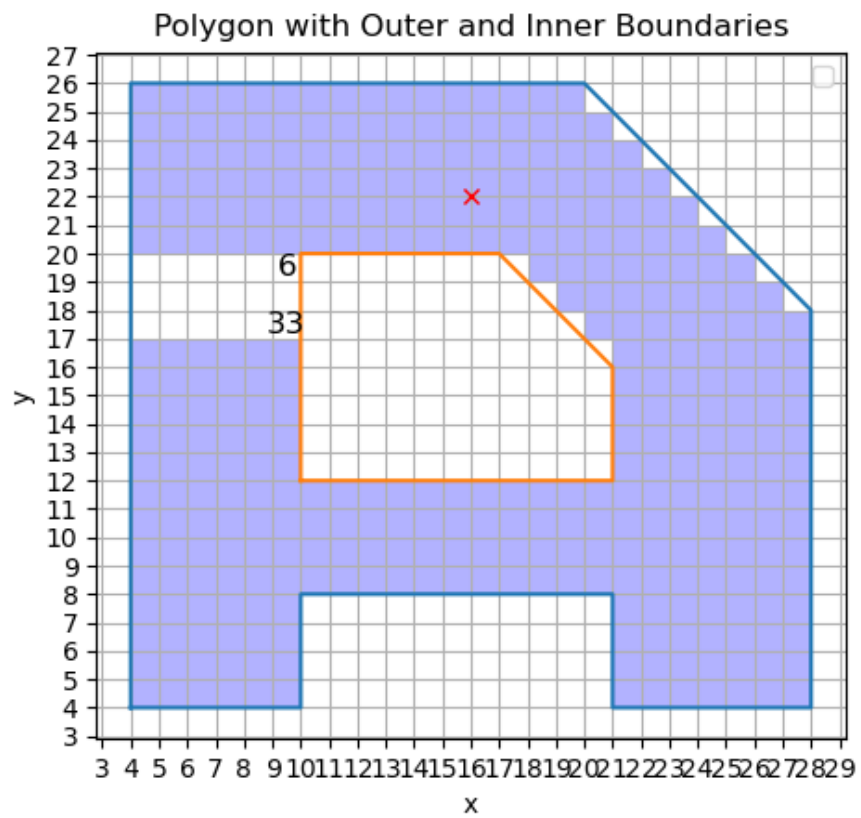
stack is [6, 31]



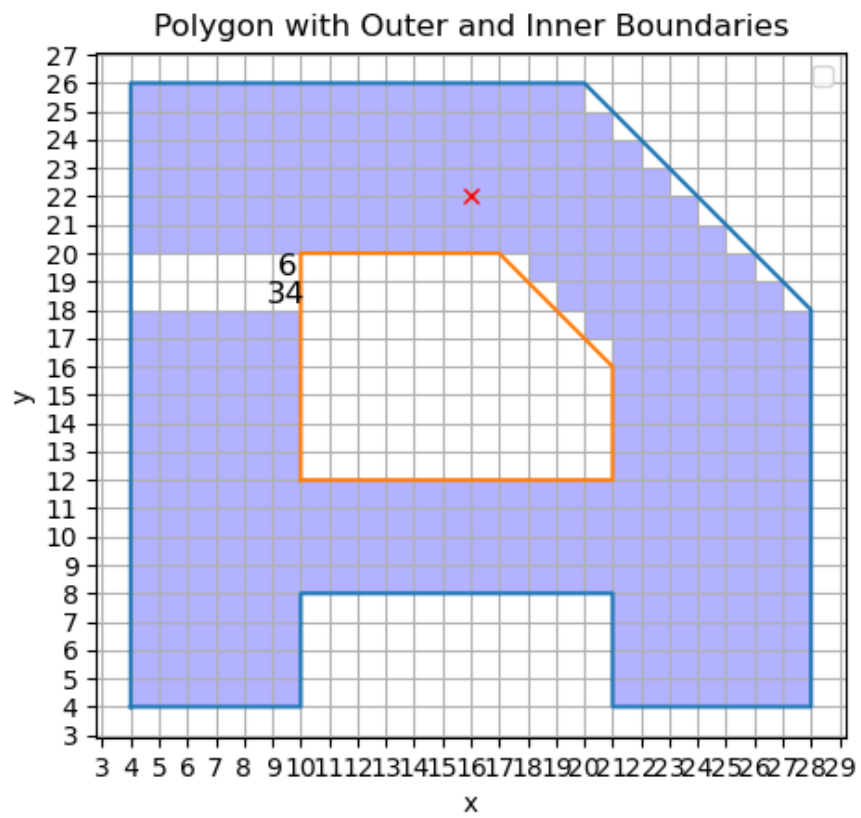
stack is [6, 32]



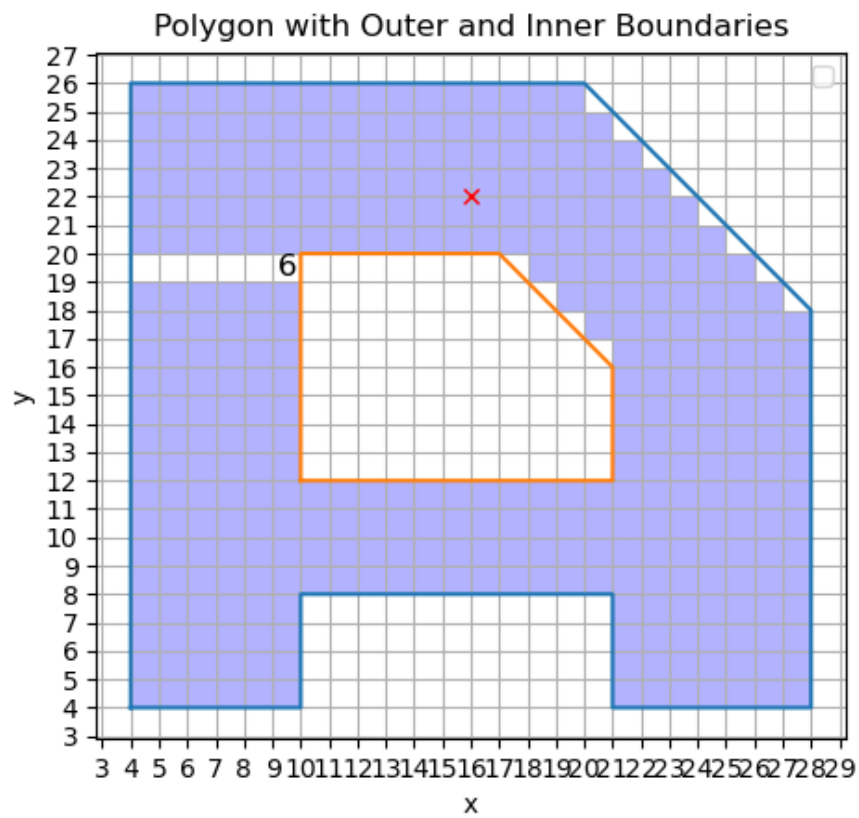
stack is [6, 33]



stack is [6, 34]



stack is [6]



stack is []

