Design Project

EEX4331

Circuit theory and Design

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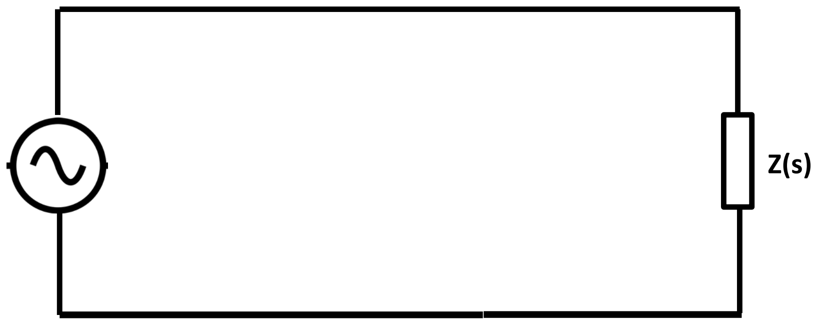
Design Requirement

A transmission line of an electrical network which carries AC current, has to be terminated by a load. It is required to maintain the following criteria after the connection of the load.

1. Frequencies at which the circuit consumes minimum current (zeros) are 100 rad/s, 200 rad/s.
2. Frequencies at which the circuit consumes maximum current (poles) are 0 rad/s, 300 rad/s, 400 rad/s.
3. Minimum impedance of the circuit is 100 Ω (Z_{min} =100 Ω)



R=100Ω

(figure 1)

Circuit theories used

Network synthesis can be viewed as the inverse operation of the network analysis. In network analysis, the response is predicted for a known network by applying various electric circuit theorems. On the other hand, in Network synthesis, a network is obtained for a desired response.

The driving point impedance Z(s) is a mathematical representation for the input impedance of a network in the frequency domain using Laplace transform (S-domain) or Fourier transform. The expression is expanded and this expansion is transformed in to a network of electrical elements. There are number of realization methods which can be used to obtain the driving point impedance. Here we use **Foster 2nd form**.

Frequencies at where the impedance becomes infinite are called Poles and frequencies where the impedance becomes zero are called Zeros. If the Poles and Zeros of desired network is given, the impedance function can be formed as follows.

Zeros – Z_1, Z_2, Z_3, ……..Z_n
Poles – P_1, P_2, P_3, ……..P_n

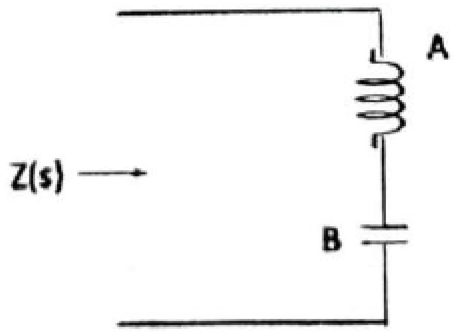
$$Z(s) = \frac{(s - Z_1)(s - Z_2) \ldots\ldots(s - Z_n)}{(s - P_1)(s - P_2) \ldots\ldots(s - P_n)}$$

Foster 2nd form: The Foster 2nd form of LC network realization is obtained by,

- Take the impedance function/ expression. [Z(s)]
- Convert it to the admittance expression. Y(s) = 1/ [Z(s)]
- Solve the expression.
- Partial fractioning of the simplified admittance expression.
- Identification of each term as the summarization as admittance.
- Obtaining the values for each element.

If $Y_k(s) = \frac{1}{As + \frac{1}{Bs}}$

$Y_k(s)$ is a series configuration of an inductor and a capacitor where the inductance and the capacitance respectively equal to A and B as shown in figure 2.



(figure 2)

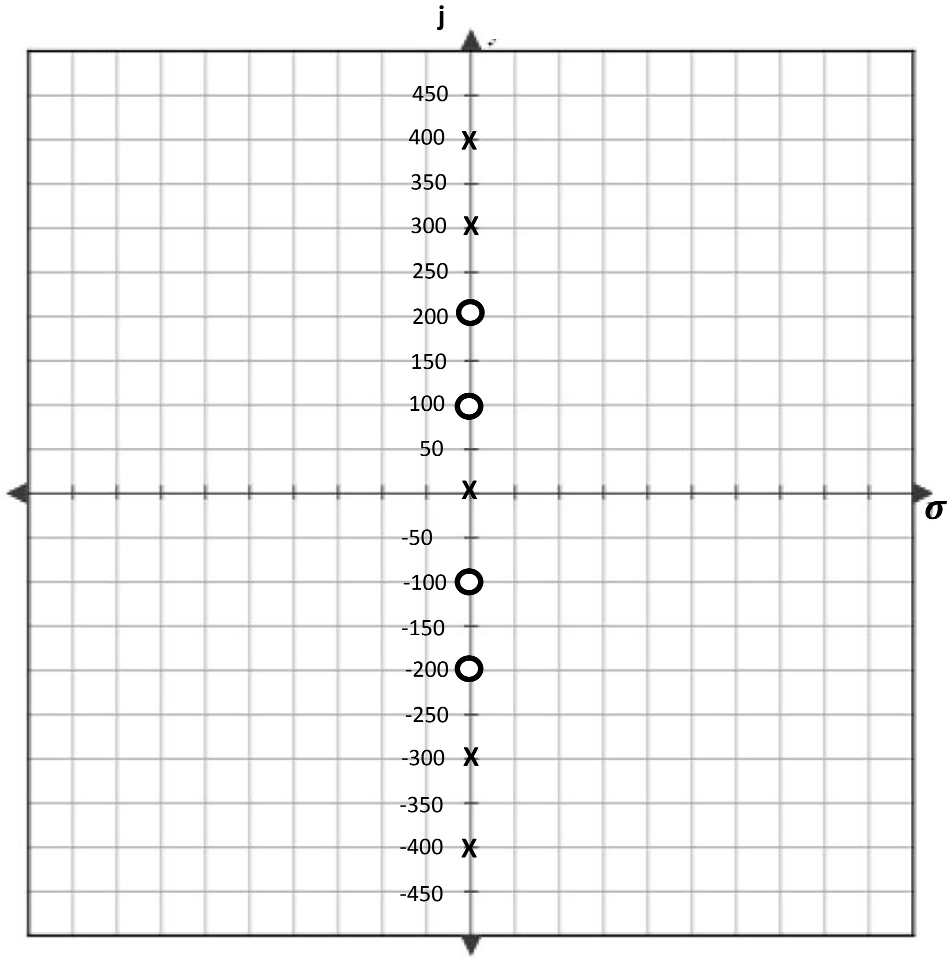
Design calculations

Zeros: 100 rad/s, 200rad/s

Poles: 0 rad/s, 300 rad/s, 400 rad/s

Z(S) = K $\frac{(s-100j)(s+100j)(s-200j)(s+200j)}{(s-0)(s-300j)(s+300j)(s-400j)(s+400j)}$; K is scaling factor.

Pole Zero diagram



Z(S) = K $\frac{(s-100j)(s+100j)(s-200j)(s+200j)}{(s-0)(s-300j)(s+300j)(s-400j)(s+400j)}$

Z(S) = K $\frac{(s^2+100^2)(s^2+200^2)}{(s)(s^2+300^2)(s^2+400^2)}$

Y(S) = $\frac{1}{Z(s)} = \frac{1}{K}\frac{(s)(s^2+300^2)(s^2+400^2)}{(s^2+100^2)(s^2+200^2)}$

$$Z_{min} = 100$$
$$Y_{max} = \frac{1}{Z_{min}}$$
$$Y_{max} = \frac{1}{100}$$
$$Y(s)_{max} = \frac{1}{100}$$
$$\frac{1}{K} = Y(s)_{max} = \frac{1}{100}$$

the driving point admittance function is

Y(S) = $\boldsymbol{\frac{1}{Z(s)} = (\frac{1}{100})\frac{(s)(s^2+300^2)(s^2+400^2)}{(s^2+100^2)(s^2+200^2)}}$

The type of the proposed circuit.

Considering the Driving point impedance function Y(s). It has only imaginary poles and zeros. So, the circuit is **LC circuit.**

Realizing the system using Foster 2nd method.

Y(S) $= (0.01)\frac{(s)(s^2+300^2)(s^2+400^2)}{(s^2+100^2)(s^2+200^2)}$

$$= (0.01)\frac{(s^5+250000s^3+14400000000s)}{(s^4+50000s^2+400000000)}$$

From long division ,

$$= s + \frac{200000s^3 + 14000000000s}{s^4 + 50000s^2 + 400000000}$$

Let's take the second part for the partial fractions,

$$\frac{200000s^3 + 14000000000s}{s^4 + 50000s^2 + 400000000} = \frac{200000s^3 + 14000000000s}{(s^2 + 100^2)(s^2 + 200^2)}$$

$$= \frac{\mathrm{As} + \mathrm{B}}{(s^2 + 100^2)} + \frac{Cs + D}{(s^2 + 200^2)}$$

$$200000s^3 + 14000000000s = (\mathrm{As} + \mathrm{B})(s^2 + 200^2) + (Cs + D)(s^2 + 100^2)$$

By equating coefficients,

<u>S^3</u> $\rightarrow$ $200000 = \mathrm{A} + \mathrm{C}$ ------------------------------------ (1)

<u>S^2</u> $\rightarrow$ $0 = \mathrm{B} + \mathrm{D}$ --- (2)

<u>S</u> $\rightarrow$ $14000000000 = 200^2\mathrm{A} + 100^2 C$--------------(3)

<u>Constants</u> $\rightarrow$ $0 = 200^2\mathrm{B} + 100^2 D$-----------------------(4)

<u>(2)</u> $\rightarrow$ $\mathrm{B} = -\mathrm{D}$ --- (5)

From (5) and (4),

$0 = 200^2(-\mathrm{D}) + 100^2\mathrm{D}$

$\mathrm{D} = 0$

<u>(5)</u> $\rightarrow$ $\mathrm{B} = 0$

<u>(3)</u> $\rightarrow$ $14000000000 = 200^2\mathrm{A} + 100^2 C$

$1400000 = 4\mathrm{A} + C$-------------------(6)

(6) – (1),

$1400000 - 200000 = 4\mathrm{A} + C - A - C$

$1200000 = 3\mathrm{A}$

$400000 = \mathrm{A}$

$\underline{(1)} \rightarrow \quad 200000 - 400000 = \mathrm{C}$

$$-200000 = \mathrm{C}$$

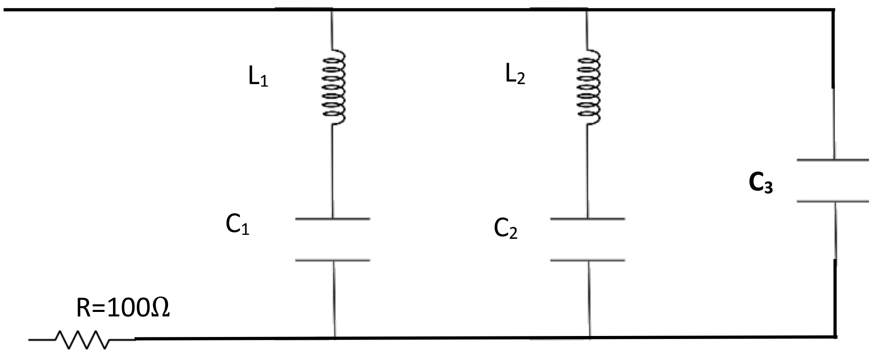
So, the partial fraction 0f Y(s) is,

$$Y(s) = (0.01)(s + \frac{400000\mathrm{s}}{(s^2 + 100^2)} + \frac{(-200000)\mathrm{s}}{(s^2 + 200^2)})$$

$$Y(s) = (\frac{1}{\frac{1}{0.01s}} + \frac{1}{\frac{s^2}{4000\mathrm{s}} + \frac{100^2}{4000\mathrm{s}}} - \frac{1}{\left(\frac{s^2}{2000\mathrm{s}} + \frac{200^2}{2000\mathrm{s}}\right)}$$

$$Y(s) = (\frac{1}{\frac{1}{0.01s}} + \frac{1}{\frac{s}{4000} + \frac{1}{\frac{4000\mathrm{s}}{100^2}}} - \frac{1}{\left(\frac{s}{2000} + \frac{1}{\frac{2000\mathrm{s}}{200^2}}\right)}$$

So the network will be,



$$L_1 = \frac{1}{4000} = 0.25mH$$

$$= 0.\boldsymbol{25mH}$$

$$C_1 = \frac{4000}{10000} = 0.4F$$

$$= \boldsymbol{400uF}$$

$$L_2 = \frac{1}{2000} = 0.5mH$$

= **0.5mH**

$$C_2 = \frac{2000}{40000} = 0.05F$$

$$= \mathbf{50}\boldsymbol{uF}$$

$$C_3 = \frac{0.01}{1} = 0.01F$$

$$= \mathbf{10}\boldsymbol{uF}$$

Scaling factor

Scaling factor(K) use to adjust the theoretical components values to practically available values.

Inductance adjustment equation = L x K

Capacitance adjustment equation = C/K

Example

K=0.5 and the theoretical capacitance 23.5uF and Inductance 2mH.

The practical values of,

Inductance =2 x 0.5
=1mH

Capacitance = 23.5/0.5
=47uF

Modifications

Scaling the calculated components to available components,

Practical inductor suitable for the L_1,L_2 is 1mH, so the scaling factors,

$K_{L1} \rightarrow \frac{1}{0.25} = \mathbf{4}$

$K_{L2} \rightarrow \frac{1}{0.5} = \mathbf{2}$

Practical capacitor suitable for the C_1 is 470uF, so the scaling factors,

$K_{C1} \rightarrow \frac{470}{400} = \mathbf{1.175}$

Practical capacitor 50uf available for the C_2, so no need to scaling,

Capacitor C_3(10uF) is practically available.

The Role of Poles and Zeros

- Zeros is the frequencies where the impedance of the circuit is maximized.
- Poles is the frequencies where the impedance of the circuit is minimized

Experimental Techniques for Identifying Poles and Zeros

- Time domain analysis
  Apply a step input to observe resonances; decay rates correspond to pole locations.

- Frequency sweep
  Measure impedance or admittance over a range of frequencies; peaks (poles) and nulls (zeros) are observed in the sweep.

- Network Analyzer
  Plots impedance or admittance vs. frequency, showing poles as peaks and zeros as troughs

Instructions to apply the design in the target environment

Ensures the minimum impedance condition is satisfied at Z_{min} = 100 Ω.