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(14/07/2015).

UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) –2015

AM 1001 – DIFFERENTIAL EQUATIONS - I

(Two Hours)

Answer all questions

No. of questions: 04

No. of pages: 12

Important Instructions to the Candidates

- Check the number of question and number of pages. If a page or a part of this question paper is not printed, please inform the Supervisor immediately.
- Enter your index Number on all pages of the answer scripts and also in the box provided in the MCQ answer sheet.
- **MCQ TYPE:** In each of these multiple choice questions mark the correct response on the given MCQ answer sheet with a pen. Write down the question paper code number in the space provided on the MCQ answer sheet.
- **STRUCTURED TYPE**: Write the answers in the space provided in the question paper.
- **ESSAY TYPE**: Write the answers to these questions on the booklet provided.
- Attach the MCQ answer sheet and the structured type question together with the answers to the essay type question and hand it over to the supervisor. **Do not attach the MCQ question paper and essay type question to the answer script.**

(1) (i) The order and the degree of the ordinary differential equation

$$x\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx} - 1\right)^2 = xy \text{ are respectively}$$

(a) 2 and 2
(b) 6 and 3
(c) 2 and 6
(d) 2 and 3
(e) 6 and 2.

(ii) $Y = A\,e^{(x+\frac{x^2}{2})} - 1$, where A is a constant, is a solution of the differential equation

(a) $Y' = (1+y)(1+x)$
(b) $Y' = (1-y)(1+x)$
(c) $Y' = (1+y)(1-x)$
(d) $Y' = (1-y)(1-x)$
(e) None of the above.

(iii) The solution of the initial value problem

$$t^2(1+y^2) - 2y\frac{dy}{dt} = 0, \quad y(0) = 1 \text{ is}$$

(a) $(2e^{\frac{t^3}{3}} + 1)^{\frac{1}{2}}$
(b) $(-2e^{\frac{t^3}{3}} + 1)^{\frac{1}{3}}$
(c) $(2e^{\frac{t^3}{3}} - 1)^{\frac{1}{2}}$
(d) $(2e^{\frac{t^3}{3}} + 1)^{\frac{1}{3}}$
(e) $(-2e^{\frac{t^3}{3}} + 1)^{\frac{1}{2}}$

(iv) The general solution of the differential equation

$$(t + y - 3) + (t + 4y - 1)\ \frac{dy}{dt}\ = 0 \text{ is}$$

(a) $2y^2 + (t\text{-}1)\ y = t\ (3 - \frac{t}{2}) + C$

(b) $2y^2 + (t\text{+}1)\ y = t\ (3 - \frac{t}{2}) + C$

(c) $2y^2 + (t\text{+}1)\ y = t\ (3 + \frac{t}{2}) + C$

(d) $2y^2 + (t\text{-}1)\ y = t\ (3 + \frac{t}{2}) + C$, where C is a constant

(e) none of the above.

For problems (v) and (vi) consider the following differential equation.

$$\frac{1}{t^2} - \frac{1}{y^2} + \frac{(at-1)}{y^3}\frac{dy}{dt} = 0.$$

(v) The differential equation is exact when a =

(a) -2

(b) 2

(c) 3

(d) -3

(e) none of the above.

(vi) Solution of the differential equation when it is exact is

(a) $-\frac{1}{t} - \frac{1}{y^2}\left(\frac{1}{4} + t\right) = C$

(b) $\frac{1}{t} + \frac{1}{y^2}\left(\frac{1}{2} - t\right) = C$

(c) $\frac{1}{t} - \frac{1}{y^2}\left(\frac{1}{4} - t\right) = C$

(d) $-\frac{1}{t} + \frac{1}{y^2}\left(\frac{1}{2} + t\right) = C$

(e) $-\frac{1}{t} + \frac{1}{y^2}\left(\frac{1}{2} - t\right) = C$, where C is a constant.

(vii) An integrating factor transforms a given differential equation to

(a) a linear equation
(b) a perfect differential
(c) a partial differential
(d) a non linear equation
(e) a homogeneous equation.

(viii) An integrating factor of the differential equation

$2xy\,dy - (x^2 + y^2)\,dx = 0$ is

(a) $\frac{1}{x^2}$
(b) $-x$
(c) x^2
(d) $\frac{1}{x}$
(e) $\frac{1}{x^3}$

(ix) An integrating factor of the differential equation

$\frac{dy}{dx} + 3xy = x$ is

(a) $e^{-\frac{3x^2}{2}}$
(b) $e^{\frac{3x}{2}}$
(c) $e^{\frac{3x^2}{2}}$
(d) $e^{-\frac{x}{2}}$
(e) $e^{\frac{x}{2}}$

(x) The orthogonal trajectories of the family of curves

$y = \frac{c}{x^2}$ is

(a) $y^2 + \frac{x^2}{2} = C$
(b) $Y = C x^2$
(c) $xy = C$
(d) $x^2 + y^2 = C$
(e) $Y^2 - \frac{x^2}{2} = C$, where C is a constant.

(2) Consider the following differential equation for the probems (i), (ii), (iii) and (iv).

$$a(x) \frac{d^2y}{dx^2} + b(x) \frac{dy}{dx} + c(x)\, y = f(x).$$

(i) When a(x) = 1, b(x) = -1, c(x) = -2 and f(x) = 3x, the general solution of the differential equation is

(a) $y = Ae^{-x} + Be^{2x} + \frac{3}{2}x + \frac{3}{4}$
(b) $y = Ae^{-x} + Be^{2x} - \frac{3}{2}x + \frac{3}{4}$
(c) $y = Ae^{-x} + Be^{2x} - \frac{3}{2}x - \frac{3}{4}$
(d) $y = Ae^{-x} + Be^{2x} - \frac{3}{2}x + \frac{3}{2}$
(e) $y = Ae^{-x} + Be^{2x} - \frac{3}{2}x - \frac{3}{2}$

(ii) When a(x) = 1, b(x) = -1, c(x) = -2 and f(x) = xe^x, the general solution of the differential equation is

(a) $y = Ae^{-x} + Be^{2x} - x\frac{e^x}{2} + \frac{e^x}{4}$

(b) $y = Ae^{-x} + Be^{2x} - x\frac{e^x}{4} + \frac{e^x}{2}$

(c) $y = Ae^{-x} + Be^{2x} - x\frac{e^x}{4} - \frac{e^x}{2}$

(d) $y = Ae^{-x} + Be^{2x} - x\frac{e^x}{2} - \frac{e^x}{4}$

(e) $y = Ae^{-x} + Be^{2x} + x\frac{e^x}{4} + \frac{e^x}{2}$

(iii) What is the differential equation after substituting $x = e^t$ into the differential equation with a(x) = x^2, b(x) = 0, c(x) = -2 and f(x) = 3 lnx ?

(a) $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3t$

(b) $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 3t$

(c) $\frac{d^2y}{dt^2} - \frac{dy}{dt} + 2y = 3t$

(d) $\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = -3t$

(e) $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = -3t$

(iv) The particular integral of the differential equation $x^2\frac{d^2y}{dx^2} - 2y = 3\ln x$ (in problem (iii)) is

(a) $y = -\frac{3}{2}\ln x + \frac{3}{4}$

(b) $y = -\frac{3}{2}\ln x + \frac{3}{2}$

(c) $y = \frac{3}{2}\ln x - \frac{3}{2}$

(d) $y = \frac{3}{2}\ln x + \frac{3}{4}$

(e) $y = \frac{3}{2}\ln x + \frac{3}{2}$

(v) The general solution of the differential equation $x + kyy' = \frac{1}{2}$, where k is a constant, is a circle when

(a) $k = 1$
(b) $k = \frac{1}{2}$
(c) $k = 2$
(d) $k = \frac{1}{4}$
(e) $k = -1$

(vi) Let $y(x)$ be the investment after x years from a deposit y_0 at an interest rate r. If the interest is compounded quarterly then $y(5)$ is

(a) $y_0(1+\frac{r}{4})^{15}$
(b) $y_0(1+\frac{r}{3})^{20}$
(c) $y_0(1+\frac{r}{4})^{20}$
(d) $y_0(1+\frac{r}{3})^{15}$
(e) none of the above.

(vii) Solution of the differential equation $\frac{dy}{dx} = \frac{x^2 - 1}{y+5}$ is

(a) $\frac{x^3}{3} + x - \frac{y^2}{2} - 5y = C$
(b) $\frac{x^3}{6} - x + \frac{y^2}{2} - 5y = C$
(c) $\frac{x^3}{3} - x - \frac{y^2}{2} - 5y = C$
(d) $\frac{x^3}{3} + x + \frac{y^2}{2} - 5y = C$
(e) $\frac{x^3}{6} + x + \frac{y^2}{2} + 5y = C$, where C is a constant.

(viii) Which of the following are singular points of the differential equation

$y\,(x-1)\,y' = x^2 - 1$?

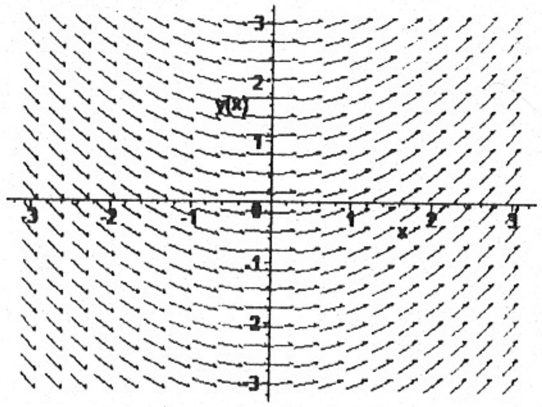
(a) (0, 1), (0, -2)
(b) (0, 1), (0, -1)
(c) (0, -1), (0, -2)
(d) (1, 0), (-1, 0)
(e) (2, 0), (-1, 0).

(ix) Let Pdx + Qdy = 0 be a non-exact equation. It has an integrating factor F depending only on y if $\frac{1}{F}\frac{dF}{dy} =$

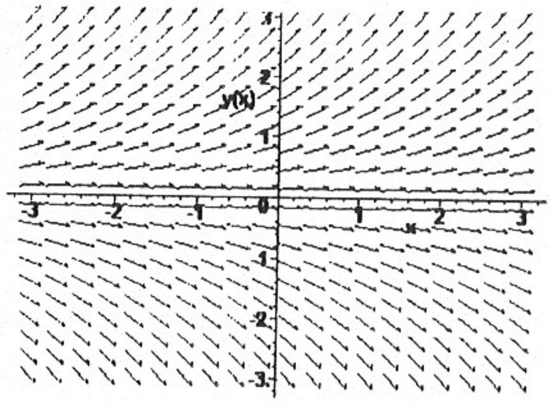
(a) $\frac{1}{P}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$
(b) $\frac{1}{P}\left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right)$
(c) $\frac{1}{Q}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$
(d) $\frac{1}{Q}\left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial y}\right)$
(e) $\frac{1}{P}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial x}\right)$

(x) The direction field of $y' = \frac{1}{2}x$ is

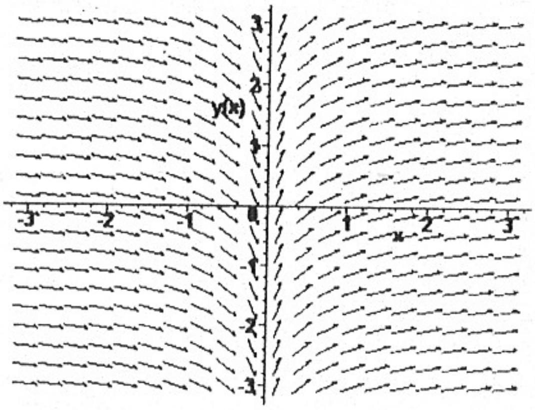
(a)



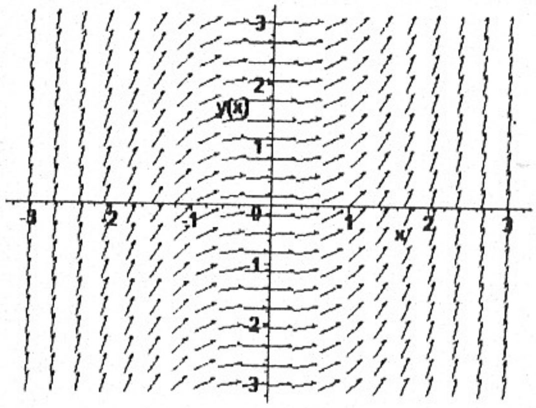
(b)



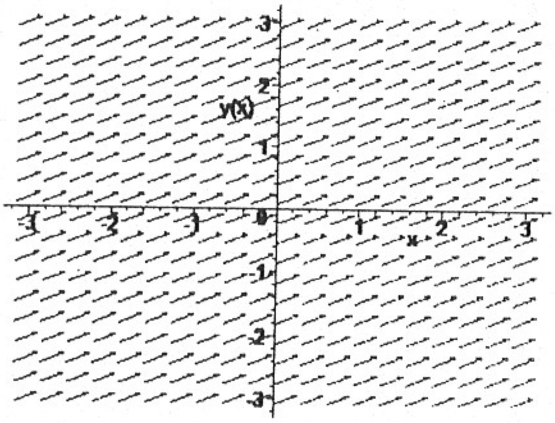
(c)



(d)



(e)



(3) Write your answers only within the given space. You may use a pencil to write answers which enables you corrections.

(a) Convert Bernoulli's Equation $\frac{dy}{dx} + p(x)y = q(x)y^2$, where p(x) and q(x) are independent of y, to the general form of the first order linear equation.

(b) Find the general solution of

$$\frac{dy}{dx} - \frac{y}{x} = xy^2$$

(c) Reduce the following differential equation to the separable form and find the solution.

$$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$$

(4) Prove that the complete primitive of the differential equation

$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c\,y = 0$, where a, b, c are real constants, is given by

$y = Ae^{\alpha x} + Be^{\beta x}$, if $\alpha \neq \beta$

$y = (Ax + B)e^{\alpha x}$, if $\alpha = \beta$,

where A, B are arbitrary constants and α, β are the roots of $a\lambda^2 + b\lambda + c = 0$.

Let $D \equiv \frac{d}{dx}$ and $\emptyset(D) = aD^2 + bD + c$. Show that a particular integral of

$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c\,y = e^{\alpha x}$ is $\frac{1}{\emptyset(\alpha)}e^{\alpha x}$, if $\emptyset(\alpha) \neq 0$..

Hence find the general solution of $(D^2 + 3D + 2)\,y = e^{2x}$.

Code: *PI*34#5JQ*

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LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) –2015

AM 1001 – DIFFERENTIAL EQUATIONS - I

MCQ Answer Sheet

Index No: ....

Code No: ....

01.	(i)	a	b	c	d	e	02.	(i)	a	b	c	d	e
	(ii)	a	b	c	d	e		(ii)	a	b	c	d	e
	(iii)	a	b	c	d	e		(iii)	a	b	c	d	e
	(iv)	a	b	c	d	e		(iv)	a	b	c	d	e
	(v)	a	b	c	d	e		(v)	a	b	c	d	e
	(vi)	a	b	c	d	e		(vi)	a	b	c	d	e
	(vii)	a	b	c	d	e		(vii)	a	b	c	d	e
	(viii)	a	b	c	d	e		(viii)	a	b	c	d	e
	(ix)	a	b	c	d	e		(ix)	a	b	c	d	e
	(x)	a	b	c	d	e		(x)	a	b	c	d	e

Correct way of shading the appropriate response is given below.

If the answer is *c*, a b ● d e