UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2014

AM 1001

DIFFERENTIAL EQUATIONS -I

(Two Hours)

Answer all questions

No. of pages: 10
No. Of Questions:04

Important Instructions to the Candidates

- Check the number of question and number of pages. If a page or a part of this question paper is not printed, please inform the Supervisor immediately.
- Enter your Index Number on all pages of the answer scripts and also in the box provided in the MCQ answer sheet.
- MCQ TYPE: In each of these multiple choice questions mark the correct response on the given MCQ answer sheet with a pen. Write down the question paper code number in the space provided on the MCQ answer sheet.
- STRUCTURED TYPE: Write the answers in the space provided in the question paper.
- ESSAY TYPE: Write the answers to these questions on the booklet provided.
- Attach the MCQ answer sheet and the structured type question together with the answers to the essay type question and hand it over to the supervisor. **Do not attach the MCQ question paper and essay type question to the answer scripts.**

Code No: 4563723418

Problem 1:

1. The ordinary differential equation $2x^2\frac{dy}{dx} = x^2+y^2$ can be transformed into the separable form

   (a) $2x\frac{dv}{dx} = v^2 - 2v + 1$

   (b) $2x\frac{dv}{dx} = \frac{1}{2}v^2 - 2v + 1$

   (c) $2x\frac{dv}{dx} = v^2 + 2v + 1$

   (d) $2x\frac{dv}{dx} = v^2 + 2v - 1$

   (e) n on of the above

2. At a singular point of $y' = f(x,y),\ x, x_0 \in [a,b], y(x_0) = y_0$,

   (a) t he solution $y(x)$ is not defined.

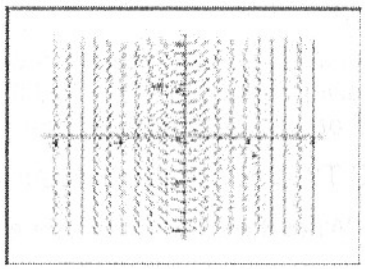
   (b) $f(x,y)$ tends to infinity.

   (c) $f(x,y)$ is not defined.

   (d) $y(x)$ tends to infinity.

   (e) n on of the above.

3. Differential equation of the following isocline plot is given by



   (a) $y' = sin(x)$

   (b) $y' = cos(x)$

   (c) $y' = 2x$

   (d) $y' = e^x$

   (e) $y' = 1/x$

4. The minimum requirement to exists a solution for the initial value question $y' = f(x,y),\ x, x_0 \in [a,b], y(x_0) = y_0$

   (a) $f(x,y)$ is Lipschitz continuous.

   (b) $f(x,y)$ is continuous.

   (c) $f(x,y)$ is differentiable.

   (d) $f(x,y)$ is bounded.

   (e) N on of the above.

5. The orthogonal trajectories of the family of curves $y = cx^2$, where c is an arbitrary constant, is given by

   (a) $y^2 = x^2 + k$

   (b) $y^2 = -\frac{1}{2}x^2 + k$

   (c) $y^2 = -2x^2 + k$

   (d) $2y^2 = x^2 + k,$

   (e) n on of the above
   where k is a constant.

6. All the singular points of the differential equation

$$(y^2 - 1)\frac{dy}{dx} = x(x^2 + 1)$$

   are

   (a) $(0,1), (0,-1)$

   (b) $(1,-1), (1,0)$

   (c) $(-1,0), (1,-1)$

   (d) $(1,1), (-1,-1)$

   (e) n on of the above

7. An integrating factor of the differential equation
   $x^2\frac{dy}{dx} - 2xy = 4x^4 - 3x^2y$ is

   (a) $\frac{1}{x^2}$

   (b) e^{2x}

   (c) e^{-2x}

   (d) $\frac{e^{3x}}{x^2}$

   (e) n on of the above

8. A solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} + 3y\frac{dy}{dx} + \frac{1}{2}y = 3$$

is

(a) $y = x^2$

(b) $y = (2x+1)^3$

(c) $y = \sqrt{x}$

(d) $y = \frac{1}{\sqrt{x}}$

(e) n on of the above

9. Existence and uniqueness theorem guarantees the existence of unique solution for the initial value question $y' = 1 + 2y^2$, $y(0) = 0$ and $R = \{(x,y)||x| < 5, |y| < 3\}$ in the interval

(a) $-5/19 < x < 5/19$

(b) $-5/26 < x < 5/26$

(c) $-3/7 < x < 3/7$

(d) $-3/19 < x < 3/19$

(e) $-8/19 < x < 8/19$

10. The order and the degree of the differential equation

$$5\frac{d^2y}{dx^2} + 3x^2\left(\frac{dy}{dx}\right)^2 + x^4y = \sin(x)$$

are respectively

(a) 2 and 2

(b) 2 and 1

(c) 1 and 2

(d) 1 and 4

(e) 4 and 1

Problem 2:

1. The initial value question

$$(x^2-4)\frac{dy}{dx} = (y-1)x, \quad y(2) = 3, x \in \mathbb{R}$$

   (a) h as no solution.

   (b) h as an infinitely many solutions.

   (c) h as a unique solution

   (d) h as some insufficient initial data.

   (e) h as only the trivial solution $y(x) = 0$.

2. Which of the following differential equations is exact ?

   (a) $x \sin y dy + \frac{1}{2}x^2 \cos y dx = 0$

   (b) $\frac{xy-1}{x^2y}dx - \frac{1}{xy^2}dy = 0$

   (c) $(2xy - 5x^3y)dx - (x^2 + y^2)dy = 0$

   (d) $(5x^2y - 3x^2y^3)dx + (3xy^2 - 5x^2y^2)dy = 0$

   (e) $(3x^4y^2 - x^2)dy + (4x^3y^3 + 2xy)dx = 0$

3. General solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ is

   (a) $y(x) = Ae^{-2x} + Be^{-x}$

   (b) $y(x) = Ae^{2x} + Be^{-x}$

   (c) $y(x) = Ae^{-2x} + Be^{x}$

   (d) $y(x) = Ae^{2x} + Be^{x}$

   (e) $y(x) = (A + 2Bx)e^{x}$

4. The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is said to be exact if

   (a) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

   (b) $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$

   (c) t here exists u such that $du = Q(x,y)dx + P(x,y)dy$

   (d) t here exists u such that $\frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$

   (e) t here exists u such that $\frac{\partial Q}{\partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Use the following information to answer the questions (iv),(v) and (vi).
The efficiency of engines of air planes depends on air pressure $p(x)$ at height x and usually is maximum at 35,000ft. Experimentally it is verified that the rate of change of $p(x)$ with respect to height x is proportional to the pressure $p(x)$ at the height x. Let k be a positive proportionality constant. At the height of 18,000ft it is observed that the pressure is half of

its value $p(0) = p_0$ at sea level.

5. Which of the following differential equation best fit with above information
   (a) $\frac{dp}{dx} = -kp(x)$
   (b) $\frac{dp}{dx} = kp(x)$
   (c) $\frac{dp}{dx} = \frac{-k}{p(x)}$
   (d) $\frac{dp}{dx} = -kxp(x)$
   (e) n on of the above
6. value of the proportionality constant k is given by
   (a) $\frac{-1}{18}\ln(\frac{1}{2}) \times 10^3$
   (b) $\frac{1}{18}\ln(\frac{1}{2}) \times 10^{-3}$
   (c) $\frac{-1}{18}\ln(\frac{1}{2}) \times 10^{-3}$
   (d) $\frac{1}{18}\ln(\frac{1}{2}) \times 10^3$
   (e) n on of the above
7. Air pressure at 35000ft is given by
   (a) $p_0 e^{-\frac{18}{35}\ln(\frac{1}{2})}$
   (b) $p_0 e^{\frac{18}{35}\ln(\frac{1}{2})}$
   (c) $p_0 \left(\frac{1}{2}\right)^{-\frac{35}{18}}$
   (d) $p_0 \left(\frac{1}{2}\right)^{\frac{35}{18}}$
   (e) n on of the above
8. An integrating factor of the equation $2\sin(y^2)dx + xy\cos(y^2)dy = 0$ is
   (a) $-1/x$
   (b) x^3
   (c) x^2y
   (d) $-x^3$
   (e) $1/x^2$

9. Which of the following equation can be transformed into separable form using the transformation $y = vx$?

   (a) $\frac{dy}{dx} = \frac{y-2x}{4y+3x}$

   (b) $\frac{dy}{dx} = \frac{(2x^2+3y^2-7)x}{(3x^2+2y^2-8)y}$

   (c) $\frac{dy}{dx} = \frac{2x^2-3y+2}{2y^2+4x-19}$

   (d) $\frac{dy}{dx} = \frac{3x-3y^2+2}{2y+4x^2-19}$

   (e) $\frac{dy}{dx} = \frac{x^2-y^2}{x+y}$

10. The orthogonal trajectories of the family of curves $x^2 + y^2 = c$,where c is a constant, is given by

    (a) $y^2 = -\frac{x^2}{2} + c$

    (b) $y = cx$

    (c) $x^2 + y^2 = c^2$

    (d) $xy = c^2$

    (e) $x^2 - y^2 = c^2$

Problem 3: Write the answers in the given booklet

1. Show that a particular integral of the differential equation

$$(aD^2 + bD + c)y = p(x)$$

,where $p(x)$ is a polynomial and a, b, c are real constants, can be written as

$$\frac{1}{(D-\alpha)(D-\beta)}p(x)$$

, where α, β are roots of the quadratic equation $a\lambda^2 + b\lambda + c = 0$.

Find complete primitives of the equations

(a) $(2D^2 - 3D - 2)y = x^2 + 4$

(b) $(D^2 - 4D + 4)y = x\sin(2x)$

Problem 4: Index No:.....

Write your answers within the given space. You may request additional papers if needed.

1. (a) Show that the equation $P(x,y)dx + Q(x,y)dy = 0$ is exact then $\frac{\partial P(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x}$

(b) Solve the equation $(y^2 + x^2 - 2x + 3)dx + (2xy - y^2 + 10)dy = 0$

(c) Show that the equation $[2(x + y^2)\cos x + \sin x]dx + 2y \sin x dy = 0$ has an integrating factor which only depends on x and hence find its solution.

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LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2014

AM 1001- DIFFERENTIAL EQUATIONS -I

MCQ Answer Sheet

Code No:. .....

Index No. .....

Problem 1)

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Problem 2)

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Guidelines for Answering

Correct *way of shading the appropriate oval is given below.*
If the answer is c *:*

a b ● d e

(i), (ii) & (iii) which are given below are the ***incorrect*** *way of shading the appropriate oval.*
If the answer is ***iii:***

(i) a b ◯ d e

(ii) a b ⊗ d e

(iii) a b ◐ d e

පාරිභාෂික ශබ්ද මාලාව

Arbitrary Constant	-	අභිමත නියතය
Complete primitive	-	පූර්ණ ආද්‍යය
Concentric circles	-	ඒක කේන්ද්‍රීය වෘත්ත
Degree	-	මාත්‍රය
Dependent Variable	-	පරායත්ත විචල්‍යය
Direction Field	-	දිශා ක්ෂේත්‍රය
Exact	-	සපිරි
Existence	-	පැවතීම
Family of curves	-	වක්‍ර කුලය
General Solution	-	සාධාරණ විසඳුම
Independent solutions	-	ස්වායත්ත විසඳුම්
Independent Variable	-	ස්වායත්ත විචල්‍යය
Initial Value Problem	-	ආරම්භක අගය ගැටලුව
Integrating factor	-	අනුකලන සාධකය
Necessary and Sufficient	-	අනිවාර්ය සහ ප්‍රමාණවත් අවශ්‍යතාව
Order	-	ගනය
Ordinary Differential Equations	-	සාමාන්‍ය අවකල සමීකරණ
Orthogonal Trajectory	-	ප්‍රලම්බ පරාවක්‍රය
Partial Differential Equation	-	ආංශික අවකල සමීකරණය
Particular Solution	-	ව්‍යක්තික විසඳුම
Singular Point	-	අපූර්ව ලක්ෂ්‍යය
Uniqueness	-	අනන්‍යතාව