UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

FIRST YEAR EXAMINATION IN SCIENCE (SEMESTER I) – 2008/2009

AM 1001 – ORDINARY DIFFERENTIAL EQUATIONS

(Two Hours)

Answer all questions

No. of questions: 04

No. of pages: 08

Important Instructions to the Candidates

- If a page or a part of this question paper is not printed, please inform the Supervisor immediately.
- MCQ TYPE: The MCQ answer sheet is provided with the question paper.
- **Check whether the same CODE is printed on all pages of the exam paper including the answer sheet, except the cover page. If not, please inform the Supervisor immediately.**

  Enter your Index Number in the space provided in the question paper and the MCQ answer sheet.

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  Choose the correct response to MCQ question and cross the appropriate box with a pen. (Each question has only one correct response.)
- Structured TYPE: Write the answers in the given space on the question paper itself.
- ESSAY TYPE: Write the answers on the answer booklets provided.
- No calculators may be used.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.
- At the end of the examination, attaché your written answers to the question paper including the MCQ answer sheet and hand it over to the supervisor. **Do not take any part of the question paper out of the examination hall**

1. i.) Which of the following statements is true ?

   (a) The general solution of an n^{th} order ordinary differential equation contains n arbitrary constants.

   (b) The general solution is not defined at singular points.

   (c) If $y_1(x)$ and $y_2(x)$ are any two solutions of a given differential equation, then $c_1y_1(x) + c_2y_2(x)$ is also a solution of the same equation, where c_1, c_2 are arbitrary constants.

   (d) A Singular solution can be obtained by setting a suitable value for the arbitrary constants of the general solution.

   (e) A Particular solution of the n^{th} degree ordinary differential equation contains n arbitrary constants.

   ii.) Which of the following statements is true ?

   (a) If a dependent variable is a function of more than one independent variable, then the rate of change of that dependent variable can be modeled by an ordinary differential equation.

   (b) In a physical system if one dependent variable is a function of more than one independent variable , then the rates of change of that dependent variable can be modeled by a system of partial differential equations.

   (c) If a dependent variable is a function of only one independent variable, then the rate of change of that dependent variable can be modeled by an ordinary differential equation.

   (d) If a dependent variable is a function of more than one independent variable, then the rate of change of that dependent variable can be modeled by a partial differential equation.

   (e) If a dependent variable is a function of more than one independent variable, then the rate of change of that dependent variable can be modeled by a system of partial differential equation.

   iii.) Which of the following statements is true ?

   (a) Any differential equation has a solution.

   (b) A solution of a differential equation is continuous.

   (c) If there is a solution for a given differential equation, then it is analytically computable.

   (d) There is a unique solution for an initial value problem of the form $y' = f(x, y),\ y(x_0) = y_0$.

   (e) If a physical process is modeled by a differential equation, then this differential equation always has a unique solution.

iv.) At a singular point of $y' = f(x, y),\ x, x_0 \in [a, b], y(x_0) = y_0$,

(a) the solution $y(x)$ is not defined.
(b) $f(x, y)$ tends to infinity.
(c) $f(x, y)$ is not defined.
(d) $y(x)$ tends to infinity.
(e) $y(x)$ is not continuous.

v.) The singular points of $(x-1)y' = x(y^2 - 4),\ x, x_0 \in [a, b], y(x_0) = y_0$ are

(a) $(0, -2), (0, 2)$.
(b) $(1, 2), (1, -2)$.
(c) $(1, 0), (1, 2)$.
(d) $(1, 4), (0, 4)$.
(e) $(1, 2), (0, 2)$.

vi.) The order and the degree of the differential equation

$$(y'')^3 - 5x(y')^4 = e^x + 1$$

are respectively

(a) 3 and 4
(b) 3 and 2
(c) 2 and 3
(d) 4 and 3
(e) 2 and 4

vii.) Two chemicals A and B react in the ratio 2:3 to form the compound Z. 45g of A and 60g of B are mixed together to form Z. If the rate of formation of Z is proportional to the product of the amounts of two components A and B which have not yet combined together, the differential equation of the reaction can be written as

(a) $\frac{dZ}{dt} = k(9 - Z)(36 - Z)$
(b) $\frac{dZ}{dt} = k(18 - Z)(6 - Z)$
(c) $\frac{dZ}{dt} = k(18 - Z)(36 - Z)$
(d) $\frac{dZ}{dt} = k(45 - \frac{2}{5}Z)(60 - \frac{3}{5}Z)$
(e) $\frac{dZ}{dt} = k(6 - Z)(12 - Z)$,

where k is constant.

viii.) The general solution of the differential equation $y'' + \omega^2 y = 0$ can be written as

(a) $y = a\cos\omega x + b\sin\omega x$, where a and b are arbitrary constants.

(b) $y = e^{i\omega x}\cos a + e^{-i\omega x}\sin b$, where a and b are arbitrary constants and $i^2 = -1$.

(c) $y = ae^{i\omega x} - be^{i\omega x}$, where a and b are arbitrary constants and $i^2 = -1$.

(d) $y = a\cos(\omega x + b)^2$, where a and b are arbitrary constants.

(e) $y = \cos a\omega x + \sin b\omega x$, where a and b are arbitrary constants.

ix.) The initial value problem $y' = f(x, y),\ x, x_0 \in [a, b], y(x_0) = y_0$ has a unique solution if

(a) $f(x, y)$ is Lipschitz continuous.

(b) $f(x, y)$ is continuous.

(c) $f(x, y)$ is differentiable.

(d) $f(x, y)$ is bounded.

(e) Non of the above.

x.) Existence and uniqueness theorem guarantees the existence of unique solution for the initial value problem $y' = 1 + y^2,\ , y(0) = 0$ and $R = \{(x, y) | |x| < 5, |y| < 3\}$ in the interval

(a) $-0.3 < x < 0.3$

(b) $-0.6 < x < 0.6$

(c) $-0.9 < x < 0.9$

(d) $-0.5 < x < 0.5$

(e) $-0.2 < x < 0.2$

2\. i.) The initial value problem

$$(x^2 - 4)\frac{dy}{dx} = (y - 1)x, \quad y(2) = 3$$

(a) has no solution.

(b) has an infinite number of solutions.

(c) has a unique solution

(d) has some insufficient initial data.

(e) has only the trivial solution $y(x) = 0$.

ii.) The orthogonal trajectories of the family of curves $x^2 + y^2 = c$,where c is a constant, is given by

(a) $y^2 = -\frac{x^2}{2} + c$
(b) $y = cx$
(c) $x^2 + y^2 = c^2$
(d) $xy = c^2$
(e) $x^2 - y^2 = c^2$

iii.) Which of the following equation is homogeneous ?

(a) $\frac{dy}{dx} = \frac{y-2x}{4y+3x}$
(b) $\frac{dy}{dx} = \frac{(2x^2+3y^2-7)x}{(3x^2+2y^2-8)y}$
(c) $\frac{dy}{dx} = \frac{2x^2-3y+2}{2y^2+4x-19}$
(d) $\frac{dy}{dx} = \frac{3x-3y^2+2}{2y+4x^2-19}$
(e) $\frac{dy}{dx} = \frac{x^2-y^2}{x+y}$

iv.) A tank contains $10m^3$ of water in which 20Kg of salt is dissolved. $0.5m^3$ of brine , each cubic meter containing 2kg of dissolved salt runs into the tank per minute, and the mixture kept uniform by stirring, runs out at the same rate. The amount of salt $y(t)Kg$ in the tank at any time t is given by.

(a) $y(t) = 18(e^{-t} + 2)$
(b) $y(t) = 20(e^{-0.5t} + 1)$
(c) $y(t) = 20(e^{-0.05t} + 2)$
(d) $y(t) = -20(e^{-0.05t} - 1)$
(e) $y(t) = 20$

v.) The differential equation $P(x, y)dx + Q(x, y)dy = 0$ is exact if

(a) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
(b) $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$
(c) there exists u such that $du = Q(x, y)dx + P(x, y)dy$
(d) there exists u such that $\frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$
(e) there exists u such that $\frac{\partial Q}{\partial y} = \frac{\partial^2 u}{\partial y \partial x}$

vi.) Which of the following differential equations is exact ?

(a) $x \sin y dy + \frac{1}{2}x^2 \cos y dx = 0$
(b) $\frac{xy-1}{x^2y}dx - \frac{1}{xy^2}dy = 0$
(c) $(2xy - 5x^3y)dx - (x^2 + y^2)dy = 0$
(d) $(5x^2y - 3x^2y^3)dx + (3xy^2 - 5x^2y^2)dy = 0$
(e) $(3x^4y^2 - x^2)dy + (4x^3y^3 + 2xy)dx = 0$

vii.) The general solution of $9yy' + 4x = 0$ is

(a) a set of concentric circles centered about the origin.
(b) a set of concentric ellipses centered about the origin with the major axis $y = 0$.
(c) a set parabolas with the axis $x = 0$.
(d) a set of concentric ellipses centered about the origin with the major axis $x = 0$.
(e) a set of hyperbolas.

viii.) An integrating factor transforms a given differential equation to

(a) a linear equation
(b) an auxiliary equation
(c) a partial differential
(d) an exact equation
(e) a homogeneous equation

ix.) Which of the following is an exact differential ?

(a) $\frac{xdy-ydx}{x^2}$
(b) $-(y-x)dx + (y+x)dy$
(c) $x^2dy + y^2dx$
(d) $\frac{ydy}{x^2+y^2} + \frac{x^2dx}{x^2+y^2}$
(e) $\frac{1}{x}dx - xdy$

x.) An integrating factor of the equation $2\sin(y^2)dx + xy\cos(y^2)dy = 0$ is

(a) $-1/x$
(b) x^3
(c) x^2y
(d) $-x^3$
(e) $1/x^2$

3. **Write your answers only within the given space.** You may use a pencil to write answers which enables you corrections.

   (a) Find the solution of the equation

$$\frac{dy}{dx} + p(x)y = q(x)y^n$$

   where $p(x), q(x)$ and $n \neq 0, 1$ are independent of y.

(b) Transform the equation $\frac{dy}{dx} - x^3(y-x)^2 = \frac{y}{x}$ to a Bernulli type equation using a suitable transformation.

(c) Hence find the general solution of $\frac{dy}{dx} - x^3(y-x)^2 = \frac{y}{x}$

4. **Write the answers in the given booklet and attach to the paper.**
Show that the general solution of the differential equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0,$$

where a, b, c are real constants, is given by

$$\begin{aligned} y(x) &= c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x},\ \lambda_1 \neq \lambda_1 \\ &= (c_1 + c_2x)e^{\lambda x},\ \lambda = \lambda_1 = \lambda_1 \end{aligned}$$

where λ_1, λ_2 are roots of $a\lambda^2 + b\lambda + c = o$ and a, b, c are real constants.
Hence show that the general solution of solution of the

$$(D^2 + 1)y = \sin 2x$$

is

$$y = c_1 \cos x + c_2 \sin x - \frac{1}{3}\sin 2x,$$

where c_1, c_2 are arbitrary constants.

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