Chapter 7

Model Question Papers

7.1 Model Paper I

Index No:**S-.....**

UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

FIRST YEAR EXAMINATION IN SCIENCE 2016-Semester I

AM 1001 - DIFFERENTIAL EQUATIONS - I(Model Paper)

(Two Hours)

1. Check the number of pages and number of questions in both Sinhala and English papers.
2. Write the index number in the given space
3. Answer **All** the questions.
4. Question 1 and 2 are MCQ questions. Correct answers to these questions should be indicated by colouring the appropriate circle in the answer sheet attached to the question paper.

5. Handover both the MCQ & structured parts of the question paper (**Taking MCQ and/or structured part/s of the question paper out of the examination hall will be considered as an examination offence**).

6. Calculators are not allowed

Number of Questions:04
Number of Pages:09

1. i.) Which of the following statements is true ?
   (a) If a dependent variable is a function of more than one independent variables, then the rate change of that dependent variable can be modelled by an ordinary differential equation.
   (b) If a dependent variable is a function of only one independent variable, then the rate change of that dependent variable can be modelled by an ordinary differential equation.
   (c) If a dependent variable is a function of more than one independent variables, then the rate change of that dependent variable can be modelled by an of ordinary differential equations.
   (d) If a dependent variable is a function of more than one independent variables, then the rate change of that dependent variable can be modelled by a partial differential equation.
   (e) If a dependent variable is a function of more than one independent variables, then the rate change of that dependent variable can be modelled by a system of partial differential equation.

   ii.) Which of the following statements is true ?
   (a) For given any differential equation has a solution.
   (b) A Solution of a differential equation is continuous.
   (c) If there is a solution for a given differential equation, then it is analytically computable.
   (d) There can't be many solution for a an initial value problem of the form $y' = f(x, y), \;\; y(x_0) = y_0$.
   (e) If a physical process is modelled by a differential equation, then it always has a unique solution.

   iii.) Which of the following statements istrue ?
   (a) General solution of the n^{th} order ordinary differential equation contains $n - 1$ arbitrary constants.
   (b) Any particular solution can be obtain by substituting suitable values for the arbitrary constants in the general solution.

(c) $y_(x)$ and $y_2(x)$ are any two solutions of any given differential equation, then $y_(x) + y_2(x)$ is also a solution of the same equation.

(d) Singular solution is a solution obtained by setting all arbitrary constants of the general solution to zero.

(e) General solution of the n^{th} degree ordinary differential equation contains n arbitrary constants.

iv.) Order and the degree of the differential equation

$$(y''')^3 - 5x(y')^4 = e^x + 1$$

are respectively

(a) 3 and 4

(b) 3 and 3

(c) 2 and 3

(d) 4 and 3

(e) Cannot be defined.

v.) $y = cx - c^2$, where c is a constant is a solution of the differential equation

(a) $y' - xy'^2 + y = 0$

(b) $y'^2 - xy' - y = 0$

(c) $y'^2 - xy' + y = 0$

(d) $y'^2 - xy' - y^2 = 0$

(e) $xy'^2 - x^2y' - xy = 0$

vi.) Let $y(x)$ be the investment resulting from a deposit y_0 after x years at an interest rate r. Which of the following statement is true ?

(a) $y(x) = y_0[1 - r/4]^{4x}$ if the interest is reinvested after each four months.

(b) $y(x) = y_0[1 + r]^x$ if the interest is compounded annually.

(c) $y(x) = y_0[1 + r/2]^{2x}$ if the interest is reinvested after each two months.

(d) $y(x) = y_0[1 + r/3]^{3x}$ if the interest is compounded quarterly.

(e) $y(x) = y_0[1 + r/2]^x$ if the interest is compounded semiannually.

vii.) Singular points of a differential equations are the points where

(a) the differential equation has no solution.

(b) the differential equation has infinite number of solution.

(c) the differential equation has a unique solution.

(d) the derivative tends to infinity.

(e) the derivative is not defined.

viii.) The general solution of the differential equation $y'' + y = 0$ can be written as

(a) $y = e^{ix}\cos a + e^{-ix}\sin b$, where a and b are constants and $i^2 = -1$.

(b) $y = ae^{ix} - be^{-ix}$, where a and b are constants and $i^2 = -1$.

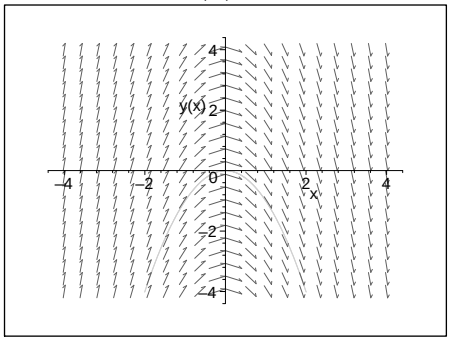
(c) $y = a\cos x + ib\sin x$, where a and b are constants, and $i^2 = -1$.

(d) $y = a + \cos(x + b)$, where a and b are constants.

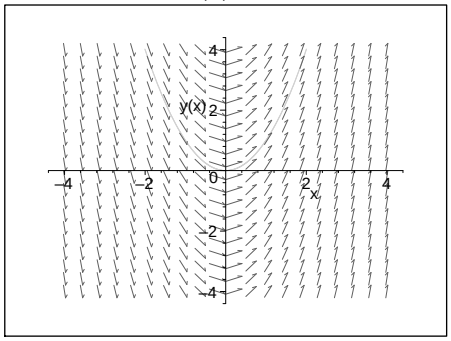
(e) $y = \cos ax + \sin bx$, where a and b are constants.

ix.) The direction field of $y' = 2x$ is

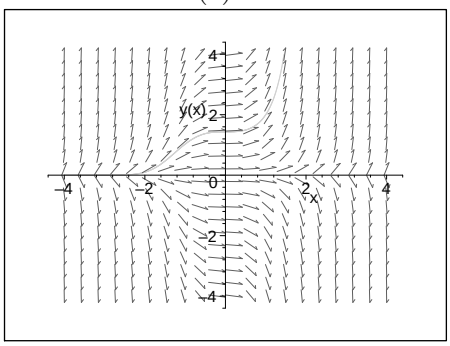
(a)


y(x)
x
4
2
0
−2
−4
−4
−2
2
4

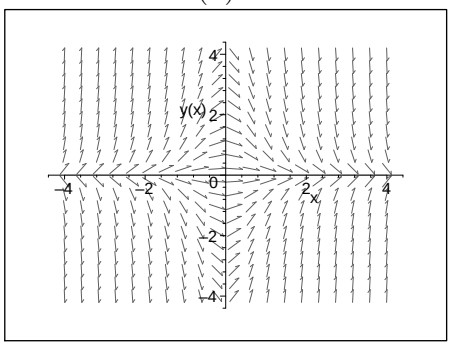
(b)


y(x)
x
4
2
0
−2
−4
−4
−2
2
4

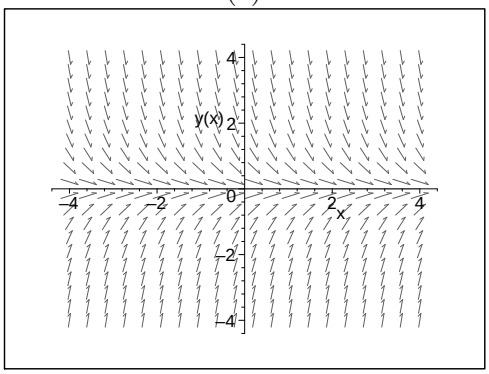
(c)


y(x)
x
4
2
0
−2
−4
−4
−2
2
4

(d)


y(x)
x
4
2
0
−2
−4
−4
−2
2
4

(e)


y(x)
x
4
2
0
−2
−4
−4
−2
2
4

x.) The initial value problem

$$(x^2 - 4)\frac{dy}{dx} = (y - 1)x, \;\; y(3) = 0$$

(a) has no solution.
(b) has infinite number of solutions.
(c) has a unique solution
(d) initial data is not enough.
(e) has only the trivial solution $y(x) = 0$.

2. i.) The general solution of $9yy' + 4x = 0$ is

(a) a set of concentric circles centered about the origin.
(b) a set of concentric ellipses centered about the origin with the major axis $y = 0$.
(c) a set parabolas with the axis $x = 0$.
(d) a set of concentric ellipses centered about the origin with the major axis $y = 0$.
(e) a set of hyperbolas.

ii.) The solution of the initial value problem $y' = -y/x, \;\; y(1) = 1$ is

(a) $y = -x^2$
(b) $y = 1/x$
(c) an ellipse.
(d) $y = 1/x^2$
(e) $y = -1/x$

iii.) The general solution of $(2x-4y+5)y'+(x-2y+3) = 0$ can be written as

(a) $4x + 8y + \ln(4x - 8y + 11) = c$, where c is an arbitrary constant.
(b) $4x + 8y + \ln|4x - 8y + 11| = c$, where c is an arbitrary constant.
(c) $e^{4x+8y}(4x - 8y + 11) = c$, where c is an arbitrary constant.
(d) $4x + 8y + \ln(4x + 8y + 11) = c$, where c is an arbitrary constant.
(e) $4x - 8y + \ln(4x - 8y + 11) = c$, where c is an arbitrary constant.

iv.) A tank containing 200gal of water in which 40 lb of salt are dissolved. Five gal of brine, each containing 2lb of dissolved salt, run into the tank per minute, and mixture, kept uniform by stirring, runs out at the same rate. If $y(t)$ is the amount of salt in the tank at time t ,then a mathematical model to describes the process can be formulated as

(a) $y' - 40 - 0.25y = 0$
(b) $y' - 10 + 0.025y = 0$
(c) $y' - 200 + 0.025y = 0$
(d) $y' - 10 - 0.025y = 0$
(e) $y' + 0.025(y - 400) = 0$

v.) The differential equation $P(x, y)dx + Q(x, y)dy = 0$ is exact if

(a) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
(b) $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$
(c) there exists u such that $du = \int P(x, y)dx + \int Q(x, y)dy$
(d) there exists u such that $\frac{\partial P}{\partial x} = \frac{\partial^2 P}{\partial x \partial y}$
(e) there exists u such that $\frac{\partial Q}{\partial y} = \frac{\partial^2 P}{\partial y \partial x}$

vi.) Which of the following differential equation is exact ?

(a) $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$
(b) $(5x^2 - 3x^2y)dx + (3xy^2 - 5xy)dy = 0$
(c) $(x^2 - 4xy^3)dx - (3x^2 - y^2)dy = 0$
(d) $2xydx - (x^2y + y^2)dy = 0$
(e) $(5x^2y - 3x^2y)dx + (3xy^2 - 5x^2y)dy = 0$

vii.) An integrating factor transforms a given equation to

(a) a linear equation
(b) a perfect differential
(c) a partial differential
(d) a non linear equaion
(e) a homogeneous equation

viii.) An integrating factor of the equation $2\sin(y^2)dx + xy\cos(y^2)dy = 0$ is

(a) $-1/x$
(b) x^3
(c) x^2y
(d) $-x^3$
(e) $1/x^2$

ix.) An integrating factor of the equation $-ydx + xdy = 0$ is

(a) $-1/x$
(b) $1/xy$
(c) x^3
(d) $-x^3$
(e) $1/x^2$

x.) Orthogonal trajectories of the family of curves $\frac{1}{2}x^2 + y^2 = c$,where c is a constant, is given by

(a) $y^2 = -\frac{x^2}{2} + c$
(b) $y = cx^2$
(c) $x^2 + y^2 = c^2$
(d) $xy = c^2$
(e) $x^2 - y^2 = c^2$

3. **Write your answers only within the given space.** You may use a pencil to write answers which enables you corrections.

   (a) Suppose a person has a P_0 amount of money in his pension fund at his retirement and it has been invested at the rate r per annum and he withdraws $w(t)$ per annum to cover his living expenses. Further it is foretasted that rate of inflation is k per annum. If $y(t)$ is the amount of money at any time t in his pension fund , construct a ordinary differential equation model to find an expression for $y(t)$.If w_0 is his initial withdrawal solve the above model and find $y(t)$.

(b) Ms. Silva retired yesterday at age 55. Her IRA account has a principal of Rs.4,500,000, which invested with a guaranteed interest rate of 12%, compounded continuously. Her budget calls for annual expenses of Rs.120,000 with projected inflation of 4.2%. Determine Ms. Silvas savings account balance ,t years after her retirement. How long will her money last?

(c) Her horoscope reader told that her life span is not more than 100 years. Then she decided to make her spending so that all her money will be finished at the age of 100. Compute her initial withdrawal amount assuming that all other parameters remain unchanged.

4. A series circuit contains resister $R = 24\Omega$, an inductor with $L = 2H$, a capacitor with $C = 0.005F$ and $12V$ battery. The initial charge is $Q = 0.001C$ and the initial current is zero.

   (a) Draw the circuit diagram and Construct the differential equation which explains the variation of the charge $Q(t)$ in the circuit.

   (b) Find expressions for the current and charge at any time t.

   (c) Sketch the graph of $Q(t)$ against t

@@@@@@@@

7.1.1 Answers sheet of the Model Paper

INDEX NO:.....

1.

Question no.	(a)	(b)	(c)	(d)	(e)
i)	▢	▢	▢	▢	▢
ii)	▢	▢	▢	▢	▢
iii)	▢	▢	▢	▢	▢
iv)	▢	▢	▢	▢	▢
v)	▢	▢	▢	▢	▢
vi)	▢	▢	▢	▢	▢
vii)	▢	▢	▢	▢	▢
viii)	▢	▢	▢	▢	▢
ix)	▢	▢	▢	▢	▢
x)	▢	▢	▢	▢	▢

2.

Question no.	(a)	(b)	(c)	(d)	(e)
i)	▢	▢	▢	▢	▢
ii)	▢	▢	▢	▢	▢
iii)	▢	▢	▢	▢	▢
iv)	▢	▢	▢	▢	▢
v)	▢	▢	▢	▢	▢
vi)	▢	▢	▢	▢	▢
vii)	▢	▢	▢	▢	▢
viii)	▢	▢	▢	▢	▢
ix)	▢	▢	▢	▢	▢
x)	▢	▢	▢	▢	▢

Bibliography

[1] Tom M. Apostol, *Calculus, Vol I*, John Willy & Sons

[2] Rechard Bronson, *Differential Equations,Schaum's Solved Problem series*

[3] Ervin Kreyszig, *Advanced Engineering Mathematics*, 8^{th} Edition.

[4] Robert .O'Malley, *Thinking about Ordinary Differential Equations*

[5] Rasinghania, *Differential Equations*

[6] J.N.Kapur, Mathematical Modelling.