# AM 1001-ORDINARY DIFFERENTIAL EQUATIONS
## INCLASS TEST II

(30 minits.)

octamil

**Answer all questions**

Code No:8322

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(i) The order and degree of the differential equation

$$a_0(x)\left(\frac{d^n y}{dx^n}\right)^{n-5} + a_1(x)\left(\frac{d^{n-1}y}{dx^{n-1}}\right)^{n} + \ldots + a_{n-1}(x)\left(\frac{dy}{dx}\right)^{n+2} + a_n(x)y = 0$$

are respectively,

(a) $n$ and $n-5$. ✓ (c) $n$ and $n+2$. (e) 1 and $n+2$.
(b) $n+2$ and 1. (d) $n-5$ and $n$.

(ii) The solution of the deferential equation $\frac{dy}{dx} = \frac{x+2y-4}{2x+y-5}$ is given by

(a) $|x+y+1| = c|x+y+3|^3$. (c) $|x+y-1|^3 = c|x+y+3|^3$. (e) $|x-y-1|^3 = c|x+y-3|$. ✓
(b) $|x+y+1|^3 = c|x+y+3|$. (d) $|x-y-1| = c|x+y-3|^3$.

where $c$ is a constant.

(iii) Which of the following is a solution of the implicit differential equation $yy' + x = 0$ on the interval $-1 < x < 1$ ?

(a) $x + y^2 - 1 = 0$. (c) $x + y - 1 = 0$. (e) $x^2 - y^2 + 1 = 0$.
(b) $x^2 + y^2 - 1 = 0$. (d) $x^2 + y - 1 = 0$.

(iv) The integrating factor of the differential equation $u' + \frac{1}{t}u = 5t$ is,

(a) $e^{\frac{1}{t}}$. (c) $e^t$. (e) none of the above.
(b) $e^{-t}$. (d) $t$. ✓

$$I = e^{\int P(x)\,dx}$$

$$= e^{\int \frac{1}{t}\,dt} \qquad \ln t$$

$$= e$$

$$= t$$

(v) A large tank initially contains $50m^3$ of brine in which there is dissolved 10 $kg$ of salt. Brine containing 2 $kg$ of dissolved salt per $1m^3$ flows into the tank at rate of $5m^3/min$. The mixture is kept uniform by stirring and the stirred mixture simultaneously flows out at the slower rate of 3 $m^3/min$. How much salt in the tank at any time $t > 0$?

(a) $4t^2 + 100 + \frac{22500\sqrt{2}}{(2t+50)^{\frac{3}{2}}}$. (c) $4t + 100 - \frac{22500\sqrt{2}}{(2t+50)^{\frac{3}{2}}}$. (e) $4t^2 - 100 - \frac{22500\sqrt{2}}{(2t+50)^{\frac{3}{2}}}$.

(b) $4t^3 + 100 - \frac{22500\sqrt{2}}{(2t+50)^{\frac{3}{2}}}$. (d) $4t^3 + 100 + \frac{22500\sqrt{2}}{(2t+50)^{\frac{3}{2}}}$.

(vi) Which of the following is Lipschitz Condition for the initial value problem

$$y' = f(x,y), y(x_0) = y_0 \ ?$$

(a) $|f(x,y_2) - f(x,y_1)| \leqslant M|y_2 - y_1|$. (d) $|f(x,y_2) - f(x,y_1)| \leqslant M|y_2 + y_1|$.

(b) $|f(x_2,y_2) + f(x_1,y_1)| \leqslant M|y_2 - y_1|$.

(c) $|f(x_2,y_2) + f(x_1,y_1)| \leqslant M|y_2 + y_1|$. (e) none of the above.

(vii) The solution of the differential equation $(3e^{3x}y - 2x)dx + e^{3x}dy = 0$ is given by

(a) $e^{3x}y - x^2 = c$. (c) $e^{-3y}y - x^3 = c$. (e) $e^{2x}y + x^3 = c$.

(b) $e^{3y}x + y^2 = c$. (d) $e^{-3x}x - y^3 = c$.

(viii) The initial value problem

$$(x^2 - 4)\frac{dy}{dx} = (y-1)x, \ \ y(2) = 3$$

(a) has a unique solution. (d) initial data is not enough.

(b) has infinite number of solutions.

(c) has only the trivial solution $y(x) = 0$. (e) has no solution.

(ix) The orthogonal trajectories of the family of curves $xy = c$, is given by

(a) $y - x^2 = k$. (c) $x^2 - y = k$. (e) $x^2 - y^2 = k$.

(b) $x^2 + y^2 = k$. (d) $y + x^2 = k$. $k$ is a constant.

(x) Which of the following is true?

(a) The equation $\frac{dy}{dx} + p(x)y = q(x)$, where $q(x) \neq 0$ is a linear, first order, homogeneous differential equation.

(b) The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is exact if and only if $\frac{\partial P(x,y)}{\partial x} = \frac{\partial Q(x,y)}{\partial y}$.

(c) The orthogonal trajectories to the family of curves given by $f_1(x,y,\lambda) = 0$ are again a family of curves that intersect those given by $f_2(x,y,\lambda) = 0$ at right angles.

(d) The initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ always has infinitely many solutions.

(e) none of the above.