UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) - **2007/2008**

AM 1001

DIFFERENTIAL EQUATIONS

(Two Hours)

Code No:*501731*

Answer all FOUR questions

No. of pages: 08

Important Instructions to the Candidates

- Check the number of questions and number of pages. If a page or a part of this question paper is not printed, please inform the Supervisor immediately.
- Enter your Index Number on all pages of the answer scripts and also in the box provided in the MCQ answer sheet.
- MCQ TYPE: In each of these multiple choice questions mark the correct response on the given MCQ answer sheet with a pen. Write down the question paper code number in the space provided on the MCQ answer sheet.
- STRUCTURED TYPE: Write the answers in the space provided in the question paper.
- ESSAY TYPE: Write the answers to these questions on the writing paper that is provided.
- Attach the MCQ answer sheet and the structured type question together with the answers to the essay type question and hand it over to the supervisor. **Do not attach the MCQ question paper and essay type question to the answer scripts.**

(1) (i) Consider the equation $y''+y=0$ with $y(0)=1$ and $y(\pi)=5$.
Which of the following is/are true?

- (a) It has an infinite number of solutions
- (b) It has a unique solution.
- (c) It has no solution.
- (d) The given data is not sufficient.
- (e) None of the above.

(ii) The isoclines of the differential equation $\dfrac{dy}{dx}=x^2+y^2$ is

(a) $x^2+y^2=C$ (b) $x^2+y^2=C$ with $C>0$
(c) $x^2+y^2\leq C$ (d) $x^2+y^2\geq C$
(e) None of the above.
Where C is any constant.

(iii) Which of the following statement/s is/are true?

- (a) Always we can find a unique solution for a differential equation.
- (b) Particular Integral is a special case of the Complete Primitive.
- (c) $y'+P(x)y=Q(x)y^2$ is called a separable differential equation.
- (d) General solution of an n^{th} degree ODE contains n arbitrary constants.
- (e) Singular solution can not be obtained from the complete primitive.

(iv) Which of the following differential equation/s is/are non-linear?

- (a) $y'+x^2y=e^x$
- (b) $x^2dy+y^2dx=0$
- (c) $y''+\sin xy=0$
- (d) $y''+x\dot{y}=5yy'$
- (e) $x^2y''+xy'+y=\sin x$

(v) The order and the degree of the differential equation $y'''+(y')^2=y^3+x^2$ are respectively

(a) 3 and 2 (b) 2 and 3 (c) 3 and 1 (d) 1 and 3 (e) 3 and 0

(vi) The complete primitive of the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+5y=0$ is

(a) $e^x[P\cos 2x+Q\sin 2x]$ (b) $e^x[P\cos 4x+Q\sin 4x]$
(c) $e^{2x}[P\cos 2x+Q\sin 2x]$ (d) $Pe^{2x}+Qe^{-2x}$
(e) None of the above.
Where P and Q are arbitrary constants.

(vii) An integrating factor/s of the equation $(y+2x-1)dx+x(x+y)dy=0$ is/are

(a) e^{-y} (b) e^{y} (c) $e^{y}y^{2}$ (d) $e^{y}y$ (e) y

(viii) The solution of the differential equation $\frac{dy}{dx}=\frac{x}{y}$ and $y(0)=1$ is

(a) $y^2=x^2-1$ (b) $y^2=-x^2+1$

(c) $y^2=x^2+1$ (d) $y=e^{x^2/2}$

(e) $y=e^{-x^2/2}$

(ix) Which of the following differential equation and initial condition satisfies the function $y(x)=3-e^{-4x}$?

(a) $y'=12-4y$ and $y(0)=3$
(b) $y'=4-12y$ and $y(0)=2$
(c) $y'=12-4y$ and $y(0)=2$
(d) $y'=3-4y$ and $y(0)=3$
(e) $y'=3-y$ and $y(0)=3$

(x) Which of the following function/s is/are homogeneous?

(a) $f(x,y)=x^4-x^3y$ (b) $f(x,y)=e^{x+y}$ (c) $f(x,y)=xy-3x^2$

(d) $f(x,y)=\frac{1}{x}+\frac{3x^3}{y^2}$ (e) $f(x,y)=xy-y$

(2) (i) The differential equation $y'=3y$ with $y(0)=a$ gives

(a) $y(x)=3e^{ax}$ (b) $y(x)=e^{ax}-e^{ax/2}$

(c) $y(x)=ae^{3x}$ (d) $y(x)=ae^{3x/2}$

(e) $y(x)=\sqrt{6x+a^2}$
where a is a constant.

(ii) The solution of the differential equation $y - \frac{dy}{dx} = y^2$ is

(a) $\frac{1}{1+Ce^{-x}}$ (b) $\frac{1}{1+Ce^{x}}$ (c) $\frac{1}{e^{x}+C}$

(d) $1+Ce^{-x}$ (e) $1+Ce^{x}$

where C is an arbitrary constant.

(iii) A particular integral of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$ is

(a) $\frac{e^{2x}}{3}$ (b) $\frac{xe^{2x}}{3}$ (c) 0 (d) $-xe^{2x}$ (e) $-(x+1)e$

(iv) Which of the following statement/s is/are true?

(a) If $P(x,y)dx + Q(x,y)dy = 0$ is exact then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ is called the sufficient condition.

(b) If $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ then $P(x,y)dx + Q(x,y)dy = 0$ is exact, is called the necessary condition.

(c) If there exists a function F such that $\frac{\partial F}{\partial x} = P(x,y)$ and $\frac{\partial F}{\partial y} = Q(x,y)$ then $P(x,y)dx + Q(x,y)dy = 0$ is exact.

(d) $\mu(x,y)$ is an integrating factor of $P(x,y)dx + Q(x,y)dy = 0$ if $\frac{\partial}{\partial y}[\mu(x,y)P(x,y)] = \frac{\partial}{\partial x}[\mu(x,y)Q(x,y)]$

(e) If $P(x,y)dx + Q(x,y)dy = 0$ is exact then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ is called the necessary condition.

(v) Which of the following statement/s is/are true?

(a) Ordinary differential equations contain one or more independent variables.
(b) The order of the differential equation is the order of the highest derivative that appears in the equation.
(c) $\frac{d^2y}{dx^2} + y = 0$ with $y(0) = 1$, $y(\frac{\pi}{2}) = 5$ is an example for an initial value problem.
(d) The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is exact if $\frac{\partial P}{\partial x} = -$
(e) None of the above.

(vi) The complete primitive of $x^2y''+xy'+y=\ell n\,x,\ \ x>0$ can be written as

(a) $Ax^{i}+Bx^{-i}+\ell n\,x$ (b) $Ae^{ix}+Be^{-ix}+x$
(c) $A\cos x+B\sin x+x$ (d) $Ax^{i}+Bx^{-i}+x$
(e) $Ae^{ix}+Be^{-ix}+\ell n\,x$
where A and B are arbitrary constants.

(vii) Which of the following equation/s is/are exact?

(a) $ydx+2xdy=0$ (b) $y^2dx-x^2dy=0$
(c) $ydx+\left(x^2y-x\right)dy=0$ (d) $\left(xy-1\right)dx+\left(x^2-xy\right)dy=0$
(e) $y^2dx+\left(2xy-y^2e^y\right)dy=0$

(viii) $\dfrac{1}{D^2-5}\{\sin x\}$ is equal to

(a) $-\dfrac{\sin x}{4}$ (b) $-\dfrac{\cos x}{6}$ (c) $\dfrac{\sin x}{4}$
(d) $-\dfrac{\sin x}{6}$ (e) $\dfrac{\sin x}{5}$,
where $D\equiv\dfrac{d}{dx}$.

(ix) Orthogonal trajectories of the family of Curves $y=Ce^x$ is given by

(a) $y^2=B+2x$ (b) $y^2=B-2x$
(c) $y=\dfrac{e^{-x}}{C}+B$ (d) $y=e^{-x}+B$
(e) None of the above,
where B and C are constants.

(x) The complementary function of the differential equation $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=e^{2x}$ is

(a) $Ae^{-x}+Be^{2x}$ (b) $Ae^{x}+Be^{2x}$
(c) $Ae^{x}+Be^{-2x}$ (d) $Ae^{-x}+Be^{-2x}$
(e) $e^{x}\left(A\cos x+B\sin x\right)$
where A and B are arbitrary constants.

(3) The experiment indicates that a radioactive substance decays at a rate directly proportional to the amount of undecayed matter remaining. Also the time req for one-half of the substance to decay is called the half-life of the radio active substance. This problem can be modeled as,

$$\frac{dx}{dt} = -\lambda x$$

where x is the amount of radioactive substance at any time and $\lambda > 0$. If x_1 and x_2 are the amount of radioactive substance present at times t_1 and t_2 respectively show that the half-time of the radioactive substance is $\frac{(t_2 - t_1)\ell n2}{\ell n\left(\frac{x_1}{x_2}\right)}$

(4) (a) If $\phi(D)y = e^{\lambda x} f(x)$, then prove that $\frac{1}{\phi(D)} e^{\lambda x} f(x) = e^{\lambda x} \cdot \frac{1}{\phi(D+\lambda)} f$

where $\phi(D) = aD^2 + bD + c$, $D \equiv \frac{d}{dx}$ and a, b, c and λ are constants.

(State the theorems which will be used to get this result.)

(b) Find the complete primitive of the differential equation

$$\frac{d^2y}{dx^2} - 4y = x^2 \sin 2x$$