UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2012/2013

AM 1001

DIFFERENTIAL EQUATIONS

(Two Hours)

Code No: *501731*

Answer all FOUR questions

No. of pages: 08

Important Instructions to the Candidates

- Check the number of questions and number of pages. If a page or a part of this question paper is not printed, please inform the Supervisor immediately.
- Enter your Index Number on all pages of the answer scripts and also in the box provided in the MCQ answer sheet.
- MCQ TYPE: In each of these multiple choice questions mark the correct response on the given MCQ answer sheet with a pen. Write down the question paper code number in the space provided on the MCQ answer sheet.
- STRUCTURED TYPE: Write the answers in the space provided in the question paper.
- ESSAY TYPE: Write the answers to these questions on the writing paper that is provided.
- Attach the MCQ answer sheet and the structured type question together with the answers to the essay type question and hand it over to the supervisor. **Do not attach the MCQ question paper and essay type question to the answer scripts.**

1. i.) Suppose that the existence theorem guarantees the existence of a solution of an initial value problem

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

and its validity in the interval (a, b). Which of the following statements is NOT correct ?

(a) A solution exists within this interval. ✓
(b) A solution may exist outside the interval .×
(c) Every solution satisfies the initial condition. ✓
(d) There is a unique solution within the interval. ✓
(e) None of the above.

ii.) Suppose that you formulate a mathematical model for a real world process with one independent variable and many dependent variables using differential equations. Then it will be

(a) an ordinary differential equation.
(b) a system of ordinary differential equations.
(c) a partial differential equation.
(d) a system of partial differential equations.
(e) None of the above.

iii.) Singular points of the ordinary differential

$$(x^2 - 4)\frac{dy}{dx} = 4(x-1)y, \; x \in \mathbb{R}$$

are

(a) $(-2, 1)$, $(2, 1)$.
(b) $(-2, 0)$, $(2, 0)$
(c) $(1, -2)$, $(1, 2)$.
(d) $(0, 0)$, $(1, 0)$.
(e) $(-2, -1)$, $(2, -1)$

iv.) Which of the following statements is true ?

(a) The general solution of an n^{th} order ordinary differential equation contains n arbitrary constants.

(b) Any singular solution can be obtained by substituting suitable values for the arbitrary constants in the general solution.

(c) If $y_1(x)$ and $y_2(x)$ are any two solutions of any given differential equation, then $y_1(x) + y_2(x)$ is also a solution of the same equation.

(d) A singular solution is a solution obtained by setting all the arbitrary constants of the general solution to zero.

(e) The general solution of an n^{th} degree ordinary differential equation contains n arbitrary constants.

v.) The order and the degree of the differential equation $4(y'')^3 - 5x(y')^4 = 3x^4$ are respectively

(a) 3 and 4
(b) 3 and 3
(c) 2 and 3 ✓
(d) 2 and 4
(e) undefined.

vi.) $y = 4e^{-3x} + e^{-x}$ is a solution of the differential equation

(a) $y' - xy'^2 + y = 0$
(b) $y'^2 - xy' - y = 0$
(c) $y'^2 - xy' + y = 0$
(d) $y'^2 - xy' - y^2 = 0$
(e) $y' + 3y - 2e^{-x} = 0$ ✓

vii.) If $y_1(x)$ and $y_2(x)$ are solutions of the differential equations $a(x)y'' + b(x)y' + c(x)y = q(x)$ and $a(x)y'' + b(x)y' + c(x)y = 0$ respectively, then

(a) $y_2(x) - y_1(x)$ is a solution of $a(x)y'' + b(x)y' + c(x)y = 0$

(b) $y_2(x) - y_1(x)$ is a solution of $a(x)y'' + b(x)y' + c(x)y = q(x)$

(c) $y_2(x) + y_1(x)$ is a solution of $a(x)y'' + b(x)y' + c(x)y = q(x)$

(d) $y_2(x) + y_1(x)$ is a solution of $a(x)y'' + b(x)y' + c(x)y = 0$

(e) $C_1y_2(x) + C_2y_1(x)$ is a solution of $a(x)y'' + b(x)y' + c(x)y = q(x)$, where C_1, C_2 are arbitrary constants viii.) A 100-liter tank is initially filled with 50 liters of pure water. A salt water solution with a concentration of 10 grams of salt per liter is added at a rate of 5 liters per minute to the tank. The solution in the tank is rapidly mixed so that the solution has a uniform concentration at all times. Uniform mixture is also drained out at a rate of 5 liters per minute to maintain the solution with constant volume of 50 liters in the tank. If $y(t)$ is the amount of salt in grams in the tank at time t, the governing equation for y(t) is given by

(a) $\frac{dy}{dt} + y = 5$
(b) $\frac{dy}{dt} - 0.1y = 5$
✓(c) $\frac{dy}{dt} + 0.1y = 5$
(d) $\frac{dy}{dt} + y = 50$
(e) $10\frac{dy}{dt} + y = 500$

ix.) The differential equation $\frac{dy}{dx} = \sqrt{|y|},\ y(0) = 0$

(a) has no solution.
✓(b) has a unique solution.
(c) has more than one solution.
(d) does not satisfy the hypothesis of the existence theorem.
(e) satisfies the hypothesis of the uniqueness theorem.

x.) The initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

where $f(x, y)$ is continuous on $R = \{(x, y) | |x - x_0| < a, |y - y_0| < b\}$ and there exists a constant K such that $|f(x, y)| < K$ in R ,a, b are two positive constants, has a solution valid in

(a) (a, b)
(b) $(\min\{a, a/K\}, b)$
(c) $(a, b/K)$
(d) $(a, \min\{b, b/K\})$
✓(e) $(x_0 - \min\{a, b/K\}, x_0 + \min\{a, b/K\})$

2. i.) The solution of the initial value problem $\frac{dy}{dx} = \frac{x}{y},\ y(1) = 1$ is

(a) $x^2 + y^2 = 1$
(b) $x^2 + y^2 = 2$
✓(c) $x^2 - y^2 = 0$
(d) $x^2y^2 = 1$
(e) $y^2 = 2/x^2$

ii.) The equation $P(x,y)dx + Q(x,y)dy = 0$ is said to be exact if

(a) $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$
(b) there exists $u(x,y)$ such that $du = P(x,y)dx + Q(x,y)dy$
✓(c) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
(d) $\frac{\partial^2 P}{\partial x \partial y} = \frac{\partial^2 Q}{\partial y \partial x}$
(e) $\frac{\partial^2 P}{\partial y \partial x} = \frac{\partial^2 Q}{\partial x \partial y}$

iii.) Let $Pdx + Qdy = 0$ be a non-exact equation. It has an integrating factor F depending only on x if $\frac{1}{F}\frac{dF}{dx} =$

(a) $\frac{1}{Q}(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y})$
✓(b) $\frac{1}{Q}(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x})$
(c) $\frac{1}{P}(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x})$
(d) $\frac{1}{P}(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y})$
(e) $\frac{1}{Q}(\frac{\partial P}{\partial y} - \frac{\partial P}{\partial x})$

iv.) Which of the following equations is exact

✓(a) $[2(x+y^2)\cos x + sinx]dx + 2y\sin x dy = 0$
(b) $(2y-1)\cos 2x dx - y^{-2}\sin 2x dy = 0$
(c) $(x^2 + 2x + 3y)dx + (y^2 - 3x^2 - 1)dy = 0$
(d) $y \sin 2x dx + (y^2 + \cos^2 x)dy = 0$
(e) $y^2 \sin 2x dx - (y^2 + y\cos 2x)dy = 0$

v.) The solution of $(y^2 + x^2 - 2x + 3)dx + (2xy - y^2 + 10)dy = 0$ is

(a) $xy^2 + \frac{1}{3}x^3 - x^2 + 3x - \frac{1}{3}y^3 + 10y = C$
(b) $xy^2 - \frac{1}{3}x^3 - x^2 + 3x - \frac{1}{3}y^3 + 10y = C$
(c) $xy^2 + \frac{2}{3}x^3 - x^2 + 3x + \frac{1}{3}y^3 + 10y = C$
(d) $xy^2 - \frac{1}{3}x^3 - x^2 + 3x - \frac{1}{3}y^3 + y = C$
(e) $xy^2 + \frac{1}{3}x^3 + x^2 + 3x - \frac{1}{3}y^3 + 10y = C$

where C is an arbitrary constant.

vi.) $I(x,y)$ is said to be an integrating factor of $P(x,y)dx+Q(x,y)dy = 0$ if

(a) $\frac{\partial}{\partial x}\{I(x,y)P(x,y)\} = \frac{\partial}{\partial y}\{I(x,y)Q(x,y)\}$
✓(b) $\frac{\partial}{\partial y}\{I(x,y)P(x,y)\} = \frac{\partial}{\partial x}\{I(x,y)Q(x,y)\}$
(c) $I(x,y) = e^{\int P(x,y)dx}$
(d) $I(x,y) = e^{\int Q(x,y)dx}$
(e) there exists $u(x,y)$ such that $du = I(x,y)P(x,y)dx + I(x,y)Q(x,y)dy$