CISC 102 (Fall 21)
Homework #2: Logic #2: Logic (25 Points)

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Question 1

A) $r \wedge \neg q$
B) $p \wedge q \wedge r$
C) $r \longrightarrow p$
D) $p \wedge \neg q \wedge r$

Question 2

A) If you are promoted, then you have washed the boss's car.
B) If there are winds coming from the south, then it is a spring thaw.
C) If Willy cheats, then he will get caught.
D) If Carol is on a boat, then she will get seasick.

Question 3

A) "If it snows tonight, then I will stay at home."
Converse: If I stay home, it will snow tonight.
Contrapositive: If I don't stay home, then it did not snow.
Inverse: If it doesn't snow tonight, I will not stay home.

B) "I go to the beach whenever it is a sunny summer day."
Converse: When I go to the beach it is a sunny summer day.
Contrapositive: When I don't go to the beach it is not a sunny summer day.
Inverse: When it's not sunny I don't go to the beach.

C) "When I stay up late, it is necessary that I sleep until noon."
Converse: Sleeping until noon means I stayed up late.
Contrapositive: If I did not sleep until noon, I did not stay up late.
Inverse: When I don't stay up late, I don't sleep in until noon.

Question 4

p	q	$p \longrightarrow q$	$\neg q \longrightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Question 5

p	q	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \longrightarrow q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Question 6

A+B)
To prove that $A \subseteq B \longrightarrow A \cap \overline{B} = \varnothing$ and that $A \cap \overline{B} = \varnothing \longrightarrow A \subseteq B$ I will prove their equivalence:
$A \cap \overline{B} = \varnothing \equiv \forall x \,(x \in A \wedge x \notin B) = False$
$\equiv \forall x \,(x \in A \wedge \neg\,(x \in B)) = False$
$\equiv \forall x \neg\,(x \in A \wedge \neg\,(x \in B))$
$\equiv \forall x \,(\neg x \in A \vee x \in B)$
$\equiv \forall x \,(x \in A \longrightarrow x \in B)$
$\equiv A \subseteq B$
QED

Question 7

p	q	r	$(p \longrightarrow q) \vee (p \longrightarrow r)$	$p \longrightarrow (q \vee r)$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

Question 8

Let x represent any student in the class, and let u represent any person.
A)
i) $\exists x \neg S(x)$, where S(x) holds if they can swim.
ii) $existsu \neg S(u) \wedge C(u)$, where S(u) holds if they can swim, and C(u) holds if they are in the class.

B)
i) $\forall x Q(x)$, where Q(x) holds if x can solve quadratic equations.
ii) $\forall u \, (C(u) \longrightarrow Q(u))$, where Q(u) holds if u can solve quadratic equations and C(u) holds if u is a student in the class.

C)
i) $\exists x \neg R(x)$, where R(x) holds if x can read.
ii) $\exists u \, (R(u) \wedge C(u))$, where R(u) holds if u can read, and C(u) holds if u is in the class.

Question 9

A) $\exists x \neg M(x)$, where x is any student in the class, and M(x) holds if x likes math. Not every student in this class likes math.
B) $\forall x S(x)$, where x is any student in the class, and S(x) holds if x has seen a computer. Every student in the class has seen a computer.
C) $\forall x \neg T(x)$, where x is any student in the class, and T(x) holds if x has taken every math course offered at this school. There is not a student in this class that has taken every math course offered at this school.

Question 10

A) $\forall x Q(x)$ is false as x = 2 does not satisfy, and $\exists x Q(x)$ is true, as $x = 1$ satisfies this condition.
B) $\exists n \forall m P(m, n)$ is false, and $\forall m \exists n P(m, n)$ is true, as there does not exist a particular n greater than all possible m, as m can be infinitely big, whereas for every m, there exists an n greater than it to satisfy P(m, n).