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Internet of Things for Smart Buildings
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Project 3: Stochastic MPC

We present a building with an energy system that consists of a solar panel for energy generation, a battery to store energy locally and a connection to the grid (see Fig. 1). It is possible to buy or sell electricity via the grid.

The states that describe the system depicted in Fig. 1 are $x^T = [T_\mathrm{r}, E_\mathrm{bat}]$, where T_r is the room temperature and E_bat is the energy stored in the battery. The available control inputs are $u^T = [P_\mathrm{hvac}, P_\mathrm{bat}, P_\mathrm{grid}]$, where P_hvac is the power needed for the HVAC, P_bat the power delivered by or charged to the battery and P_grid the power sold or bought from the grid., If $P_\mathrm{grid} < 0$ energy is sold to the grid and if $P_\mathrm{grid} \geq 0$ energy is bought. For $P_\mathrm{hvac} < 0$ the building is cooled whereas for $P_\mathrm{hvac} \geq 0$ the building is heated. If $P_\mathrm{bat} \geq 0$ energy stored in the battery is used to power the HVAC, if $P_\mathrm{bat} < 0$ the battery is being charged.

The external disturbances acting on the building are $d = [d_\mathrm{T}, d_\mathrm{sr}, d_\mathrm{int}]$, where d_T is the external temperature, d_sr is the solar radiation and d_int are the internal gains.

The system can be described by a linear dynamic system

$$x_{k+1} = Ax_k + Bu_k + Ed_k + w_k, \tag{1}$$

where

$$A = \begin{bmatrix} \alpha_1 \cdot 0.8511 & 0 \\ 0 & \alpha_2 \cdot 1 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.0035 & 0 & 0 \\ 0 & -5 & 0 \end{bmatrix}, E = 10^{-3} \begin{bmatrix} 22.217 & 1.7912 & 42.212 \\ 0 & 0 & 0 \end{bmatrix}.$$

You will receive data containing reasonable values for the external disturbances d_k. You should consider these values as exact. The uncertainty enters the system by the additive disturbances $w_k \in \mathbb{R}^2$ which summarize errors in the weather predictions as well as by the uncertain parameters α_1, α_2 which summarize model mismatches. You should consider that the additive disturbances w_k follow some probability distribution between:

$$\begin{bmatrix} -0.1 \\ -100 \end{bmatrix} \leq \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix} \leq \begin{bmatrix} 0.1 \\ 100 \end{bmatrix}$$

and that the uncertain parameters follow a uniform distribution between 0.7 and 1.0. The goal of the energy management system is to fulfill several constraints for all possible values of the uncertainty. The state constraints are given by

$$20.0 \leq T_\mathrm{r} \leq 23.0°\mathrm{C}, \tag{2}$$

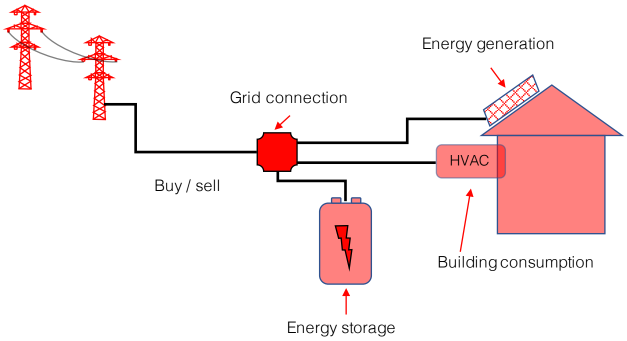

Figure 1: A simplified energy management system.

which gives a band of comfortable room temperatures and

$$0.0 \leq SoC \leq 200000, \tag{3}$$

where SoC is the state of charge of the battery.

Regardless of whether conditions, the power required by the HVAC for cooling or heating should be provided by the energy stored in the battery, the power currently generated by the solar panels P_{PV} and the power bought from the grid:

$$|P_{\mathrm{hvac}}| \leq P_{\mathrm{bat}} + P_{\mathrm{grid}} + P_{\mathrm{PV}}, \tag{4}$$

where $P_{\mathrm{PV}} = 0.5\frac{1}{m^2} \cdot d_{\mathrm{sr}}$, meaning that we assume that 50% of the solar radiation can be converted to electrical energy.

The *economic* control goal is to maximize the energy sold to the grid. Additionally, we assume that it is desired to have a desired energy level in battery to ensure flexibility, which is modeled by a small tracking term. The resulting QP that should be solved at each sampling time of the MPC can be written as:

$$
\begin{aligned}
& \underset{(x,u)}{\text{minimize}} && \sum_{k=0}^{N-1} (P^k_{\mathrm{grid}} + \gamma(E^k_{\mathrm{bat}} - E^{\mathrm{ref}}_{\mathrm{bat}})^2) && \text{(5a)}\\
& \text{subject to} && x_{\mathrm{lb}} \leq x_k \leq x_{\mathrm{ub}}, && \text{(5b)}\\
& && u_{\mathrm{lb}} \leq u_k \leq u_{\mathrm{ub}}, && \text{(5c)}\\
& && m_{\mathrm{lb}} \leq Du_k + Gd_k \leq m_{\mathrm{ub}}, && \text{(5d)}\\
& && x_{k+1} = Ax_k + Bu_k + Ed_k, && \text{(5e)}
\end{aligned}
$$

where N is the prediction horizon, γ is a penalty parameter, and x_{lb}, x_{ub}, u_{lb}, u_{ub}, m_{lb} and m_{ub} are the lower and upper bounds of the states, inputs and mixed constraints. The mixed constraints explained above can be defined by the matrices:

$$D = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0.5 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}.$$

The input constraints are given by

$$u_{\mathrm{lb}} = \begin{bmatrix} -1000 \\ -500 \\ -500 \end{bmatrix}, \quad u_{\mathrm{ub}} = \begin{bmatrix} 1000 \\ 500 \\ 500 \end{bmatrix}.$$

The mixed constraints bounds are given by

$$m_{\mathrm{lb}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad m_{\mathrm{ub}} = \begin{bmatrix} \infty \\ \infty \end{bmatrix}.$$

To deal with the uncertainties. You will explore and implement two different approaches.

Tasks

- Implement nominal MPC of of the system assuming no disturbances and evaluate the performance with disturbances.
- Implement stochastic MPC [1] by exploiting convexity of the optimization problem. Compare the closed-loop violations with the chance constraints that you decide in the formulation of the stochastic MPC.
- Implement stochastic MPC [2] for general nonlinear systems.
- You are free to adapt the ranges of the uncertain parameters in order to obtain scenarios that illustrate better the potential of stochastic MPC.

References

[1] G. Calafiore and M. Campi. The scenario approach to robust control design. *IEEE Transactions on Automatic Control*, 51:742–753, 2006.

[2] M. C. Campi, S. Garatti, and F. A. Ramponi. A general scenario theory for non-convex optimization and decision making. *IEEE Transactions on Automatic Control*, 9286(c), 2018.