#### Lecture 12

Lect. PhD. Arthur Molnar

#### Searching

The searching problem
Searching algorithms
Binary search
Search in Pyth

#### Sorting

The sorting problem Selection sort Insertion sort Bubble Sort Quick Sort

Lambda Expression

# Searching. Sorting. Lambdas

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### Overview

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#### Searching

The searching problem Searching algorithms Binary search Search in Pyth

#### Sortir

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### 1 Searching

- The searching problem
- Searching algorithms
- Binary search
- Search in Python

### 2 Sorting

- The sorting problem
- Selection sort
- Insertion sort
- Bubble Sort
- Quick Sort
- 3 Lambda Expressions

### Feedback for the course

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- You can write feedback at academicinfo.ubbcluj.ro
- It is both important as well as anonymous
- Write both what you like (so we keep&improve it) and what you don't
- Best if you write about all activities (lecture, seminar and laboratory)

# Searching

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# Searching The searching problem Searching algorithms Binary search Search in Pyth

### Sorting

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- Data are available in the internal memory, as a sequence of records  $(k_1, k_2, ..., k_n)$
- Search a record having a certain value for one of its fields, called the search key.
- If the search is successful, we have the position of the record in the given sequence.
- We approach the search problem's two possibilities separately:
  - Searching with unordered keys
  - Searching with ordered keys

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Lambda Expression

### Problem specification

- **Data**:  $a, n, (k_i, i = 0, ..., n 1)$ , where  $n \in \mathbb{N}, n \ge 0$ .
- **Results**: p, where  $(0 \le p \le n-1, a = k_p)$  or p = -1, if key is not found.

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Lambda Everession

```
searchSeq(el, I):
def
    , , ,
    Search for an element in list
    el - element
    I - list of elements
    Return the position of the element, -1 if not
        found
    , , ,
    poz = -1
    for i in range(0,len(1)):
        if el == |[i]:
             poz = i
    return poz
```

Computational complexity is 
$$T(n) = \sum_{i=0}^{n-1} 1 = n \in \Theta(n)$$

```
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```

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Lambda Expressions

```
def searchSeq(el, l):
    Search for an element in list
    el - element
    I - list of elements
    Return the position of the element, -1 if not
        found
    , , ,
    i = 0
    while i < len(1) and el! = I[i]:
        i += 1
    if i < len(1):
        return i
    return -1
```

What is the difference between this and the previous version?

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Lambda Expression ■ Best case: the element is at the first position,  $T(n) \in \Theta(1)$ .

- Worst case: the element is in the n-1 position,  $T(n) \in \Theta(n)$ .
- Average case: if distributing the element uniformly, the loop can be executed 0, 1, ..., n-1 times, so  $T(n) = \frac{1+2+...+n-1}{2} \in \Theta(n)$ .
- Overabll complexity is O(n)

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Lambda Expression:

### Problem specification

- **Data**:  $a, n, (k_i, i = 0, ..., n 1)$ , where  $n \in \mathbb{N}, n \ge 0$ , and  $k_0 < k_1 < ... < k_{n-1}$ ;
- **Results**: p, where  $(p = 0 \text{ and } a \le k_0)$  or  $(p = n \text{ and } a > k_{n-1})$  or  $(0 and <math>(k_{p-1} < a \le k_p)$ .

```
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```

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### Sorting

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```
def searchSeq(el, l):
    Search for an element in list
    el - element
    I - list of ordered elements
    Return the position of the first occurrence, or
        position where element can be inserted
    , , ,
    if len(1) = 0: return 0
    poz = -1
    for i in range(0,len(1)):
        if el<=l[i]:
            poz = i
    if poz = -1: return len(I)
    return poz
```

Computational complexity is 
$$T(n) = \sum_{i=0}^{\infty} 1 = n \in \Theta(n)$$

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#### Sorting

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```
def searchSuccesor(el, I):
    , , ,
    Search for an element in list
    el - element
    I - list of ordered elements
    Return the position of the first occurrence, or
        position where element can be inserted
    , , ,
    if len(I)==0 or eI <= I[0]:
        return 0
    if el>=l[-1]:
        return len(I)
    i = 0
    while i < len(1) and el > l[i]:
        i += 1
    return i
```

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- Best case: the element is at the first position,  $T(n) \in \Theta(1)$ .
- Worst case: the element is in the n-1 position,  $T(n) \in \Theta(n)$ .
- Average case: if distributing the element uniformly, the loop can be executed 0, 1, ..., n-1 times, so  $T(n) = \frac{1+2+...+n-1}{2} \in \Theta(n)$ .
- Overabll complexity is O(n)

# Searching algorithms

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#### Searching

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Sorting

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- Sequential search
  - Keys are successively examined
  - Keys may not be ordered
- Binary search
  - Uses the divide and conquer technique
  - Keys are ordered

### Recursive binary-search algorithm

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```
def
    binarySearch(key, data, left, right):
    Search for an element in an ordered list
    key - element to search
    left, right — bounds of the search
    Return insertion position of key that keeps list
         ordered
    , , ,
    if left >= right - 1:
        return right
    middle = (left + right) // 2
    if key < data[middle]:</pre>
        return binarySearch(key, data, left, middle)
    else:
        return binarySearch (key, data, middle, right
print(binarySearch(2000, data, 0, len(data)))
```

### Recursive binary-search function

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### Search in Pyth

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```
def search(key, data):
    , , ,
    Search for an element in an ordered list
    key - element to search
    data — the list
    Return insertion position of key that keeps list
         ordered
    , , ,
    if len(data) = 0 or key < data[0]:
        return O
    if key > data[-1]:
        return len (data)
    return binarySearch(key, data, 0, len(data))
```

# Binary-search recurrence

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Binary search

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Lambda Expression ■ The recurrence:  $\mathsf{T}(\mathsf{n}) = \begin{cases} 1, n = 1 \\ T(\frac{n}{2}) + 1, n > 1 \end{cases}$ 

# Iterative binary-search function

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#### Searching

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Sorting
The sorting problem
Selection sort

```
def binarySearch(key, data):

    specification -

    if len(data) = 0 or key < data[0]:
        return 0
    if key > data[-1]:
        return len (data)
    left = 0
    right = len(data)
    while right - left > 1:
        middle = (left + right) // 2
         if key <= data[middle]:</pre>
             right = middle
        else:
             left = middle
    return right
```

# Search problem runtime complexity

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#### Searchin

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Selection so Insertion sort

Lambda

Algorithm	Best case	Average	Worst case	Overall
Sequential	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Succesor	Θ(1)	$\Theta(n)$	$\Theta(n)$	O(n)
Binary-search	Θ(1)	$\Theta(\log_2 n)$	$\Theta(\log_2 n)$	$O(\log_2 n)$

### Demo

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Lambda Expression

### Collections and search

Examine the source code in ex34\_search.py

## The sorting problem

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### The sorting

problem
Selection sort
Insertion sort
Bubble Sort
Quick Sort

Lambda Expression

### Sorting

Rearrange a data collection in such a way that the elements of the collection verify a given order.

- Internal sort data to be sorted are available in the internal memory
- External sort data is available as a file (on external media)
- In-place sort transforms the input data into the output, only using a small additional space. Its opposite is called out-of-place.
- Sorting stability we say that sorting is stable when the original order of multiple records having the same key is preserved

# The sorting problem

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#### Sorting

#### The sorting problem Selection sort Insertion sort Bubble Sort Quick Sort

- Elements of the data collection are called records
- A record is formed by one or more components, called fields
- A key K is associated to each record, and is usually one of the fields.
- We say that a collection of n records is:
  - Sorted in increasing order by the key K: if  $K(i) \le K(j)$  for  $0 \le i < j < n$
  - Sorted in decreasing order: if  $K(i) \ge K(j)$  for  $0 \le i < j < n$

# Internal sorting

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#### Sorting

### The sorting problem

Insertion sort
Bubble Sort
Quick Sort

Lambda Expression

### Problem specification

- **Data**: n, K, where  $K = (k_1, k_2, ..., k_n), k_i \in \mathbb{R}, i = 1, n$
- **Results**: K', where K' is a permutation of K, having sorted elements:  $k'_1 \leq k'_2 \leq ... \leq k'_n$ .

# Sorting algorithms

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#### Searchin

problem
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#### Sortin

# The sorting problem

Insertion sort Bubble Sort Quick Sort

Lambda Expression A few algorithms that we will study:

- Selection sort
- Insertion sort
- Bubble sort
- Quick sort

### Selection Sort

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problem
Selection sort
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- Determine the element having the minimal key, and swap it with the first element.
- Resume the procedure for the remaining elements, until all elements have been considered.

### Selection sort algorithm

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Selection sort Insertion sort Bubble Sort

# Selection sort - time complexity

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problem

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Lambda Expression ■ The total number of comparisons is

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1 = \frac{n(n-1)}{2} \in \Theta(n^2)$$

• Independent of the input data size, what are the best, average, worst-case computational complexities?

# Selection sort - space complexity

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#### Sortii

problem
Selection sort
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Quick Sort

- **In-place** algorithms. Algorithms that use a small (constant) quantity of additional memory.
- Out-of-place or not-in-space algorithms. Algorithms that use a non-constant quantity of extra-space.
- The additional memory required by selection sort is O(1).
- Selection sort is an in-place sorting algorithm.

### Direct selection sort

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### Direct selection sort

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Lambda

• Overall time complexity:  $\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1 = \frac{n(n-1)}{2} \in \Theta(n^2)$ 

### Insertion Sort

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The sorting problem Selection sort Insertion sort Bubble Sort Quick Sort

- Traverse the elements.
- Insert the current element at the right position in the subsequence of already sorted elements.
- The sub-sequence containing the already processed elements is kept sorted, so that, at the end of the traversal, the whole sequence is sorted.

### Insertion Sort - Algorithm

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```
def insertSort(data):
    for i in range(1, len(data)):
        index = i - 1
        elem = data[i]
    # Insert into correct position
    while index >= 0 and elem < data[index]:
        data[index + 1] = data[index]
        index -= 1
        data[index + 1] = elem</pre>
```

# Insertion Sort - time complexity

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Lambda Expression Maximum number of iterations (worst case) happens if the initial array is sorted in a descending order:

$$T(n) = \sum_{i=2}^{n} (i-1) = \frac{n(n-1)}{2} \in \Theta(n^2)$$

# Insertion Sort - time complexity

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Lambda Expression Minimum number of iterations (best case) happens if the initial array is already sorted:

$$T(n) = \sum_{i=2}^{n} 1 = n - 1 \in \Theta(n)$$

# Insertion Sort - Space complexity

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- Time complexity The overall time complexity of insertion sort is  $O(n^2)$ .
- lacksquare Space complexity The complexity of insertion sort is heta(1)
- Insertion sort is an in-place sorting algorithm.

### **Bubble Sort**

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#### Sorting

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- Compares pairs of consecutive elements that are swapped if not in the expected order.
- The comparison process ends when all pairs of consecutive elements are in the expected order.

### Bubble Sort - Algorithm

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# **Bubble Sort - Complexity**

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\_ambda =xpression

- **Best-case** running time complexity order is  $\theta(n)$
- Worst-case running time complexity order is  $\theta(n^2)$
- **Average** running-time complexity order is  $\theta(n^2)$
- **Space complexity**, additional memory required is  $\theta(1)$
- Bubble sort is an *in-place* sorting algorithm.

## Quick Sort

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Lambda Expression Based on the divide and conquer technique

**1 Divide:** partition array into 2 sub-arrays such that elements in the lower part  $\leq$  elements in the higher part.

### **Partitioning**

Re-arrange the elements so that the element called pivot occupies the final position in the sub-sequence. If i is that position:  $k_j \le k_i \le k_l$ , for  $Left \le j < i < l \le Right$ 

- **2 Conquer:** recursively sort the 2 sub-arrays.
- **3 Combine:** trivial since sorting is done in place.

## Quick Sort - partitioning algorithm

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Lambda Expressions

```
def partition(data, left, right):
    pivot = data[left]
    i = left
    i = right
    while i != i:
        # Find an element smaller than the pivot
        while data[j] >= pivot and i < j:
            i -= 1
        data[i] = data[i]
        # Find an element larger than the pivot
        while data[i] <= pivot and i < j:
            i += 1
        data[j] = data[i]
   # Place the pivot in position
    data[i] = pivot
    return i
```

## Quick Sort - algorithm

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Lambda Expressions

```
def quickSort(data, left, right):
    # Partition the list
    pos = partition(data, left, right)
    # Order left side
    if left < pos - 1:
        quickSort(data, left, pos - 1)
    # Order right side
    if pos + 1 < right:
        quickSort(data, pos + 1, right)</pre>
```

# Quick Sort - time complexity

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Lambda Expressions

- The run time of quick-sort depends on the distribution of splits
- The partitioning function requires linear time
- **Best case**, the partitioning function splits the array evenly:  $T(n) = 2T(\frac{n}{2}) + \Theta(n), T(n) \in \Theta(n \log_2 n)$

# Quick Sort - best partitioning

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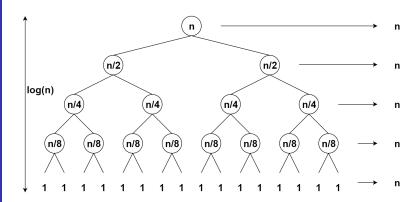
### Searching

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Lambda Expressions



■ We partition n elements  $\log_2 n$  times, so  $T(n) \in \Theta(n \log_2 n)$ 

# Quick Sort - worst partitioning

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Problem
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Lambda Expression In the worst case, function Partition splits the array such that one side of the partition has only one element:

$$T(n) = T(1) + T(n-1) + \Theta(n) = T(n-1) + \Theta(n) =$$

$$\sum_{k=1}^{n} \Theta(k) \in \Theta(n^2)$$

## Quick Sort - Worst case

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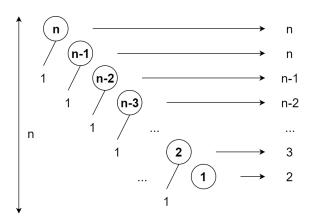
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### Sorting

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■ Worst case partitioning appears when the input array is sorted or reverse sorted, so n elements are partitioned n times,  $T(n) \in \Theta(n^2)$ 

# Sorting runtime complexity

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Algorithm	Worst case	Average
Selection sort	$\Theta(n^2)$	$\Theta(n^2)$
Insertion sort	$\Theta(n^2)$	$\Theta(n^2)$
Bubble sort	$\Theta(n^2)$	$\Theta(n^2)$
Quick sort	$\Theta(n^2)$	$\Theta(n\log_2 n)$

### Demo

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### Searching

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### Sorting

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## Sorting

Examine the source code in ex35\_sort.py

# Lambda expressions

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Lambda Expressions

### Lambda expressions

Small anonymous functions, that you define and use in the same place.

- Syntactically restricted to a single expression.
- Can reference variables from the containing scope (just like nested functions).
- They are *syntactic sugar* for a function definition.

### Demo

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Lambda Expressions

## Lambda Expressions

Examine the source code in ex36\_lambdas.py