Lab 9

Conditioning number of a linear system

1. a) Solve the following linear system:

$$\begin{bmatrix} 400 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}.$$

b) Solve the linear system obtained by changing an element of the coefficient matrix:

$$\begin{bmatrix} 401 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}.$$

c) Find the conditioning number. (Use Matlab function *cond*.)

2. Consider the system:

$$Ax = b,$$

with

$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 32 \\ 23 \\ 33 \\ 31 \end{bmatrix}.$$

a) Solve this system and find its conditioning number. (Use Matlab function *cond*.)

b) Solve the system

$$A\tilde{x} = \tilde{b},$$

where

$$\tilde{b} = \begin{bmatrix} 32.1 \\ 22.9 \\ 33.1 \\ 30.9 \end{bmatrix}.$$

Compute the input relative error $\frac{\|b-\tilde{b}\|}{\|b\|}$ and the output relative error $\frac{\|x-\tilde{x}\|}{\|x\|}$. Compute $\frac{\|x-\tilde{x}\|}{\|x\|} \Big/ \frac{\|b-\tilde{b}\|}{\|b\|}$

c) Solve the system

$$\bar{A}\bar{x} = b,$$

with

$$\bar{A} = \begin{bmatrix} 10 & 7 & 8.1 & 7.2 \\ 7.08 & 5.04 & 6 & 5 \\ 8 & 5.98 & 9.89 & 9 \\ 6.99 & 4.99 & 9 & 9.98 \end{bmatrix}.$$

Compute the input relative error $\frac{\|A-\bar{A}\|}{\|A\|}$ and the output relative error $\frac{\|x-\bar{x}\|}{\|x\|}$. Compute $\frac{\|x-\bar{x}\|}{\|x\|} \Big/ \frac{\|A-\bar{A}\|}{\|A\|}$.

3. Find the conditioning numbers of the Hilbert matrices $H_n = (h_{ij})_{\substack{1\leq i\leq n \\ 1\leq j\leq n}}$, with $h_{ij} = \frac{1}{i+j-1}$, for $n = 10 : 15$.

4. a) Find the conditioning numbers of the Vandermonde matrices $V(t_k)$ for the points $t_k = \frac{1}{k},\ k = \overline{1,n}$, for $n = \overline{10,15}$.

b) Find the conditioning numbers of the Vandermonde matrices $V(t_k)$ for the points $t_k = -1 + \frac{2}{n}k \in [-1,1]$, with $k = \overline{1,n}$, for $n = \overline{10,15}$.