Lab 8

Quadrature formulas (2)

The rectangle (midpoint) quadrature formula is

$$\int_a^b f(x)dx = (b-a)f\left(\frac{a+b}{2}\right) + R_1(f). \tag{1}$$

The repeated rectangle (midpoint) quadrature formula is

$$\int_a^b f(x)dx = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i) + R_n(f),$$

with $x_1 = a + \frac{b-a}{2n}$, $x_i = x_1 + (i-1)\frac{b-a}{n},\ i = 2, ..., n.$

Problems:

1. a) Use the rectangle formula to evaluate the integral

$$\int_1^{1.5} e^{-x^2} dx.$$

   b) Plot the graph of the function f and the graph of the rectangle which area approximates the integral by formula (1).

   c) Use the repeated rectangle formula, for $n = 150$ and 500, to evaluate the integral

$$\int_1^{1.5} e^{-x^2} dx.$$

   (*Result:* 0.1094)

2. Use two Romberg schemes (see [Course 7, formulas (2) and (5)]) for approximating the integral

$$\int_0^1 \frac{2}{1+x^2} dx,$$

   with precision $\varepsilon = 10^{-4}$.

3. Plot the graph of $f : [1,3] \to \mathbb{R}$, $f(x) = \frac{100}{x^2} \sin \frac{10}{x}$. Use an adaptive quadrature algorithm for Simpson formula to approximate the integral

$$\int_1^3 f(x)dx,$$

with precision $\varepsilon = 10^{-4}$. Compare the obtained result with the one obtained applying repeated Simpson formula for $n = 50$ and 100. (The exact value is -1.4260247818.)

Facultative problem

1. Use the Romberg algorithm for rectangle formula to approximate the integral

$$\int_1^{1.5} e^{-x^2} dx,$$

with precision $\varepsilon = 10^{-4}$.