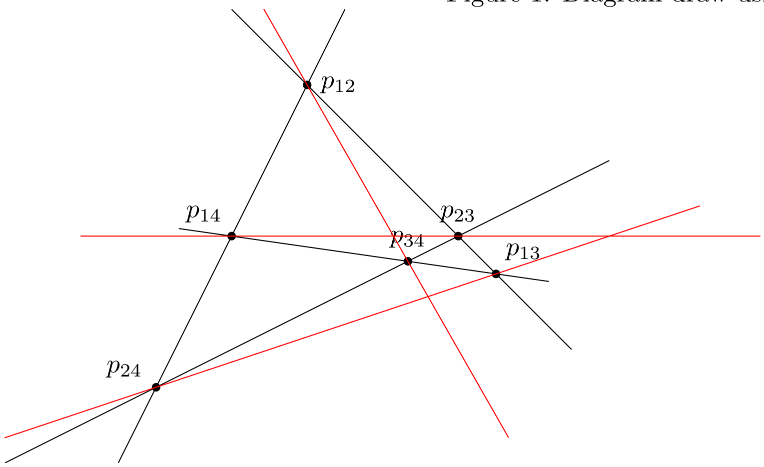
Figure 1: Diagram draw using explicitly calculated end points.



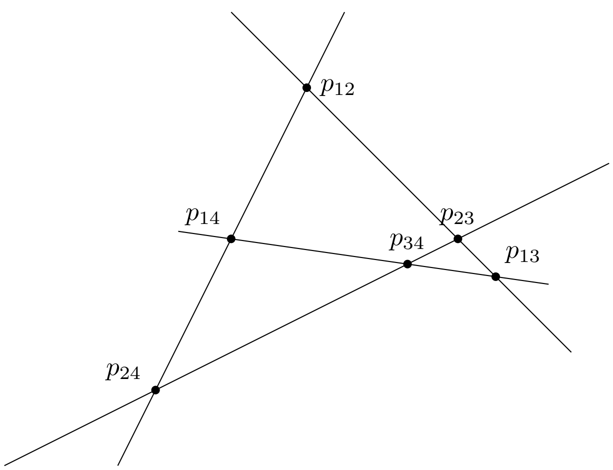
This document describes the construction of harmonic points in the (Desarguesian) projective plane. The motivation is an exer learning to use the LaTeX drawing software *tikz*

The construction makes use of Desargues' Theorem:

Definition. *Two triangles ABC and DEF are in perspective from a point if the lines AD, BE and CF joining corresponding verti concurrent.*
Two triangles ABC and DEF are in perspective from a line if the points of intersection of corresponding sides $AB \cdot DE, BC \cdot E$ $AC \cdot DF$ are co-linear.

Theorem (Desagues)**.** *If two triangles ABC and DEF are in perspective from a point they are in perspective from a line.*
Dually:
If two triangles ABC and DEF are in perspective from a line they are in perspective from a point.

Definition. *Four lines in a projective plane with their six points of intersection form a **complete quadrilateral**.*



Definition. *The three lines joining the pairs of point not ona commom line form the **diagonals** of the complete quadrilatal.*

The code above created by explicitly calculating end points of lines.

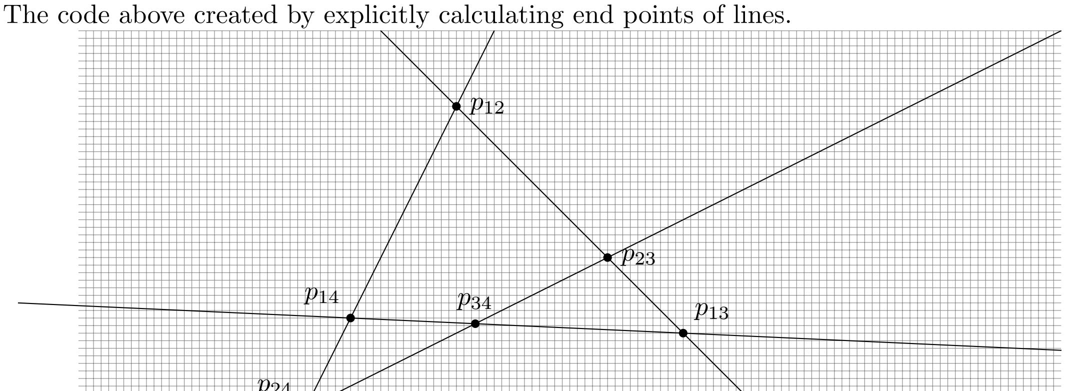
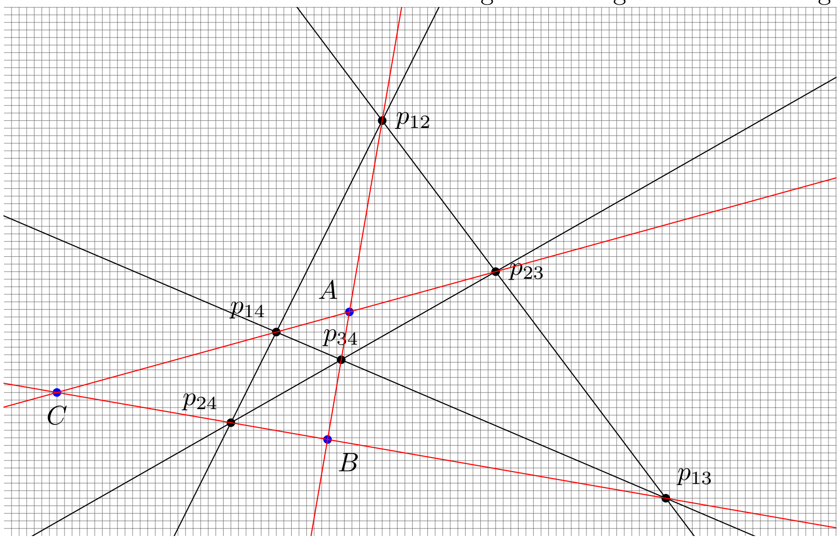


Figure 2: Diagonal Point Triangle of a Complete Quadrangle



There are three pairs of points p_{14}, p_{23}, $p12, p_{34}$ and p_{24}, p_{13} that are not on a commom line. The lines through these points are r
to as the **diagonals**. Together they form the **diagonal point triangle**.

There are four points on each line of the quadrilateral. These points are said to be in **harmonic range**.

Theorem. *Given points A and B with a third point C on the line AB there is a unique point D the **harmonic conjugate** of C
$ABCD$ form an harmonic range.*

Construction. *Choose a point F not on the line ABC and a point H on CF.*