Why log over normal returns?

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Why are logarithmic returns better to use in pairs trading than normal rate of returns: additivity, linearity, and stability over variance.

Take wealth at 'n' w.r.t. wealth 'x' days ago to be:

$$t_n = t_{n-x}(1 + r_{n-x})(1 + r_{n-x+1}) \cdots (1 + r_n) \tag{1}$$

Eqn.1 depicts a compunding effect of the returns. To reach the above equation, we take 'y' to be some price at t_0:

$$t_0 \implies y$$

and r_1 to be the rate of return from $P(t_0)$ to $P(t_1)$:

$$r_1 = \frac{t_1 - t_0}{t_0}$$

Thus,

$$r_n = \frac{t_n - t_{n-1}}{t_{n-1}}$$

Thus, wealth at t_2 is:

$$t_1 \implies y(1 + r_1)$$

$$t_2 \implies t_1(1 + r_2)$$

$$t_2 \implies y(1 + r_1)(1 + r_2)$$

$$\vdots$$

$$t_n \implies t_{n-1}(1 + r_n)$$

$$t_n \implies t_{n-x}(1 + r_{n-x})(1 + r_{n-x+1}) \cdots (1 + r_n)$$

So eqn.1 becomes: $t_n = t_{n-x} \coprod_{k=n-x+1}^{n} (1 + r_k)$

Now we can define log return R_t to be log (1+r_t)

The wealth recursion in terms of log returns becomes:

$$\log t_n = \log t_{n-x} + \sum_{k=n-x+1}^{n} (1 + R_k)$$

$$t_n = t_{n-x} \times \exp(\sum_{k=n-x+1}^{n} R_k) \tag{2}$$

Comparing eqn.(s) 1 and 2, we can see it is easier to keep log returns due to additivity, linearity, and stability over variance. This also heps ensure that the spread stays within bound and doesn't explode.