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Computational Physics – Exercise 6

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1 Bias of the correlation function

The two-point correlation function for spins is defined as.

$$C_{ij} = \langle s_i s_j \rangle = \frac{1}{\Lambda} \sum_x \tag{1}$$

when we have translation in-variance, we can rewrite this to be,

$$C_r = C_{i,i+r} \tag{2}$$

For a large lattice and $J > J_c$, we expect C to be biased as in this regime the system is highly correlated and is mostly likely in a ferromagnetic phase, thus leading to $C = 1$.

2 C at $r = 0$

At $r = 0$, we expect,

$$C_r = 1 \tag{3}$$

as, the correlation function measures on average how similar a spin at site i is to it's r^{th} neighbour. For the case of, $r = 0$ we are merely examining it's correlation with itself. Which should be 1 irrespective of whether the site has a spin pointing up or down.

3 Implementing C with FFTs and convolution

We implement the correlation function by using Fast Fourier Transforms and the convolution theorem at [1]. We also show that this produces what we expect at $r = 0$.

4 C as a function of r

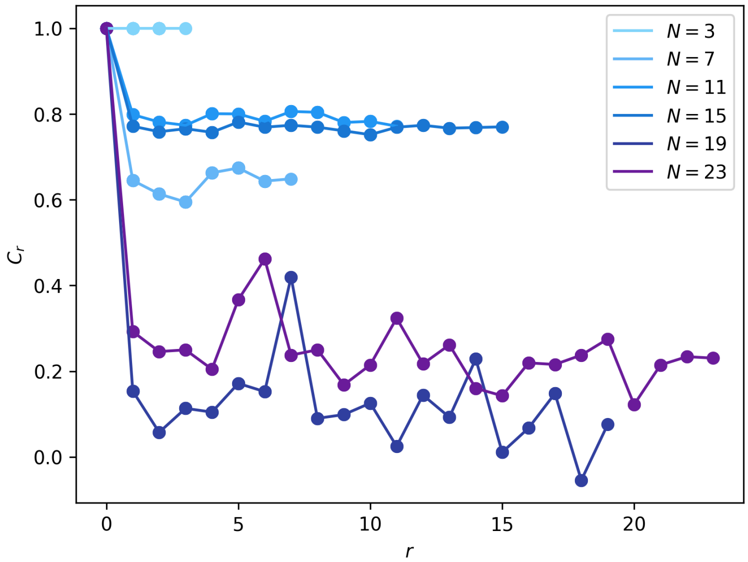

Figure 1: Here we plot the correlation function, C versus r

5 Autocorrelation time for absolute magnetization

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6 Dynamical exponent

We have,

$$\tau = N^z \tag{4}$$

where τ is the autocorrelation time, N is the number of sites on one side and z is the dynamical exponent. In the log-log plot of τ versus N, the line traced would follow an equation

$$\log \tau = z \log N \tag{5}$$

We can see that the dynamical exponent z appears as the gradient of the plot.

References

[1] P. Anancia Devaneyan and R. K. Senthilkumar. Exercises for the Physics 760: Computational Phyics course during the WS 22/23 Term.