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Computational Physics – Exercise 4

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1 Equations of Motion

We have the Hamiltonian,

$$\mathcal{H}[\boldsymbol{p}, \boldsymbol{\phi}] = \frac{p_0^2}{2} + \frac{p_1^2}{2} + \frac{p_2^2}{2} + \beta\chi^2(\phi) \tag{1}$$

Where,

$$\chi^2(\phi) = \frac{1}{2}\sum_{i=1}^{5} \frac{\left(f_i - f\left(x_i, \phi\right)\right)^2}{\delta f_i^2} \tag{2}$$

$$f\left(m_\pi, \boldsymbol{\phi}\right) = \phi_0 + \phi_1 m_\pi + \phi_2 m_\pi^2 \tag{3}$$

m_π(GeV)	f_i(data in MeV)	δf_i(error in MeV)
.176	960	25
.234	1025	20
.260	1055	15
.284	1085	10
.324	1130	8

Table 1: The data we want to fit to.

Using Hamilton's equations of motion,

$$\dot{\phi}_i = \frac{\partial}{\partial p_i}\mathcal{H} \tag{4}$$

$$\dot{p}_i = -\frac{\partial}{\partial \phi_i}\mathcal{H} \tag{5}$$

we get

$$\dot{\phi}_i = p_i \tag{6}$$

$$\dot{p}_0 = -\frac{\beta}{2}.\sum_{j=1}^{5} \frac{2f(m_{\pi_j}, \phi) - f_j}{\delta f_j} \tag{7}$$

$$\dot{p}_1 = -\frac{\beta}{2}.\frac{(1 - 2m_{\pi_j})f_j - 2m_{\pi_j} f(m_{pi_i,\phi})}{\delta f_j} \tag{8}$$

$$\dot{p}_2 = -\frac{\beta}{2}.\frac{(1 - 2m_{\pi_j}^2)f_j - 2m_{\pi_j}^2 f(m_{pi_i,\phi})}{\delta f_j} \tag{9}$$

2 Modified Leapfrog Algorithm

We modify the Leapfrog algorithm to integrate the EOMs. We plot for the convergence in figure 1.

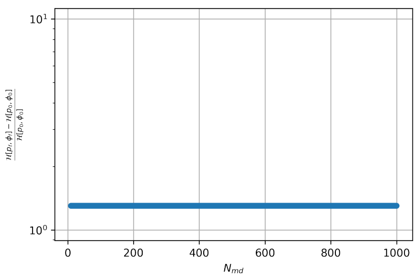

Figure 1: Convergence of the leap-frog in in terms of integration steps N_{md}.

3 Modified Hybrid Monte Carlo

We coded the HMC for the following case, it can be found at [1].

4 Markov Chain as a function of the HMC trajectory

5 Finding the best fit

6 Mass of the neutron in this chiral limit

In the chiral limit, the neutron mass corresponds to ϕ_0 from Eq. (3), i.e. the y-intercept.

References

[1] P. Anancia Devaneyan and R. K. Senthilkumar. Exercises for the Physics 760: Computational Phyics course during the WS 22/23 Term.