Exercises for Introduction to Quantum Computing

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In [ ]:
```
import numpy as np
import qiskit as qi
from qiskit import IBMQ, Aer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, execute
from qiskit.visualization import plot_histogram
```

1. Quantum Fourier Transform (continued)

For a state,

$$|\psi(x=1)\rangle = \frac{1}{\sqrt{8}}\sum_{k=0}^{7} e^{2\pi i \frac{k}{8}}|k\rangle$$

applying the inverse qFT,

$$qFT^{\dagger} = \frac{1}{\sqrt{8}}\sum_{j=0}^{7} e^{-2\pi i \frac{kj}{8}}|j\rangle$$

gives us,

$$qFT^{\dagger}|\psi(x=1)\rangle = \frac{1}{8}\sum_{j=0}^{7}\sum_{k=0}^{7} e^{-2\pi i \frac{kj}{8}} e^{2\pi i \frac{k}{8}}|j\rangle$$

$$qFT^{\dagger}|\psi(x=1)\rangle = \frac{1}{8}\sum_{j=0}^{7}\sum_{k=0}^{7} e^{2\pi i \frac{k(1-j)}{8}}|j\rangle$$

For a state,

$$|\psi(x=\frac{1}{2})\rangle = \frac{1}{\sqrt{8}}\sum_{k=0}^{7} e^{2\pi i \frac{k}{16}}|k\rangle$$

applying the inverse qFT gives us,

$$qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle = \frac{1}{8}\sum_{j=0}^{7}\sum_{k=0}^{7} e^{-2\pi i \frac{kj}{8}} e^{2\pi i \frac{k}{16}}|j\rangle$$

We can compute the probability of measuring 0 after measurement by computing $|\langle 000|qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle|^2$,

$$|\langle 000|qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle|^2 = |\frac{1}{8}\sum_{k=0}^{7} e^{\pi i \frac{k}{8}}|^2$$

Evaluating the sum, we get,

$$|\langle 000|qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle|^2 = 0.41$$

Now, to compute the probability of measuring 0 or 1, we simply need to compute

$$|\langle 001|qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle|^2 = |\frac{1}{8}\sum_{k=0}^{7} e^{-\pi i \frac{k}{8}}|^2$$

and add it to the previous result. Computing the sum,

$$|\langle 001|qFT^{\dagger}|\psi(x=\frac{1}{2})\rangle|^2 = 0.41$$

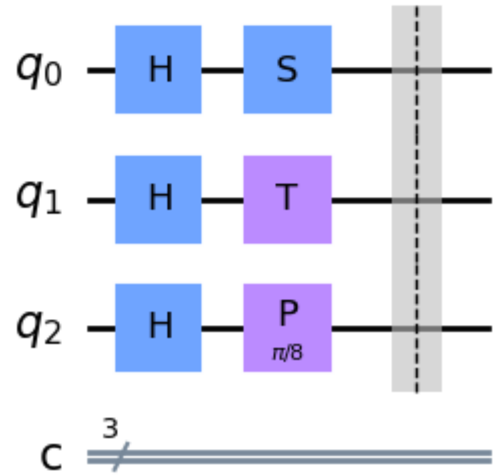
Thus, the pobability of observing 0 or 1 = 0.82

We shall now evaluate the $qFT^{\dagger}$ using Qiskit. We begin by intialising the state,

In [ ]:

```
inv_qFT = QuantumCircuit(3,3)
inv_qFT.h(0)
inv_qFT.h(1)
inv_qFT.h(2)
inv_qFT.s(0)
inv_qFT.t(1)
inv_qFT.p(np.pi/8,2)
inv_qFT.barrier()
inv_qFT.draw('mpl')
```

Out[ ]:

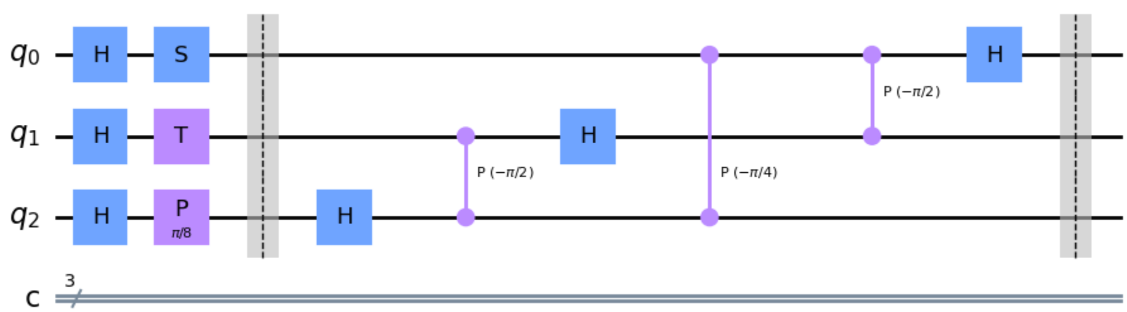


Now we implement the $qFT^\dagger$

In [ ]:
```
inv_qFT.h(2)
inv_qFT.cp(-np.pi/2, 2, 1)
inv_qFT.h(1)
inv_qFT.cp(-np.pi/4, 2, 0)
inv_qFT.cp(-np.pi/2, 1, 0)
inv_qFT.h(0)
inv_qFT.barrier()

inv_qFT.draw('mpl')
```

Out[ ]:

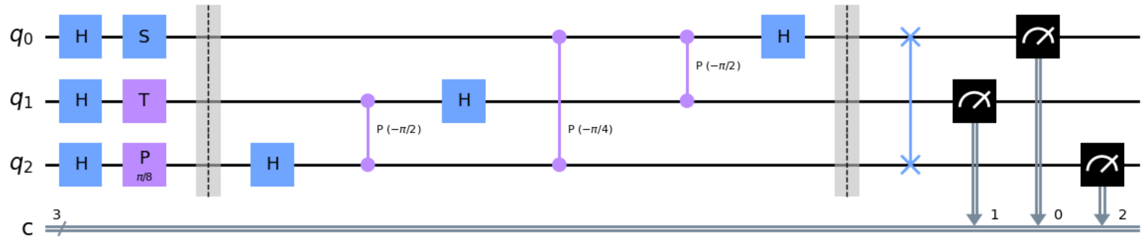


We now need to perform a set of swap operations,

In [ ]:
```
def swap_registers(circuit, n):
    for qubit in range(n//2):
        circuit.swap(qubit, n-qubit-1)
    return circuit
```

In [ ]:
```
inv_qFT = swap_registers(inv_qFT,3)
inv_qFT.measure([0,1,2],[0,1,2])
inv_qFT.draw('mpl')
```

Out[ ]:



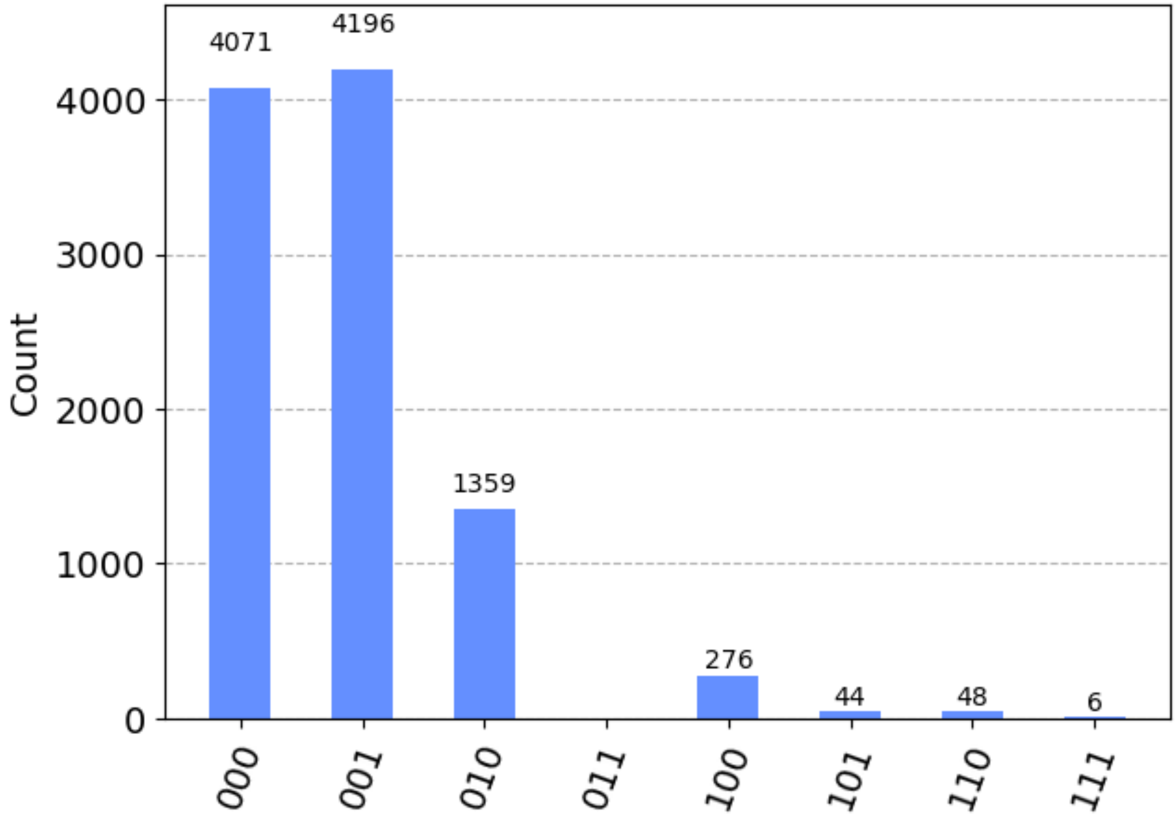
We now have our inverse-qFT circuit acting on the prepared state. Let's now run the simulation!

In [ ]:
```
simulator = Aer.get_backend('qasm_simulator')
result = execute(inv_qFT,backend=simulator, shots = 10000).result()
results = result.get_counts()
display(results)
```

```
{'000': 4071,
 '100': 4196,
 '010': 1359,
 '001': 276,
 '011': 48,
 '101': 44,
 '111': 6}
```

In [ ]:
```
NQ_odering = {}
for q0 in range(2):
    for q1 in range(2):
        for q2 in range(2):
            myin = str(q2)+ str(q1) + str(q0)
            myout = str(q0) + str(q1) + str(q2)
            age = results.get(myin)
            if age:
                NQ_odering[myout] = results[myin]
            else:
                results[myin] = 0
                NQ_odering[myout] = results[myin]
plot_histogram(NQ_odering)
```

Out[ ]:



We can see that the simulation results agree very well with the theory predictions that there would be a high probability to observe 0 or 1!