Exercises for Introduction to Quantum Computing

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1. Iterative Phase Estimation (IPE)

a) We are to compute the probability of measuring ϕ_m correctly, that is the probability that the outcome of the measurement is $x_m = \phi_m$ after the first iteration $k = 1$. Right before the measurement, the state of the system is,

$$\frac{1}{2}\left[\left(1+e^{i2\pi\phi}\right)|0\rangle + \left(1-e^{i2\pi\phi}\right)|1\rangle\right]|u\rangle$$

this gives the probability

$$P_0 = \cos^2(\pi\frac{\delta}{2})$$

to measure phi_m.

b) Assuming that was measured correctly after the first iteration, then the probability of measuring for $k = 2$ is given by,

$$\cos^2(\pi\delta/4)$$

c) From (a) and (b) we saw that the probability to measure is ,

$$\cos^2(\pi\delta/2)$$

and if was measured correctly, the probability to measure for $k = 2$ is give by

$$\cos^2(\pi\delta/4)$$

Thus, the conditional probability for each bit to be measured correctly individually is,

$$P_i = \cos^2\left(\pi 2^{k-m-1}\delta\right)$$

The total probability is the product of this,

$$P(\delta) = \prod_{i=1}^{m} P_i$$

The hint in the question reads as,

$$\prod_{k=1}^{m}\cos^2\left(\frac{\alpha}{2^k}\right) = \frac{\sin^2(\alpha)}{2^{2m}\sin^2\left(2^{-m}\alpha\right)}$$

Thus, we have,

$$P(\delta) = \frac{\sin^2(\pi\delta)}{2^{2m}\sin^2\left(\pi 2^{-m}\delta\right)}$$

d) Using the formula we derived from (c), the probability of measuring

$$\hat{\phi}' = \hat{\phi} + 2^{-m} = 0.\phi_1\ldots\phi_m + 2^{-m}$$

is simply,

$$P(1-\delta)$$

e) Simillarly, the probability of measuring a phase which differs from ϕ by less than 2^{-m} implies thgat we accept both $\hat{\phi} + 2^{-m}$ amd $\hat{\phi}$ to be the correct phase. Thus, the success probability is simply given by,

$$P(\delta) + P(1-\delta)$$

f) We are to compute,

$$min\left[\lim_{m\to\infty} P_0(\delta = 1/2)\right]$$

we know that like P, P_0 is monotonically decreasing in m and in the limit of $m \to \infty$, it's minimum is reached for $\delta = \frac{1}{2}$. This limit is takes the value,

$$\frac{8}{\pi^2}$$

This can be seen by computing $P(0.5)$ and then using the formula from (e) and computing the limit.