Exercises for Introduction to Quantum Computing

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In [ ]:
```
import numpy as np
import qiskit as qi
import qiskit.quantum_info as quantu_info
from qiskit import IBMQ, Aer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, transpile, execute
import math
from qiskit.visualization import plot_histogram
from qiskit.circuit.library import QFT
```

1. Phase estimation algorithm

The unitary that we have is given by,

$$U = Z \otimes S$$

thus, their eigenvalues and eigenvectors must simply be the respective products, thus we have the eigenvalues given by

$$\lambda_1 = -i, \lambda_2 = i, \lambda_3 = -1, \lambda_4 = 1$$

and their respective eigenvectors

$$v_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

b) What is the minimal number of qubits t in the first register that is required to exactly measure the eigenvalues (2 points)?

To represent a phase ϕ exactly by means of a $t-bit$ binary number as follows,

$$\varphi = \frac{\varphi_1}{2} + \frac{\varphi_2}{4} + \ldots + \frac{\varphi_t}{2^t}$$

here, $\varphi_i \in \{0, 1\}$ In our case, we computed the eigenvalues exacly and found them to lie in the set

$$\{1, -1, i, -i\}$$

these can be represented as

$$e^{2\pi i\phi}$$

where, the phase

$$\phi \in \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$$

These can be represented using a $t = 2$ bit binary number. Thus, $t = 2$ is the minimal number of qubits required in the first register to exactly represent the eigenvalues.

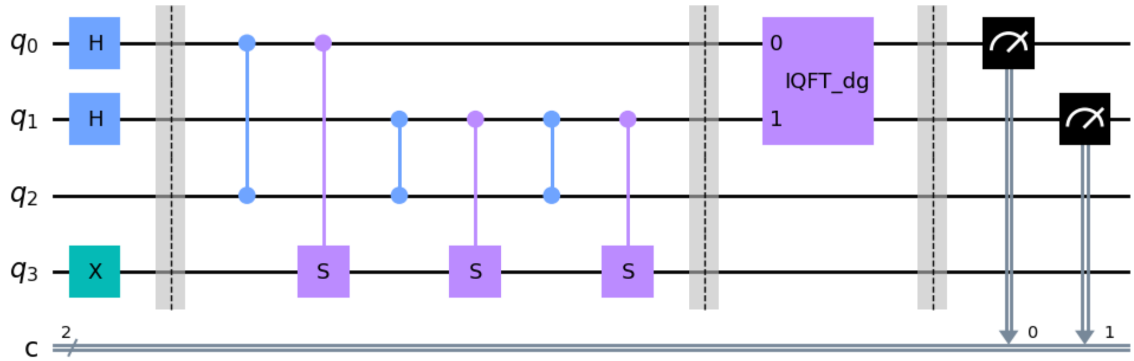
Now, we can define the circuit for the phase estimation algorithm. We begin by defining the unitary in Qiskit

In [ ]:
```
def unitary(control_qubit,t):
    test.cz(control_qubit,t)
    test.cs(control_qubit,t+1)
```

Now we define the circuit

In [ ]:
```
t = 2
#initializing the circuit and the state
test = QuantumCircuit(t+2,t)
test.x(t+1)

for i in range(t):
    test.h(i)
test.barrier()

repeats = 1
for control_qubit in range(t):
    for i in range(repeats):
        unitary(control_qubit,t)
    repeats *= 2
test.barrier()

test = test.compose(QFT(t, inverse=True), np.linspace(0,t-1,t,dtype=int))

test.barrier()
for n in range(t):
    test.measure(n,n)

test.draw('mpl')
```

Out[ ]:

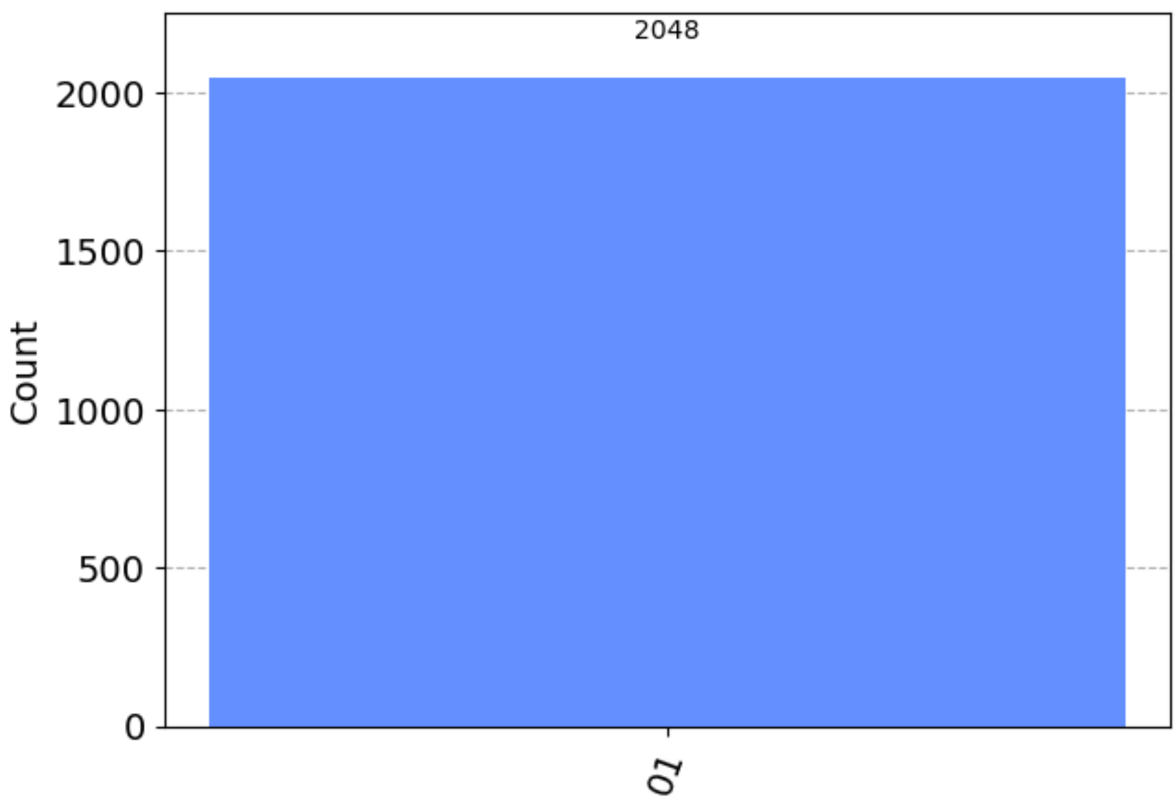


We can simulate the outcome of the circuit in Qiskit

In [ ]:

```
aer_sim = Aer.get_backend('aer_simulator')
shots = 2048
t_qpe = transpile(test, aer_sim)
results = aer_sim.run(t_qpe, shots=shots).result()
answer = results.get_counts()

plot_histogram(answer)
```

Out[ ]:



We measure the phase $\phi = \frac{1}{2^2}$ with certainty.

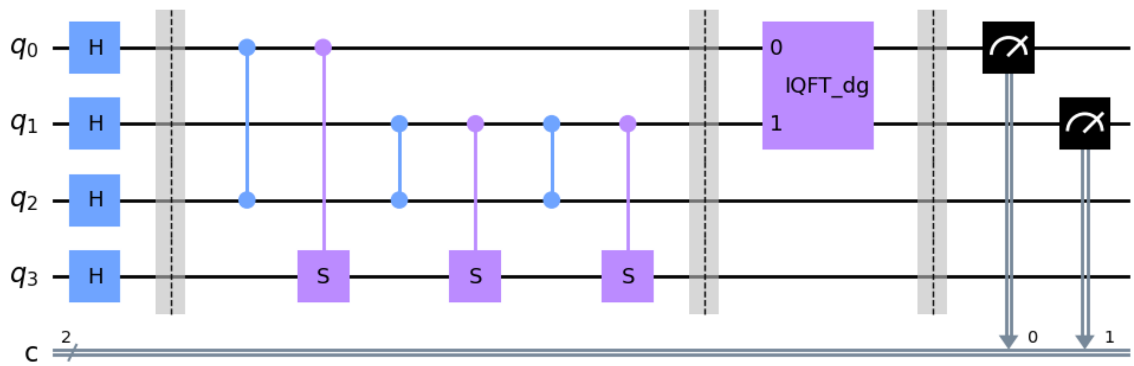
In [ ]:

```
def unitary(control_qubit,t):
    qpe_2.cz(control_qubit,t)
    qpe_2.cs(control_qubit,t+1)
```

In [ ]:

```
t = 2
#initializing the circuit and the state
qpe_2 = QuantumCircuit(t+2,t)
qpe_2.h(t)
qpe_2.h(t+1)

for i in range(t):
    qpe_2.h(i)
qpe_2.barrier()

repeats = 1
for control_qubit in range(t):
    for i in range(repeats):
        unitary(control_qubit,t)
    repeats *= 2
qpe_2.barrier()

qpe_2 = qpe_2.compose(QFT(t, inverse=True), np.linspace(0,t-1,t,dtype=int))

qpe_2.barrier()
for n in range(t):
    qpe_2.measure(n,n)

qpe_2.draw('mpl')
```

Out[ ]:



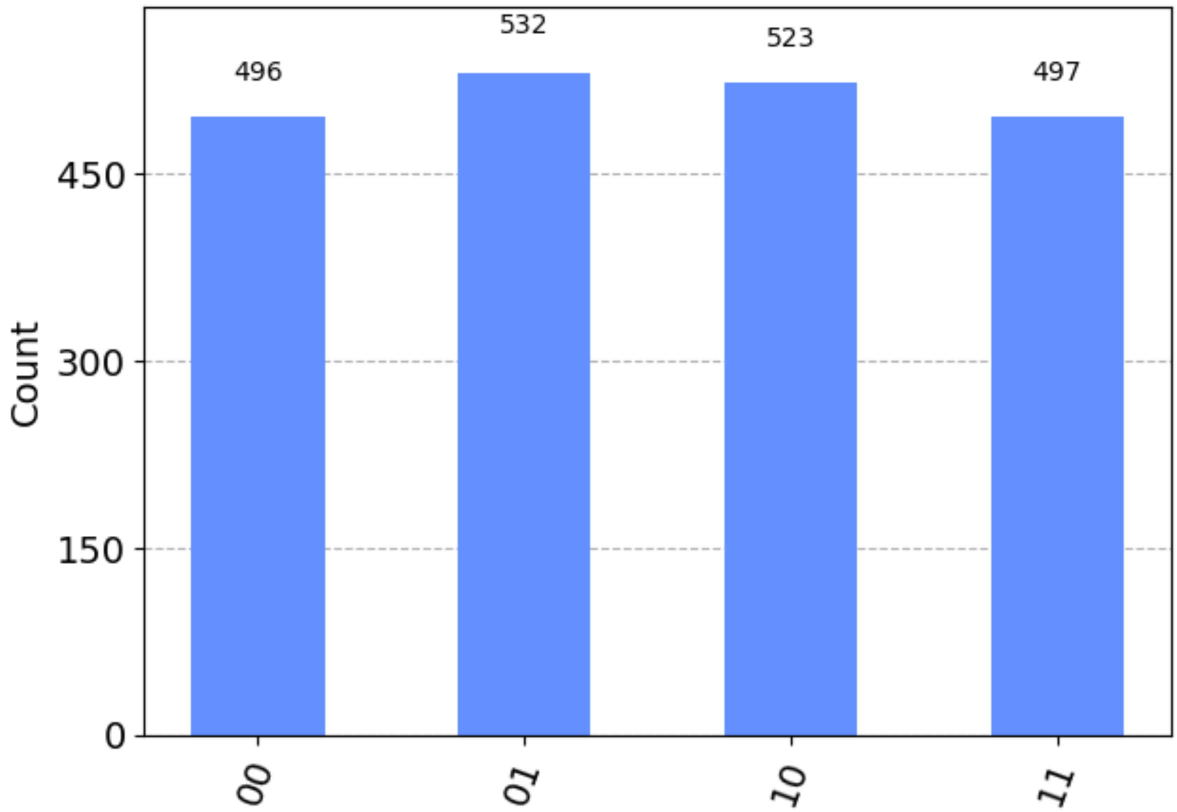
In [ ]:

```
aer_sim = Aer.get_backend('aer_simulator')
shots = 2048
t_qpe = transpile(qpe_2, aer_sim)
result = aer_sim.run(t_qpe, shots=shots).result()
results = result.get_counts()
display(results)
```

```
{'00': 496, '11': 497, '01': 523, '10': 532}
```

In [ ]:
```python
NQ_odering = {}
for q0 in range(2):
    for q1 in range(2):
            myin = str(q1) + str(q0)
            myout = str(q0) + str(q1)
            age = results.get(myin)
            if age:
                NQ_odering[myout] = results[myin]
            else:
                results[myin] = 0
                NQ_odering[myout] = results[myin]
plot_histogram(NQ_odering)
```

Out[ ]:



From this, we can say that the probability to measure the phase $\phi = \frac{1}{4}$ is,

$$p_\phi = \frac{523}{2048} \approx \frac{1}{4}$$

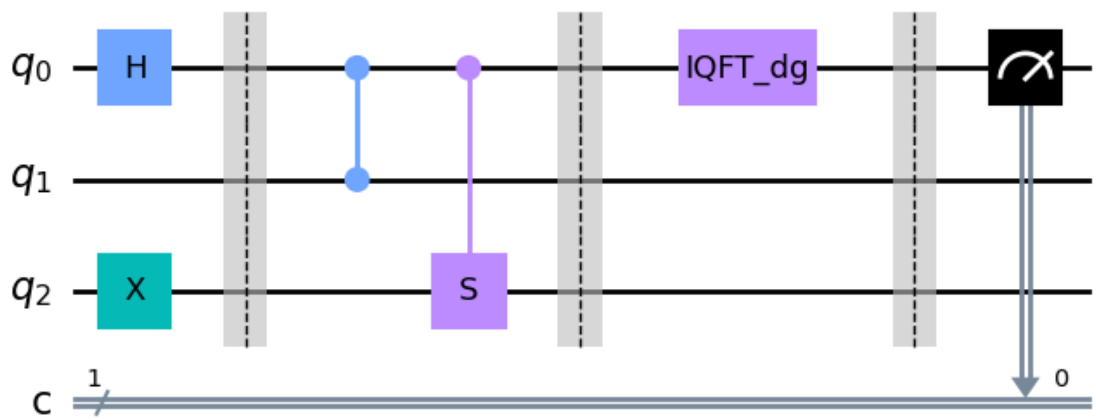
Now, we proceed with implementing the circuit for which $t = 1$

In [ ]:
```python
def unitary(control_qubit,t):
    qpe_3.cz(control_qubit,t)
    qpe_3.cs(control_qubit,t+1)
```

In [ ]:

```
t = 1
#initializing the circuit and the state
qpe_3 = QuantumCircuit(t+2,t)
qpe_3.x(t+1)

for i in range(t):
    qpe_3.h(i)
qpe_3.barrier()

repeats = 1
for control_qubit in range(t):
    for i in range(repeats):
        unitary(control_qubit,t)
    repeats *= 2
qpe_3.barrier()

qpe_3 = qpe_3.compose(QFT(t, inverse=True), np.linspace(0,t-1,t,dtype=int))

qpe_3.barrier()
for n in range(t):
    qpe_3.measure(n,n)

qpe_3.draw('mpl')
```

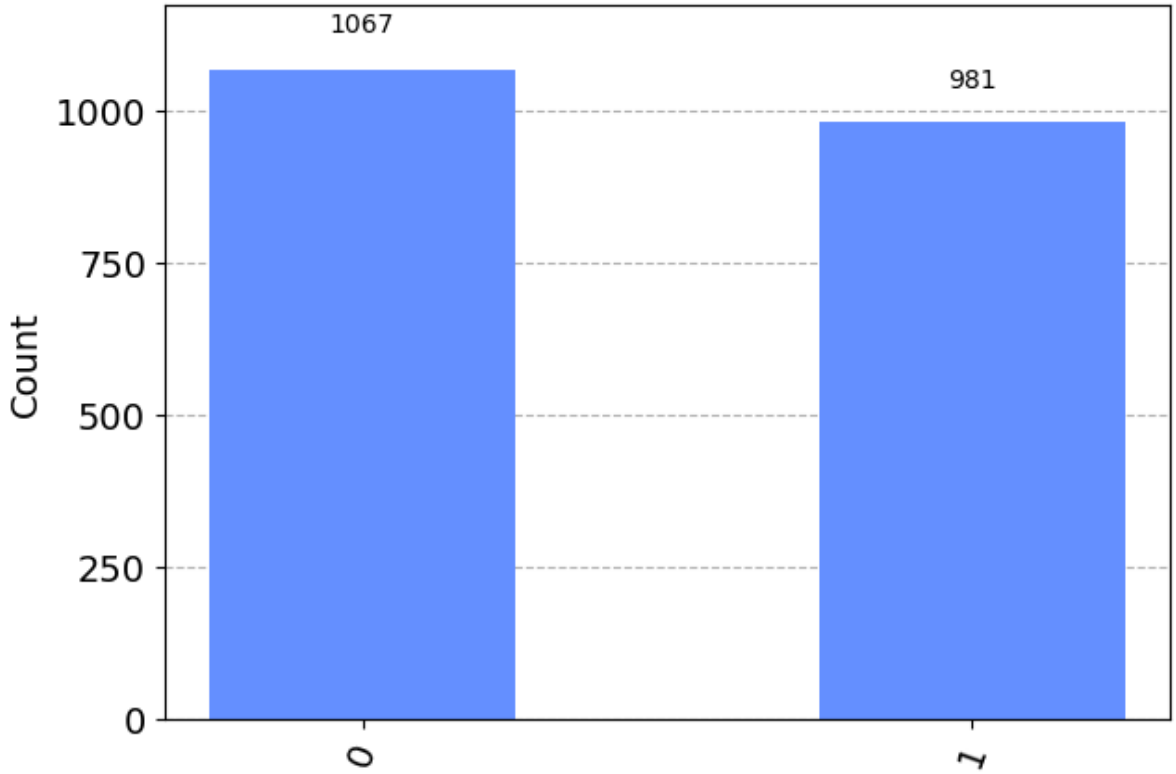
Out[ ]:



In [ ]:

```
aer_sim = Aer.get_backend('aer_simulator')
shots = 2048
t_qpe = transpile(qpe_3, aer_sim)
result = aer_sim.run(t_qpe, shots=shots).result()
results = result.get_counts()
plot_histogram(results)
```

Out[ ]:



From this, we can conclude that the probability to measure 0 is,

$$p_0 = \frac{1067}{2048} \approx \frac{1}{2}$$

and the probability to measure 1 is,

$$p_1 = \frac{981}{2048} \approx \frac{1}{2}$$