Exercises for Introduction to Quantum Computing

Name: Pugazharasu Anancia Devaneyan (s6puanan)
Matriculation number: 3300280

In [ ]:
```
#Importing the required libraries
import matplotlib.pyplot as plt
import numpy as np
from qiskit import *
```

1 Grover's algorithm

a)

b) We know from the way the Grover's agorithm is designed, the probability of finding the item after t iterations is given by,

$$P(\text{ success }) = |\langle x^* \mid \psi \rangle|^2 = \sin^2\left(\frac{2t+1}{\sqrt{N}}\right)$$

We are to compute the largest value of $N = 2^n$ such that $P(\text{ success }) = 1$ for a single iteration i.e. $t = 1$, thus we are to solve

$$P(\text{ success }) = |\langle x^* \mid \psi \rangle|^2 = \sin^2\left(\frac{3}{\sqrt{N}}\right) = 1$$

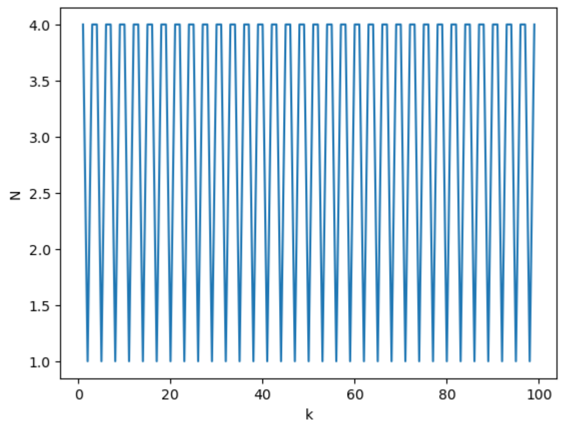
for N. Solving this, we find,

$$N = 4$$

This is exemplified by the following plot for N vs k where k signifies the multivalued nature of the sine function.

In [ ]:
```
n = np.arange(1,100, dtype = int)
plt.xlabel("k")
plt.ylabel("N")
plt.plot(n,[1/((np.sin((np.pi * (i-0.5) )/3))**2) for i in n])
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x18364b177c0>]
```



2 Error mitigation

a) The Hamiltonian,

$$H = X_0 \otimes Y_1$$

when exponentiated, takes the form,

$$U = e^{-iH\delta t} = (\mathbb{I}_n \otimes \mathbb{I}_n) \cdot \cos(\delta t) - i\sin(\delta t) \cdot (X_0 \otimes Y_1)$$

Computing the expectation value

$$\langle \psi | Z_0 \otimes Z_1 | \psi \rangle$$

for $\delta t = 1$ and

$$|\psi\rangle = e^{-iH\delta t}|00\rangle$$

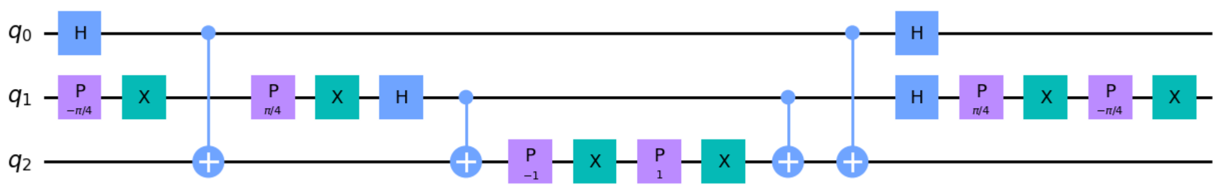
, we have,

$$\langle \psi | Z_0 \otimes Z_1 | \psi \rangle = 1$$

b) Running the code in the given Juypter notebook, we find that probabilistic error cancellation gives the best result (see below for details).

In [ ]:

```python
# e^i theta Z
def Rz(qc, theta,i):
    qc.p(theta,i)
    qc.x(i)
    qc.p(-theta,i)
    qc.x(i)
    return qc


def expiHdt(circuit,dt, r):
    qc=QuantumCircuit(3)
    qc.h(0)
    qc.cx(0,2)
    qc = Rz(qc, -np.pi/4,1)
    qc.h(1)
    qc.cx(1,2)
    ##
    qc = Rz(qc, -dt,2)
    # ##
    qc.cx(1,2)
    qc.h(1)
    qc = Rz(qc, np.pi/4,1)
    qc.cx(0,2)
    qc.h(0)
    circuit.append(   qc, [0,1,2])
    return circuit

from qiskit_ibm_runtime import QiskitRuntimeService, Sampler, Estimator, Session, Options
service = QiskitRuntimeService(channel="ibm_quantum")
backend = "ibmq_qasm_simulator"
from qiskit.providers.fake_provider import FakeManila
from qiskit_aer.noise import NoiseModel

# Make a noise model
fake_backend = FakeManila()
noise_model = NoiseModel.from_backend(fake_backend)

from qiskit.quantum_info import SparsePauliOp
ZZ =  SparsePauliOp("IZZ")

qc=QuantumCircuit(3)
qc=expiHdt(qc,1,0)
qc = qc.remove_final_measurements(inplace=False)
display(qc.decompose().draw('mpl'))

noise_options = Options()
noise_options.simulator = {
    "noise_model": noise_model,
    "basis_gates": fake_backend.configuration().basis_gates,
    "coupling_map": fake_backend.configuration().coupling_map,
    "seed_simulator": 42
}

noise_options.execution.shots = 1000

with Session(service=service, backend=backend):
    noise_options.resilience_level = 0
    estimator = Estimator(options=noise_options)
    job = estimator.run( circuits=[qc],
        # parameter_values=individual_phases,
        observables=[ZZ]    )
    print("level 0: ",job.result() )
    noise_options.resilience_level = 1
    estimator = Estimator(options=noise_options)
    job = estimator.run( circuits=[qc],
        # parameter_values=individual_phases,
        observables=[ZZ]    )
    print("level 1: ",job.result() )

    noise_options.resilience_level = 2
    estimator = Estimator(options=noise_options)
    job = estimator.run( circuits=[qc],
        # parameter_values=individual_phases,
        observables=[ZZ]    )
    print("level 2: ",job.result() )

    noise_options.resilience_level = 3
    estimator = Estimator(options=noise_options)
    job = estimator.run( circuits=[qc],
        # parameter_values=individual_phases,
        observables=[ZZ]    )
    print("level 3: ",job.result() )
```




```
level 0:  EstimatorResult(values=array([0.822]), metadata=[{'variance': 0.32431600000000005, 'shots': 1000}])
level 1:  EstimatorResult(values=array([0.8407225]), metadata=[{'variance': 1.5997098182629468, 'shots': 1008, 'readout_mitigation_num_twirled_circuits': 16, 'readout_mitiga
tion_shots_calibration': 8192}])
level 2:  EstimatorResult(values=array([0.60933333]), metadata=[{'zne': {'noise_amplification': {'noise_amplifier': "<TwoQubitAmplifier:{'noise_factor_relative_tolerance':
0.01, 'random_seed': None, 'sub_folding_option': 'from_first'}>", 'noise_factors': [1, 3, 5], 'values': [0.52, 0.788, 0.52], 'variance': [0.7296, 0.37905599999999995, 0.729
6], 'shots': [1000, 1000, 1000]}, 'extrapolation': {'extrapolator': 'LinearExtrapolator'}}}])
level 3:  EstimatorResult(values=array([0.99423044]), metadata=[{'standard_error': 0.007806756644913258, 'confidence_interval': [0.9083033453137671, 1.0801575320869066], 'co
nfidence_level': 0.95, 'shots': 157184, 'samples': 1228, 'sampling_overhead': 1.2289498202368314, 'total_mitigated_layers': 4}])
```