Exercises for Introduction to Quantum Computing

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1. Installation of Qiskit

In [ ]:
```
%pip install qiskit
```

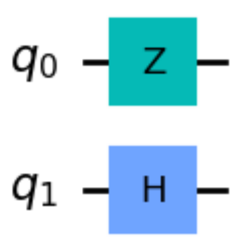
In [ ]:
```
import numpy as np
import qiskit as qi
from qiskit import IBMQ, Aer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, transpile

from qiskit.visualization import plot_histogram
```

2. Quantum Gates

In [ ]:
```
circ_1 = QuantumCircuit(2)
circ_1.z(0)
circ_1.h(1)
circ_1.draw('mpl')
```

Out[ ]:



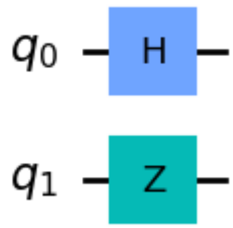
In the standard computational basis, this two-qubit system can be written as,

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \\ \frac{\gamma}{\sqrt{2}} + \frac{\delta}{\sqrt{2}} \\ \frac{\gamma}{\sqrt{2}} - \frac{\delta}{\sqrt{2}} \end{pmatrix}$$

Now, for the next two-qubit system,

In [ ]:
```
circ_2 = QuantumCircuit(2)
circ_2.z(1)
circ_2.h(0)
circ_2.draw('mpl')
```

Out[ ]:



In the standard computational basis, this two-qubit system can be written as,

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}} \\ \frac{\alpha}{\sqrt{2}} - \frac{\beta}{\sqrt{2}} \\ \gamma \\ -\delta \end{pmatrix}$$

3. Quantum Circuits

A well known identity involving CNOT gates is that,

$$H \otimes H \; CNOT_{1,2} \; H \otimes H = CNOT_{2,1}$$

Applying this to the circuit in the RHS, we have,

$$(\mathbb{I} \otimes H) CNOT_{2,1} (\mathbb{I} \otimes H) = (\mathbb{I} \otimes H)(H \otimes H \; CNOT_{1,2} \; H \otimes H)(\mathbb{I} \otimes H)$$

Since we know that the Hadarmard gate is self-inverse i.e. squares to identity,

$$H^2 = \mathbb{I}$$

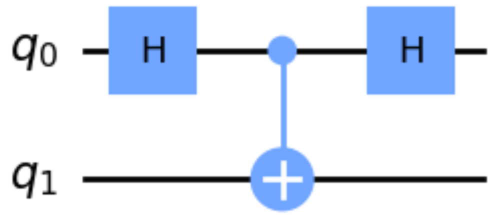
thus we have,

$$(\mathbb{I} \otimes H) CNOT_{2,1} (\mathbb{I} \otimes H) = (H \otimes \mathbb{I}) CNOT_{1,2} (H \otimes \mathbb{I})$$

We can also prove this equivalence using qiskit,

In [ ]:
```
circ_3 = QuantumCircuit(2)
circ_3.h(0)
circ_3.cx(0,1)
circ_3.h(0)
circ_3.draw('mpl')
```

Out[ ]:

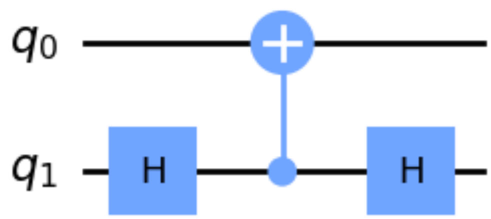


Now to evaluate the RHS,

In [ ]:

```
circ_4 = QuantumCircuit(2)
circ_4.h(1)
circ_4.cx(1,0)
circ_4.h(1)
circ_4.draw('mpl')
```

Out[ ]:



Proving their equivalence upto the equivalence of statevectors and unitary matrices using qiskit,

In [ ]:

```
#checking if both the circuits produce the same state vectors
from qiskit.quantum_info import Statevector
Statevector.from_instruction(circ_3).equiv(Statevector.from_instruction(circ_4))
```

Out[ ]: True

In [ ]:

```
#checking if both the circuits produce the unitary matrix
backend_sim = Aer.get_backend('unitary_simulator')
job_sim = qi.execute([circ_3, circ_4], backend_sim)
result_sim = job_sim.result()
unitary1 = result_sim.get_unitary(circ_3)
unitary2 = result_sim.get_unitary(circ_4)

np.allclose(unitary1, unitary2)
```

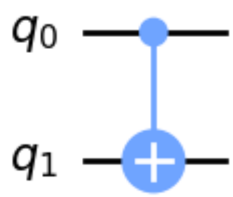
Out[ ]: True

4. No-Cloning Theorem

We shall now create a quantum circuit that copies the states $|0\rangle$ and $|1\rangle$ into a target qubit, where the latter is initialized in the state $|0\rangle$

In [ ]:
```
circ_5 = QuantumCircuit(2)
circ_5.cx(0,1)
circ_5.draw('mpl')
```

Out[ ]:



We can check using Qiskit if this circuit acts as a copying circuit,

In [ ]:
```
state = qi.quantum_info.Statevector.from_label('01')
state.draw(output='latex')
```

Out[ ]:

$$|01\rangle$$

In [ ]:
```
state = state.evolve(circ_5)
state.draw(output='latex')
```

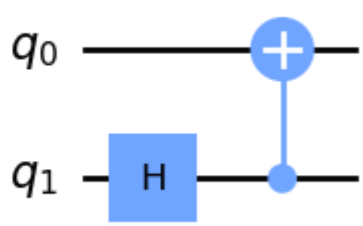
Out[ ]:

$$|11\rangle$$

We shall now create a quantum circuit that copies the states $|+\rangle$ and $|-\rangle$ into a target qubit, where the latter is initialized in the state $|0\rangle$

In [ ]:
```
circ_6 = QuantumCircuit(2)
circ_6.h(1)
circ_6.cx(1,0)
circ_6.draw('mpl')
```

Out[ ]:



In [ ]:
```
state_2 = qi.quantum_info.Statevector.from_label('0-')
state_2.draw(output='latex')
```

Out[ ]:

$$\frac{\sqrt{2}}{2}|00\rangle - \frac{\sqrt{2}}{2}|01\rangle$$

In [ ]:
```
state_2 = state_2.evolve(circ_6)
state_2.draw(output='latex')
```

Out[ ]:

$$\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$