CPI index forecasting

Author: Joana Simões, joanasimoes@student.dei.uc.pt

In [1]:
```
import pandas as pd
import matplotlib.pyplot as plt
import os
import numpy as np
import scipy.signal as scs
import seaborn as sns
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import statsmodels.tsa.stattools as st
from sklearn.model_selection import train_test_split

from statsmodels.tsa.stattools import acf, pacf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima.model import ARIMA
from statsmodels.tsa.holtwinters import ExponentialSmoothing
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.statespace.varmax import VARMAX

from sklearn.preprocessing import MinMaxScaler
from sklearn import metrics

from utils import *

# recurrent model
from keras.models import Sequential
from keras import layers
from keras.callbacks import EarlyStopping
from keras.optimizers import Adam
```

In [2]:
```
# disable warnings
import warnings
warnings.filterwarnings('ignore')
```

Introduction

This notebook presents an analysis of the CPI Index from Norway from 1990 until the most recent update (August, 2022).

NOTE: Throughout the analysis, a significance level of 95% was chosen for all the tests developed.

Read data

In [3]:
```
df = pd.read_csv(os.path.join('CPITimeSeries', 'time_series_data.csv'), i
```

In [4]:
```
df.index = pd.to_datetime(df.index)
```

In [5]:
```
# select data from 1990 until now
df = df.loc[df.index >= '1990-01-01']
```

Select the data from Norway

In [6]:
```
country = 'Norway'
```

Add European Union CPI Index

In [7]:
```
european_union_countries_in_1990 = [
    "Belgium",
    "Denmark",
    "France",
    "Germany",
    "Greece",
    "Ireland",
    "Italy",
    "Luxembourg",
    "Netherlands, The",
    "Portugal",
    "Spain",
    "United Kingdom"
]
```

In [8]:
```
df['European Union'] = df[european_union_countries_in_1990].mean(axis=1)
```

In a first analysis of the time series (TS), it's clearly seen a increasing trend of the CPI values over time. On a closer look, the TS appears to have a repetition (seasonality) every 6 months. With the increasing trend and seasonality every 6-months, it can be concluded that the original TS is not stationary. Therefore, it should be transformed to become stationary in order to apply forecasting models.

From the TS, it can also be concluded that there is and rapid increasing in the last 2-3 years, which will impact the final results as this exponential increasing is just the final part of the TS

Period

In [9]:
```
period = 12
```

Divide the time series into train and test

In [10]:
```
ts_stationary = df[country].diff().diff(period)
```

In [11]:
```
train_ratio = 0.8
split_index = int(len(df) * train_ratio)

# Split the time series data
train_data = df[:split_index]
test_data = df[split_index:]
```

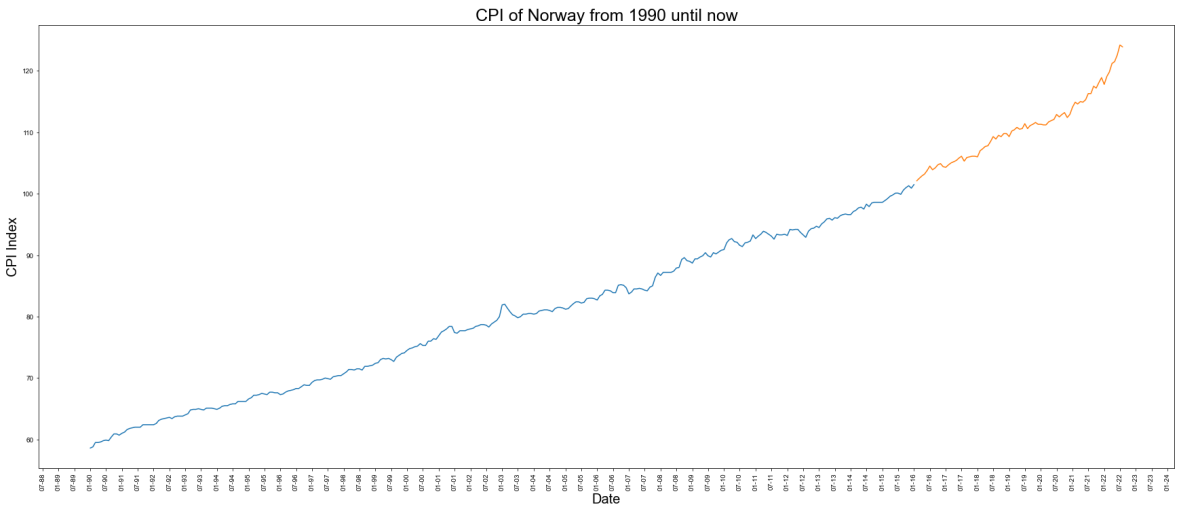
```
months_dates = months = np.arange(len(df))
months = months_dates[:split_index]
months_test = months_dates[split_index:]
```

In [12]:
```
stationary_ts_train = ts_stationary[:split_index].dropna()
stationary_ts_test = ts_stationary[split_index:]
```

In [13]:
```
ts = train_data[country]
ts_test = test_data[country]
```

Time Series Overview

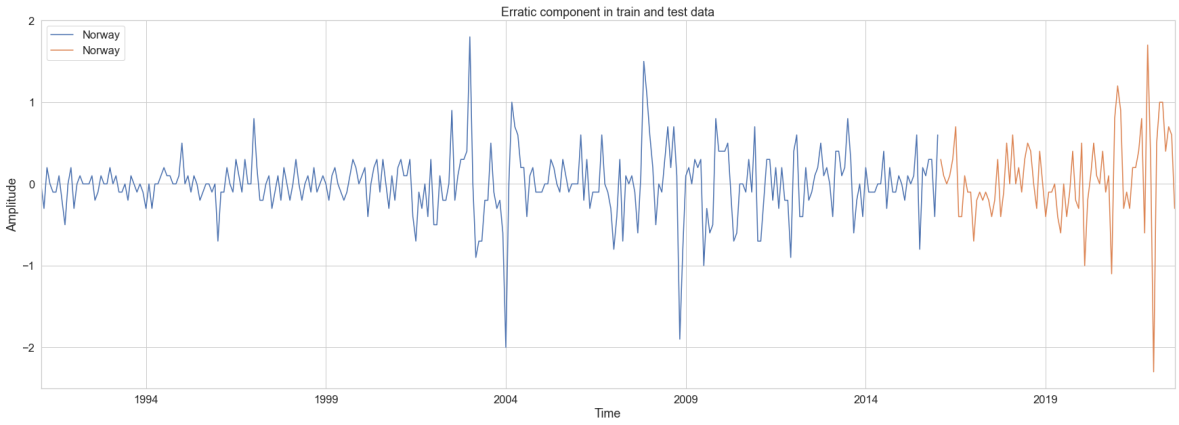
In [14]:
```
fig, ax = plt.subplots(figsize=(30, 12))
sns.set(font_scale=1.5, style="whitegrid")
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')

# Ensure a major tick for each week using (interval=1)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
date_form = DateFormatter("%m-%y")
ax.xaxis.set_major_formatter(date_form)

# Ensure a major tick for each week using (interval=1)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
plt.xlabel('Date', fontdict=dict(size=20))
plt.ylabel('CPI Index', fontdict=dict(size=20))
plt.title(f'CPI of {country} from 1990 until now', fontdict=dict(size=25))
plt.savefig(os.path.join('images', 'original-ts.png'))
plt.show()
```



Linear models for stationary Time series

In [15]:
```
fig, ax = plt.subplots(figsize=(30, 10))
stationary_ts_train.plot(ax=ax, legend=True)
stationary_ts_test.plot(ax=ax, legend=True)
plt.xlabel('Time')
plt.ylabel('Amplitude')
```

```
plt.title('Erratic component in train and test data')
plt.savefig(os.path.join('images', 'train_test_erratic.png'))
```



Trend and seasonal adjusted by model-fitting and filtering

In [16]:
```
adf_test(stationary_ts_train)
```

```
ADF Statistic: -7.240584
p-value: 0.000000
Critical Values:
        1%: -3.454
        5%: -2.872
        10%: -2.572
```

Based on the ADF test, given the p-value of 0.0, the transformed TS is stationary.

Trend and seasonal adjusted test

In [17]:
```
adf_test(stationary_ts_test)
```

```
ADF Statistic: -2.990176
p-value: 0.035822
Critical Values:
        1%: -3.534
        5%: -2.906
        10%: -2.591
```

By the Dickey-Fuller test, after differencing the resulted TS is stationary. However, the ADF statistic most closer to the critical values than when the TS is decomposed by model-fitting and filtering.

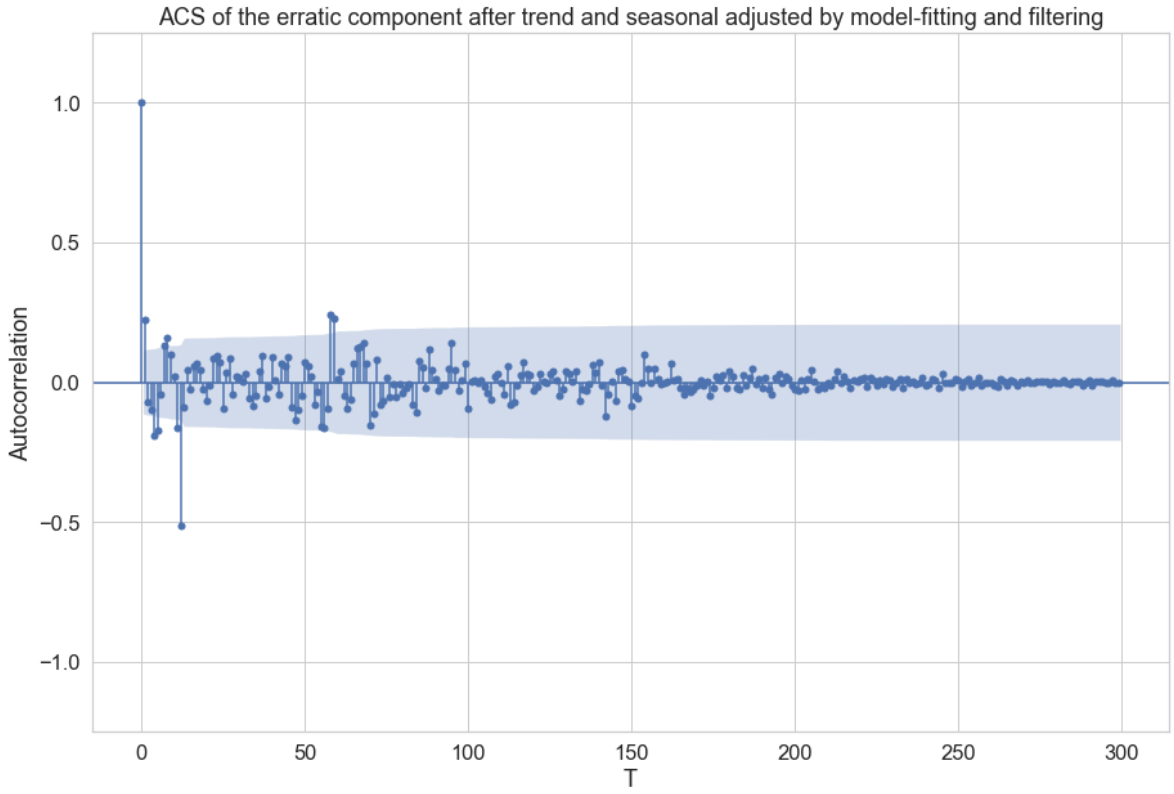
ACS to see stationary

In [18]:
```
def auto_covarience_aux(ts, T=0):
    N = len(ts)
    mean = ts.mean()
    cov_coef = 0
    for n in range(N-T-1): ## Não estaremos a retirar um a mais
        cov_coef += (ts[n] - mean) * (ts[n+T]-mean)
    return cov_coef / N


def auto_covarience(ts, T=0):
```

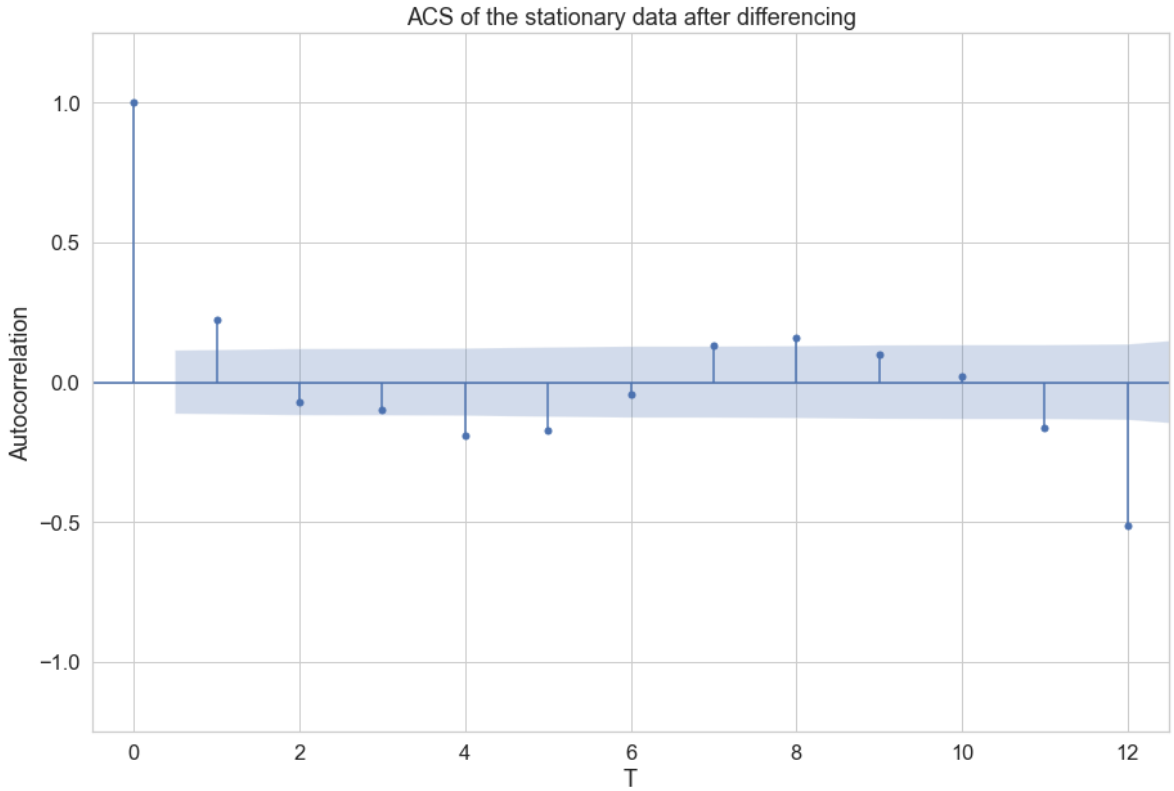
```
    return auto_covarience_aux(ts, T) / auto_covarience_aux(ts)


def correlogram(ts, max_T, twoside=False):
    N = len(ts)
    if twoside:
        corrl = np.zeros(2 * max_T + 1)
        index = np.arange(max_T + 1)
        index = np.concatenate((-np.flip(index[1:]), index), axis=0)
    else:
        corrl = np.zeros(max_T)
        index = np.arange(max_T)

    for i in range(max_T):
        if twoside:
            corrl[max_T + i] = auto_covarience(ts, i)
            corrl[max_T - i] = corrl[max_T + 1]
        else:
            corrl[i] = auto_covarience(ts, i)

    d = {'ACS':corrl, 'upper_CB':np.ones(max_T)*(1.96/np.sqrt(N)),'lower_
    return pd.DataFrame(data=d, index=index)
```

In [19]:
```
def plot_correlogram(ts, title, period=12):
    corrl = correlogram(ts, len(ts))
    fig, ax = plt.subplots(1, 1, figsize=(15, 10))
    ax.stem(corrl.index, corrl.ACS, label='ACS')
    ax.plot(corrl.index, corrl.upper_CB, linestyle='--', color='r', linew
    ax.plot(corrl.index, corrl.lower_CB, linestyle='--', color='r', linew
    plt.title(title, fontdict=dict(size=25))
    plt.legend()
    ax.set_xlim([-0.5, period+0.5])
    plt.ylabel('Autocorrelation')
    plt.xlabel('T')
```

In [20]:
```
# plot_correlogram(trend_seasonal_adjusted, 'Erratic Component after tren
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(stationary_ts_train, alpha=0.05, ax=ax, lags=len(stationary_ts_t
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
plt.title('ACS of the erratic component after trend and seasonal adjusted
plt.savefig(os.path.join('images', 'acs_additive_model.png'))
plt.show()
```



In [21]:
```
# plot_correlogram(trend_seasonal_adjusted, 'Erratic Component after tren
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(stationary_ts_train, alpha=0.05, ax=ax, lags=len(stationary_ts_t
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
ax.set_xlim([-0.5, period + 0.5])
plt.title('ACS of the stationary data after differencing')
plt.savefig(os.path.join('images', 'acs_stationary_data_diff.png'))
plt.show()
```

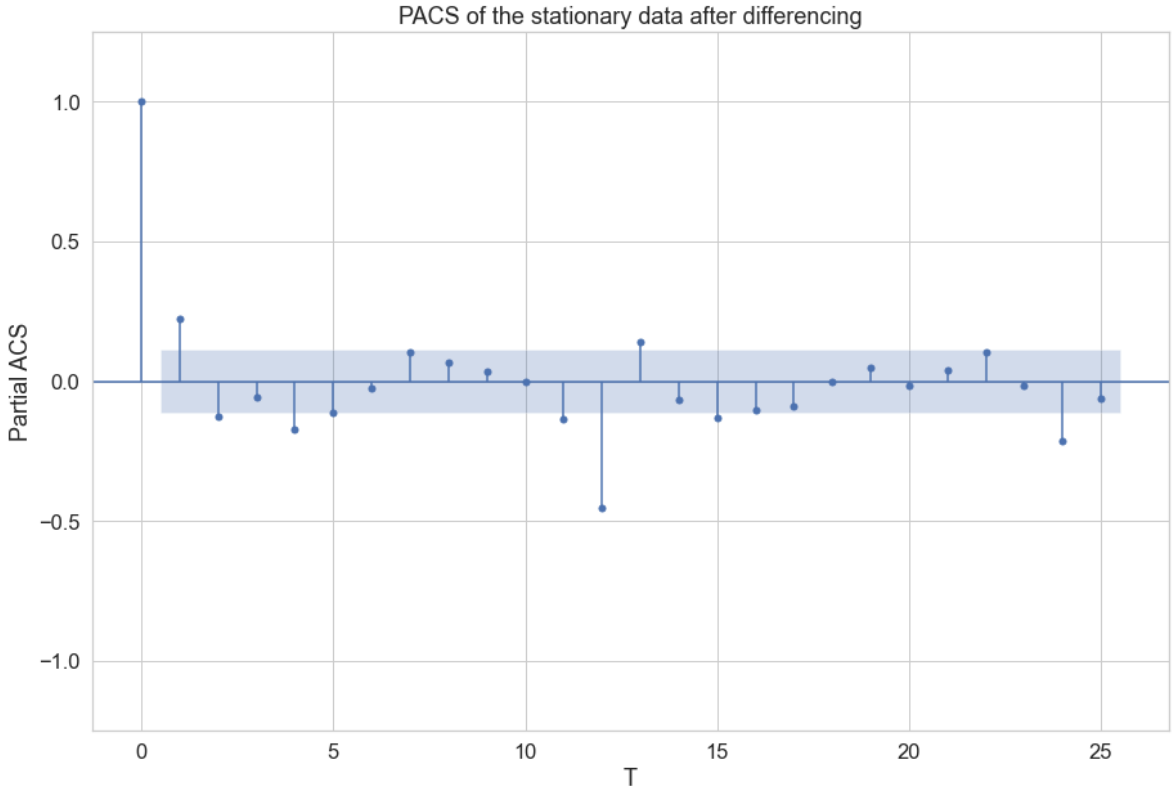


It can be seen that there is a rapid decreasing the the ACS, which indicates that the erratic component is stationary. Looking at the ACS for the first 12 lags (period=12), there are 2 significant value outside the unit root, therefore, a MA of order 2 can be used to describe the process.

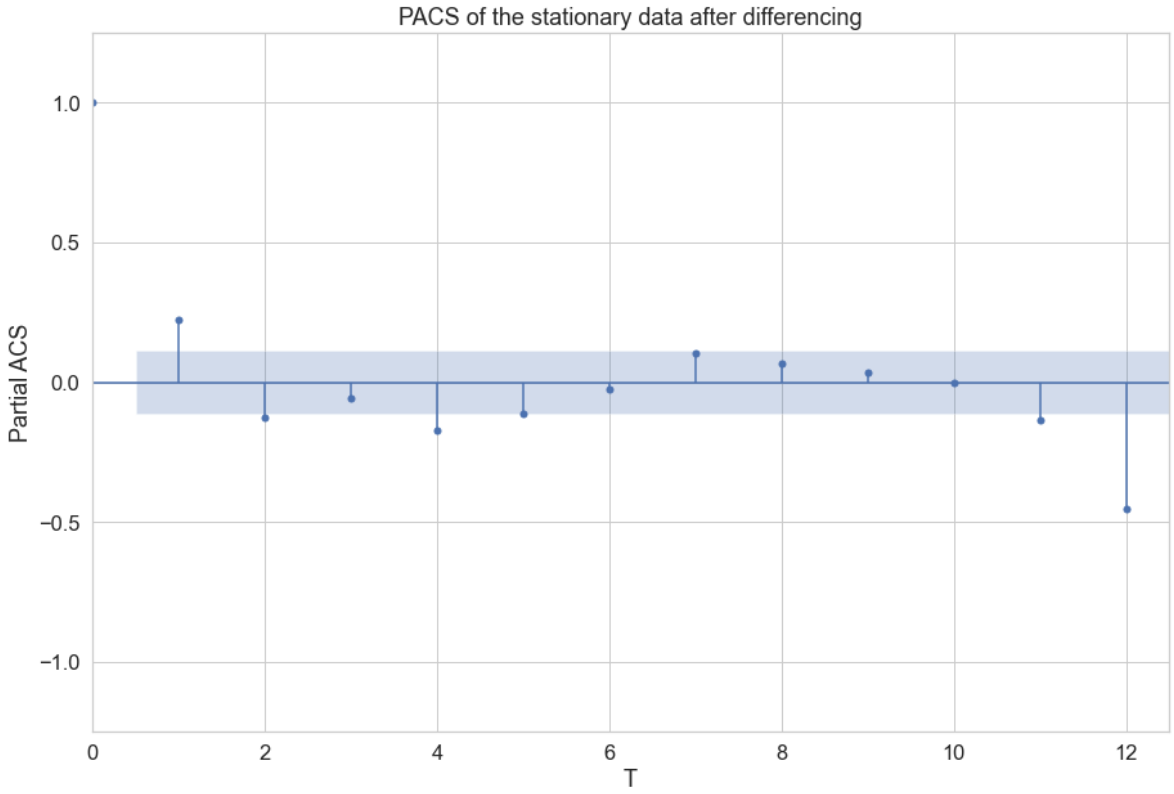
Assessing the underlying process of the stationary TS

PACS

In [22]:
```
fog, ax = plt.subplots(1, figsize=(15, 10))
plot_pacf(stationary_ts_train, alpha=0.05, ax=ax)
ax.set_ylim([-1.25, 1.25])
plt.xlabel('T')
plt.ylabel('Partial ACS')
plt.title('PACS of the stationary data after differencing')
plt.savefig(os.path.join('images', 'pacs_stationary_data_diff.png'))
plt.show()
```



In [23]:
```
fog, ax = plt.subplots(1, figsize=(15, 10))
plot_pacf(stationary_ts_train, alpha=0.05, ax=ax)
ax.set_ylim([-1.25, 1.25])
ax.set_xlim([0, 12.5])
plt.xlabel('T')
plt.ylabel('Partial ACS')
plt.title('PACS of the stationary data after differencing')
plt.savefig(os.path.join('images', 'pacs_until_lag_12.png'))
plt.show()
```

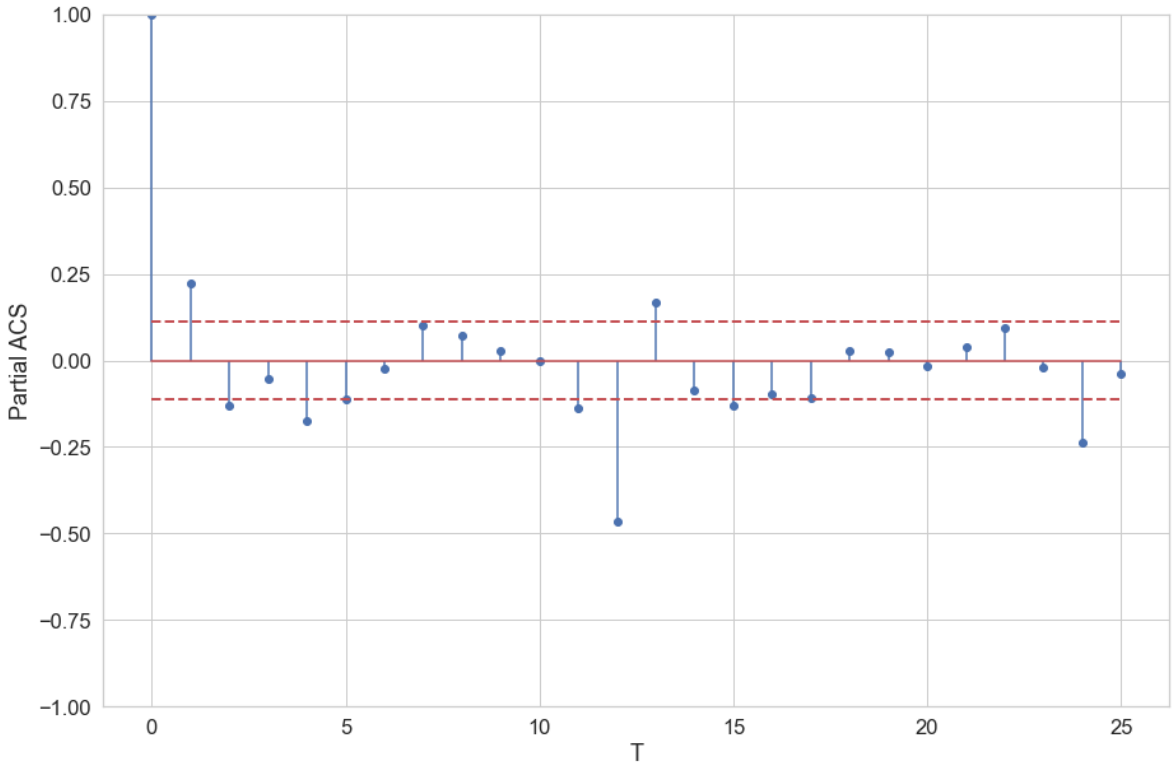


Looking at the PACS graph, it can be concluded that it should be use a AR model of order 6 from the problem, as there are clearly 6 significant autocorrelation values.

In [24]:
```
def pacs(serie,max_lag=25):
    N=serie.size
    pcorrl=np.zeros(max_lag+1)
    ix=np.array(range(max_lag+1))
    pcorrl[0]=1
    for p in range(1,max_lag+1):
        model = ARIMA(serie, order=(p,0,0))
        model_fit = model.fit()
        pcorrl[p]=model_fit.arparams[-1]

    d = {'PACS':pcorrl, 'upsig':np.ones(max_lag+1)*(1.96/np.sqrt(N)),'dns
    corrl=pd.DataFrame(data=d,index=ix)
    return corrl
```

In [25]:
```
pcorrl = pacs(stationary_ts_train)
```

In [26]:
```
fig, ax = plt.subplots(1, figsize=(15, 10))
ax.stem(pcorrl.index,pcorrl['PACS'])
ax.plot(pcorrl.index,pcorrl['upsig'],linestyle='--', color='r', linewidth
ax.plot(pcorrl.index,pcorrl['dnsig'],linestyle='--', color='r', linewidth
ax.set_ylim([-1, 1])
plt.xlabel('T')
plt.ylabel('Partial ACS')
plt.show()
```



From the PACS plot, it can also be concluded that the time series presents a autoregressive component, concluding that should be use a ARMA model.

In order to find the best order for the model, it will be use grid search, varying the q order from 0 to 2 and the p order from 0 to 11. To evaluate the different models, it will be use the Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) metrics as using the residuals error does not penalize the complexity of the model.

Looking for the best Linear model for stationary time series

In [27]:
```
max_q = 4
max_p = 4
```

In [28]:
```
residuals = pd.DataFrame(columns=['p', 'q', 'residual', 'aic', 'bic'])
for order_p in range(0,  max_p+1):
    for order_q in range(0,  max_q+1):
        model = ARIMA(stationary_ts_train, order=(order_p, 0, order_q))
        model_fit = model.fit()
        residual = np.sum(model_fit.resid**2)
        aux = {
            'p' : [order_p],
            'q' : [order_q],
            'residual' : [residual],
            'aic' : [model_fit.aic],
            'bic' : [model_fit.bic]
        }
        print('P-', order_p, '\tQ-', order_q, '\tResiduals-', residual, '
        residuals = pd.concat([residuals, pd.DataFrame(aux)], ignore_inde
```

```
P- 0    Q- 0    Residuals- 44.28916667358771    AIC- 281.45040838965366
BIC- 288.8579733389661
P- 0    Q- 1    Residuals- 41.630057404268726   AIC- 264.94632175686314
BIC- 276.05766918083174
P- 0    Q- 2    Residuals- 41.483608088866006   AIC- 265.88916558587965
BIC- 280.7042954845044
P- 0    Q- 3    Residuals- 41.33009711759758    AIC- 266.7841784137908  BI
C- 285.3030907870718
P- 0    Q- 4    Residuals- 40.58311127801609    AIC- 263.3659391659437  BI
C- 285.58863401388095
P- 1    Q- 0    Residuals- 42.09397910946183    AIC- 268.25070353814095
BIC- 279.36205096210955
P- 1    Q- 1    Residuals- 41.51419993588624    AIC- 266.1106728798744  BI
C- 280.92580277849925
P- 1    Q- 2    Residuals- 38.440724586565864   AIC- 246.37642114648472
BIC- 264.8953335197657
P- 1    Q- 3    Residuals- 38.404299482314286   AIC- 248.11941361979564
BIC- 270.3421084677328
P- 1    Q- 4    Residuals- 38.137685266454426   AIC- 248.16301842859946
BIC- 274.0894957511929
P- 2    Q- 0    Residuals- 41.39599312540633    AIC- 265.25751770197496
BIC- 280.0726476005998
P- 2    Q- 1    Residuals- 38.44262956656755    AIC- 246.50243870849044
BIC- 265.02135108177146
P- 2    Q- 2    Residuals- 38.461895156789105   AIC- 248.48141085474475
BIC- 270.70410570268194
P- 2    Q- 3    Residuals- 37.49286840917644    AIC- 243.61496480960804
BIC- 269.54144213220144
P- 2    Q- 4    Residuals- 36.14514503826069    AIC- 234.83102977558462
BIC- 264.46128957283423
P- 3    Q- 0    Residuals- 41.27219114813725    AIC- 266.3639985551152  BI
C- 284.8829109283962
P- 3    Q- 1    Residuals- 38.395935956529094   AIC- 248.08899577382368
BIC- 270.3116906217609
P- 3    Q- 2    Residuals- 37.28923532153435    AIC- 242.16076541833263
BIC- 268.08724274092606
P- 3    Q- 3    Residuals- 37.203291015170365   AIC- 242.6236133803612  BI
C- 272.2538731776108
P- 3    Q- 4    Residuals- 37.30144279780058    AIC- 244.86929123593677
BIC- 278.20333350784256
P- 4    Q- 0    Residuals- 40.032122077492424   AIC- 259.2944605015297  BI
C- 281.5171553494669
P- 4    Q- 1    Residuals- 39.67823963814028    AIC- 258.65888029962485
BIC- 284.5853576222183
P- 4    Q- 2    Residuals- 37.29247406252688    AIC- 244.157272317819   BI
C- 273.78753211506864
P- 4    Q- 3    Residuals- 36.52446289872482    AIC- 240.1263968795769  BI
C- 273.4604391514827
P- 4    Q- 4    Residuals- 36.73250984927225    AIC- 242.66311845526567
BIC- 279.70094320182767
```

In [29]:
```
residuals
```

Out[29]:

	p	q	residual	aic	bic
0	0	0	44.289167	281.450408	288.857973
1	0	1	41.630057	264.946322	276.057669
2	0	2	41.483608	265.889166	280.704295
3	0	3	41.330097	266.784178	285.303091
4	0	4	40.583111	263.365939	285.588634
5	1	0	42.093979	268.250704	279.362051
6	1	1	41.514200	266.110673	280.925803
7	1	2	38.440725	246.376421	264.895334
8	1	3	38.404299	248.119414	270.342108
9	1	4	38.137685	248.163018	274.089496
10	2	0	41.395993	265.257518	280.072648
11	2	1	38.442630	246.502439	265.021351
12	2	2	38.461895	248.481411	270.704106
13	2	3	37.492868	243.614965	269.541442
14	2	4	36.145145	234.831030	264.461290
15	3	0	41.272191	266.363999	284.882911
16	3	1	38.395936	248.088996	270.311691
17	3	2	37.289235	242.160765	268.087243
18	3	3	37.203291	242.623613	272.253873
19	3	4	37.301443	244.869291	278.203334
20	4	0	40.032122	259.294461	281.517155
21	4	1	39.678240	258.658880	284.585358
22	4	2	37.292474	244.157272	273.787532
23	4	3	36.524463	240.126397	273.460439
24	4	4	36.732510	242.663118	279.700943

In [30]:
```
residuals.loc[(residuals.aic == residuals.aic.min())]
```

Out[30]:

	p	q	residual	aic	bic
14	2	4	36.145145	234.83103	264.46129

In [31]:
```
lower_p = residuals.loc[residuals.p <= 6]
```

In [32]:
```
lower_p.loc[lower_p.aic == lower_p.aic.min()]
```

Out[32]:

	p	q	residual	aic	bic
14	2	4	36.145145	234.83103	264.46129

In [33]:
```
best = residuals.loc[residuals.aic == residuals.aic.min()]
best
```

Out[33]:

	p	q	residual	aic	bic
14	2	4	36.145145	234.83103	264.46129

In [34]:
```
best_p = best.p.values[0]
best_q = best.q.values[0]
```

In [35]:
```
model = ARIMA(stationary_ts_train, order=(best_p, 0, best_q))
model = model.fit()
```

In [36]:
```
for h in [3, 6, 12]:
    preds = get_predictions_from_horizon(
        model_fitted = model,
        forecast_horizon = h,
        ts = stationary_ts_train,
        ts_test = stationary_ts_test
    )
    reversed_preds = reverse_diff(ts, preds, split_index, period)
    print('\nFORECAST HORIZON:', h)
    metrics_summary(ts_test, reversed_preds)
```

```
FORECAST HORIZON: 3
MAPE: 0.010514784820015157
MAE: 1.1871961360582644
MSE: 2.5598431106277664
RMSE: 1.599950971319986
R2: 0.9117647758917181

FORECAST HORIZON: 6
MAPE: 0.010276146780927916
MAE: 1.1599107387973604
MSE: 2.449466743396495
RMSE: 1.565077232406278
R2: 0.9155693385457633

FORECAST HORIZON: 12
MAPE: 0.014837112586071594
MAE: 1.6729110613636782
MSE: 4.412636146689359
RMSE: 2.1006275602041784
R2: 0.8479008585741186
```

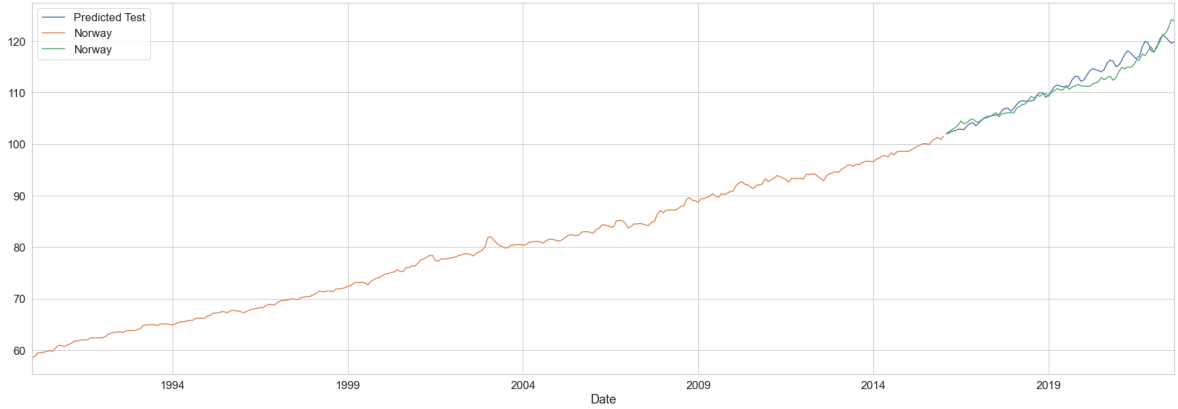
From testing different forecast horizons, it can be concluded that the best on is for 6 months.

In [37]:
```
preds = get_predictions_from_horizon(
        model_fitted = model,
        forecast_horizon = 6,
        ts = stationary_ts_train,
```

```
        ts_test = stationary_ts_test
    )
preds = reverse_diff(ts, preds, split_index, period)
preds.name = 'Predicted Test'
```

In [38]:
```
ax = preds.plot(figsize=(30, 10), legend=True)
ts.plot(ax=ax, legend=True)
ts_test.plot(ax=ax, legend=True)
```

Out[38]: <Axes: xlabel='Date'>



In [39]:
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.010276146780927916
MAE: 1.1599107387973604
MSE: 2.449466743396495
RMSE: 1.565077232406278
R2: 0.9155693385457633
```

The results achieved by the ARIMA(2, 4) have a lower MAPE Error, indicating that the results are good. Looking for the graphic, it can be seen that the model has some difficulty in the final curve. This was previously imagined when doing the initial plot of the TS, because it follows a linear trend in the begin but the Out-Sample data has a curve.

Linear models from non-stationary time series

For non-stationary time series, and since the TS has a seasonality, it will be use a SARIMA model. To found the best parameters of the model, it will be used grid-search and the models will be evaluated by the AIC and BIC values.

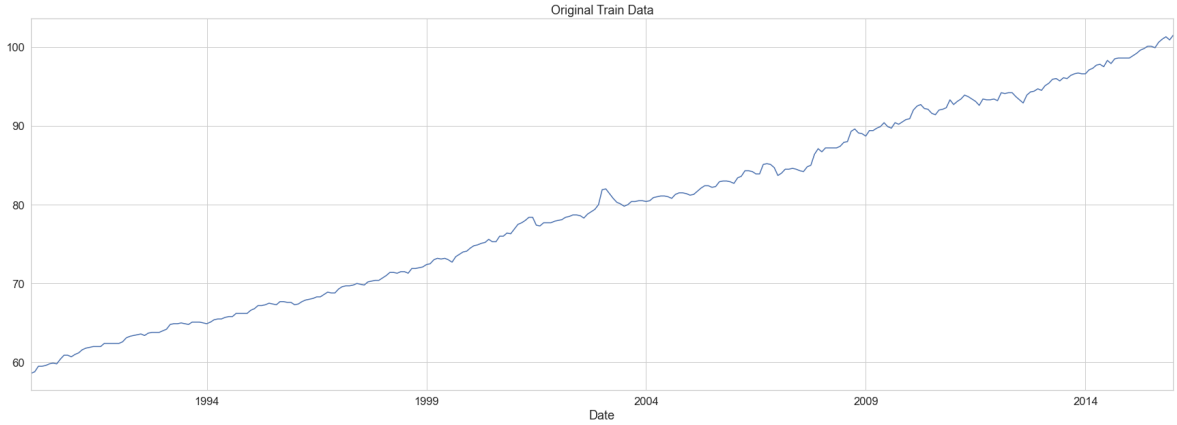
SARIMA model

(p, d, q) X (P, D, Q) S

S = seasonality period d = for applying differencing - since it has a trend, use a d = 1
D = 1 -> apply first differencing for seasonal component based on the period

In [40]:
```
ts.plot(figsize=(30, 10), title='Original Train Data')
```

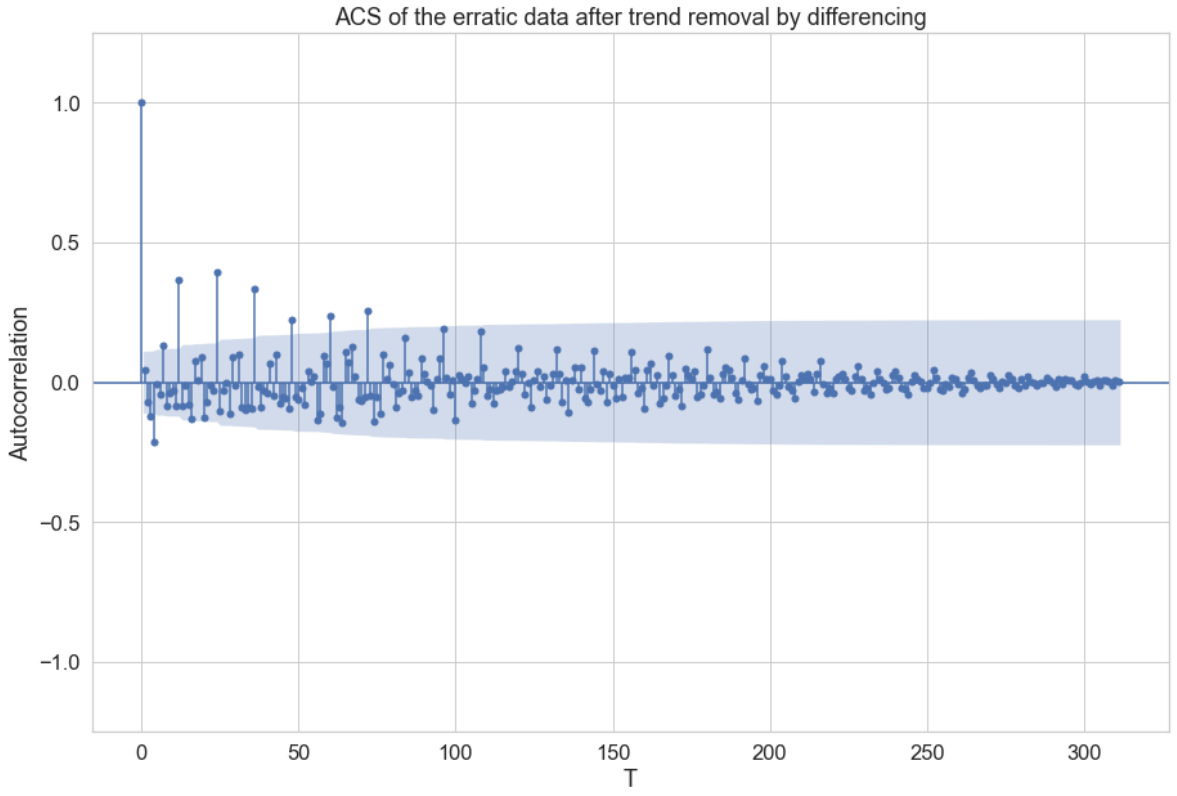
Out[40]: <Axes: title={'center': 'Original Train Data'}, xlabel='Date'>



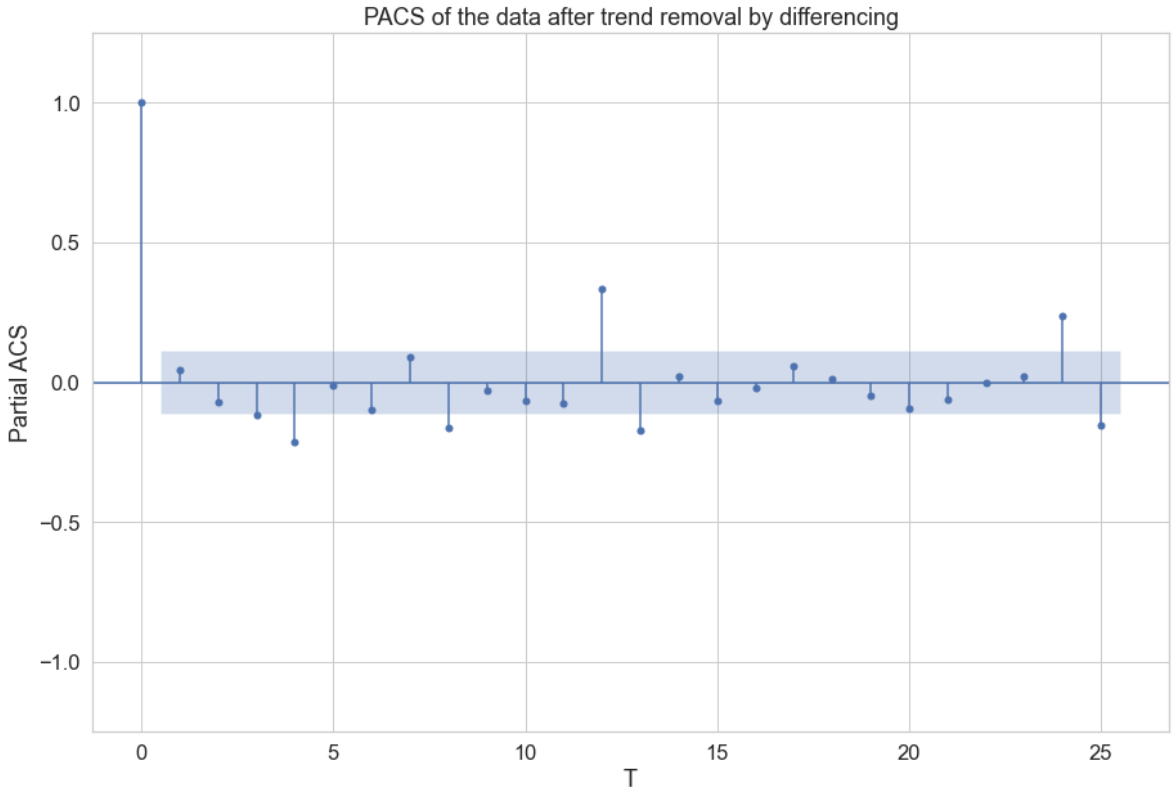
As there is a clear trend, it will be applied a first order simple differencing.

In [41]:
```
trend_diff_adjusted = ts.diff()
```

In [42]:
```
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(trend_diff_adjusted.dropna(), alpha=0.05, ax=ax, lags=len(trend_
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
plt.title('ACS of the erratic data after trend removal by differencing')
plt.savefig(os.path.join('images', 'acs_trend_diff.png'))
plt.show()
```



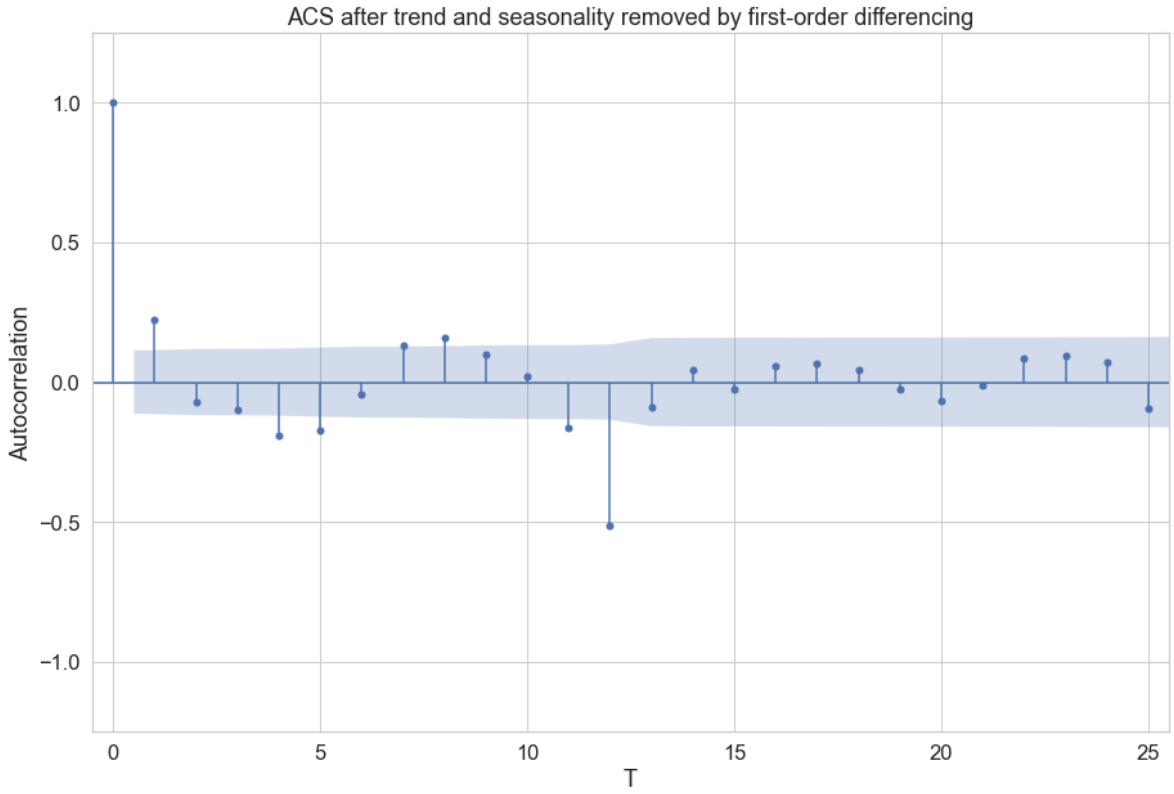
In [43]:
```
fog, ax = plt.subplots(1, figsize=(15, 10))
plot_pacf(trend_diff_adjusted.dropna(), alpha=0.05, ax=ax)
ax.set_ylim([-1.25, 1.25])
plt.xlabel('T')
plt.ylabel('Partial ACS')
plt.title('PACS of the data after trend removal by differencing')
plt.savefig(os.path.join('images', 'pacs_trend_diff.png'))
plt.show()
```



From the ACS it can be confirmed that seasonality is present, since there are ACS peaks at 12, 24, 36, etc. With this in mind, first-order seasonal differencing with S=12 will be applied as previously referenced.

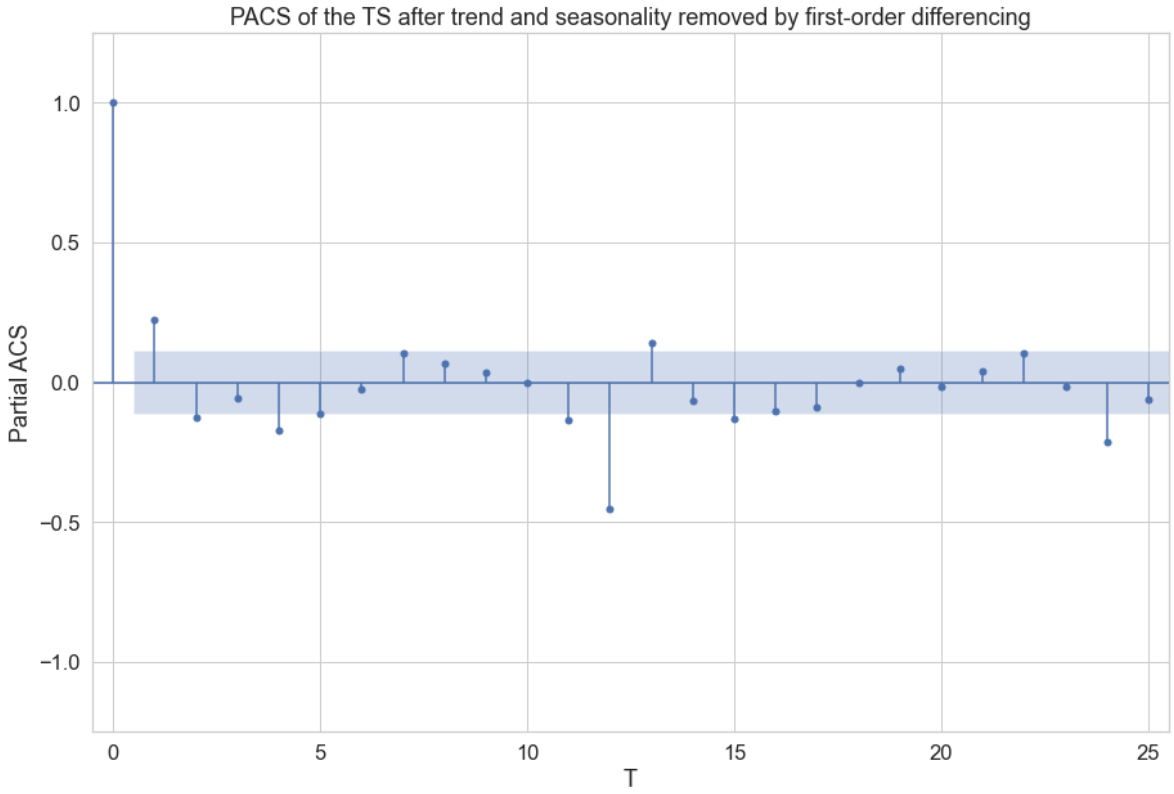
In [44]:
```
seasonal_adjusted = trend_diff_adjusted.diff(period)
```

In [45]:
```
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(seasonal_adjusted.dropna(), alpha=0.05, ax=ax, lags=len(seasonal
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
ax.set_xlim([-0.5, 0.5+25])
plt.title('ACS after trend and seasonality removed by first-order differe
plt.savefig(os.path.join('images', 'acs_ts_diff.png'))
plt.show()
```



Looking at the ACS for T < 12, at lag 1, the ACS fall outside the unit root, meaning a q = 1.

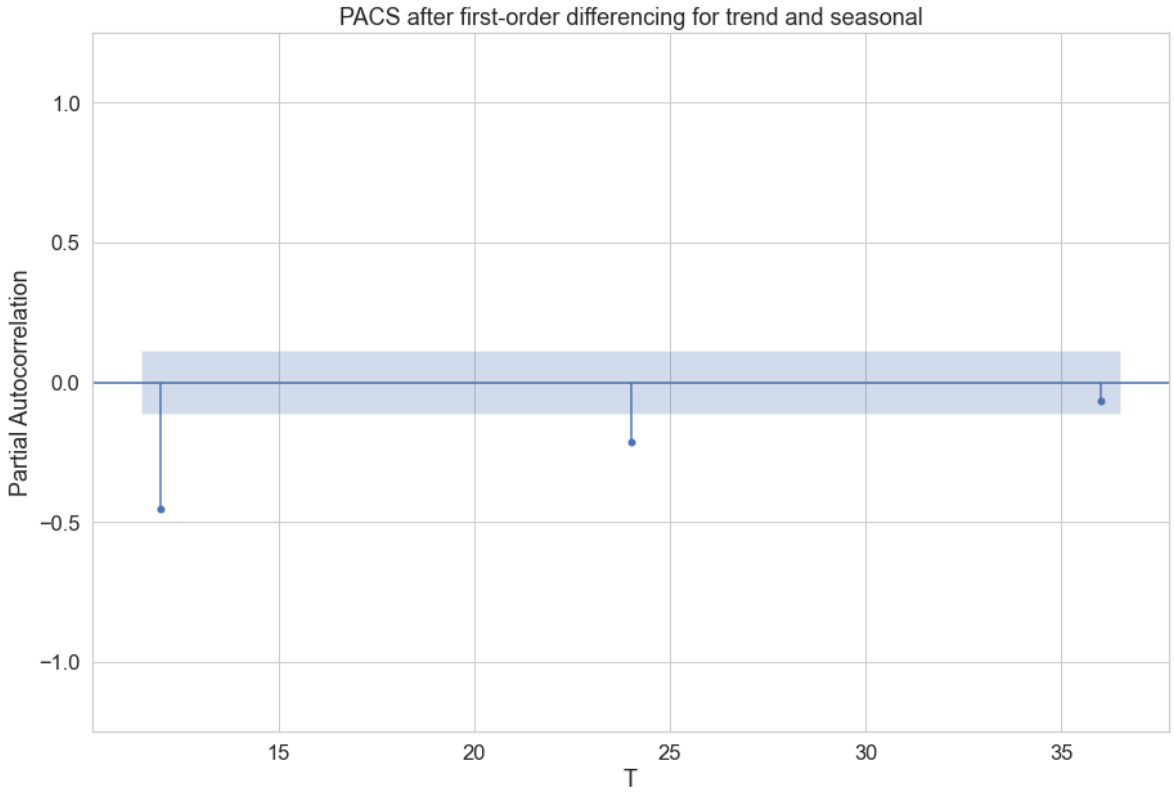
In [46]:
```
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_pacf(seasonal_adjusted.dropna(), alpha=0.05, ax=ax)
ax.set_ylim([-1.25, 1.25])
ax.set_xlim([-0.5, 0.5+25])
plt.xlabel('T')
plt.ylabel('Partial ACS')
plt.title('PACS of the TS after trend and seasonality removed by first-or
plt.savefig(os.path.join('images', 'pacs_ts_diff_plot.png'))
plt.show()
```



The first-order seasonal differencing seems appropriated to transform the TS into stationary, as previously seen in the ts-analysis notebook and confirmed by the ADF test. Therefore, a D=1 should be applied. Looking at the PACS for T<12, for lag 1 and 2, the PACS fall out the unit root, so we can estimate as p=2.

Looking at the lags 12, 24, and 36, we can estimate the Q=1, as there is one significant value at lag 12.

In [37]:
```
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_pacf(seasonal_adjusted.dropna(), alpha=0.05, ax=ax, lags=[12, 24, 36
plt.ylabel('Partial Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
plt.title('PACS after first-order differencing for trend and seasonal')
plt.show()
```



From the PACS for lags 12, 24, and 36, it can be estimated that P=2 as there is two significant values

With all values estimated, the SARIMA model must be :

(p, d, q) X (P, D, Q) S with d = 1, D = 1, S = 12, p = 2, q = 1, Q = 1, P=2

In [49]:
```
d = 1
D = 1
S = period
p = 2
q = 1
Q = 1
P = 2
```

In [50]:
```
model = ARIMA(ts, order=(p, d, q), seasonal_order=(P, D, Q, S))
model_fitted = model.fit()
```

In [52]:
```
for h in [3, 6, 12]:
    print('\nForecast Horizon', h)
    preds = get_predictions_from_horizon(
        model_fitted,
        forecast_horizon=h,
        ts = ts,
        ts_test = ts_test
    )
    metrics_summary(ts_test, preds)
```

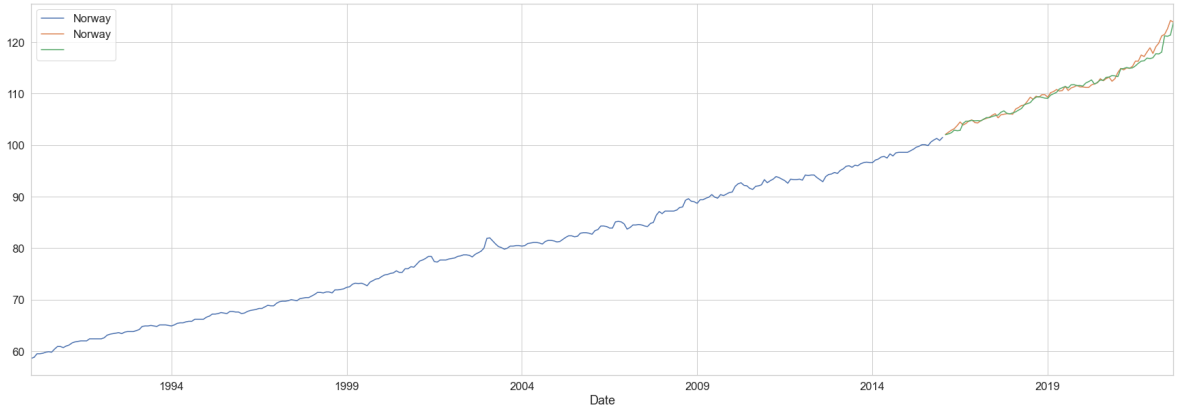
```
Forecast Horizon 3
MAPE: 0.004867748233760922
MAE: 0.551450753701392
MSE: 0.668188152857302
RMSE: 0.8174277661404108
R2: 0.9769682246661576

Forecast Horizon 6
MAPE: 0.006550264462472748
MAE: 0.7476055871498042
MSE: 1.4266714005106793
RMSE: 1.1944335061068403
R2: 0.9508240679945166

Forecast Horizon 12
MAPE: 0.009009989469651548
MAE: 1.0244815227993638
MSE: 2.262407001450068
RMSE: 1.5041299815674403
R2: 0.9220171002010598
```

It can be seen that the best forecast horizon is at 3 months while using a SARIMA (p, d, q) X (P, D, Q) S with d = 1, D = 1, S = 12, p = 2, q = 1, Q = 1, P=2

In [53]:
```
preds = get_predictions_from_horizon(
    model_fitted,
    forecast_horizon=3,
    ts = ts,
    ts_test = ts_test
)
```

In [54]:
```
ax = ts.plot(figsize=(30, 10), legend=True)
ts_test.plot(ax=ax, legend=True)
preds.plot(ax=ax, legend=True)
```

Out[54]: <Axes: xlabel='Date'>



In [55]:
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.004867748233760922
MAE: 0.551450753701392
MSE: 0.668188152857302
RMSE: 0.8174277661404108
R2: 0.9769682246661576
```

As the model may not have the best appropriate parameters, it will be used grid-search to find the best parameters. The evaluation will take into consideration the

AIC wit the train data and the MAPE error in the validation set.

In [56]:
```
# split train data to evaluate the models
ts_train, ts_val, months_train, months_val = train_test_split(ts, months)
```

In [57]:
```
values_p = 5
values_q = 3
values_P = 4
values_Q = 2
D = 1
d = 1
S = period
```

In [ ]:
```
results = pd.DataFrame()
count_models = 1
total_models = values_P * values_p * values_Q * values_q
for p in range(values_p):
    for q in range(values_q):
        for P in range(values_P):
            for Q in range(values_Q):
                print(count_models, ' out of ', total_models)
                print(p, q, P, Q)
                model = ARIMA(ts_train, order=(p, d, q), seasonal_order=(
                model.initialize_approximate_diffuse()
                model_fitted = model.fit()
                preds = model_fitted.predict(start=len(ts_train), end=len
                res = {
                    'd' : [d],
                    'D' : D,
                    'S' : S,
                    'p' : p,
                    'q' : q,
                    'P' : P,
                    'Q' : Q,
                    'AIC' : model_fitted.aic,
                    'MAPE' : metrics.mean_absolute_percentage_error(ts_va

                }
                results = pd.concat([results, pd.DataFrame(res)], ignore_
                count_models += 1
results.to_csv('results_grid_ARIMA.csv', index=False)
```

In [244…
```
results.loc[results.AIC == results.AIC.min()]
```

Out[244…

	d	D	S	p	q	P	Q	AIC	MAPE
9	1	1	12	0	1	0	1	1762.051898	0.149979

In [245…
```
best = results.loc[results.MAPE == results.MAPE.min()]
best
```

Out[245…

	d	D	S	p	q	P	Q	AIC	MAPE
118	1	1	12	4	2	3	0	1995.274633	0.144232

From the grid-search the best SARIMA model is:

(p, d, q) X (P, D, Q) S with d = 1, D = 1, S = 12, p = 2, q = 2, Q = 1, P=3

In [60]:
```
p = 4#best.p.values[0]
q = 2#best.q.values[0]
P = 3#best.P.values[0]
Q = 0#best.Q.values[0]
```

Re-train best model and test it on the test data

In [61]:
```
model = ARIMA(ts, order=(p, d, q), seasonal_order=(P, D, Q, S))
model.initialize_approximate_diffuse()
model_fitted = model.fit()
```

In [63]:
```
for h in [3, 6, 12]:
    print('\nForecast Horizon:', h)
    preds = get_predictions_from_horizon(
        model_fitted=model_fitted,
        forecast_horizon=h,
        ts=ts,
        ts_test=ts_test
    )
    metrics_summary(ts_test, preds)
```

```
Forecast Horizon: 3
MAPE: 0.004873835659738448
MAE: 0.5533269482453366
MSE: 0.6585123629811567
RMSE: 0.811487746661129
R2: 0.9773017394368878

Forecast Horizon: 6
MAPE: 0.006488141440914774
MAE: 0.7425027357577877
MSE: 1.3843466612087623
RMSE: 1.1765826197971658
R2: 0.9522829593000519

Forecast Horizon: 12
MAPE: 0.008874341970103783
MAE: 1.0088090667108347
MSE: 1.9869778191368939
RMSE: 1.409602007354166
R2: 0.9315108678177024
```

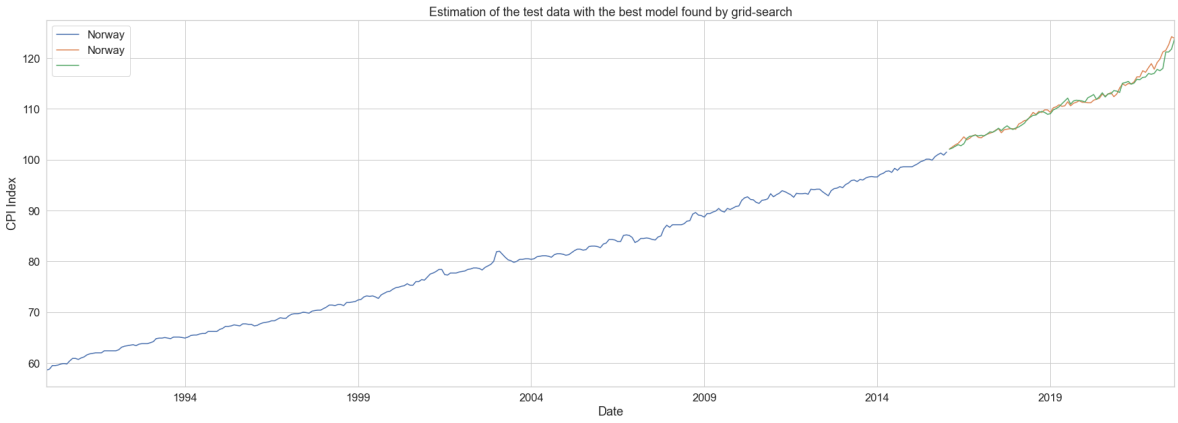
The best SARIMA model achieved the best perfornace using a forecast horizon of 3 months

In [64]:
```
preds = get_predictions_from_horizon(
        model_fitted=model_fitted,
        forecast_horizon=3,
        ts=ts,
        ts_test=ts_test
    )
```

In [65]:
```
ax = ts.plot(figsize=(30, 10), legend=True)
ts_test.plot(ax=ax, legend=True)
preds.plot(ax=ax, legend=True)
ax.set_title('Estimation of the test data with the best model found by gr
```

```
ax.set_xlabel('Date')
ax.set_ylabel('CPI Index')
```

Out[65]: Text(0, 0.5, 'CPI Index')



In [66]:
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.004873835659738448
MAE: 0.5533269482453366
MSE: 0.6585123629811567
RMSE: 0.811487746661129
R2: 0.9773017394368878
```

In the prediction plot, it is concluded that the model has a good forecasting property. However, it still exists some error, as previously referenced, because the last 2/2.5 years, the CPI Index goes from more or less linear trend to a quadratic one. As this change only appears in the test data, the model will always have difficulty in forecasting.

From the metrics assessed, the model seems to also have a good forecasting power, as the result MAPE error is low and and the MAE error has a value of almost one, which is small for the range of values presented in the CPI Index. Additionally, the R Squared metric is nearly at 1, meaning that the forecasted values almost perfectly fit the data.

Exponential Smoothing

In [67]:
```
alphas = np.arange(0.05, 1, 0.05)
gammas = np.arange(0.05, 1, 0.05)
deltas = np.arange(0.05, 1, 0.05)
```

In [ ]:
```
results_es = pd.DataFrame()
count = 0
for alpha in alphas:
    for gamma in gammas:
        for delta in deltas:
            count += 1
            print(count, 'out of', len(alphas) * len(gammas) * len(deltas
            model = ExponentialSmoothing(endog=ts_train, seasonal_periods
            model_fitted = model.fit(
                smoothing_level=alpha,
                smoothing_trend=gamma,
                smoothing_seasonal=delta
```

```
            )
            preds = model_fitted.forecast(len(ts_val))
            mape = metrics.mean_absolute_percentage_error(ts_val, preds)
            res = {
                'alpha' : [alpha],
                'gamma' : gamma,
                'delta' : delta,
                'AIC' : model_fitted.aic,
                'MAPE' : mape
            }
            results_es = pd.concat([results_es, pd.DataFrame(res)], ignor

results_es.to_csv('results_grid_es.csv', index=False)
```

In [253…
```
best_es = results_es.loc[results_es.MAPE == results_es.MAPE.min()]
best_es
```

Out[253…

	alpha	gamma	delta	AIC	MAPE
255	0.05	0.7	0.45	1299.546618	0.145161

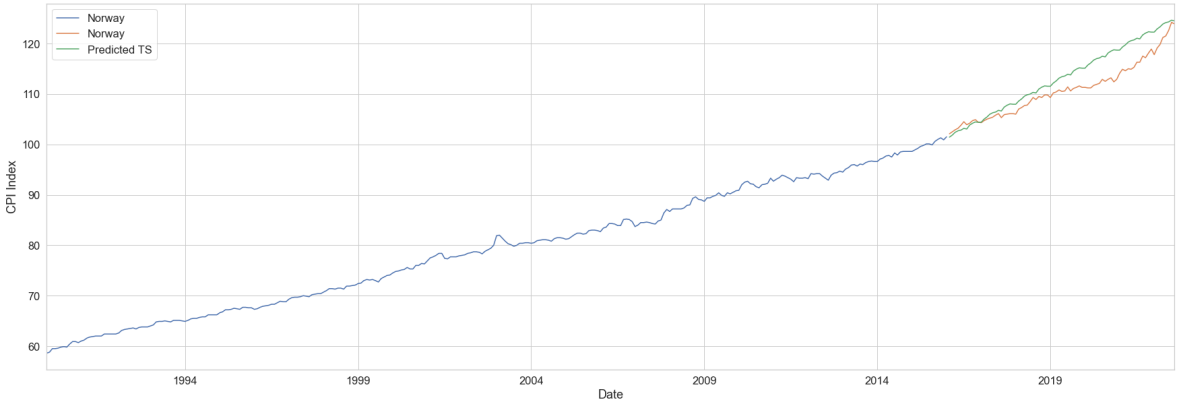
In [68]:
```
alpha = 0.05
gamma = 0.7
delta = 0.45
```

In [69]:
```
model = ExponentialSmoothing(endog=ts, seasonal_periods=period, trend='ad
model_fitted = model.fit(
                smoothing_level=alpha,
                smoothing_trend=gamma,
                smoothing_seasonal=delta,
                optimized=True
            )
```

In [71]:
```
preds = model_fitted.forecast(len(ts_test))
preds.name = 'Predicted TS'
```

In [72]:
```
ax = ts.plot(figsize=(30, 10), legend=True)
ts_test.plot(ax=ax, legend=True)
preds.plot(ax=ax, legend=True)
ax.set_ylabel('CPI Index')
```

Out[72]: Text(0, 0.5, 'CPI Index')



In [73]:
```
metrics_summary(ts_test, preds)
```

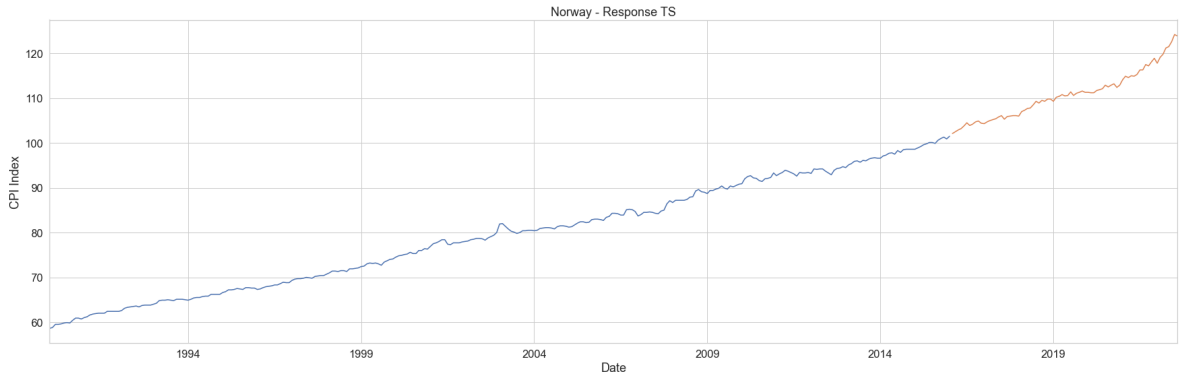
```
MAPE: 0.023680560615546496
MAE: 2.665777807176654
MSE: 10.211985703275278
RMSE: 3.1956197682570555
R2: 0.6480030969952324
```

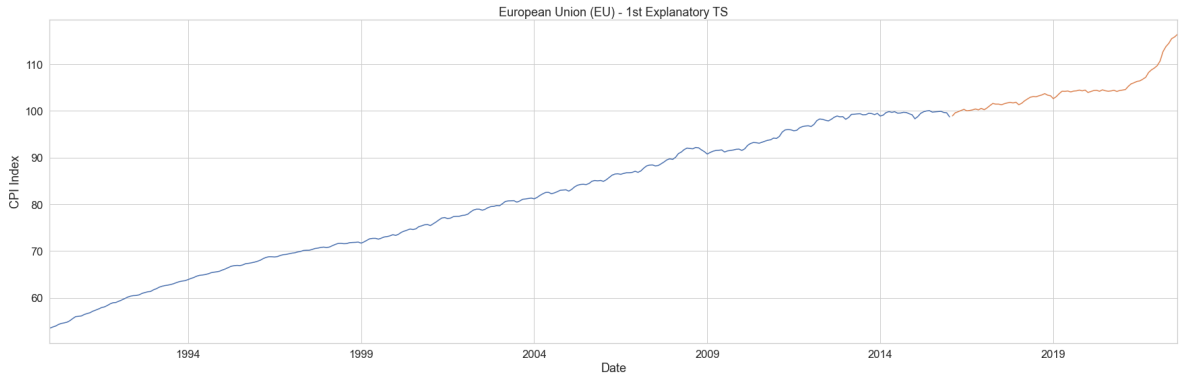
From the exponential smoothing model's predictions image and the metrics attained, it is evident that the model has less predictive power than the SARIMA model. However, the metrics show that the model learns the data's major pattern, although not as much as the SARIMA model does.

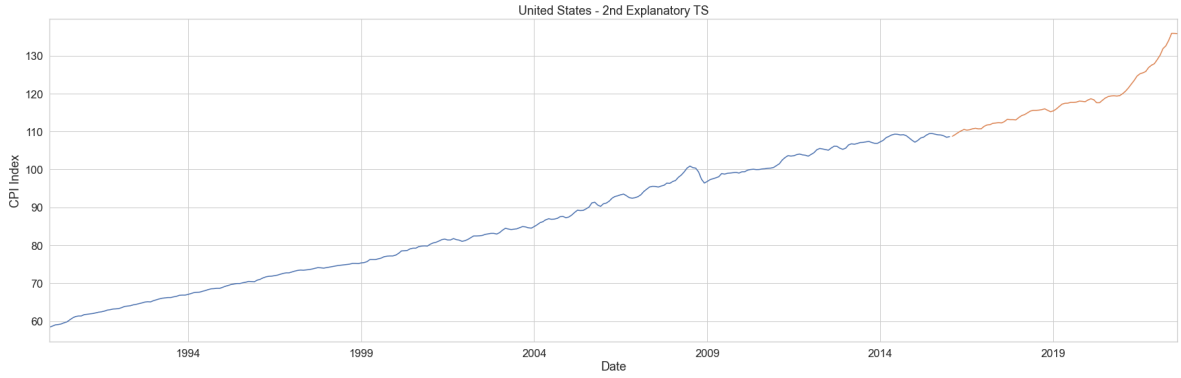
Multivariate TS

Plot time series

In [15]:
```
fig, axs = plt.subplots(3, figsize=(30, 30))

ts.plot(ax=axs[0])
ts_test.plot(ax=axs[0])
axs[0].set_ylabel('CPI Index')
axs[0].set_title('Norway - Response TS')

train_data['European Union'].plot(ax=axs[1])
test_data['European Union'].plot(ax=axs[1])
axs[1].set_ylabel('CPI Index')
axs[1].set_title('European Union (EU) - 1st Explanatory TS')

train_data['United States'].plot(ax=axs[2])
test_data['United States'].plot(ax=axs[2])
axs[2].set_ylabel('CPI Index')
axs[2].set_title('United States - 2nd Explanatory TS')

plt.savefig(os.path.join('images', 'multivariate-ts.png'))
plt.show()
```







For the multivariate part of this work, it will be used the European Union (EU) CPI Index and the United States (USA) CPI Index. The European Union CPI Index was calculated using the mean of CPI Index of the countries that belong to it in 1990.

Looking at the results achieved by the plot os the different Time Series, it is observed that the three TS are similar.

In [38]:
```
def crosscov(timeserie1,timeserie2,T=0):
    N=np.array([timeserie1.size,timeserie2.size]).min()
    mu1=timeserie1.mean()
    mu2=timeserie2.mean()
    covCoef=0
    for n in range(0,N-T-1):
        covCoef=covCoef+(timeserie1[n]-mu1)*(timeserie2[n+T]-mu2)
    return covCoef/float(np.abs(N))
```

In [39]:
```
def crosscoef(timeserie1,timeserie2,T=0):
    s1=timeserie1.std()
    s2=timeserie2.std()
    return crosscov(timeserie1,timeserie2,T)/(s1*s2)
```

In [40]:
```
def ccs(timeserie1,timeserie2,maxT,twoside=False):
    N=np.array([timeserie1.size,timeserie2.size]).min()
```

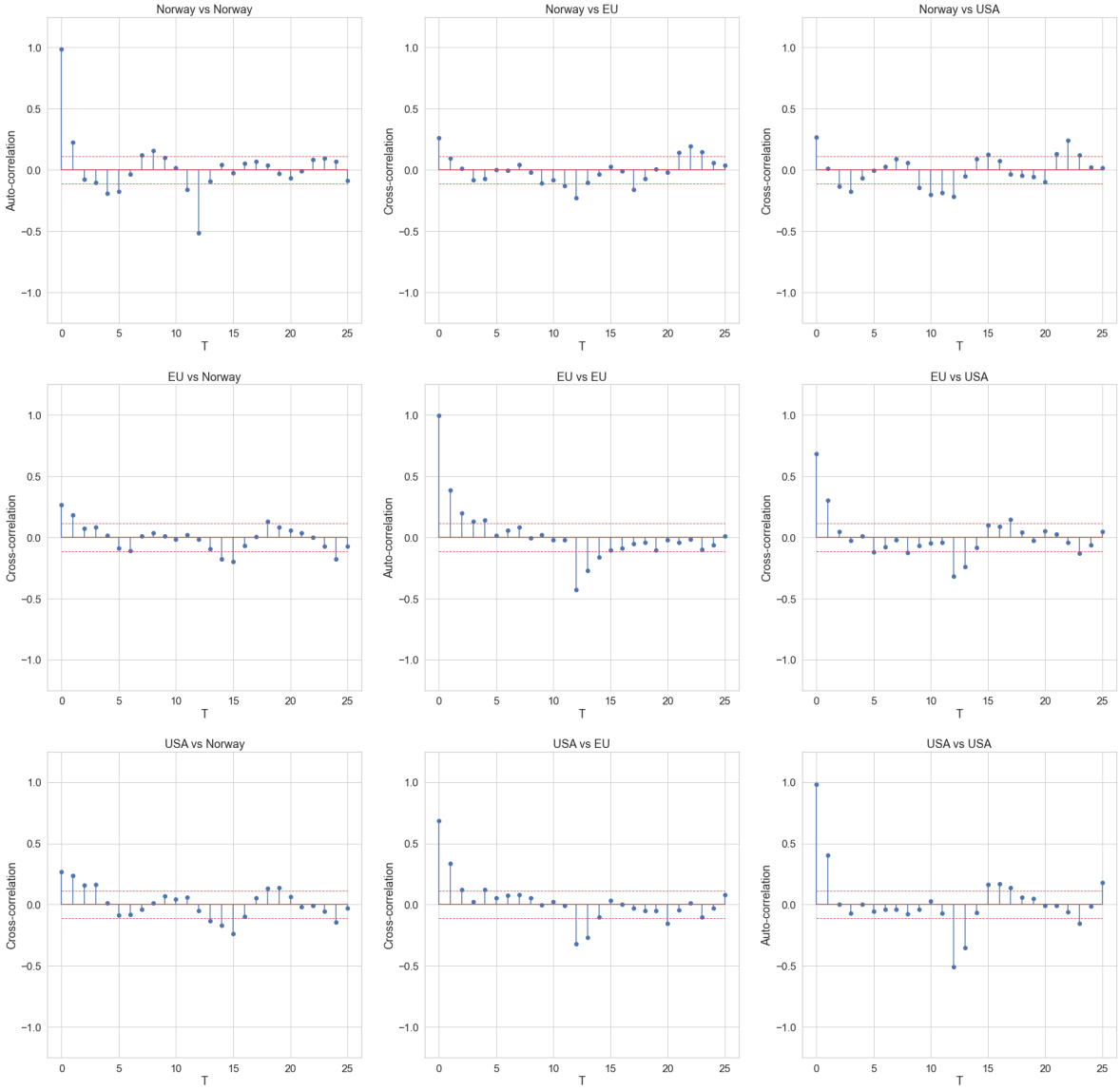
```
    if twoside:
        corrl=np.zeros(2*maxT+1)
        ix=np.array(range(maxT+1))
        ix=np.concatenate((-np.flip(ix[1:]),ix),axis=0)
    else:
        corrl=np.zeros(maxT+1)
        ix=np.array(range(maxT+1))

    for i in range(maxT+1):
        if twoside:
            if i==0:
                corrl[i+maxT]=crosscoef(timeserie1,timeserie2,i);
            else:
                corrl[i+maxT]=crosscoef(timeserie1,timeserie2,i);
                corrl[maxT-i]=crosscoef(timeserie2,timeserie1,i);
        else:
            corrl[i]=crosscoef(timeserie1,timeserie2,i);
    d = {'CCS':corrl, 'Upper_Bound':np.ones(maxT+1)*(1.96/np.sqrt(N)),'Lo
    corrl=pd.DataFrame(data=d,index=ix)
    return corrl
```

All the multivariate TS were transformed into stationary using differencing as described in notebook ts-analysis.ipynb

In [41]:
```
ts_diff = ts.diff().diff(12)
eu_ts_diff = train_data['European Union'].diff().diff(12)
usa_ts_diff = train_data['United States'].diff().diff(12)
```

In [42]:
```
ts_names = ['Norway', 'EU', 'USA']
ts_values = [ts_diff.dropna(), eu_ts_diff.dropna(), usa_ts_diff.dropna()]
```

In [43]:
```
fig, axs = plt.subplots(len(ts_names), len(ts_names), figsize=(30, 30))

for i in range(len(ts_names)):
    for j in range(len(ts_names)):
        corrl = ccs(ts_values[i], ts_values[j], maxT=25)
        axs[i, j].stem(corrl.index, corrl.CCS)
        axs[i, j].plot(corrl.index, corrl.Upper_Bound, linestyle='--', co
        axs[i, j].plot(corrl.index, corrl.Lower_Bound, linestyle='--', co
        axs[i, j].set_xlabel('T')
        axs[i, j].set_ylim([-1.25, 1.25])
        if i == j:
            axs[i, j].set_ylabel('Auto-correlation')
        else:
            axs[i, j].set_ylabel('Cross-correlation')
        axs[i, j].set_title(f'{ts_names[i]} vs {ts_names[j]}')

plt.savefig(os.path.join('images', 'multivariate-cross-correlation.png'))
```



From the cross-correlation plots, it is observed that there are few unidirectional relations between our TS of interest and the explanatory TS, except at the exact point that they are.

Other relations appear when T=12, meaning that the CPI Index on the previous year for the EU and USA influence the TS in a point. It was also found that for a given time point, the CPI index of USA in the previous 3 months influences the current time stamp.

SARIMAX

As the value from the explanatory time series that most impact the results is at lag T=0, we will test a SARIMAX model for multivariate purpose, as it only uses the current value from the explanatory TS on the forecasting. Similar to the previous test, we will use grid-search to find the best parameters

In [44]:
```
tss_train, tss_val, month_train, months_val = train_test_split(train_data
```

In [23]:
```
values_p = 5
values_q = 3
```

```
values_P = 4
values_Q = 2
D = 1
d = 1
S = period
```

In [24]:
```
explanatory_ts = ['United States', 'European Union']
```

In [ ]:
```
results = pd.DataFrame()
count_models = 1
total_models = values_P * values_p * values_Q * values_q
for p in range(values_p):
    for q in range(values_q):
        for P in range(values_P):
            for Q in range(values_Q):
                print(count_models, ' out of ', total_models)
                print(p, q, P, Q)
                model = SARIMAX(tss_train[country], exog=tss_train[explan
                model.initialize_approximate_diffuse()
                model_fitted = model.fit(disp=False)
                preds = model_fitted.forecast(len(tss_val), exog=tss_val[
                res = {
                    'd' : [d],
                    'D' : D,
                    'S' : S,
                    'p' : p,
                    'q' : q,
                    'P' : P,
                    'Q' : Q,
                    'AIC' : model_fitted.aic,
                    'MAPE' : metrics.mean_absolute_percentage_error(tss_v

                }
                results = pd.concat([results, pd.DataFrame(res)], ignore_
                count_models += 1

results.to_csv('results_grid_sarimax.csv', index=False)
```

In [276…
```
best = results.loc[results.MAPE == results.MAPE.min()]
best
```

Out[276…

	d	D	S	p	q	P	Q	AIC	MAPE
27	1	1	12	1	0	1	1	854.650643	0.010268

In [88]:
```
p = 1#best.p.values[0]
q = 0#best.q.values[0]
P = 1#best.P.values[0]
Q = 1#best.Q.values[0]
```

In [91]:
```
model = SARIMAX(ts, exog=train_data[explanatory_ts], order=(p, d, q), sea
fitted_model = model.fit()
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            6     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.33158D-01    |proj g|=  8.78321D-01

At iterate    5    f=  1.98754D-01    |proj g|=  4.42110D-02
 This problem is unconstrained.
At iterate   10    f=  1.88973D-01    |proj g|=  5.75567D-02

At iterate   15    f=  1.87679D-01    |proj g|=  8.42842D-03

At iterate   20    f=  1.87454D-01    |proj g|=  5.79890D-03

At iterate   25    f=  1.87446D-01    |proj g|=  3.19483D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    6     25     30      1     0     0   3.195D-05   1.874D-01
  F =  0.18744644002996877

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```

In [92]:
```python
for h in [3, 6, 12]:
    print('\nForecast Horizon:', h)
    preds = get_multi_predictions_from_horizon(
        fitted_model=fitted_model,
        train_data = train_data,
        test_data = test_data,
        explanatory_ts=explanatory_ts,
        ts = ts,
        ts_test = ts_test,
        forecast_horizon=h
    )
    metrics_summary(ts_test, preds)
```

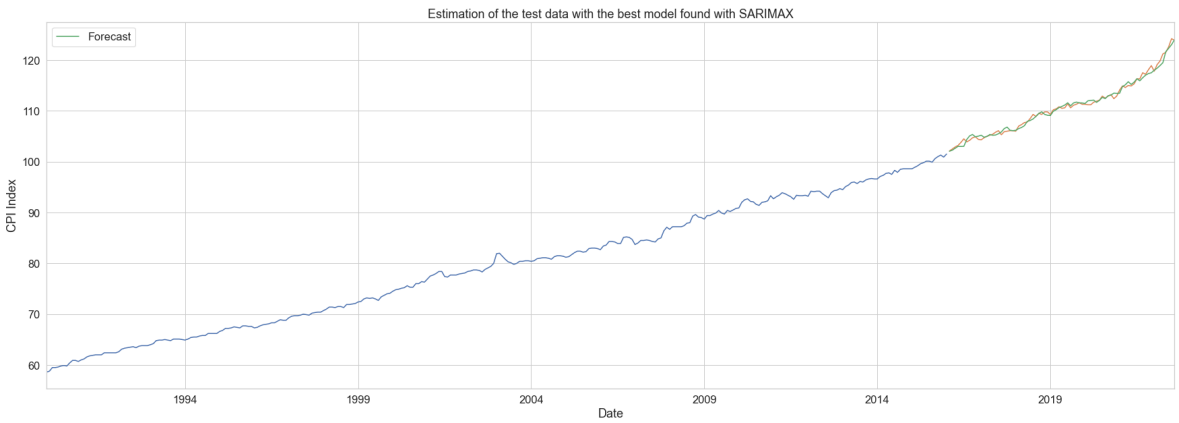
```
Forecast Horizon: 3
MAPE: 0.003972687111463143
MAE: 0.4416176768505722
MSE: 0.3281554452930404
RMSE: 0.5728485360835274
R2: 0.988688810990966

Forecast Horizon: 6
MAPE: 0.005493545223151229
MAE: 0.6145254742608667
MSE: 0.6732757500586829
RMSE: 0.8205338201796943
R2: 0.9767928602942659

Forecast Horizon: 12
MAPE: 0.006274985482756155
MAE: 0.7042573287135092
MSE: 0.89430519435119
RMSE: 0.945677108928407
R2: 0.9691741970759191
```

From the metrics, it can be concluded that the best SARIMAX model obtains the best results with a forecast horizon of 3 months

In [93]:
```
preds = get_multi_predictions_from_horizon(
        fitted_model=fitted_model,
        train_data = train_data,
        test_data = test_data,
        explanatory_ts=explanatory_ts,
        ts = ts,
        ts_test = ts_test,
        forecast_horizon=3
    )
preds.name = 'Forecast'
```

In [94]:
```
ax = ts.plot(figsize=(30, 10))
ts_test.plot(ax=ax)
preds.plot(ax=ax, legend=True)
ax.set_title('Estimation of the test data with the best model found with
ax.set_xlabel('Date')
ax.set_ylabel('CPI Index')
```

Out[94]: Text(0, 0.5, 'CPI Index')



In [95]:
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.003972687111463143
MAE: 0.4416176768505722
MSE: 0.3281554452930404
RMSE: 0.5728485360835274
R2: 0.988688810990966
```

After grid-search, it was found that the best model is (1, 1, 2) x (1, 1, 1, 12).

From the results achieved by the the best model found with SARIMAX, it is concluded that it is the best model found so far, surpassing the ARIMA model.

Vector Autoregressive (VAR) and Vector Autoregressive Moving Average (VARMA)

In [45]:
```
inputs = df[['Norway', 'United States', 'European Union']].copy()
```

In [46]:
```
for col in inputs.columns:
    inputs[col] = inputs[col].diff().diff(12)
```

In [47]:
```
train_multi_data = inputs[:split_index]
test_multi_data = inputs[split_index:]
```

Get the train data without the null values

In [48]:
```
train_multi_data = pd.DataFrame(train_multi_data.loc[train_multi_data.isn
```

In [31]:
```
x_train, x_val = train_test_split(train_multi_data)
```

In [ ]:
```
values_orders = pd.DataFrame()
values = [
        (0, 1), (1, 0), (1, 1), (1, 2), (2, 1), (2, 2), (0, 2), (2, 0)
]
for (p,q) in values:
    model = VARMAX(x_train, order=(p, q))
    fitted_model = model.fit(maxiter=1000, disp=False)
    preds = fitted_model.forecast(len(x_val))[country]

    res = {
        'p' : [p],
        'q' : [q],
        'MAPE' : [metrics.mean_absolute_percentage_error(x_val[country],
        'AIC' : fitted_model.aic
    }
    print(res)
    values_orders = pd.concat([values_orders, pd.DataFrame(res)])
```

In [102…
```
best = values_orders.loc[values_orders.MAPE == values_orders.MAPE.min()]
best
```

Out[102…

	p	q	MAPE	AIC
0	1	2	9.847576e+12	86.713922

In [25]:
```
best_p = 1 #best.p.values[0]
best_q = 2 #best.q.values[0]
```

In [32]:
```
model = VARMAX(train_multi_data, order=(best_p, best_q))
fitted_model = model.fit(maxiter=1000, disp=False)
```

In [33]:
```
for h in [3, 6, 12]:
    print('\nForecast Horizon:', h)
    preds = get_prediction_from_horizon_varmax(
        model_fitted = fitted_model,
        train_multi_data=train_multi_data,
        test_multi_data=test_multi_data,
        forecast_horizon=h
    )[country]
    preds = reverse_diff(ts, preds, split_index, period)
    metrics_summary(ts_test, preds)
```

```
Forecast Horizon: 3
MAPE: 0.08387650681026598
MAE: 9.608239294544932
MSE: 145.95949859704913
RMSE: 12.081369897368806
R2: -4.0310774968879794

Forecast Horizon: 6
MAPE: 0.07726561699069148
MAE: 8.844648049353983
MSE: 121.78780328412206
RMSE: 11.035751142723456
R2: -3.1979034073672397

Forecast Horizon: 12
MAPE: 0.03871535703582291
MAE: 4.458596563152331
MSE: 36.53296299920562
RMSE: 6.044250408380316
R2: -0.2592545864203213
```

The overall results are now great compared with the previously achieved. HOwever, from the results achieved the best forecast horizon for the VARMAX model is 1 year (12 months)

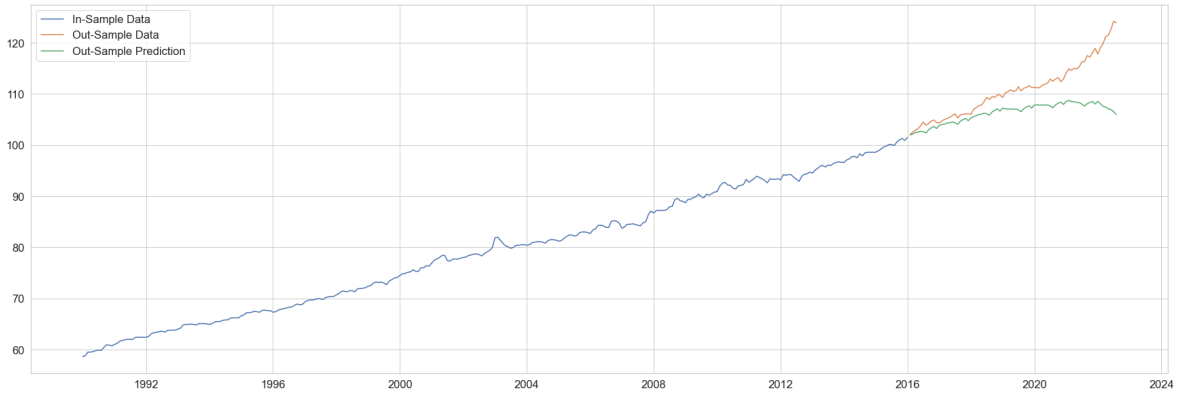
In [34]:
```
preds = get_prediction_from_horizon_varmax(
        model_fitted = fitted_model,
        train_multi_data=train_multi_data,
        test_multi_data=test_multi_data,
        forecast_horizon=12
    )
preds = preds.add_suffix(' [Predicted]')
```

In [35]:
```
predicted = preds['Norway [Predicted]']
```

In [36]:
```
predicted = reverse_diff(ts, predicted, split_index, period)
```

In [37]:
```
plt.figure(figsize=(30, 10))
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')
```

```
plt.plot(predicted.index, predicted, label='Out-Sample Prediction')
plt.legend()
plt.show()
```



In [38]:
```
metrics_summary(ts_test, predicted)
```

```
MAPE: 0.03871535703582291
MAE: 4.458596563152331
MSE: 36.53296299920562
RMSE: 6.044250408380316
R2: -0.2592545864203213
```

Even after doig a grid-search to find the best parameters for the VARMAX model, the results are poor, far below the results obtained with both SARIMA and SARIMAX.

Machine Learning Models

In this section, it will be tested Machine Learning (ML) model to forecast the Norwegian CPI Index.

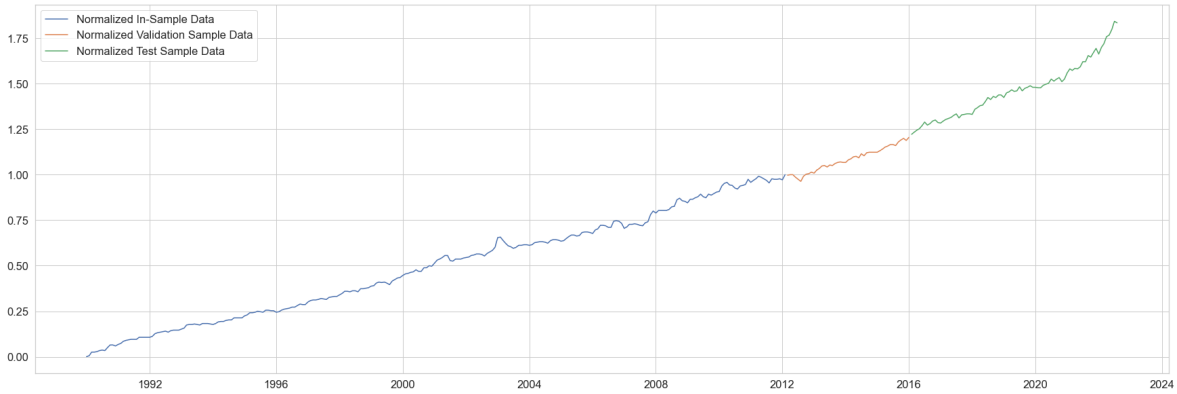
Univariate model

Normalized data

In [191…
```
x_train, x_val = train_test_split(ts, shuffle=False, test_size=0.15)
```

In [192…
```
scaler_ts = MinMaxScaler()
normalized_train = pd.Series(scaler_ts.fit_transform(x_train.values.resha
normalized_train.name = 'Normalized Train Data'
normalized_val = pd.Series(scaler_ts.transform(x_val.values.reshape(-1, 1
normalized_val.name = 'Normalized Validation Data'
normalized_test = pd.Series(scaler_ts.transform(ts_test.values.reshape(-1
normalized_test.name = 'Normalized Test Data'
```

In [193…
```
plt.figure(figsize=(30, 10))
plt.plot(normalized_train.index, normalized_train, label='Normalized In-S
plt.plot(normalized_val.index, normalized_val, label='Normalized Validati
plt.plot(normalized_test.index, normalized_test, label='Normalized Test S
plt.legend()
plt.show()
```



In [71]:
```
def generate_IO_data(outData,inData=pd.DataFrame(),delayEndog=[1],delayEx
    N=outData.shape[0]
    inShape=inData.shape
    if len(inShape)==1:
        M=1
    else:
        M=inShape[1]# The number of explanatory TS (Exogenous inputs)

    laggedData=pd.DataFrame(index=inData.index)
    for i in range(M):
        for j in range(len(delayExog[i])):
            if isinstance(inData, pd.DataFrame):
                laggedData[inData.columns[i]+'_lag'+str(delayExog[i][j]+(
            else:
                laggedData[inData.name+'_lag'+str(delayExog[i][j]+(h-1))]

    for j in range(len(delayEndog)):
        laggedData[outData.name+'_lag'+str(delayEndog[j]+(h-1))] = outDat

    laggedData[outData.name]=outData

    return laggedData.dropna(axis=0)
```

In [70]:
```
def generate_tensor(data,T,M):
    dataMat=data.to_numpy()
    N=data.shape[0]
    Ti=0
    tensor=np.zeros((N,T,M))
    print(tensor.shape)
    for i in range(M):
        tensor[:,:,i]=dataMat[:,Ti:Ti+T]
        Ti=Ti+T

    return tensor
```

In [196…
```
max_lag = 12
delays = range(1, max_lag+1)
```

In [197…
```
# train data
train_delays = generate_IO_data(normalized_train, delayEndog=delays)
train_input = train_delays.iloc[:, 0:-1]
train_output = np.array([train_delays.iloc[:, -1].to_numpy()]).T
train_input_tensor = generate_tensor(train_input, max_lag, 1)
```

```
(254, 12, 1)
```

In [198…
```
# validation data
val = pd.concat([normalized_train.iloc[len(normalized_train)-max_lag:], n
val.name = 'Validation'
val_delays = generate_IO_data(val, delayEndog=delays)
val_input = val_delays.iloc[:, 0:-1]
val_output = np.array([val_delays.iloc[:, -1].to_numpy()]).T
val_input_tensor = generate_tensor(val_input, max_lag, 1)
```

```
(47, 12, 1)
```

In [199…
```
test = pd.concat([normalized_val.iloc[len(normalized_val)-max_lag:], norm
test.name = 'Validation'
test_delays = generate_IO_data(test, delayEndog=delays)
test_input = test_delays.iloc[:, 0:-1]
test_output = np.array([test_delays.iloc[:, -1].to_numpy()]).T
test_input_tensor = generate_tensor(test_input, max_lag, 1)
```

```
(79, 12, 1)
```

In [200…
```
LATENT_DIM = 50
BATCH_SIZE = 1
EPOCHS = 100
HORIZON = 1
```

In [201…
```
model = Sequential()
model.add(layers.GRU(LATENT_DIM, input_shape=(max_lag, 1)))
model.add(layers.Dense(HORIZON))
```

In [202…
```
model.compile(optimizer='RMSprop', loss='mse')
model.summary()
```

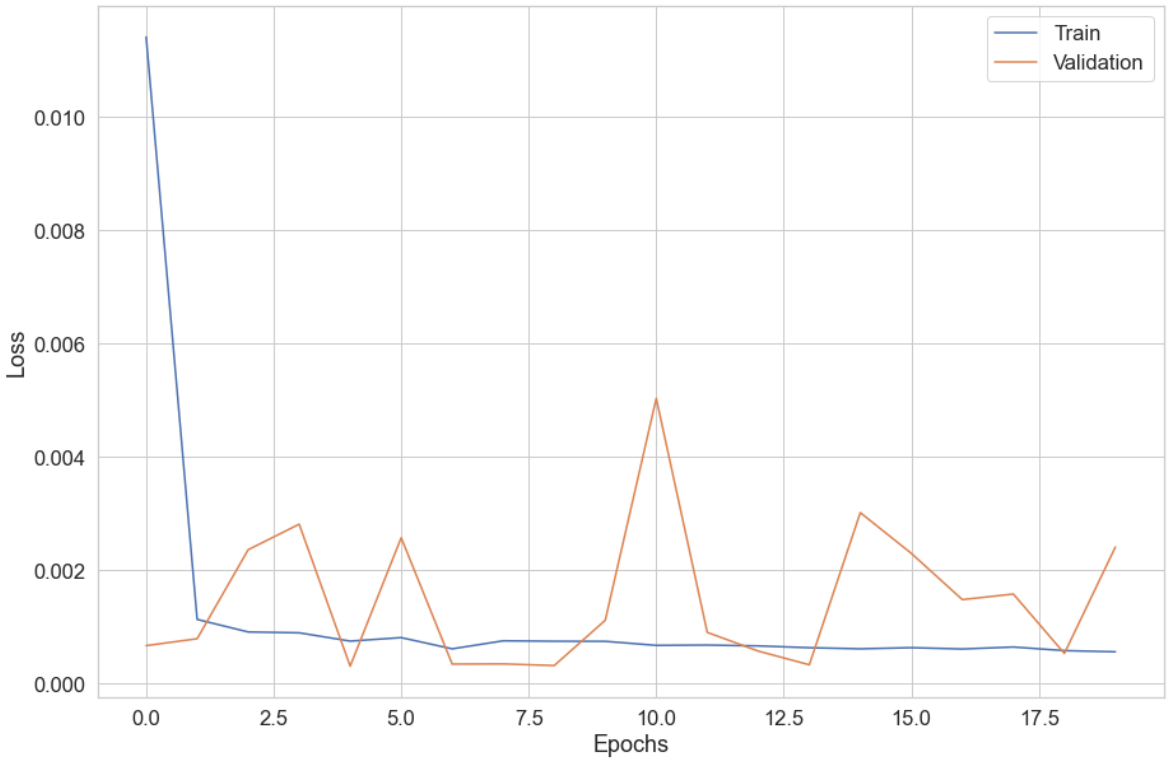
```
Model: "sequential_13"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 gru_10 (GRU)                (None, 50)                7950

 dense_13 (Dense)            (None, 1)                 51

=================================================================
Total params: 8001 (31.25 KB)
Trainable params: 8001 (31.25 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

In [203…
```
early_stopping = EarlyStopping(
    monitor='val_loss',
    patience=15,
    restore_best_weights=True)
```

In [204…
```
history = model.fit(
    train_input_tensor,
    train_output,
    batch_size = BATCH_SIZE,
    epochs = EPOCHS,
    validation_data = (val_input_tensor, val_output),
    callbacks = [early_stopping],
    verbose = 1
)
```

```
Epoch 1/100
254/254 [==============================] - 4s 13ms/step - loss: 0.0114 - v
al_loss: 6.6561e-04
Epoch 2/100
254/254 [==============================] - 3s 11ms/step - loss: 0.0011 - v
al_loss: 7.8798e-04
Epoch 3/100
254/254 [==============================] - 3s 11ms/step - loss: 9.0697e-04
- val_loss: 0.0024
Epoch 4/100
254/254 [==============================] - 3s 11ms/step - loss: 8.9212e-04
- val_loss: 0.0028
Epoch 5/100
254/254 [==============================] - 3s 11ms/step - loss: 7.4578e-04
- val_loss: 3.0355e-04
Epoch 6/100
254/254 [==============================] - 3s 11ms/step - loss: 8.0742e-04
- val_loss: 0.0026
Epoch 7/100
254/254 [==============================] - 3s 11ms/step - loss: 6.0939e-04
- val_loss: 3.3988e-04
Epoch 8/100
254/254 [==============================] - 3s 10ms/step - loss: 7.5091e-04
- val_loss: 3.4285e-04
Epoch 9/100
254/254 [==============================] - 3s 10ms/step - loss: 7.4326e-04
- val_loss: 3.1445e-04
Epoch 10/100
254/254 [==============================] - 3s 11ms/step - loss: 7.4189e-04
- val_loss: 0.0011
Epoch 11/100
254/254 [==============================] - 3s 11ms/step - loss: 6.7068e-04
- val_loss: 0.0050
Epoch 12/100
254/254 [==============================] - 3s 11ms/step - loss: 6.7608e-04
- val_loss: 8.9928e-04
Epoch 13/100
254/254 [==============================] - 3s 11ms/step - loss: 6.6105e-04
- val_loss: 5.6979e-04
Epoch 14/100
254/254 [==============================] - 3s 11ms/step - loss: 6.2915e-04
- val_loss: 3.2750e-04
Epoch 15/100
254/254 [==============================] - 3s 11ms/step - loss: 6.0985e-04
- val_loss: 0.0030
Epoch 16/100
254/254 [==============================] - 3s 11ms/step - loss: 6.2992e-04
- val_loss: 0.0023
Epoch 17/100
254/254 [==============================] - 3s 11ms/step - loss: 6.0694e-04
- val_loss: 0.0015
Epoch 18/100
254/254 [==============================] - 3s 11ms/step - loss: 6.4163e-04
- val_loss: 0.0016
Epoch 19/100
254/254 [==============================] - 3s 11ms/step - loss: 5.7829e-04
- val_loss: 5.2795e-04
Epoch 20/100
254/254 [==============================] - 3s 11ms/step - loss: 5.5696e-04
- val_loss: 0.0024
```

In [205…
```
plt.figure(figsize=(15, 10))
plt.plot(history.history['loss'], label='Train')
plt.plot(history.history['val_loss'], label='Validation')
plt.legend()
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.show()
```

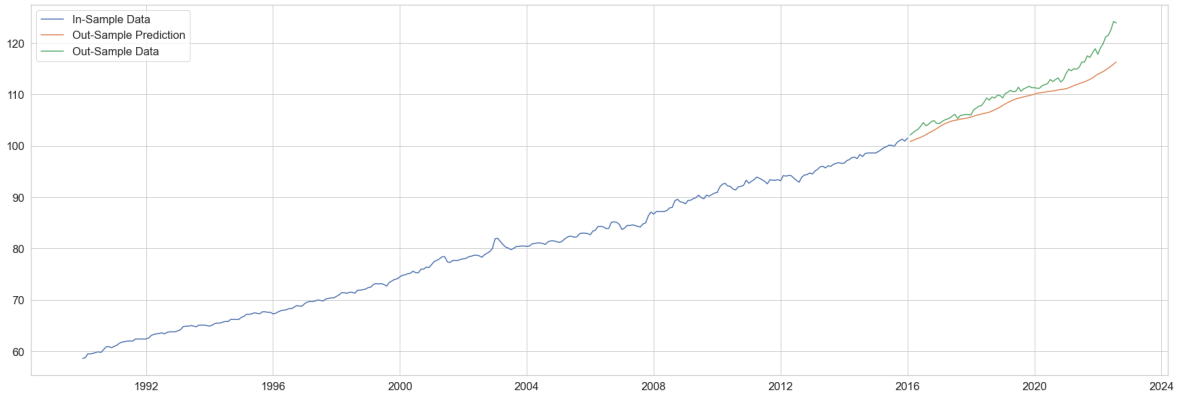


In [206…
```
preds = model.predict(test_input_tensor)
```

```
3/3 [==============================] - 0s 11ms/step
```

In [207…
```
preds = pd.Series(scaler_ts.inverse_transform(preds)[:, 0], name='Test Pr
```

In [208…
```
plt.figure(figsize=(30, 10))
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(preds.index, preds, label='Out-Sample Prediction')
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')
plt.legend()
plt.show()
```



In [209…
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.020931489512703537
MAE: 2.379853434502324
MSE: 8.596280592665035
RMSE: 2.931941437454888
R2: 0.7036948313580581
```

The results achieved, using a Simple univariate Recurrent Neural Network with a unique GRU Layer with a forecast horizon of 1 and a maximum delay of 12 month before, are not great compared with other predicting achieved previously.

Multivariate ML model

Now, it will be tested a ML model using the CPI index of US and the European Union as explanatory TS.

In [172…
```
explanatory_ts = ['United States', 'European Union']
```

In [173…
```
max_lag = 12
delays = range(1, max_lag + 1)
```

In [174…
```
# split and normalize Data
X_train, X_val = train_test_split(train_data, shuffle=False, test_size=0.

scaler = MinMaxScaler()

normalized_train = pd.DataFrame(scaler.fit_transform(X_train), columns=X_
normalized_val = pd.DataFrame(scaler.transform(X_val), columns=X_train.co
normalized_test = pd.DataFrame(scaler.transform(test_data), columns=X_tra
```

In [175…
```
# train data
train_delays = generate_IO_data(normalized_train[country], inData=normali
train_input = train_delays.iloc[:, 0:-1]
train_output = np.array([train_delays.iloc[:, -1].to_numpy()]).T
train_input_tensor = generate_tensor(train_input, max_lag, 3)
```

```
(254, 12, 3)
```

In [176…
```
# validation data
val = pd.concat([normalized_train.iloc[len(normalized_train)-max_lag:], n
val_delays = generate_IO_data(val[country], inData=val[explanatory_ts], d
val_input = val_delays.iloc[:, 0:-1]
val_output = np.array([val_delays.iloc[:, -1].to_numpy()]).T
val_input_tensor = generate_tensor(val_input, max_lag, 3)
```

```
(47, 12, 3)
```

In [177…
```
test = pd.concat([normalized_val.iloc[len(normalized_val)-max_lag:], norm
test_delays = generate_IO_data(test[country], inData=test[explanatory_ts]
test_input = test_delays.iloc[:, 0:-1]
test_output = np.array([test_delays.iloc[:, -1].to_numpy()]).T
test_input_tensor = generate_tensor(test_input, max_lag, 3)
```

```
(79, 12, 3)
```

In [178…
```
LATENT_DIM = 50
BATCH_SIZE = 1
```

```
EPOCHS = 100
HORIZON = 1
```

In [179…
```
model = Sequential()
model.add(layers.GRU(LATENT_DIM, input_shape=(max_lag, 3)))
model.add(layers.Dense(HORIZON))
```

In [180…
```
model.compile(optimizer='Adam', loss='mse')
model.summary()
```

```
Model: "sequential_12"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 gru_9 (GRU)                 (None, 50)                8250

 dense_12 (Dense)            (None, 1)                 51

=================================================================
Total params: 8301 (32.43 KB)
Trainable params: 8301 (32.43 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

In [181…
```
early_stopping = EarlyStopping(
    monitor='val_loss',
    patience = 15,
    restore_best_weights=True
)
```

In [184…
```
history = model.fit(
    train_input_tensor,
    train_output,
    batch_size = BATCH_SIZE,
    epochs = EPOCHS,
    validation_data = (val_input_tensor, val_output),
    callbacks = [early_stopping],
    verbose=1
)
```

```
Epoch 1/100
254/254 [==============================] - 4s 11ms/step - loss: 0.0197 - v
al_loss: 0.0012
Epoch 2/100
254/254 [==============================] - 3s 10ms/step - loss: 6.4254e-04
- val_loss: 8.4700e-04
Epoch 3/100
254/254 [==============================] - 2s 10ms/step - loss: 8.3872e-04
- val_loss: 0.0017
Epoch 4/100
254/254 [==============================] - 2s 10ms/step - loss: 6.2376e-04
- val_loss: 8.1190e-04
Epoch 5/100
254/254 [==============================] - 2s 10ms/step - loss: 5.3990e-04
- val_loss: 0.0018
Epoch 6/100
254/254 [==============================] - 2s 10ms/step - loss: 7.0921e-04
- val_loss: 0.0072
Epoch 7/100
254/254 [==============================] - 2s 10ms/step - loss: 8.0184e-04
- val_loss: 9.1442e-04
Epoch 8/100
254/254 [==============================] - 2s 10ms/step - loss: 7.6897e-04
- val_loss: 7.8288e-04
Epoch 9/100
254/254 [==============================] - 2s 10ms/step - loss: 5.4944e-04
- val_loss: 0.0019
Epoch 10/100
254/254 [==============================] - 2s 10ms/step - loss: 6.6887e-04
- val_loss: 6.4925e-04
Epoch 11/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6347e-04
- val_loss: 6.0676e-04
Epoch 12/100
254/254 [==============================] - 2s 10ms/step - loss: 7.0973e-04
- val_loss: 9.5522e-04
Epoch 13/100
254/254 [==============================] - 2s 10ms/step - loss: 6.6618e-04
- val_loss: 0.0028
Epoch 14/100
254/254 [==============================] - 3s 10ms/step - loss: 6.0271e-04
- val_loss: 8.7220e-04
Epoch 15/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6071e-04
- val_loss: 0.0011
Epoch 16/100
254/254 [==============================] - 2s 10ms/step - loss: 5.8379e-04
- val_loss: 6.4532e-04
Epoch 17/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6607e-04
- val_loss: 5.6260e-04
Epoch 18/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6402e-04
- val_loss: 6.7845e-04
Epoch 19/100
254/254 [==============================] - 2s 10ms/step - loss: 6.1143e-04
- val_loss: 0.0015
Epoch 20/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6504e-04
- val_loss: 0.0029
```

```
Epoch 21/100
254/254 [==============================] - 2s 10ms/step - loss: 6.1914e-04
- val_loss: 6.8115e-04
Epoch 22/100
254/254 [==============================] - 2s 10ms/step - loss: 6.8850e-04
- val_loss: 6.4603e-04
Epoch 23/100
254/254 [==============================] - 2s 10ms/step - loss: 4.7427e-04
- val_loss: 6.1050e-04
Epoch 24/100
254/254 [==============================] - 2s 10ms/step - loss: 5.4315e-04
- val_loss: 0.0013
Epoch 25/100
254/254 [==============================] - 2s 10ms/step - loss: 6.1517e-04
- val_loss: 0.0048
Epoch 26/100
254/254 [==============================] - 2s 10ms/step - loss: 5.1808e-04
- val_loss: 5.9256e-04
Epoch 27/100
254/254 [==============================] - 2s 10ms/step - loss: 4.6616e-04
- val_loss: 6.9822e-04
Epoch 28/100
254/254 [==============================] - 2s 10ms/step - loss: 4.5259e-04
- val_loss: 0.0014
Epoch 29/100
254/254 [==============================] - 2s 10ms/step - loss: 5.6962e-04
- val_loss: 9.6082e-04
Epoch 30/100
254/254 [==============================] - 2s 10ms/step - loss: 5.4052e-04
- val_loss: 9.7694e-04
Epoch 31/100
254/254 [==============================] - 2s 10ms/step - loss: 5.4709e-04
- val_loss: 6.3888e-04
Epoch 32/100
254/254 [==============================] - 2s 10ms/step - loss: 4.5832e-04
- val_loss: 4.7022e-04
Epoch 33/100
254/254 [==============================] - 3s 10ms/step - loss: 4.4991e-04
- val_loss: 5.5736e-04
Epoch 34/100
254/254 [==============================] - 2s 10ms/step - loss: 4.9404e-04
- val_loss: 0.0038
Epoch 35/100
254/254 [==============================] - 2s 10ms/step - loss: 4.4211e-04
- val_loss: 5.4132e-04
Epoch 36/100
254/254 [==============================] - 2s 10ms/step - loss: 5.5109e-04
- val_loss: 0.0052
Epoch 37/100
254/254 [==============================] - 3s 10ms/step - loss: 5.7319e-04
- val_loss: 5.3898e-04
Epoch 38/100
254/254 [==============================] - 3s 10ms/step - loss: 3.9001e-04
- val_loss: 8.2938e-04
Epoch 39/100
254/254 [==============================] - 3s 10ms/step - loss: 4.3704e-04
- val_loss: 4.0845e-04
Epoch 40/100
254/254 [==============================] - 2s 10ms/step - loss: 4.2547e-04
- val_loss: 3.4490e-04
```

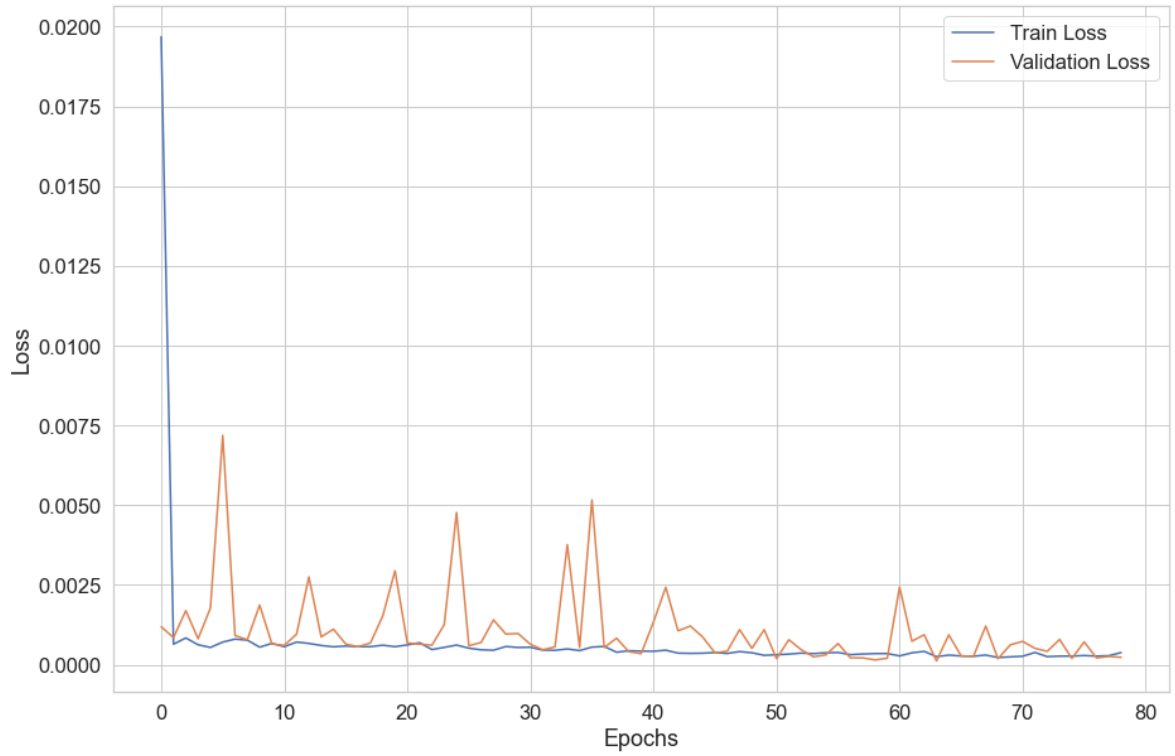
```
Epoch 41/100
254/254 [==============================] - 2s 10ms/step - loss: 4.2081e-04
- val_loss: 0.0013
Epoch 42/100
254/254 [==============================] - 2s 10ms/step - loss: 4.5432e-04
- val_loss: 0.0024
Epoch 43/100
254/254 [==============================] - 2s 10ms/step - loss: 3.6195e-04
- val_loss: 0.0011
Epoch 44/100
254/254 [==============================] - 2s 10ms/step - loss: 3.5477e-04
- val_loss: 0.0012
Epoch 45/100
254/254 [==============================] - 2s 10ms/step - loss: 3.5987e-04
- val_loss: 8.6725e-04
Epoch 46/100
254/254 [==============================] - 2s 10ms/step - loss: 3.8031e-04
- val_loss: 3.7071e-04
Epoch 47/100
254/254 [==============================] - 2s 10ms/step - loss: 3.5215e-04
- val_loss: 4.2701e-04
Epoch 48/100
254/254 [==============================] - 2s 10ms/step - loss: 4.1081e-04
- val_loss: 0.0011
Epoch 49/100
254/254 [==============================] - 2s 10ms/step - loss: 3.7379e-04
- val_loss: 5.1225e-04
Epoch 50/100
254/254 [==============================] - 2s 10ms/step - loss: 2.9273e-04
- val_loss: 0.0011
Epoch 51/100
254/254 [==============================] - 2s 10ms/step - loss: 3.1080e-04
- val_loss: 1.8698e-04
Epoch 52/100
254/254 [==============================] - 2s 10ms/step - loss: 3.3172e-04
- val_loss: 7.8124e-04
Epoch 53/100
254/254 [==============================] - 2s 10ms/step - loss: 3.6281e-04
- val_loss: 4.7111e-04
Epoch 54/100
254/254 [==============================] - 2s 10ms/step - loss: 3.4652e-04
- val_loss: 2.4729e-04
Epoch 55/100
254/254 [==============================] - 2s 10ms/step - loss: 3.7173e-04
- val_loss: 3.0391e-04
Epoch 56/100
254/254 [==============================] - 2s 10ms/step - loss: 3.8229e-04
- val_loss: 6.6397e-04
Epoch 57/100
254/254 [==============================] - 2s 10ms/step - loss: 3.1505e-04
- val_loss: 2.1545e-04
Epoch 58/100
254/254 [==============================] - 2s 10ms/step - loss: 3.3460e-04
- val_loss: 2.1349e-04
Epoch 59/100
254/254 [==============================] - 2s 10ms/step - loss: 3.4789e-04
- val_loss: 1.4872e-04
Epoch 60/100
254/254 [==============================] - 2s 10ms/step - loss: 3.4867e-04
- val_loss: 2.0224e-04
```

```
Epoch 61/100
254/254 [==============================] - 2s 10ms/step - loss: 2.7438e-04
- val_loss: 0.0024
Epoch 62/100
254/254 [==============================] - 2s 10ms/step - loss: 3.7133e-04
- val_loss: 7.3615e-04
Epoch 63/100
254/254 [==============================] - 3s 10ms/step - loss: 4.1686e-04
- val_loss: 9.3811e-04
Epoch 64/100
254/254 [==============================] - 2s 10ms/step - loss: 2.5158e-04
- val_loss: 1.1723e-04
Epoch 65/100
254/254 [==============================] - 2s 10ms/step - loss: 3.0029e-04
- val_loss: 9.3490e-04
Epoch 66/100
254/254 [==============================] - 2s 10ms/step - loss: 2.7192e-04
- val_loss: 2.7546e-04
Epoch 67/100
254/254 [==============================] - 2s 10ms/step - loss: 2.6460e-04
- val_loss: 2.6049e-04
Epoch 68/100
254/254 [==============================] - 2s 10ms/step - loss: 3.0156e-04
- val_loss: 0.0012
Epoch 69/100
254/254 [==============================] - 3s 10ms/step - loss: 2.2515e-04
- val_loss: 1.8587e-04
Epoch 70/100
254/254 [==============================] - 2s 10ms/step - loss: 2.4534e-04
- val_loss: 6.2454e-04
Epoch 71/100
254/254 [==============================] - 2s 10ms/step - loss: 2.6437e-04
- val_loss: 7.2971e-04
Epoch 72/100
254/254 [==============================] - 2s 10ms/step - loss: 3.8333e-04
- val_loss: 5.1298e-04
Epoch 73/100
254/254 [==============================] - 2s 10ms/step - loss: 2.5208e-04
- val_loss: 4.1891e-04
Epoch 74/100
254/254 [==============================] - 2s 10ms/step - loss: 2.6757e-04
- val_loss: 7.9112e-04
Epoch 75/100
254/254 [==============================] - 2s 10ms/step - loss: 2.6971e-04
- val_loss: 1.9634e-04
Epoch 76/100
254/254 [==============================] - 2s 10ms/step - loss: 2.8749e-04
- val_loss: 7.1381e-04
Epoch 77/100
254/254 [==============================] - 2s 10ms/step - loss: 2.6882e-04
- val_loss: 2.1240e-04
Epoch 78/100
254/254 [==============================] - 2s 10ms/step - loss: 2.8121e-04
- val_loss: 2.5987e-04
Epoch 79/100
254/254 [==============================] - 2s 10ms/step - loss: 3.7946e-04
- val_loss: 2.3069e-04
```

In [185…

```
plt.figure(figsize=(15, 10))
plt.plot(history.history['loss'], label='Train Loss')
```

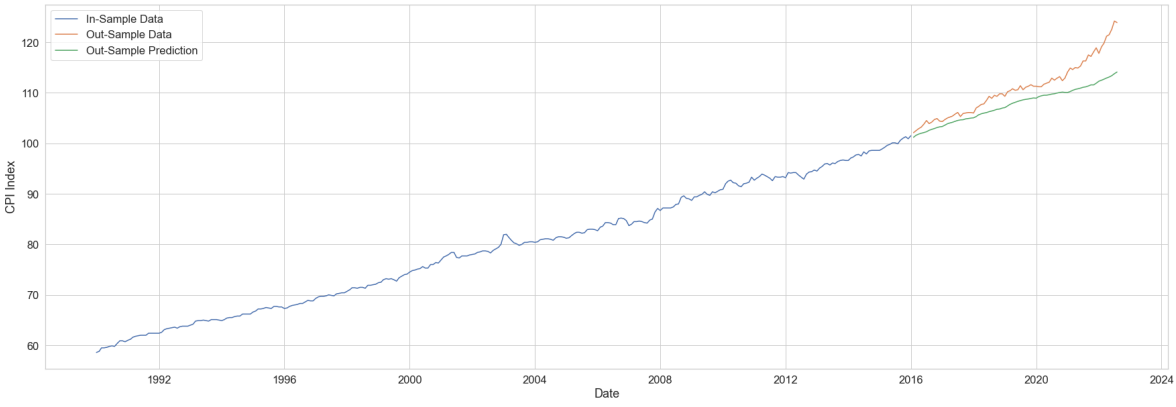
```
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.legend()
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.show()
```



In [186…
```
preds = pd.Series(scaler_ts.inverse_transform(model.predict(test_input_te
```

```
3/3 [==============================] - 0s 14ms/step
```

In [187…
```
plt.figure(figsize=(30, 10))
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')
plt.plot(preds.index, preds, label='Out-Sample Prediction')
plt.legend()
plt.ylabel('CPI Index')
plt.xlabel('Date')
plt.show()
```



In [188…
```
metrics_summary(ts_test, preds)
```

```
MAPE: 0.027057776380951734
MAE: 3.0851858404618278
MSE: 14.418693655857734
RMSE: 3.7971954987671803
R2: 0.503002093842669
```

Train ML models on stationary data

In [111…
```
inputs = df[['Norway' ,'United States', 'European Union']].copy()
```

In [112…
```
for col in inputs.columns:
    inputs[col] = inputs[col].diff().diff(period)
```

In [113…
```
train_multi_data = inputs[:split_index]
test_multi_data = inputs[split_index:]
```

In [114…
```
train_multi_data = pd.DataFrame(train_multi_data.loc[train_multi_data.isn
```

In [115…
```
x_train, x_val = train_test_split(train_multi_data, shuffle=False, test_s
```

In [116…
```
x_train_ts = x_train[country]
x_val_ts = x_val[country]
x_test_ts = test_multi_data[country]
```

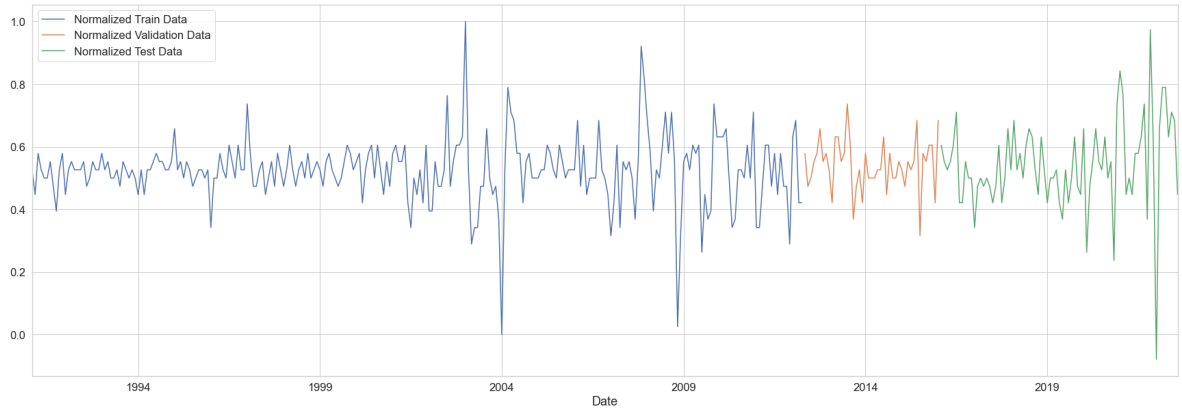
In [117…
```
scaler_ts = MinMaxScaler()
normalized_train = pd.Series(scaler_ts.fit_transform(x_train_ts.values.re
normalized_train.name = 'Normalized Train Data'
normalized_val = pd.Series(scaler_ts.transform(x_val_ts.values.reshape(-1
normalized_val.name = 'Normalized Validation Data'
normalized_test = pd.Series(scaler_ts.transform(x_test_ts.values.reshape(
normalized_test.name = 'Normalized Test Data'
```

In [65]:
```
normalized_train
```

Out[65]:
```
Date
1991-02-01    0.526316
1991-03-01    0.447368
1991-04-01    0.578947
1991-05-01    0.526316
1991-06-01    0.500000
                ...
2011-12-01    0.289474
2012-01-01    0.631579
2012-02-01    0.684211
2012-03-01    0.421053
2012-04-01    0.421053
Name: Normalized Train Data, Length: 255, dtype: float64
```

In [67]:
```
fig, ax = plt.subplots(figsize=(30, 10))
normalized_train.plot(ax=ax, legend=True)
normalized_val.plot(ax=ax, legend=True)
normalized_test.plot(ax=ax, legend=True)
```

Out[67]:
```
<Axes: xlabel='Date'>
```



In [68]:
```
max_lag = 12
delays = range(1, max_lag+1)
```

In [72]:
```
# train data
train_delays = generate_IO_data(normalized_train, delayEndog=delays)
train_input = train_delays.iloc[:, 0:-1]
train_output = np.array([train_delays.iloc[:, -1].to_numpy()]).T
train_input_tensor = generate_tensor(train_input, max_lag, 1)
```

```
(243, 12, 1)
```

In [73]:
```
# validation data
val = pd.concat([normalized_train.iloc[len(normalized_train)-max_lag:], n
val.name = 'Validation'
val_delays = generate_IO_data(val, delayEndog=delays)
val_input = val_delays.iloc[:, 0:-1]
val_output = np.array([val_delays.iloc[:, -1].to_numpy()]).T
val_input_tensor = generate_tensor(val_input, max_lag, 1)
```

```
(45, 12, 1)
```

In [74]:
```
test = pd.concat([normalized_val.iloc[len(normalized_val)-max_lag:], norm
test.name = 'Validation'
test_delays = generate_IO_data(test, delayEndog=delays)
test_input = test_delays.iloc[:, 0:-1]
test_output = np.array([test_delays.iloc[:, -1].to_numpy()]).T
test_input_tensor = generate_tensor(test_input, max_lag, 1)
```

```
(79, 12, 1)
```

In [75]:
```
LATENT_DIM = 50
BATCH_SIZE = 1
EPOCHS = 100
HORIZON = 1
```

In [76]:
```
model = Sequential()
model.add(layers.GRU(LATENT_DIM, input_shape=(max_lag, 1)))
model.add(layers.Dense(HORIZON))

model.compile(optimizer='RMSprop', loss='mse')
model.summary()
```

```
2023-12-07 10:04:55.895451: I metal_plugin/src/device/metal_device.cc:115
4] Metal device set to: Apple M2 Max
2023-12-07 10:04:55.895479: I metal_plugin/src/device/metal_device.cc:296]
systemMemory: 32.00 GB
2023-12-07 10:04:55.895485: I metal_plugin/src/device/metal_device.cc:313]
maxCacheSize: 10.67 GB
2023-12-07 10:04:55.895555: I tensorflow/core/common_runtime/pluggable_dev
ice/pluggable_device_factory.cc:306] Could not identify NUMA node of platf
orm GPU ID 0, defaulting to 0. Your kernel may not have been built with NU
MA support.
2023-12-07 10:04:55.895807: I tensorflow/core/common_runtime/pluggable_dev
ice/pluggable_device_factory.cc:272] Created TensorFlow device (/job:local
host/replica:0/task:0/device:GPU:0 with 0 MB memory) -> physical Pluggable
Device (device: 0, name: METAL, pci bus id: <undefined>)
```

```
Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 gru (GRU)                   (None, 50)                7950

 dense (Dense)               (None, 1)                 51

=================================================================
Total params: 8001 (31.25 KB)
Trainable params: 8001 (31.25 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

In [77]:
```
early_stopping = EarlyStopping(
    monitor='val_loss',
    patience=15,
    restore_best_weights=True)
```

In [78]:
```
history = model.fit(
    train_input_tensor,
    train_output,
    batch_size = BATCH_SIZE,
    epochs = EPOCHS,
    validation_data = (val_input_tensor, val_output),
    callbacks = [early_stopping],
    verbose = 1
)
```

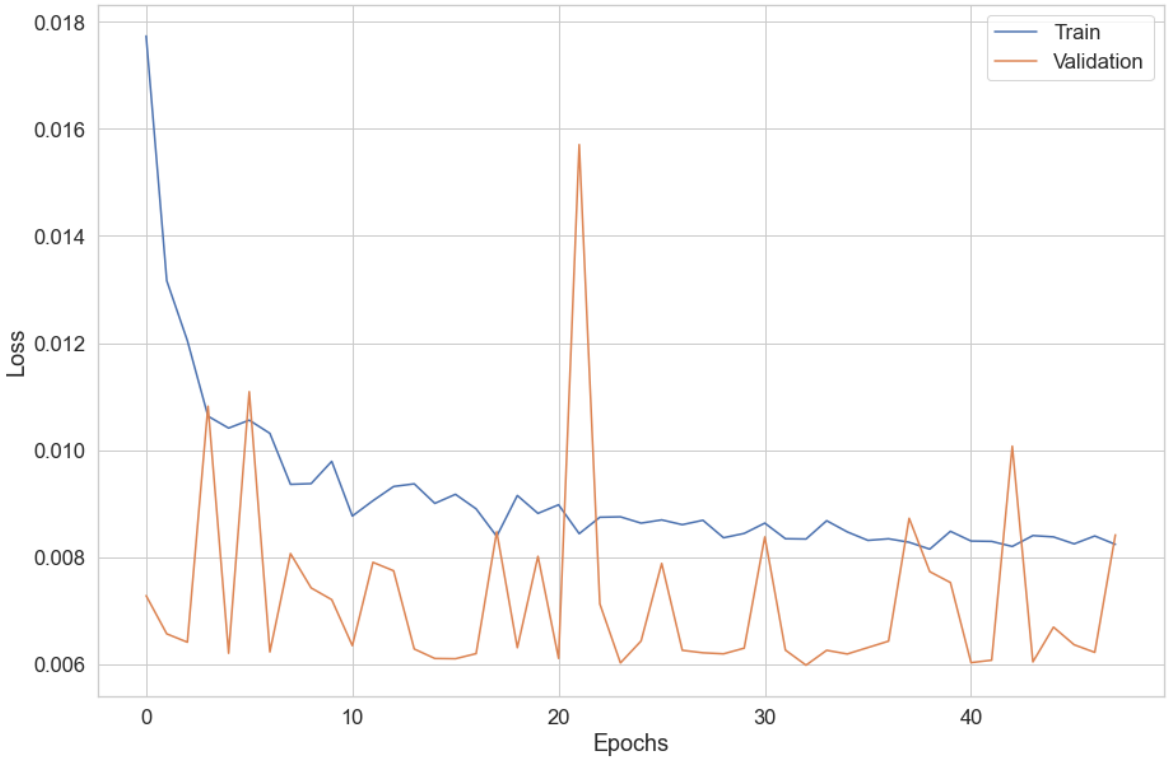
```
Epoch 1/100
2023-12-07 10:05:21.404556: I tensorflow/core/grappler/optimizers/custom_g
raph_optimizer_registry.cc:117] Plugin optimizer for device_type GPU is en
abled.
```

```
243/243 [==============================] - 4s 12ms/step - loss: 0.0177 - v
al_loss: 0.0073
Epoch 2/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0132 - v
al_loss: 0.0066
Epoch 3/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0120 - v
al_loss: 0.0064
Epoch 4/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0106 - v
al_loss: 0.0108
Epoch 5/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0104 - v
al_loss: 0.0062
Epoch 6/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0106 - v
al_loss: 0.0111
Epoch 7/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0103 - v
al_loss: 0.0062
Epoch 8/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0094 - v
al_loss: 0.0081
Epoch 9/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0094 - v
al_loss: 0.0074
Epoch 10/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0098 - v
al_loss: 0.0072
Epoch 11/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0088 - v
al_loss: 0.0063
Epoch 12/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0091 - v
al_loss: 0.0079
Epoch 13/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0093 - v
al_loss: 0.0077
Epoch 14/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0094 - v
al_loss: 0.0063
Epoch 15/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0090 - v
al_loss: 0.0061
Epoch 16/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0092 - v
al_loss: 0.0061
Epoch 17/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0089 - v
al_loss: 0.0062
Epoch 18/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0085
Epoch 19/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0092 - v
al_loss: 0.0063
Epoch 20/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0088 - v
al_loss: 0.0080
Epoch 21/100
```

```
243/243 [==============================] - 3s 11ms/step - loss: 0.0090 - v
al_loss: 0.0061
Epoch 22/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0084 - v
al_loss: 0.0157
Epoch 23/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0087 - v
al_loss: 0.0071
Epoch 24/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0088 - v
al_loss: 0.0060
Epoch 25/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0086 - v
al_loss: 0.0064
Epoch 26/100
243/243 [==============================] - 3s 10ms/step - loss: 0.0087 - v
al_loss: 0.0079
Epoch 27/100
243/243 [==============================] - 3s 10ms/step - loss: 0.0086 - v
al_loss: 0.0063
Epoch 28/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0087 - v
al_loss: 0.0062
Epoch 29/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0084 - v
al_loss: 0.0062
Epoch 30/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0084 - v
al_loss: 0.0063
Epoch 31/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0086 - v
al_loss: 0.0084
Epoch 32/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0063
Epoch 33/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0060
Epoch 34/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0087 - v
al_loss: 0.0063
Epoch 35/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0085 - v
al_loss: 0.0062
Epoch 36/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0063
Epoch 37/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0064
Epoch 38/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0087
Epoch 39/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0077
Epoch 40/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0085 - v
al_loss: 0.0075
Epoch 41/100
```

```
243/243 [==============================] - 3s 12ms/step - loss: 0.0083 - v
al_loss: 0.0060
Epoch 42/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0061
Epoch 43/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0101
Epoch 44/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0060
Epoch 45/100
243/243 [==============================] - 3s 10ms/step - loss: 0.0084 - v
al_loss: 0.0067
Epoch 46/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0064
Epoch 47/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0062
Epoch 48/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0084
```

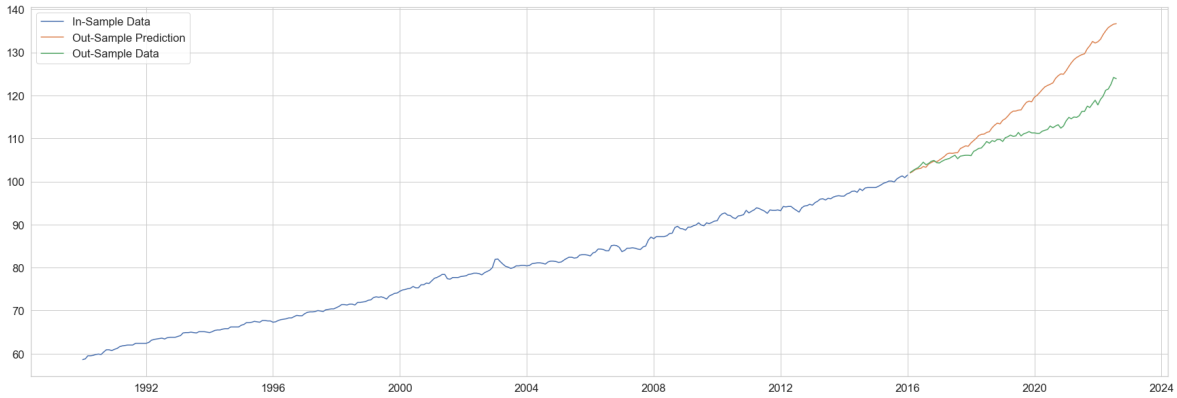
In [79]:
```
plt.figure(figsize=(15, 10))
plt.plot(history.history['loss'], label='Train')
plt.plot(history.history['val_loss'], label='Validation')
plt.legend()
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.show()
```



In [80]:
```
preds = model.predict(test_input_tensor)
preds = pd.Series(scaler_ts.inverse_transform(preds)[:, 0], name='Test Pr
reversed_preds = reverse_diff(ts, preds, split_index, period)
```

```
3/3 [==============================] - 0s 15ms/step
```

In [81]:
```
plt.figure(figsize=(30, 10))
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(reversed_preds.index, reversed_preds, label='Out-Sample Predicti
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')
plt.legend()
plt.show()
```



In [82]:
```
metrics_summary(ts_test, reversed_preds)
```

```
MAPE: 0.05731746090691856
MAE: 6.5597349508866944
MSE: 69.52752110847014
RMSE: 8.338316443291784
R2: -1.3965439058469058
```

Compared to the metrics achieved by the same model for non-stationary data the results indicate that trying to use stationary data as inputs to the NN leads to worst results, i.e, remove the trend and seasonality removed important information that the model could be using.

Multivariate ML prediction with stationary data

In [94]:
```
explanatory_ts = ['United States', 'European Union']
```

In [84]:
```
inputs = df[['Norway' ,'United States', 'European Union']].copy()
```

In [85]:
```
for col in inputs.columns:
    inputs[col] = inputs[col].diff().diff(period)
```

In [86]:
```
train_multi_data = inputs[:split_index]
test_multi_data = inputs[split_index:]
```

In [87]:
```
train_multi_data = pd.DataFrame(train_multi_data.loc[train_multi_data.isn
```

In [88]:
```
x_train, x_val = train_test_split(train_multi_data, shuffle=False, test_s
```

In [92]:
```
scaler = MinMaxScaler()
normalized_train = pd.DataFrame(scaler.fit_transform(x_train), columns=x_
normalized_val = pd.DataFrame(scaler.transform(x_val), columns=x_train.co
normalized_test = pd.DataFrame(scaler.transform(test_multi_data), columns
```

In [95]:
```
# train data
train_delays = generate_IO_data(normalized_train[country], inData=normali
```

```
train_input = train_delays.iloc[:, 0:-1]
train_output = np.array([train_delays.iloc[:, -1].to_numpy()]).T
train_input_tensor = generate_tensor(train_input, max_lag, 3)
```

```
(243, 12, 3)
```

In [96]:
```
# validation data
val = pd.concat([normalized_train.iloc[len(normalized_train)-max_lag:], n
val_delays = generate_IO_data(val[country], inData=val[explanatory_ts], d
val_input = val_delays.iloc[:, 0:-1]
val_output = np.array([val_delays.iloc[:, -1].to_numpy()]).T
val_input_tensor = generate_tensor(val_input, max_lag, 3)
```

```
(45, 12, 3)
```

In [97]:
```
test = pd.concat([normalized_val.iloc[len(normalized_val)-max_lag:], norm
test_delays = generate_IO_data(test[country], inData=test[explanatory_ts]
test_input = test_delays.iloc[:, 0:-1]
test_output = np.array([test_delays.iloc[:, -1].to_numpy()]).T
test_input_tensor = generate_tensor(test_input, max_lag, 3)
```

```
(79, 12, 3)
```

In [98]:
```
LATENT_DIM = 50
BATCH_SIZE = 1
EPOCHS = 100
HORIZON = 1
```

In [99]:
```
model = Sequential()
model.add(layers.GRU(LATENT_DIM, input_shape=(max_lag, 3)))
model.add(layers.Dense(HORIZON))
```

In [100…
```
model.compile(optimizer='RMSprop', loss='mse')
model.summary()
```

```
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 gru_1 (GRU)                 (None, 50)                8250

 dense_1 (Dense)             (None, 1)                 51

=================================================================
Total params: 8301 (32.43 KB)
Trainable params: 8301 (32.43 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

In [101…
```
early_stopping = EarlyStopping(
    monitor='val_loss',
    patience = 15,
    restore_best_weights=True
)
```

In [102…
```
history = model.fit(
    train_input_tensor,
    train_output,
    batch_size = BATCH_SIZE,
    epochs = EPOCHS,
    validation_data = (val_input_tensor, val_output),
```

```
    callbacks = [early_stopping],
    verbose=1
)
```

```
Epoch 1/100
243/243 [==============================] - 4s 13ms/step - loss: 0.0183 - v
al_loss: 0.0110
Epoch 2/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0146 - v
al_loss: 0.0091
Epoch 3/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0132 - v
al_loss: 0.0123
Epoch 4/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0119 - v
al_loss: 0.0063
Epoch 5/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0110 - v
al_loss: 0.0062
Epoch 6/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0106 - v
al_loss: 0.0083
Epoch 7/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0102 - v
al_loss: 0.0072
Epoch 8/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0100 - v
al_loss: 0.0072
Epoch 9/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0097 - v
al_loss: 0.0097
Epoch 10/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0095 - v
al_loss: 0.0064
Epoch 11/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0096 - v
al_loss: 0.0072
Epoch 12/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0090 - v
al_loss: 0.0059
Epoch 13/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0091 - v
al_loss: 0.0061
Epoch 14/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0086 - v
al_loss: 0.0068
Epoch 15/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0087 - v
al_loss: 0.0083
Epoch 16/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0088 - v
al_loss: 0.0058
Epoch 17/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0088 - v
al_loss: 0.0059
Epoch 18/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0087 - v
al_loss: 0.0079
Epoch 19/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0088 - v
al_loss: 0.0077
Epoch 20/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0089 - v
al_loss: 0.0068
```

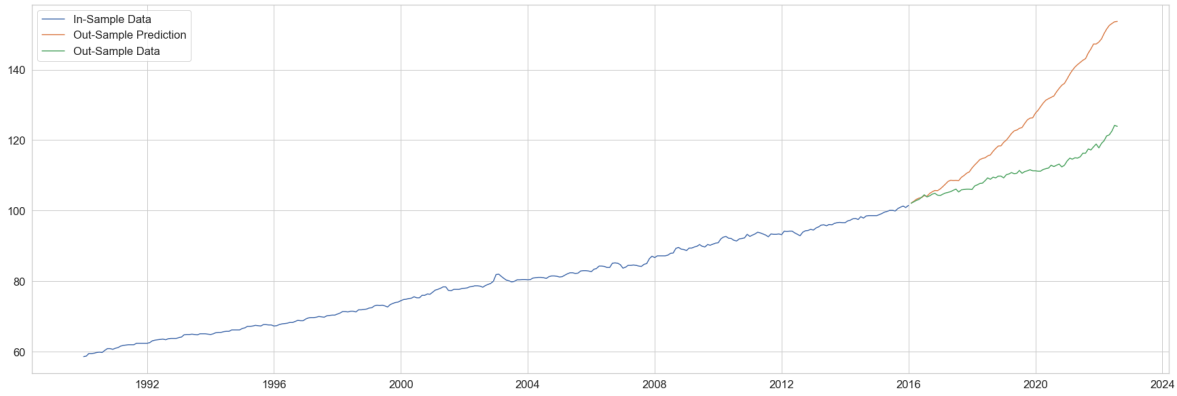
```
Epoch 21/100
243/243 [==============================] - 3s 12ms/step - loss: 0.0079 - v
al_loss: 0.0067
Epoch 22/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0086 - v
al_loss: 0.0061
Epoch 23/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0087 - v
al_loss: 0.0077
Epoch 24/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0056
Epoch 25/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0083 - v
al_loss: 0.0058
Epoch 26/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0056
Epoch 27/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0080 - v
al_loss: 0.0090
Epoch 28/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0059
Epoch 29/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0073
Epoch 30/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0079 - v
al_loss: 0.0083
Epoch 31/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0084 - v
al_loss: 0.0056
Epoch 32/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0081 - v
al_loss: 0.0066
Epoch 33/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0079 - v
al_loss: 0.0079
Epoch 34/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0081 - v
al_loss: 0.0063
Epoch 35/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0081 - v
al_loss: 0.0132
Epoch 36/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0081 - v
al_loss: 0.0093
Epoch 37/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0080 - v
al_loss: 0.0060
Epoch 38/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0075 - v
al_loss: 0.0109
Epoch 39/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0076 - v
al_loss: 0.0079
Epoch 40/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0080 - v
al_loss: 0.0057
```

```
Epoch 41/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0082 - v
al_loss: 0.0056
Epoch 42/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0078 - v
al_loss: 0.0061
Epoch 43/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0080 - v
al_loss: 0.0079
Epoch 44/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0078 - v
al_loss: 0.0075
Epoch 45/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0075 - v
al_loss: 0.0065
Epoch 46/100
243/243 [==============================] - 3s 11ms/step - loss: 0.0078 - v
al_loss: 0.0082
```

In [119…
```
preds = model.predict(test_input_tensor)
preds = pd.Series(scaler_ts.inverse_transform(preds)[:, 0], index=ts_test
reversed_preds = reverse_diff(ts, preds, split_index, period)
```

```
3/3 [==============================] - 0s 6ms/step
```

In [120…
```
plt.figure(figsize=(30, 10))
plt.plot(ts.index, ts, label='In-Sample Data')
plt.plot(reversed_preds.index, reversed_preds, label='Out-Sample Predicti
plt.plot(ts_test.index, ts_test, label='Out-Sample Data')
plt.legend()
plt.show()
```



In [121…
```
metrics_summary(ts_test, reversed_preds)
```

```
MAPE: 0.11858862465748707
MAE: 13.575607798291133
MSE: 292.05823404795706
RMSE: 17.089711350633078
R2: -9.066954348452562
```

Similar to the univariate ML model for the stationary data, the results indicate that removing the trend and seasonality components of the data also removes important information that the model should be using, leading to worst results than the obtained previously.