CPI index analysis

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In [227…
```
import pandas as pd
import matplotlib.pyplot as plt
import os
import numpy as np
import scipy.signal as scs
import seaborn as sns
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import statsmodels.tsa.stattools as st
from sklearn.model_selection import train_test_split

from statsmodels.tsa.stattools import acf, pacf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima.model import ARIMA

from utils import *
```

In [228…
```
# disable warnings
import warnings
warnings.filterwarnings('ignore')
```

Introduction

This notebook presents an analysis of the CPI Index from Norway from 1990 until the most recent update (August, 2022).

NOTE: Throughout the analysis, a significance level of 95% was chosen for all the tests developed.

Read data

In [229…
```
df = pd.read_csv(os.path.join('CPITimeSeries', 'time_series_data.csv'), i
```

In [230…
```
df.index = pd.to_datetime(df.index)
```

In [231…
```
# select data from 1990 until now
df = df.loc[df.index >= '1990-01-01']
```

In [232…
```
df
```

Out[232…

Date	Brazil	France	Bulgaria	Honduras	Colombia	Canada	Côt d'Ivoir
1990-01-01	0.005411	66.42	NaN	NaN	5.967753	76.7	Nal
1990-02-01	0.009508	66.56	NaN	NaN	6.191108	77.2	Nal
1990-03-01	0.017343	66.72	NaN	NaN	6.372583	77.5	Nal
1990-04-01	0.020034	67.09	NaN	NaN	6.547079	77.5	Nal
1990-05-01	0.021555	67.19	NaN	NaN	6.679696	77.9	Nal
...	...	...	...	...	...	...	.
2022-04-01	6382.880000	110.97	8331.527222	384.2	117.708900	149.8	114.
2022-05-01	6412.880000	111.72	8432.652791	387.6	118.703200	151.9	114.
2022-06-01	6455.850000	112.55	8506.693988	392.7	119.305300	152.9	117.
2022-07-01	6411.950000	112.87	8601.912089	396.2	120.273600	153.1	117.
2022-08-01	NaN	113.29	8702.171328	396.1	121.502500	NaN	Nal

392 rows × 190 columns

Select the data from Norway

In [233…

```
country = 'Norway'
```

Time series overview

In [234…

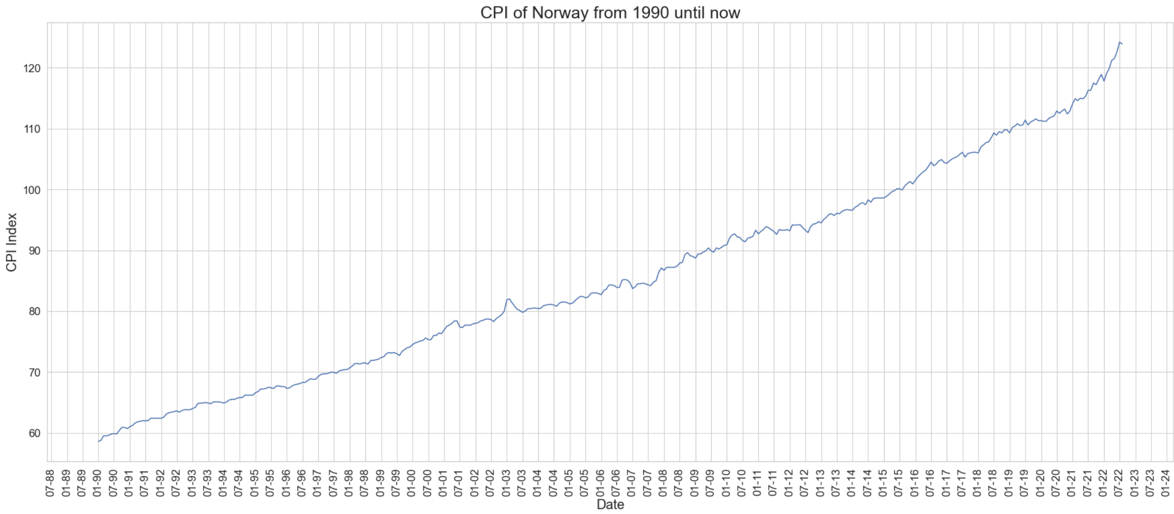
```
df.index
```

Out[234…

```
DatetimeIndex(['1990-01-01', '1990-02-01', '1990-03-01', '1990-04-01',
               '1990-05-01', '1990-06-01', '1990-07-01', '1990-08-01',
               '1990-09-01', '1990-10-01',
               ...
               '2021-11-01', '2021-12-01', '2022-01-01', '2022-02-01',
               '2022-03-01', '2022-04-01', '2022-05-01', '2022-06-01',
               '2022-07-01', '2022-08-01'],
              dtype='datetime64[ns]', name='Date', length=392, freq=Non
e)
```

In [235…

```
fig, ax = plt.subplots(figsize=(30, 12))
sns.set(font_scale=1.5, style="whitegrid")
plt.plot(df.index, df[country])

# Ensure a major tick for each week using (interval=1)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
date_form = DateFormatter("%m-%y")
ax.xaxis.set_major_formatter(date_form)

# Ensure a major tick for each week using (interval=1)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
plt.xlabel('Date', fontdict=dict(size=20))
plt.ylabel('CPI Index', fontdict=dict(size=20))
plt.title(f'CPI of {country} from 1990 until now', fontdict=dict(size=25)
plt.savefig(os.path.join('images', 'original-ts.png'))
plt.show()
```



In a first analysis of the time series (TS), it's clearly seen a increasing trend of the CPI values over time. On a closer look, the TS appears to have a repetition (seasonality) every 6 months. With the increasing trend and seasonality every 6-months, it can be concluded that the original TS is not stationary. Therefore, it should be transformed to become stationary in order to apply forecasting models.

From the TS, it can also be concluded that there is and rapid increasing in the last 2-3 years, which will impact the final results as this exponential increasing is just the final part of the TS

Divide the time series into train and test

In [236…

```
train_ratio = 0.8
split_index = int(len(df) * train_ratio)

# Split the time series data
train_data = df[:split_index]
test_data = df[split_index:]
months_dates = months = np.arange(len(df))
```

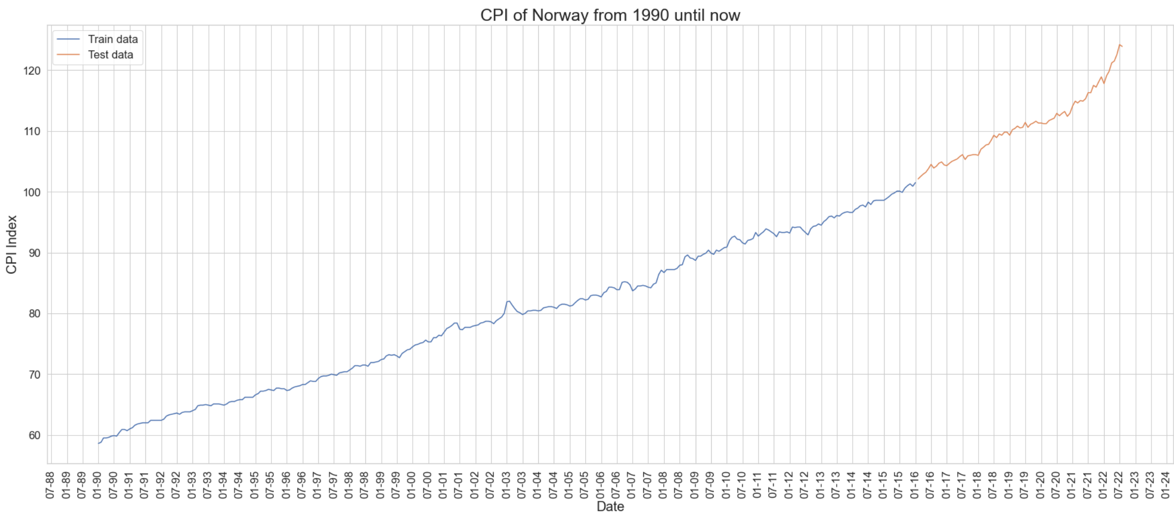
```
months = months_dates[:split_index]
months_test = months_dates[split_index:]
```

In [237…

```
ts = train_data[country]
ts_test = test_data[country]
```

In [238…

```
fig, ax = plt.subplots(figsize=(30, 12))
sns.set(font_scale=1.5, style="whitegrid")
plt.plot(ts.index, ts, label='Train data')
plt.plot(ts_test.index, ts_test, label='Test data')
plt.legend()
date_form = DateFormatter("%m-%y")
ax.xaxis.set_major_formatter(date_form)

# Ensure a major tick for each week using (interval=1)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
plt.xlabel('Date', fontdict=dict(size=20))
plt.ylabel('CPI Index', fontdict=dict(size=20))
plt.title(f'CPI of {country} from 1990 until now', fontdict=dict(size=25)
plt.savefig(os.path.join('images', 'ts-train-test.png'))
plt.show()
```



Check if it has missing values

In [239…

```
print('Null values [Train data] -', ts.isnull().any())
print('Null values [Test data] -', ts_test.isnull().any())
```

```
Null values [Train data] - False
Null values [Test data] - False
```

First look at stationary by statistical testing

The Dickey-Fuller (DF) unit root test is a statistical test that assesses the existence of this unit root. The test null and alternative hypotheses are:

- H0 ($\phi = 1$): a unit root is present in a time series sample (non-stationary TS)
- H1 ($\phi < 1$): a unit root is not present in a time series sample (stationary TS)

In [240…

```
adf_test(ts)
```

```
ADF Statistic: 0.998852
p-value: 0.994254
Critical Values:
	1%: -3.452
	5%: -2.871
	10%: -2.572
```

Since the p-value from the DF test is higher than 0.05, the null hypothesis cannot be rejected, meaning that the original TS is non-stationary.

Decomposition models

As the magnitude of the time series (TS) does not seem to change over time, it will be used an additive model

```
x(n) = tr(n) + sn(n) + e(n)
```

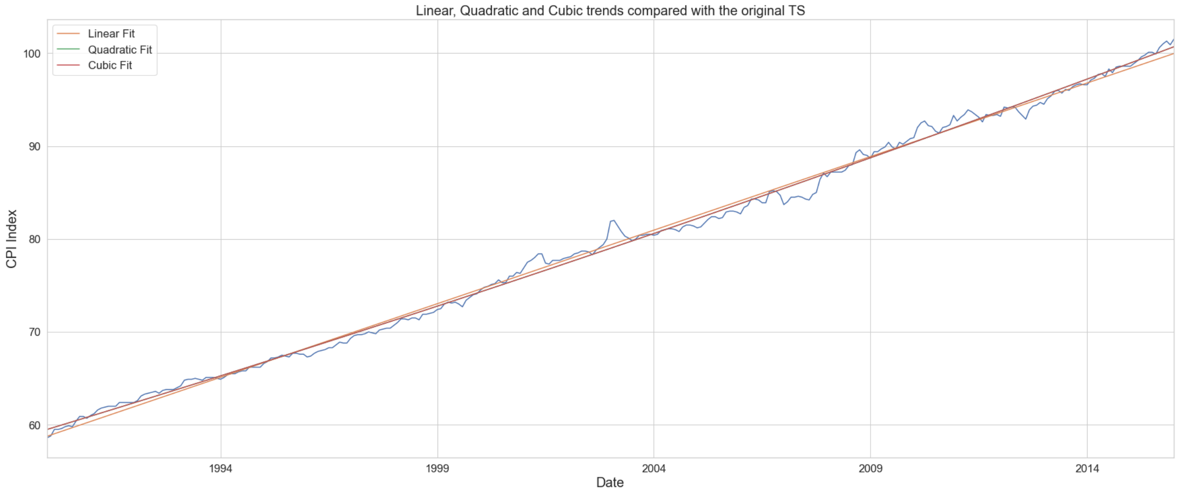
tr - trend component sn - seasonal component e = erratic component

Check trend

In [241…
```
T = 12 # period
```

In [242…
```
## trend
coefs_1d = np.polyfit(months, np.array(ts), deg=1)
values_1d = np.polyval(coefs_1d, months)
ts_1d = pd.Series(values_1d, index=ts.index, name='Linear Fit')

coefs_2d = np.polyfit(months, np.array(ts), deg=2)
values_2d = np.polyval(coefs_2d, months)
ts_2d = pd.Series(values_2d, index=ts.index, name='Quadratic Fit')

coefs_3d = np.polyfit(months, np.array(ts), deg=3)
values_3d = np.polyval(coefs_3d, months)
ts_3d = pd.Series(values_3d, index=ts.index, name='Cubic Fit')
```

In [243…
```
ax = ts.plot(figsize=(30, 12))
ts_1d.plot(ax=ax, legend=True)
ts_2d.plot(ax=ax, legend=True)
ts_3d.plot(ax=ax, legend=True)
ax.set_xlabel('Date', fontdict=dict(size=20))
ax.set_ylabel('CPI Index', fontdict=dict(size=20))
ax.set_title('Linear, Quadratic and Cubic trends compared with the origin
plt.savefig(os.path.join('images', 'trends-plot.png'))
plt.show()
```



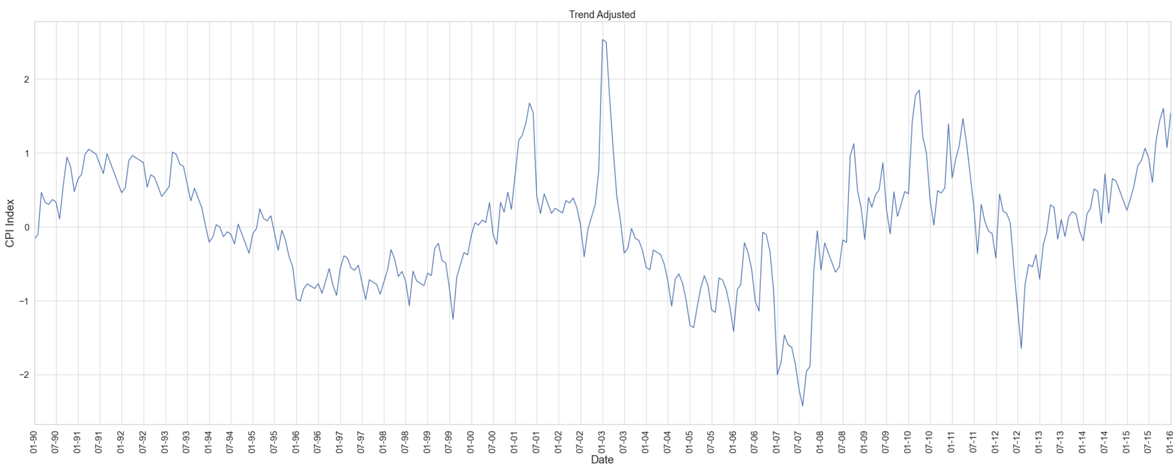
After analyzing the results of plotting the original time series (TS) against the linear, quadratic and cubic trends, it was observed that there isn't a great difference in them, so it was chosen the linear trend since it is simpler.

In [244…

```
trend = ts_1d
trend_adjusted = ts - trend
trend_adjusted.name = 'Trend Adjusted'
```

In [245…

```
fig, axs = plt.subplots(1,figsize=(30, 12))

axs.plot(trend_adjusted.index, trend_adjusted)
axs.set_ylabel('CPI Index')
axs.set_title('Trend Adjusted TS')
date_form = DateFormatter("%m-%y")
axs.xaxis.set_major_formatter(date_form)
axs.grid(True)
axs.grid(which='minor', alpha=0.3)
axs.grid(which='major', alpha=0.8)
axs.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
axs.set_xlim(ts.index[0], ts.index[-1])
plt.xticks(rotation='vertical')
plt.xlabel('Date', fontdict=dict(size=20))
plt.ylabel('CPI Index', fontdict=dict(size=20))
axs.set_title('Trend Adjusted')
plt.savefig(os.path.join('images', 'trend-adjusted.png'))
plt.tight_layout()
plt.show()
```



After adjusting the TS to the trend, the seasonal patterns appear more clearly. Each 6-month interval contains a peak more or less in the middle. At the same time, it can be seen a bigger pattern that repeat each +-7 years.

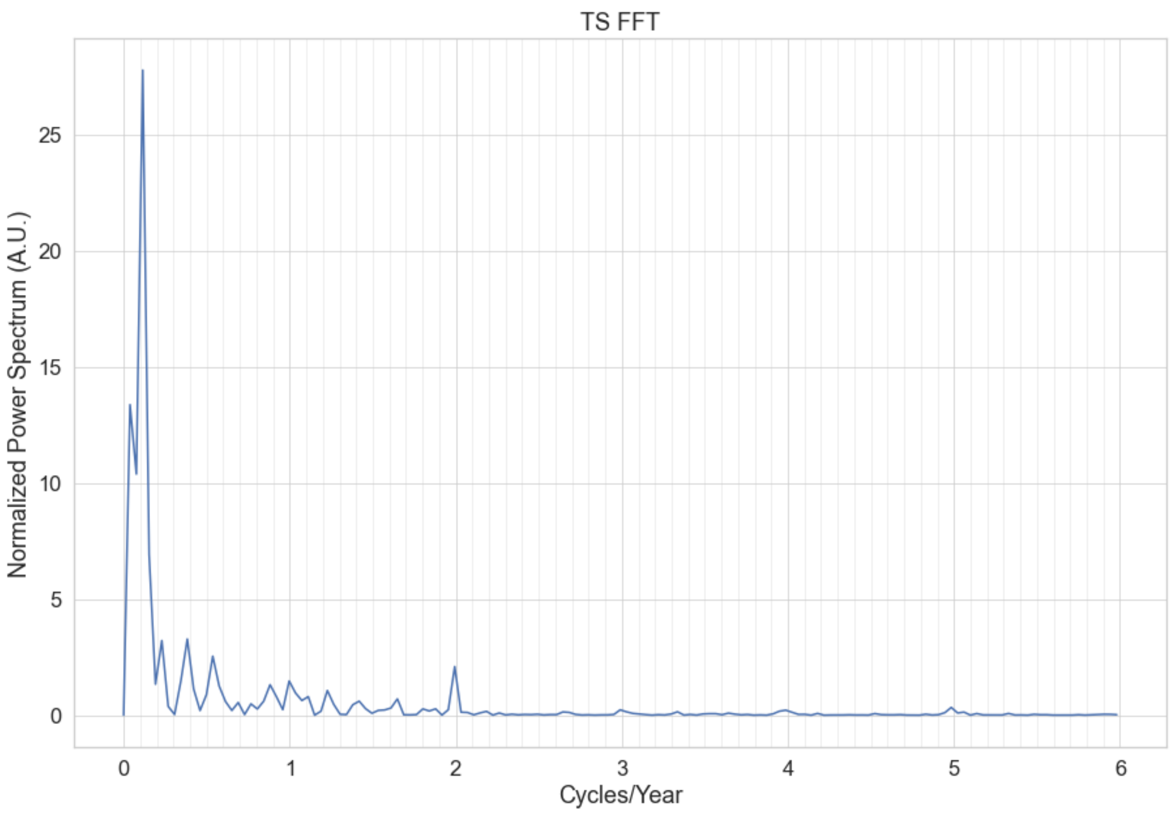
Seasonality

In [246…

```
minor_ticks = np.linspace(0, 6, 61)
```

In [247…

```
fTS=(np.abs(np.fft.rfft(trend_adjusted-trend_adjusted.mean()))**2/trend_a
sample_freq = 12 # 12 samples per year

f = np.fft.rfftfreq(trend_adjusted.size, d=1/sample_freq)

fig, ax = plt.subplots(1, 1, figsize=(15, 10))
ax.plot(f,fTS)
ax.set_xticks(minor_ticks, minor=10)
ax.grid(which='both')
ax.grid(which='minor', alpha=0.3)
ax.grid(which='major', alpha=0.7)
plt.xlabel("Cycles/Year")
plt.ylabel("Normalized Power Spectrum (A.U.)")
plt.title('TS FFT')
plt.savefig(os.path.join('images', 'original_ts_fft.png'))
plt.show()
```

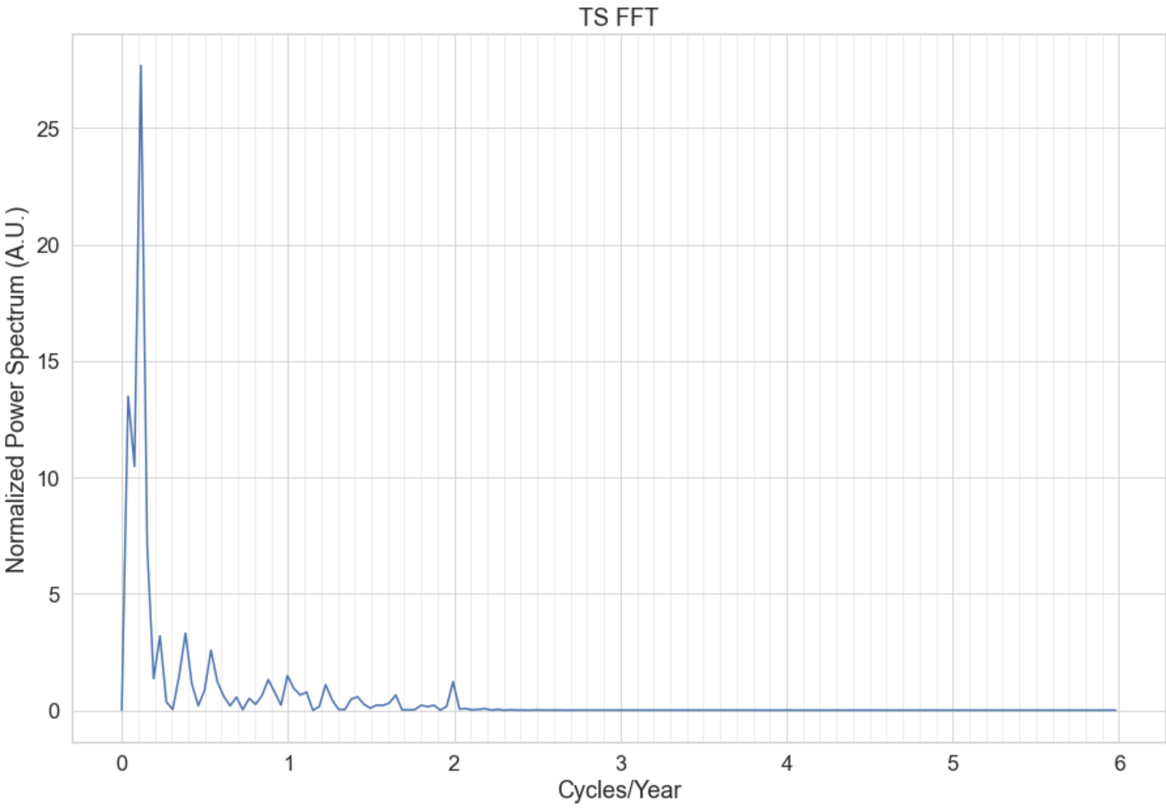


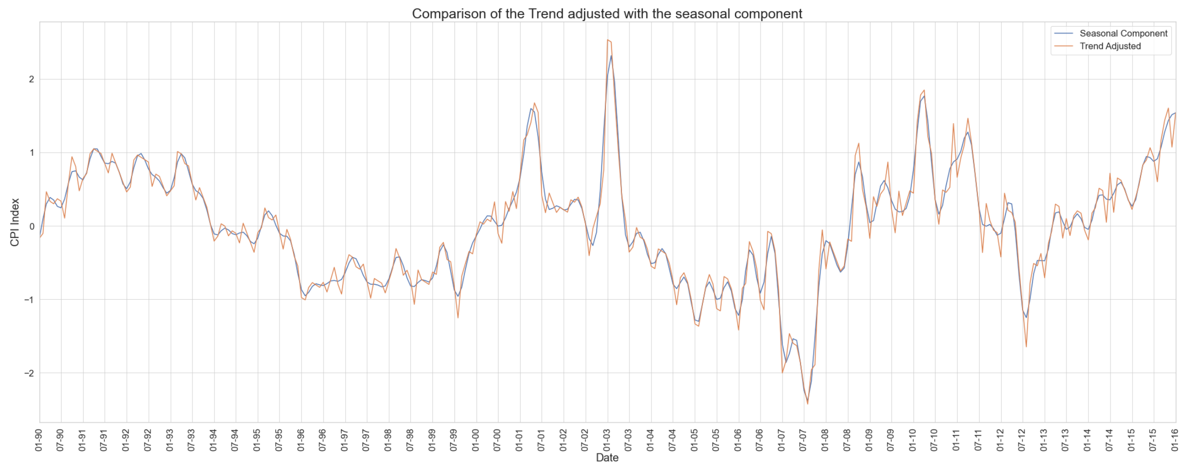
Analyzing the frequency distributions, it can confirm the previous seasonality patterns observed ever each 6 months and every 5 years, not the previous 7 years. However, it also appeared new patters every month and every 3 months and every 2 years.

Based in the frequencies presented by the Fourier Transform, and after some tests, it was chosen a cutoff value of 2.2 for the frequencies. It was used an N=5, as it is a medium value for higher order or lower order.

In [248…

```
sos = scs.butter(N=5, fs=sample_freq, Wn=[2.2], btype='lowpass', output='
seasonal = scs.sosfiltfilt(sos, trend_adjusted)
#Lets look at the filter effect
fTS=(np.abs(np.fft.rfft(seasonal-np.mean(seasonal)))**2/trend_adjusted.si

f = np.fft.rfftfreq(trend_adjusted.size, d=1/sample_freq)

fig, ax = plt.subplots(1, 1, figsize=(15, 10))
ax.plot(f,fTS)
ax.set_xticks(minor_ticks, minor=10)
ax.grid(which='both')
ax.grid(which='minor', alpha=0.3)
ax.grid(which='major', alpha=0.8)
plt.xlabel("Cycles/Year")
plt.ylabel("Normalized Power Spectrum (A.U.)")
plt.title('TS FFT')
plt.savefig(os.path.join('images', 'filtered_ts_fft.png'))
plt.show()


seasonal_ts = pd.Series(data=seasonal, index=ts.index, name='Seasonal Com

fig, ax = plt.subplots(1, figsize=(30, 12))
ax.plot(seasonal_ts.index, seasonal_ts, label='Seasonal Component')
ax.plot(trend_adjusted.index, trend_adjusted, label='Trend Adjusted')
plt.legend()
date_form = DateFormatter("%m-%y")
ax.xaxis.set_major_formatter(date_form)
ax.grid(which='minor', alpha=0.3)
ax.grid(which='major', alpha=0.8)
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
ax.set_xlim(seasonal_ts.index[0], seasonal_ts.index[-1])
plt.xlabel('Date', fontdict=(dict(size=20)))
plt.ylabel('CPI Index', fontdict=(dict(size=20)))
plt.title('Comparison of the Trend adjusted with the seasonal component',
plt.tight_layout()
plt.savefig(os.path.join('images', 'seasonality_adjusted.png'))
plt.show()
```





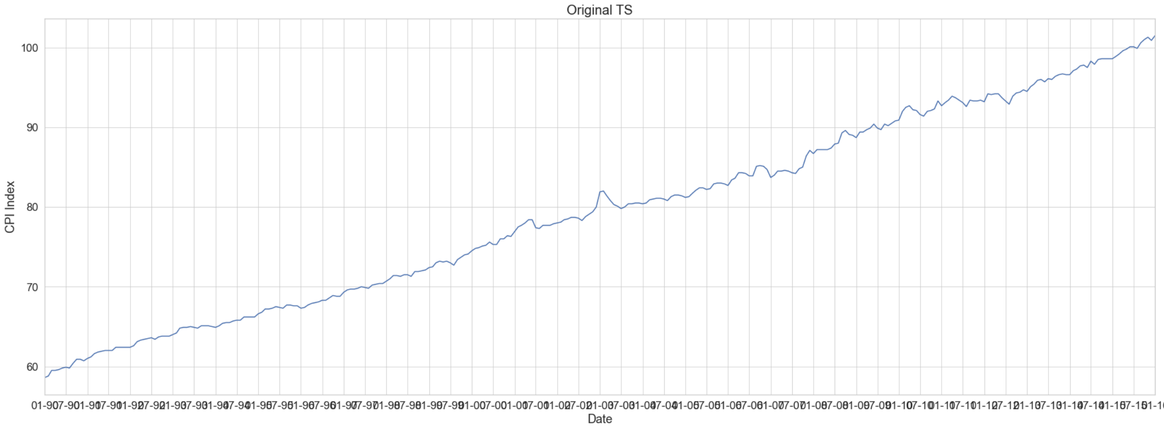
In [249…

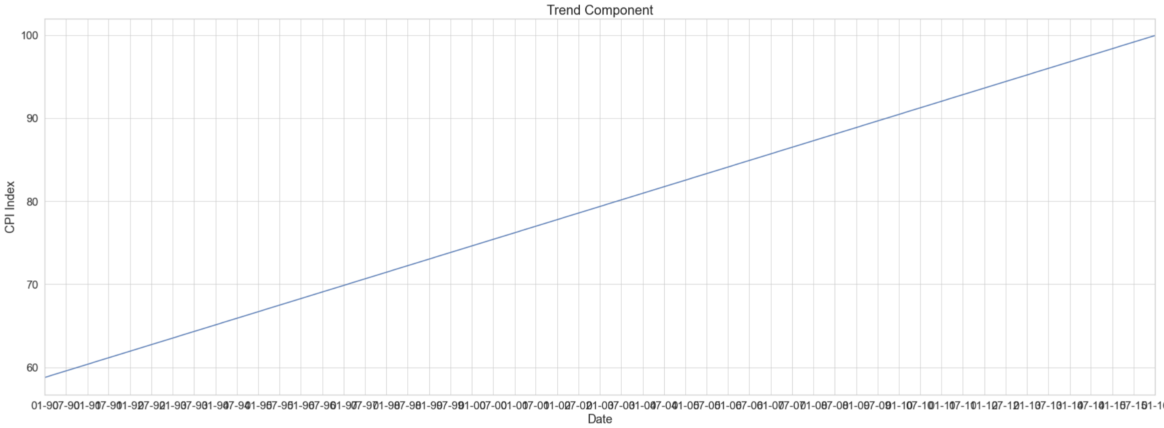
```
# remove seasonality
trend_seasonal_adjusted = trend_adjusted - seasonal_ts
trend_seasonal_adjusted.name = 'Trend and seasonality adjusted'
```

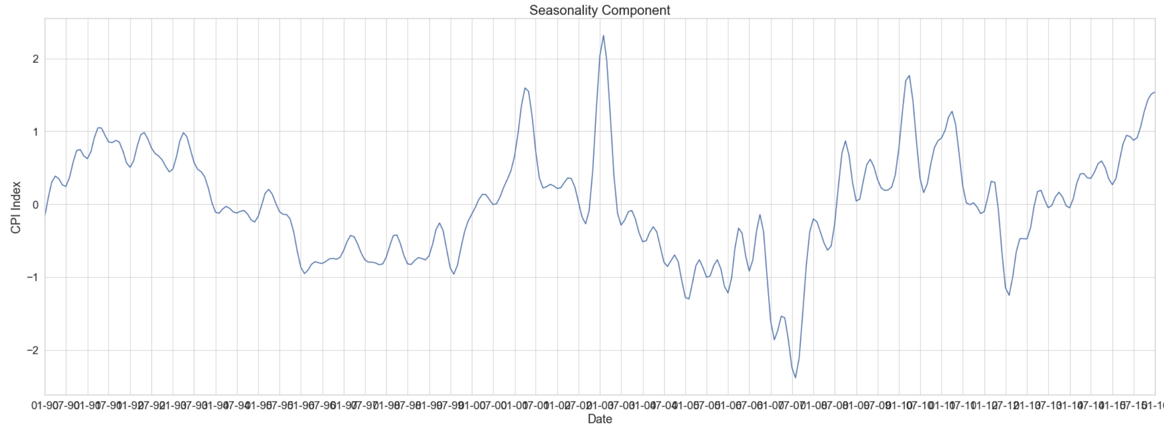
In [250…

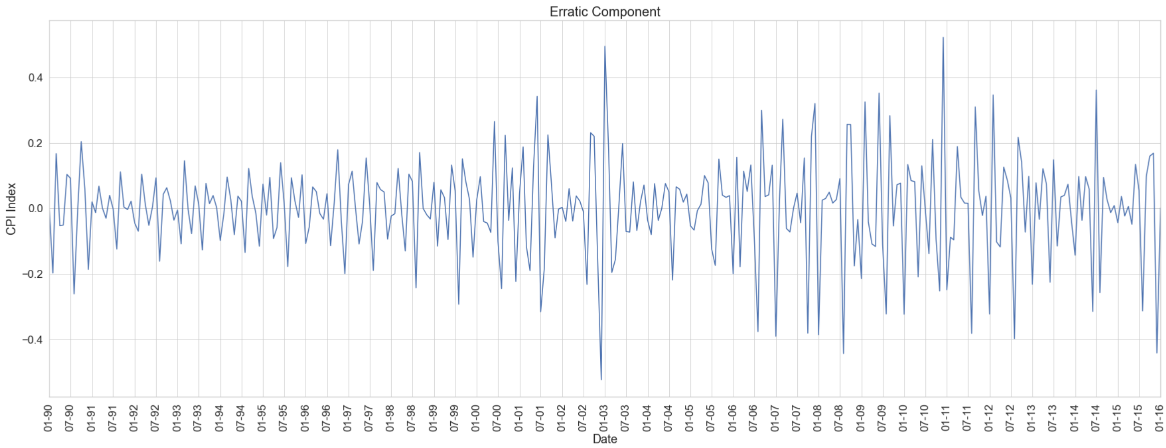
```
fig, axs = plt.subplots(4, 1, figsize=(30, 12*4))
axs[0].plot(ts.index, ts)
axs[0].set_xlabel('Date')
axs[0].set_ylabel('CPI Index')
axs[0].set_title('Original TS', fontdict=dict(size=20))
date_form = DateFormatter("%m-%y")
axs[0].xaxis.set_major_formatter(date_form)
axs[0].grid(which='minor', alpha=0.3)
axs[0].grid(which='major', alpha=0.8)
axs[0].xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
axs[0].set_xlim(ts.index[0], ts.index[-1])

axs[1].plot(trend.index, trend)
axs[1].set_xlabel('Date')
axs[1].set_ylabel('CPI Index')
axs[1].set_title('Trend Component', fontdict=dict(size=20))
```

```
date_form = DateFormatter("%m-%y")
axs[1].xaxis.set_major_formatter(date_form)
axs[1].grid(which='minor', alpha=0.3)
axs[1].grid(which='major', alpha=0.8)
axs[1].xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
axs[1].set_xlim(trend.index[0], trend.index[-1])

axs[2].plot(seasonal_ts.index, seasonal_ts)
axs[2].set_xlabel('Date')
axs[2].set_ylabel('CPI Index')
axs[2].set_title('Seasonality Component', fontdict=dict(size=20))
date_form = DateFormatter("%m-%y")
axs[2].xaxis.set_major_formatter(date_form)
axs[2].grid(which='minor', alpha=0.3)
axs[2].grid(which='major', alpha=0.8)
axs[2].xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
axs[2].set_xlim(seasonal_ts.index[0], seasonal_ts.index[-1])

axs[3].plot(trend_seasonal_adjusted.index, trend_seasonal_adjusted)
axs[3].set_xlabel('Date')
axs[3].set_ylabel('CPI Index')
axs[3].set_title('Erratic Component', fontdict=dict(size=20))
date_form = DateFormatter("%m-%y")
axs[3].xaxis.set_major_formatter(date_form)
axs[3].grid(which='minor', alpha=0.3)
axs[3].grid(which='major', alpha=0.8)
axs[3].xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.xticks(rotation='vertical')
axs[3].set_xlim(trend_seasonal_adjusted.index[0], trend_seasonal_adjusted
plt.savefig(os.path.join('images', 'additive_model_components.png'))
plt.show()
```







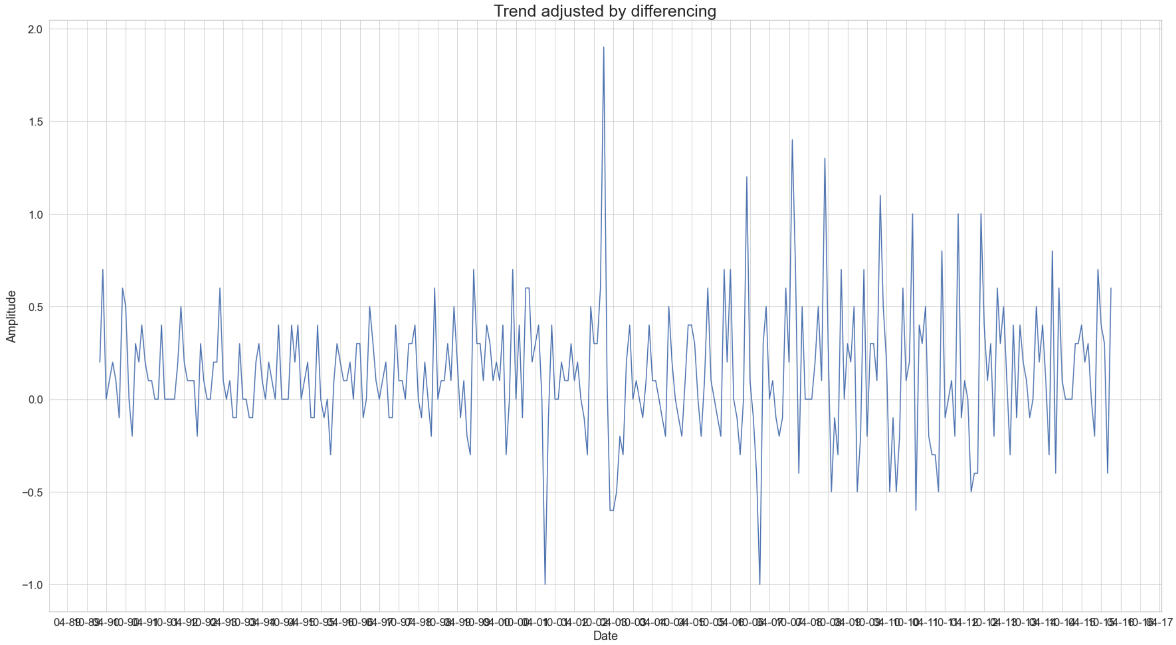


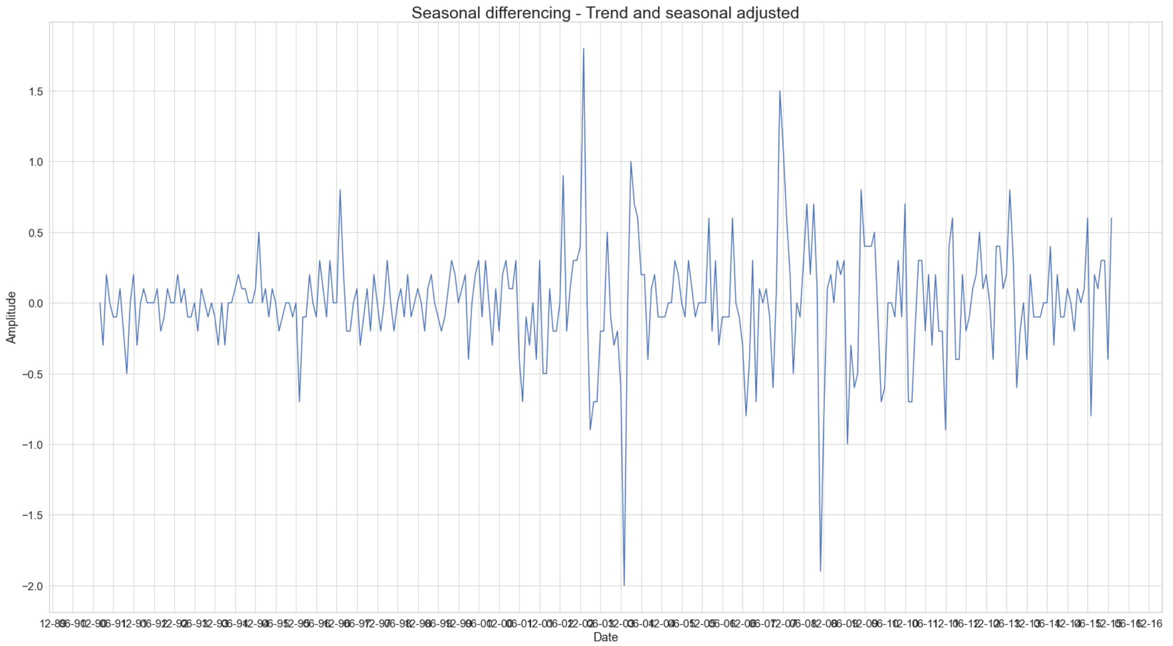
Trend and seasonal removal by differentiation

Another approach tested for transforming the TS into stationary was by differencing. As the time series presents monthly data, the period used for removing the seasonality was T=12

In [251…

```
trend_adjusted_diff = ts.diff()
# TODO: check which period to use

trend_seasonal_adjusted_diff = trend_adjusted_diff.diff(12) # because T =

fig, axs = plt.subplots(2, 1, figsize=(30, 12*3))

axs[0].plot(trend_adjusted_diff.index, trend_adjusted_diff)
axs[0].set_xlabel('Date')
axs[0].set_ylabel('Amplitude')
axs[0].set_title('Trend adjusted by differencing', fontdict=dict(size=25)
date_form = DateFormatter("%m-%y")
axs[0].xaxis.set_major_formatter(date_form)
axs[0].grid(which='minor', alpha=0.3)
axs[0].grid(which='major', alpha=0.8)
axs[0].xaxis.set_major_locator(mdates.MonthLocator(interval=6))

axs[1].plot(trend_seasonal_adjusted_diff.index, trend_seasonal_adjusted_d
axs[1].set_xlabel('Date')
axs[1].set_ylabel('Amplitude')
axs[1].set_title('Seasonal differencing - Trend and seasonal adjusted', f
date_form = DateFormatter("%m-%y")
axs[1].xaxis.set_major_formatter(date_form)
axs[1].grid(which='minor', alpha=0.3)
axs[1].grid(which='major', alpha=0.8)
axs[1].xaxis.set_major_locator(mdates.MonthLocator(interval=6))
plt.savefig(os.path.join('images', 'trend_seasonal_by_diff.png'))
plt.show()
```





The final results appears to be stationary. However, to confirm the results an compare the two approaches (model fitting + filtering and differencing), a more comprehensive analysis was carried out using the DF test.

Statistical test to check stationary

Trend and seasonal adjusted by model-fitting and filtering

In [252…

```
adf_test(trend_seasonal_adjusted)
```

```
ADF Statistic: -17.221312
p-value: 0.000000
Critical Values:
	1%: -3.453
	5%: -2.871
	10%: -2.572
```

Based on the ADF test, given the p-value of 0.0, the transformed TS is stationary.

Trend and seasonal adjusted by differencing

In [253…
```
adf_test(trend_seasonal_adjusted_diff.dropna())
```

```
ADF Statistic: -7.240584
p-value: 0.000000
Critical Values:
	1%: -3.454
	5%: -2.872
	10%: -2.572
```

By the Dickey-Fuller test, after differencing the resulted TS is stationary. However, the ADF statistic is most closer to the critical values than when the TS is decomposed by model-fitting and filtering. Having this is mind, it was selected the first approach to continue with the experiment.

ACS to see stationary

Despite the statistical tests, the stationarity of the time series was also evaluated using the Autocorrelation Sequence values (ACS).

In [254…

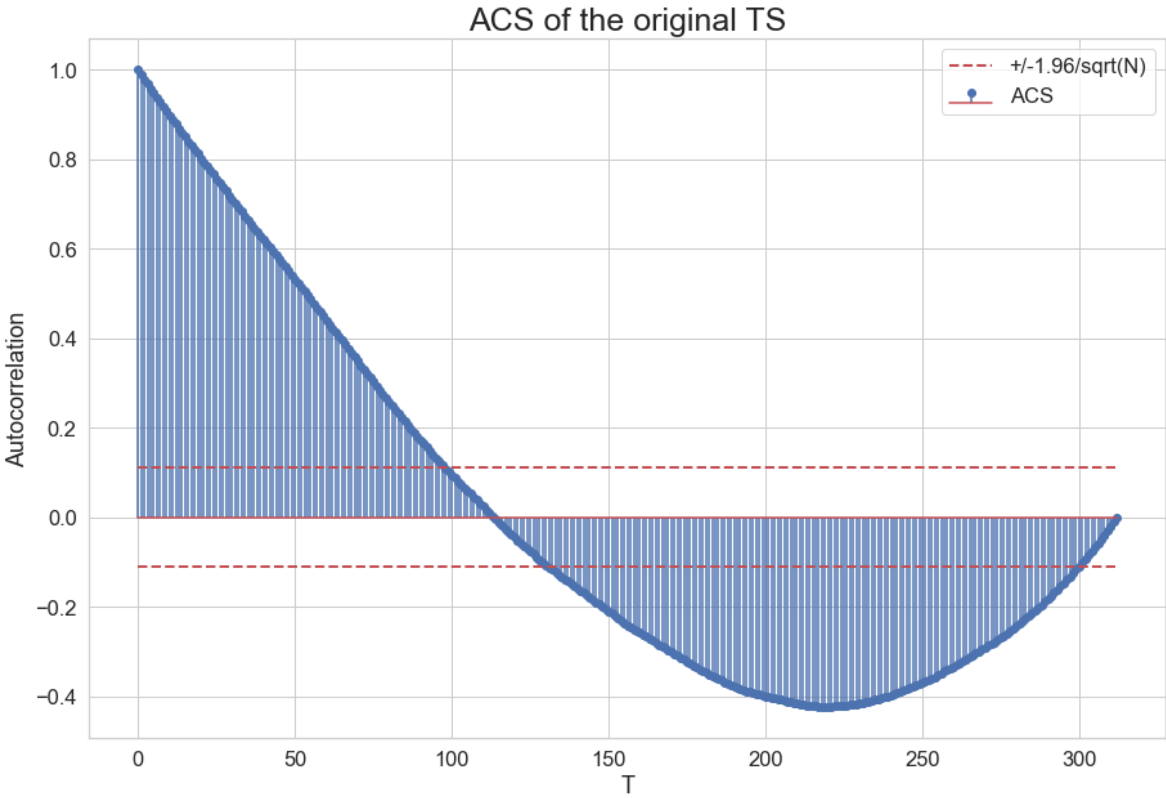
```
def auto_covarience_aux(ts, T=0):
    N = len(ts)
    mean = ts.mean()
    cov_coef = 0
    for n in range(N-T-1): ## Não estaremos a retirar um a mais
        cov_coef += (ts[n] - mean) * (ts[n+T]-mean)
    return cov_coef / N


def auto_covarience(ts, T=0):
    return auto_covarience_aux(ts, T) / auto_covarience_aux(ts)


def correlogram(ts, max_T, twoside=False):
    N = len(ts)
    if twoside:
        corrl = np.zeros(2 * max_T + 1)
        index = np.arange(max_T + 1)
        index = np.concatenate((-np.flip(index[1:]), index), axis=0)
    else:
        corrl = np.zeros(max_T)
        index = np.arange(max_T)

    for i in range(max_T):
        if twoside:
            corrl[max_T + i] = auto_covarience(ts, i)
            corrl[max_T - i] = corrl[max_T + 1]
        else:
            corrl[i] = auto_covarience(ts, i)

    d = {'ACS':corrl, 'upper_CB':np.ones(max_T)*(1.96/np.sqrt(N)),'lower_
    return pd.DataFrame(data=d, index=index)
```

In [255…

```
def plot_correlogram(ts, title):
    corrl = correlogram(ts, len(ts))
    fig, ax = plt.subplots(1, 1, figsize=(15, 10))
    ax.stem(corrl.index, corrl.ACS, label='ACS')
    ax.plot(corrl.index, corrl.upper_CB, linestyle='--', color='r', linew
    ax.plot(corrl.index, corrl.lower_CB, linestyle='--', color='r', linew
    plt.title(title, fontdict=dict(size=25))
    plt.legend()
    plt.ylabel('Autocorrelation')
    plt.xlabel('T')
```

In [256…

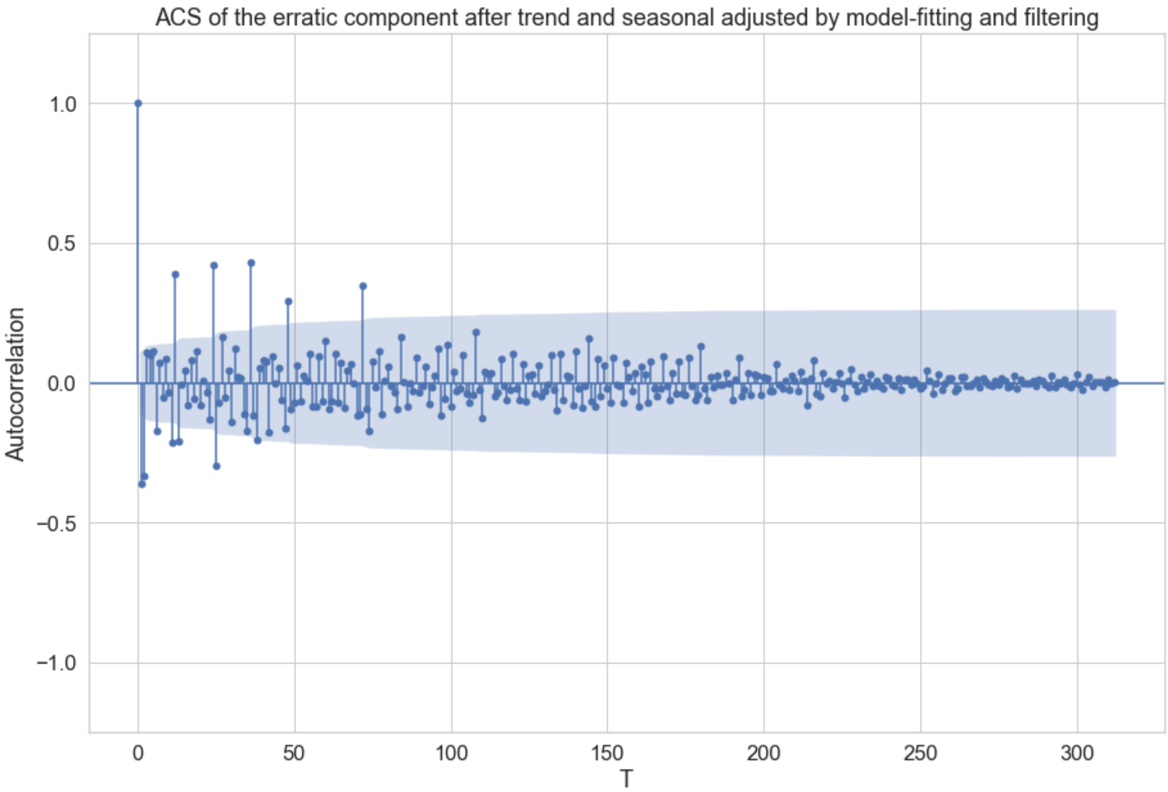
```
plot_correlogram(ts, 'ACS of the original TS')
```



From the ACS plot, it can be confirm the previous claim that the original time series isn't stationary as the ACS have significant values for T > 0, using a significance level of 95%.

In [257…

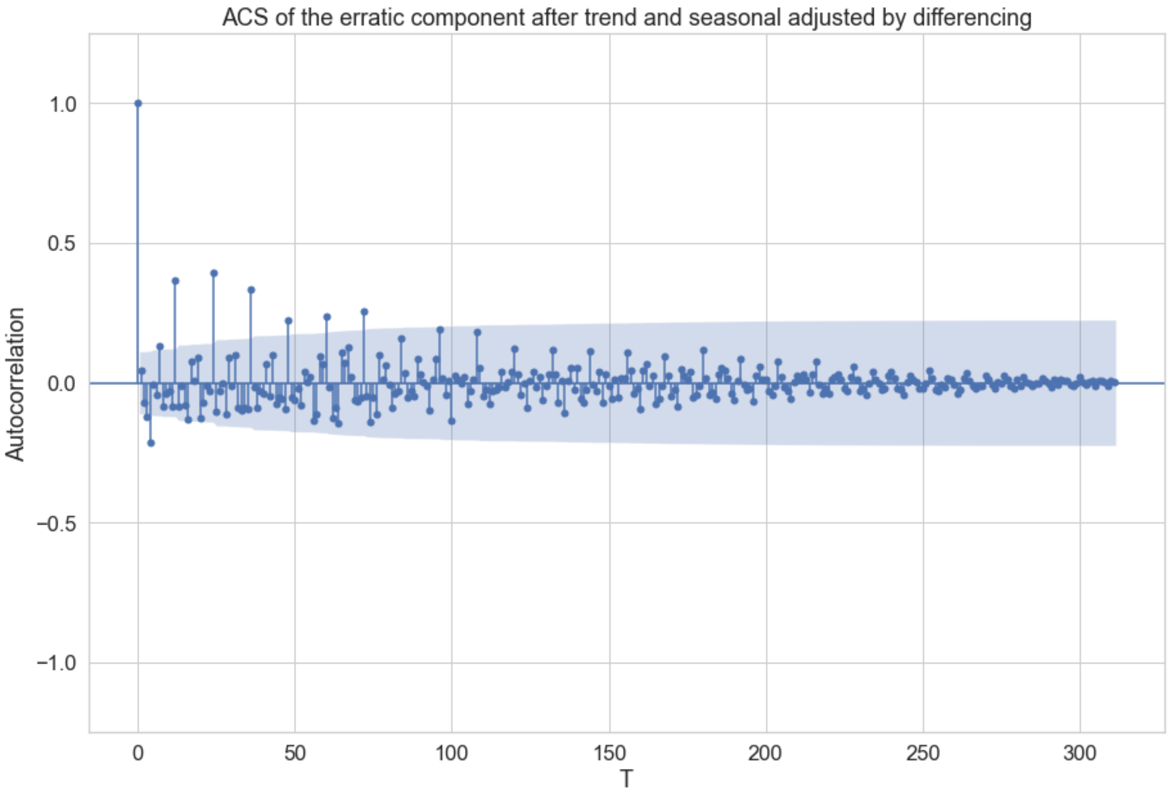
```
# plot_correlogram(trend_seasonal_adjusted, 'Erratic Component after tren
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(trend_seasonal_adjusted, alpha=0.05, ax=ax, lags=len(trend_seaso
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
plt.title('ACS of the erratic component after trend and seasonal adjusted
plt.savefig(os.path.join('images', 'acs_additive_model.png'))
plt.show()
```



For the resulted TS if the 1st approach, i can be seen that there is a rapid decreasing the the ACS, which indicates that the erratic component is stationary. From the graph, it's clear that the data was not generated with a Moving-Average (MA) linear process, as the first lags does not decay to non-significant autocorrelation values. As the ACS seem to exponential decay to zero, the Autoregressive (AR) Process could be the winner. However, it should be noted that its difficult with this plot to determine which linear-process to use (AR or ARMA) and its order, so a more extensive study should be used to determine it.

In [258…

```
# plot_correlogram(trend_seasonal_adjusted_diff.dropna(), 'Erratic Compon
fig, ax = plt.subplots(1, figsize=(15, 10))
plot_acf(trend_adjusted_diff.dropna(), alpha=0.05, ax=ax, lags=len(trend_
plt.ylabel('Autocorrelation')
plt.xlabel('T')
ax.set_ylim([-1.25, 1.25])
plt.title('ACS of the erratic component after trend and seasonal adjusted
plt.savefig(os.path.join('images', 'acs_differencing.png'))
plt.show()
```



Similar to the model-fitting and filtering transformations that transform the TS into stationary, using differencing, the ACS also decay rapid to non-significant values, which indicates the TS is not stationary.