國立成功大學 109 學年度碩士班招生考試試題

系　所：電機資訊學院-資訊聯招

考試科目：計算機數學

考試日期：0210，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一. Linear Algebra (50%)

1. True or False (25%. 5 pts each)

For each of the statements that follows, answer **true** if the statement is always true and **false** otherwise.

(a) If A and B are $n \times n$ matrices that have the same rank, then the rank of A^2 must equal the rank of B^2.

(b) Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator, and let A be the standard matrix representation of L. If L^2 is defined by

$$L^2(\mathbf{x}) = L(L(\mathbf{x})) \text{ for all } \mathbf{x} \in \mathbb{R}^2$$

then L^2 is a linear operator and its standard matrix representation is A^2.

(c) If L_1 and L_2 are both linear operators on a vector space V, then $L_1 + L_2$ is also a linear operator on V, where $L_1 + L_2$ is the mapping defined by

$$(L_1 + L_2)(\mathbf{v}) = L_1(\mathbf{v}) + L_2(\mathbf{v}) \text{ for all } \mathbf{v} \in V$$

(d) If $N(A) = \{\mathbf{0}\}$, then the system $A\mathbf{x} = \mathbf{b}$ will have a unique least squares solution.

(e) If $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is an orthonormal set of vectors in $\mathbb{R}^n$ and

$$U = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$$

then $UU^T = I_n$ (the $n \times n$ identity matrix).

2. Given $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -4 & -3 \\ -2 & 0 & -1 \end{bmatrix}$. Find the Gram-Schmidt QR factorization of $\mathbf{A}$. (10%)

3. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -8 & 6 \end{bmatrix}$ and t be a scalar. Compute $e^{\mathbf{A}t}$. (15%)

二、Discrete Mathematics (50%) (請說明如何求解過程,只寫答案不予計分)

4. (10%) Please check if the following statement is true and explain the reason.
"During the first 49 days after John graduates from NCKU, he sends his resume out to different companies. If he sends out at least one resume every day, but no more than 70 resumes in total. Then, there is a period of consecutive days during which he sends out exactly 27 resumes."

5. (20%) For x,y belonging to the set of positive real numbers, consider the determinant D_n of the n by n matrix A_{nxn}.
(a) (6%) Find the recurrent relation for the value of D_n.
(b) (7%) Find the value of D_n as a function of n, *when* $y=x$
(c) (7%) Find the value of D_n as a function of n, *when* $y=2x$

$$A_{nxn}=\begin{bmatrix} y & x & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 \\ x & y & x & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 \\ 0 & x & y & x & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & y & x & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & x & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & x & y & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & x & y & x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & x & y & x \\ 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & x & y \end{bmatrix}$$

6. (20%) If G and $\overline{G}$ are two complementary graphs with the number of vertices greater than or equal to X, then either G or $\overline{G}$ is nonplanar.
(a) (10%) Please find the value of X
(b) (10%) Please explain the reason.