※注意：請於答案卷上依序作答，並標明題號。

Problem 1. (10%)
Let S be the subspace of R^4 containing all vectors with

$X_1+X_2+X_3+X_4=0$ and $X_1+X_2-X_3-X_4=0$, find a basis for the

space $S^\perp$ ($S^\perp$=containing all vectors orthogonal to S)

Problem 2. (10%)
Let $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find two invertible matrices B, C such that
$A = B + C$

Problem 3. (15%)
Suppose the Matrix A has eigenvalues 0,1,2 with eigenvectors V_0, V_1, V_2, Solve the following equation for X
(a) $AX = V_0$
(b) $AX = V_1 + V_2$

Problem 4. (15%)
Suppose we have a matrix A with eigenvalues 0, 1, 2, 3, 4, and the corresponding eigenvectors V_0, V_1, V_2, V_3, V_4. Prove or disprove that

$\{V_0, V_1, V_2, V_3, V_4\}$ is linearly independent.

Problem 5. (5%) ___ Stirling's formula for $n!$ is

(a) $(2\pi n)^{-0.5}(n/e)^n$,
(b) $(2\pi n)^{0.5}n^n$,
(c) $(2\pi n)^{0.5}e^n$,
(d) $(2\pi n)^{-0.5}e^n$,
(e) $(2\pi n)^{0.5}(n/e)^n$,
(f) $(2\pi n)^{-0.5}n^n$.

Problem 6. (5%) $\sum_{i=0}^{n}\binom{n}{i}2^i3^{n-i}$ equals ________________.

Problem 7. (10%) The number of *integer* solutions of

$$x_1 + x_2 + \cdots + x_n = k,$$

where $x_i \geq 0$ for $1 \leq i \leq n$, is ________________.

Problem 8. (5%)

A function $f : A \to B$ is called **one-to-one** if each element of B appears at most once as the image of an element of A. If $|A| = m$ and $|B| = n \geq m$, then there are ________________ one-to-one functions from A to B.

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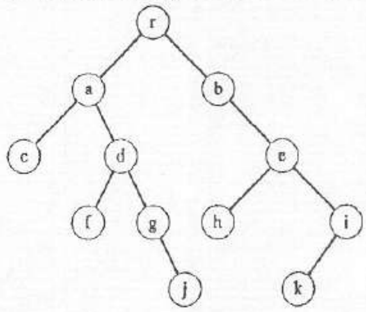
Problem 9. (10%) Let $\phi(n)$ denote the number of positive integers $m \in \{1, 2, \ldots, n-1\}$ such that $\gcd(m, n) = 1$, where $n \geq 2$. Let $n = p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t}$ be the prime factorization of n. Then $\phi(n) =$ ______________.

Problem 10. (5%) The inorder traversal of the following rooted binary tree



is ______________.

Problem 11. (10%) 75^{384} divided by 97 has remainder ______________.