1~8 題爲填充題,請依題號,將答案填寫於**答案卷**上。

1. $\begin{bmatrix} 2 & 8 \\ 8 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 2 \\ 2 & 7 \end{bmatrix} =$ __________(5%)

2. $\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} =$ __________(5%)

3. $\begin{bmatrix} \cos\frac{\pi}{100} & -\sin\frac{\pi}{100} \\ \sin\frac{\pi}{100} & \cos\frac{\pi}{100} \end{bmatrix}^{100} =$ ____________(5%)

4. If $A \in \mathbf{R}^{n\times n}$ and $A^2 - A + I_n = 0$, then $(A + 2I_n)^{-1} =$ __________(5%)

5.. If $P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13 \end{bmatrix}$, then $rank(P) =$ _____________.(5%)

6. If $A, B \in \mathbf{R}^{3\times 3}$, $\det(A) = 2$, and $\det(B) = -1$, then

$\det\begin{bmatrix} 2I_3 & B \\ 0 & AB \end{bmatrix} =$ __________.(5%)

7. If $\mathbf{u} \in \mathbf{R}^n$ and $\mathbf{u}^t\mathbf{u} = [3]$, then there exist $k \in \mathbf{R}$ such that

$(I_n + \mathbf{u}\mathbf{u}^t)^{10} = I_n + k\mathbf{u}\mathbf{u}^t$, where $k =$ ___________.(5%)

8. Let $B = \begin{bmatrix} 4 & 1 & 0 & 0 & 1 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 1 & 0 & 0 & 1 & 4 \end{bmatrix}$ *and* $\mathbf{x} = \begin{bmatrix} \cos(\theta+\phi) \\ \cos(2\theta+\phi) \\ \cos(3\theta+\phi) \\ \cos(4\theta+\phi) \\ \cos(5\theta+\phi) \end{bmatrix}$

(a) $\exists\, a, b, c \in \mathbf{R}$ such that $B\mathbf{x} - a\mathbf{x} = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \\ c \end{bmatrix}$, where $(a, b, c) =$ ___________.(6%)

(Please express a, b, and c in terms of θ and ϕ).

(b) All the eigenvalues of B are ____________(9%)

※以下第 9~12 題全對得 10 分，錯一個選項得 5 分，錯二個(含)以上選項得 0 分。

9. 以下敘述何者爲正確? 請清楚標示答案，不需說明。

(a) Suppose that A and B are two finite sets. It is possible that the number of different binary relations from A to B is not equal to the number of different binary relations from B to A.
(b) Define a relation $\Re$ on the set of integers as follows: $x \Re y$ if and only if $xy \geq 0$. Then, $\Re$ is an equivalence relation.
(c) Suppose that $\Re$ is an equivalence relation on A. Then the set of all equivalence classes induced by $\Re$ forms a partition of A.
(d) The Hasse diagram for a total ordering is a chain.
(e) It is possible that there are multiple topological orders for a partially ordered set.

10. 以下敘述何者爲正確? 請清楚標示答案，不需說明。

(a) If $(R, +, \cdot)$ is a ring, then $(R, +)$ is a commutative group.
(b) Every ring $(R, +, \cdot)$ has the identities for both + and $\cdot$.
(c) $(R, +, \div)$ forms a ring, where R is the set of real numbers, + is the ordinary addition, and $\div$ is the ordinary division.
(d) The principle of duality holds for both Boolean algebras and rings.
(e) If $(G, \cdot)$ is a finite group and $H \subseteq G$ is a subgroup of G, then $|H|$ can divide $|G|$.

11. 以下敘述何者爲正確? 請清楚標示答案，不需說明。

(a) There are 26 integers in $\{1, 2, \ldots, 100\}$ that are not divisible by 2 or 3 or 5.
(b) It is known that there are n^r r-permutations of n distinct objects with unlimited repetitions. The answer (i.e., n^r) can be expressed as the coefficient of x^r in $f(x) = (1+x+x^2+x^3+\ldots)^n$.
(c) The general solution to $a_{n+2} = 4a_{n+1} - 4a_n$, where $n \geq 0$ and $a_0 = 1$, is $a_n = 2^n$.
(d) The recurrence equation $b_n = b_0 b_{n-1} + b_1 b_{n-2} + \ldots + b_{n-2} b_1 + b_{n-1} b_0$, where b_0 is a given constant, can be solved by the aid of generation functions.
(e) Every nonlinear recurrence equation of order three with constant coefficients is solvable.

12. 以下敘述何者爲正確? 請清楚標示答案，不需說明。

(All graphs below are simple and undirected.)

(a) Two graphs are isomorphic, if they have the same number of vertices, the same number of edges, and the same nonincreasing vertex degree sequence.
(b) If a graph has vertex connectivity 3, then it is biconnected.
(c) There exists a graph that contains both a Hamiltonian path and an Euler trail.
(d) There exists a planar graph of 10 vertices and 14 edges whose planar drawing divides the plane into 8 regions.
(e) There exist sufficient-and-necessary conditions for a graph to be planar.

13. Suppose that $p_1, p_2, \ldots, p_6$ denote six people, where every two people are either familiar with or strange to each other. Prove that at least three of $p_1, p_2, \ldots, p_6$ can be found so that they are either familiar with or strange to one another. (10%)