本試題是否可以使用計算機： □可使用 ，☑不可使用 （請命題老師勾選）

一、Algorithms (50%)

1. (10%) Answer TRUE or FALSE for the following statement: For all real constants a and b such that $a > 1, n^b = o(a^n)$.

2. (15%) Solving the recurrence $T(n) = 4T(\frac{n}{2}) + n^2\sqrt{n}$ using Θ notation.

3. (15%) (a) (5%) Define the strongly connected component. (b) (10%) Give an algorithm as fast as possible to find strongly connected components and analyze its complexity.

4. (10%) (a) (5%) Describe the Floyd-Warshall algorithm to solve the all-pairs shortest paths problem. (b) (5%) Analyze the time complexity the Floyd-Warshall algorithm.

(背面仍有題目，請繼續作答)

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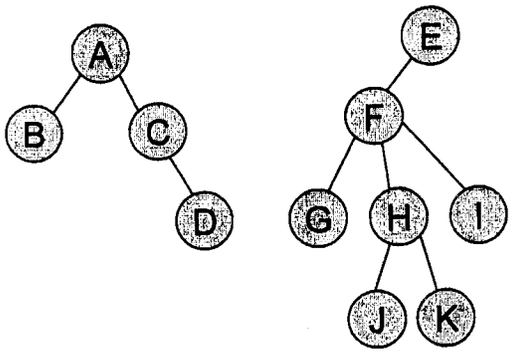
二、Data Structures

1. (10%) We define Fibonacci polynomials by the recurrence relation $F_n(Y)=Y*F_{n-1}(Y)+F_{n-2}(Y)$, where $F_0(Y)=1$, $F_1(Y)=Y$ and $n\geq 2$. How many memory spaces are actually needed to store all the Fibonacci polynomials including F_0, F_1, ….,F_{1000}? Please discuss your implementation if using a two-dimension array.

2. (10%) True or False. Please indicate the answer by T or F.
   (1) A complete binary tree of depth k has 2^k-1 nodes.
   (2) A tree must have at least one node.
   (3) The number of subtrees of a node is called its level.
   (4) A forest is a set of $n\geq 1$ disjoint trees.
   (5) A binary tree is a finite set of nodes which comprising a root and two disjoint binary trees.
   (6) The maximum number of nodes in a binary tree of depth k is 2^k-1, $k\geq 1$.

   Let G be an undirected graph with each two vertices connected by at most one edge.
   (7) All spanning tree of G has the same number of edges.
   (8) Minimal cost spanning tree of G is unique.
   (9) The path from vertex A to vertex B on a minimal cost spanning tree of G is shortest path from A to B.
   (10) Any two spanning trees of G should have a common edge.

3. (15%) For the following forest , please answer:



   (1) Write the result of the postorder traversal of the forest.
   (2) Draw the binary tree corresponding to the forest.
   (3) Write the result of the preorder traversal of the binary tree.
   (4) Write the result of the inorder traversal of the binary tree.
   (5) Draw the completed threaded version of the binary tree.

4. (15%) The behavior of a sorting method can be illustrated pictorially by displaying the contents of the array to be sorted using the so-called characteristic diagram. In displaying the contents of an array, e.g. A[1..N], a symbol "•" is placed at position (i, j) for A[i]=j. Clearly in a sorted array, each symbol "•" appears above the one to its left. Identify the corresponding characteristic diagram for each of the following five sorting schemes.
   (a) merge sort (b) insertion sort (c) heap sort (d) quick sort (e) selection sort.
   In addition, please show the sorting procedure (using C/C++/Java/Pseudo code) corresponding to the characteristic diagram Fig. 6.

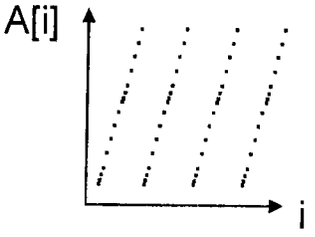


Fig. 1

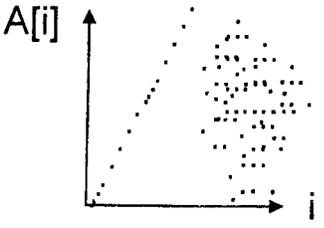


Fig. 2

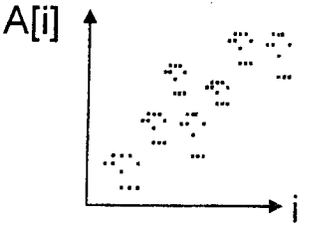


Fig. 3

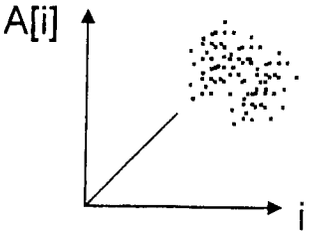


Fig. 4

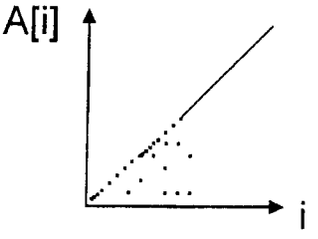


Fig. 5

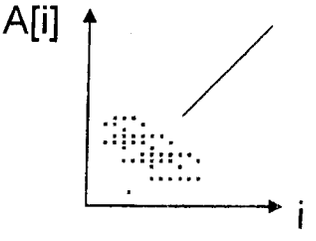


Fig. 6