請於答案卷上標明題號後依序作答

1. (15 points) Consider the following algorithm.

compute(n, t)

- **if** $n \le 1$
-     **return** *atom*$(n - t)$
- **else**
-     compute X, Y, and Z based on n
-     $s \leftarrow 0$
-     **for** $x \leftarrow 1$ **to** X
-         **do** $s \leftarrow s + compute(Y, x \cdot t)$
-     **for** $z \leftarrow 1$ **to** Z
-         **do** $s \leftarrow s + atom(z + t)$
-     **return** s

Assume that the function *atom* as well as all the arithmetic operations in the algorithm takes $\Theta(1)$ running time. In addition, assume that computing the values of X, Y, and Z takes $\Theta(1)$ running time. Determine the tightest asymptotic upper bound for the running time of *compute*$(n, 1)$ for different cases of (X, Y, Z).

X	Y	Z	running time of *compute*$(n, 1)$
5	$\lfloor \frac{n}{4} \rfloor$	$\lfloor n\sqrt{n} \rfloor$	(1)
4	$\lceil \frac{n}{2} \rceil$	n^2	(2)
3	$\lfloor \frac{n}{3} \rfloor$	$\lfloor n \log n \rfloor$	(3)
2	$n - 2$	n	(4)
n	$\lfloor \sqrt{n} \rfloor$	$\lfloor n^2 \log n \rfloor$	(5)

Please choose from the following items as the answers:

[A] $O(1)$	[B] $O(n)$	[C] $O(\log n)$	[D] $O(n \log n)$
[E] $O(2^n)$	[F] $O(n^2)$	[G] $O(n\sqrt{n})$	[H] $O(n^2 \log n)$
[I] $O(n \cdot 2^n)$	[J] $O(\sqrt{n})$	[K] $O(n^{\log_4 5})$	[L] $O(n \log^2 n)$

2. (25 points) Consider a binary search tree for storing a set of integer values. Answer the following questions.

   (1) (5 points) Show the structure of the binary search tree after inserting eight integer values $\{34, 51, 23, 11, 89, 39, 77, 21\}$ in this order as a diagram.

   (2) (5 points) Show the structure of the above binary search tree after deleting two integer values $\{51, 23\}$ in this order as a diagram.

   (3) (5 points) In what situations does the height of the binary search tree become n after inserting n integer values?

   (4) (10 points) Prove that the average height of the binary search tree after inserting n integer values $\{1, 2, \cdots, n\}$ in a random order is $O(\log n)$.

見背面

3. (14 points) A list L consists of n objects with one root pointer and one tail pointer pointing to its first and last object. As shown in the following table, there are four possible data structures that can be used to form the list L. What is the asymptotic worst-case running time for each dynamic-set operation listed?

	unsorted, singly linked	sorted, singly linked	unsorted, doubly linked	sorted, doubly linked
$Search(L, k)$	(1)	(2)	(3)	(4)
$Insert(L, p)$	(5)	(6)	(7)	(8)
$Delete(L, p)$	(9)	(10)	(11)	(12)
$Successor(L, p)$	(13)	(14)	(15)	(16)
$Predecessor(L, p)$	(17)	(18)	(19)	(20)
$Minimum(L)$	(21)	(22)	(23)	(24)
$Maximum(L)$	(25)	(26)	(27)	(28)

where $Search(L, k)$ returns the object whose key equals k in L; $Insert(L, p)$ inserts an object pointed by p into L; $Delete(L, p)$ deletes an object pointed by p from L; $Successor(L, p)/Predecessor(L, p)$ returns the next/previous object of the current one pointed by p in L; $Minimum(L)/Maximum(L)$ returns the minimum/maximum key in L.

Please choose from the following items as the answers:

[A] $O(1)$	[B] $O(n)$	[C] $O(\log n)$	[D] $O(n \log n)$

4. (10 points) In the red-black tree insertion algorithm, all the cases for handling a red-parent-with-red-child conflict (we'll refer to this as a "red-red conflict") assume that there is another level above the red-red conflict. That is, they assume that the upper red node in the red-red conflict has a parent. We did not handle the situation where you have a red node, and the parent of that node happens to be red, but that red parent node has no parent itself. Explain convincingly why we do not need such a case.

5. (11 points) Huffman encoding is an entropy encoding algorithm used for lossless data compression. Answer the following questions:

   (1) (4 points) True or false: "*The Huffman encoding algorithm employs a priority queue of binary search trees.*" Please choose from the following items as the answers:

[A] True	[B] False

   (2) (7 points) Given this set of 5 symbols and their frequencies,

symbol	frequency	symbol	frequency	symbol	frequency
a	1k	d	3k	e	4.5k
n	5k	t	6k		

   which of the following is a valid Huffman encoding of the message "attend"?

[A]	0001001000010011001
[B]	00011110110001
[C]	100000010111
[D]	None of the above.

接次頁

6. (25 points) Consider a strongly connected directed graph $G = (V, E)$, which has negative-length edges, but has no negative-length cycles. Let $\ell(u, v)$ denote the length of an edge $(u, v) \in E$, and $d(u, v)$ denote the shortest path distance from vertex u to vertex v. Answer the following questions.

   (1) (10 points) Prove that the inequality $\ell(u, w) + d(v, u) - d(v, w) \geq 0$ holds for any vertex $v \in V$ and any edge $(u, w) \in E$ on the graph G.

   (2) (10 points) Assume that a value $s(v)$ is attached to each vertex $v \in V$ on the graph G. Consider a new graph G' that comes from transforming G by replacing the length of each edge $(u, v) \in E$ with $\ell(u, v) + s(u) - s(v)$. Prove that the shortest path on the graph G' between $w \in V$ and $x \in V$ is also the shortest path between w and x on the graph G.

   (3) (5 points) To compute the shortest path tree from a vertex v to other vertex $u \in V - \{v\}$ on the graph, the following two algorithms are considered.

$A(G, v)$

- $I(G, v)$
- **for** $i \leftarrow 1$ **to** $|V| - 1$
-     **do for** each edge $(u, w) \in E$
-         **do** $R(v, u, w)$

$B(G, v)$

- $I(G, v)$
- $S \leftarrow \emptyset$
- $Q \leftarrow V$, where Q is a min-priority queue
- **while** $Q \neq \emptyset$
-     **do** $v \leftarrow$ *Extract-Min*(Q)
-     $S \leftarrow S \cup \{u\}$
-     **for** each vertex w that $(u, w) \in E$
-         **do** $R(v, u, w)$

where $I(G, v)$ sets $d(v, v) = 0$ and $d(v, u) = \infty$ for $u \in V - \{v\}$, and $R(v, u, w)$ sets $d(v, w) = \min(d(v, u) + \ell(u, w), d(v, w))$.

Answer that which one is suitable and why.

試題隨卷繳回