

系所組別：電機資訊學院-資訊聯招

考試科目：機率統計

考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A delivery truck travels from point A to point B and back using the same route each day. There are four traffic lights on the route. Let X_1 denote the number of red lights the truck encounters going from A to B and X_2 denote the number encountered on the return trip. Data collected over a long period suggest that the joint probability distribution for (X_1, X_2) is given by:

		X_2				
		0	1	2	3	4
X_1	0	0.01	0.01	0.03	0.07	0.01
	1	0.03	0.05	0.08	0.03	0.02
	2	0.03	0.11	0.15	0.01	0.01
	3	0.02	0.07	0.10	0.03	0.01
	4	0.01	0.06	0.03	0.01	0.01

- (1) Give the conditional density distribution of X_1 given $X_2 = 3$. (5%)
- (2) Give $E(X_1)$. (5%)
- (3) Give $E(X_1|X_2 = 3)$. (5%)
- (4) Give the standard deviation of X . (5%)
2. A chemical system that results from a chemical reaction has two important components among others in a blend. The joint distribution describing the proportion X_1 and X_2 of these two components is given by
- $$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$
- (1) What is the probability that component proportions produce the results $X_1 < 0.2$ and $X_2 > 0.5$ (10%)
- (2) Give the conditional distribution $f_{x_1|x_2}(x_1|x_2)$ (10%)
3. The density function of the time Z in minutes between calls to an electrical supply store is given by
- $$f(z) = \begin{cases} \frac{1}{10} e^{-z/10}, & 0 < z < \infty \\ 0, & \text{otherwise} \end{cases}$$

- (1) What is the mean time between calls? (5%)
- (2) What is the variance in the time between calls? (7%)
- (3) What is the probability that the time between calls exceeds the mean? (8%)

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4. Suppose a filling machine is used to fill cartons with a liquid product. The specification that is strictly enforced for the filling machine is 9 ± 1.5 oz. If any carton is produced with weight outside these bounds, it is considered by the supplier to be defective. It is hoped that at least 99% of cartons will meet these specifications. With the conditions $\mu = 9$ and $\sigma = 1$,
- What proportion of cartons from the process are defective (See table A.3)? (10%)
 - If changes are made to reduce variability, what must σ be reduced to in order to meet specifications with probability 0.99? Assume a normal distribution for the weight. (10%)
5. The random variables X and Y , representing the weights of creams and toffees, respectively, in 1-kilogram boxes of chocolates containing a mixture of creams, toffees, and cordials, have the joint density function

$$f(x,y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, x+y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- Find the probability density function of the random variable $Z=X+Y$. (10%)
- Using the density function of Z , find the probability that in a given box, the sum of the weight of creams and toffees accounts for at least $1/2$ but less than $3/4$ of the total weight. (10%)

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Table A.3 Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319
-3.3	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
-2.8	0.0026	0.0025	0.0024	0.0023	0.0022	0.0021	0.0020	0.0019	0.0019	0.0019	0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7791	0.7822
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106
-2.5	0.0062	0.0061	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162
-2.0	0.0218	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306
-1.9	0.0257	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9701
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2540	0.2514	0.2483	0.2451	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980
-0.5	0.3095	0.3050	0.3015	0.2981	0.2946	0.2877	0.2843	0.2810	0.2776	0.2743	2.9	0.9981	0.9982	0.9983	0.9984	0.9985	0.9985	0.9985	0.9986	0.9986
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	3.1	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247	3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997

Table A.3 (continued) Areas under the Normal Curve

