# 國立臺灣大學96學年度碩士班招生考試試題



※ 注意：務必依照題號順序作答。

(1) (10%) Let matrices $A=\begin{bmatrix}1 & 0 & 0 & 0\\ -2 & 3 & 0 & 0\\ 0 & -4 & 5 & 0\\ 0 & 0 & -6 & 7\end{bmatrix}$, $I=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$ and $B=(I+A)^{-1}(I-A)$, calculate the matrix $(I+B)^{-1}$.

(2) (10%) If the rank of the set of vectors $\boldsymbol{b}_1 = (0, 1, -1)$, $\boldsymbol{b}_2 = (a, 2, 1)$, $\boldsymbol{b}_3 = (b, 1, 0)$ is equal to the rank of the set of vectors $\boldsymbol{a}_1 = (1, 2, -3)$, $\boldsymbol{a}_2 = (3, 0, 1)$, $\boldsymbol{a}_3 = (9, 6, -7)$ and $\boldsymbol{b}_3$ can be represented as the linear combination of $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$, find the values of $a$, $b$.

(3) (10%) Given 3×3 matrix $A$ and four vectors $\boldsymbol{a}=\begin{bmatrix}1\\1\\0\end{bmatrix}, \boldsymbol{b}=\begin{bmatrix}5\\3\\2\end{bmatrix}, \boldsymbol{c}=\begin{bmatrix}1\\3\\-1\end{bmatrix}, \boldsymbol{d}=\begin{bmatrix}-2\\2\\-3\end{bmatrix}$ satisfying $A\boldsymbol{a}=\boldsymbol{b}$, $A\boldsymbol{b}=\boldsymbol{c}$, $A\boldsymbol{c}=\boldsymbol{d}$, find $A\boldsymbol{d}$.

(4) (20%) (a) If $a_0 = 2, a_1 = 3,$ and $a_{n+1} = 3a_n - 2a_{n-1}$, for all $n \geq 1$, use generating function method to find the formula for $a_n$.
(b) Redo part (a) using Eigen value method.

(5) (10%) Suppose that $\mathfrak{R}$ is an equivalence relation on {1, 2, 3, 4, 5} and the equivalence classes induced by $\mathfrak{R}$ are {1, 5}, {2, 4} and {3}. What is the value of $|\mathfrak{R}|$, i.e., the size of $\mathfrak{R}$?

(6) (20%) Suppose that $(K, \cdot, +)$ is a Boolean algebra. An element $e$ (or $z$) of $K$ is called the *identity* (or *zero*) if $e \cdot a = a \cdot e = a$ (or $z + a = a + z = a$) for all $a \in K$. Prove that the identity and zero of $K$ are unique.

(7) (20%) Suppose that $G=(V, E)$ is a connected planar graph. It is known that $|V| - |E| + r = 2$ holds for any planar drawing of $G$, where $r$ is the number of regions. Assume $r > 1$. Prove that $|E| \leq 3 \times |V| - 6$ also holds for $G$. (Hints: every region is bounded by at least three edges and every edge is shared by at most two regions.)

試題隨卷繳回