※ 注意：請於答案卷上標明題號，並依序作答。

Problem 1 (15%) Determine the dimension of each of the following subspaces V.

a. $V = \text{Span}([1,2,3],[3,4,7],[5,-2,3]) \subseteq \mathbb{R}^3$,
b. $V = \{[x_1,x_2,x_3,x_4] \in \mathbb{R}^4 : x_1+x_2+x_3+x_4 = 0, x_2+x_4 = 0\} \subseteq \mathbb{R}^4$,
c. $V = U + W$, where $U = \text{Span}([1,0,1,1],[2,1,1,2]) \subseteq \mathbb{R}^4$ and $W = \text{Span}([0,1,1,0],[2,0,1,2]) \subseteq \mathbb{R}^4$.

Problem 2 (15%) Find the point on the plane spanned by two vectors $[1,1,1]$ and $[-1,0,2]$ that is closest to the point $(1,4,3)$.

Problem 3 (20%) Let

$$A = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}.$$

Derive A^k for integer $k \geq 1$.

Problem 4 (5%) ___ Which of the following is the largest as $n \to \infty$:

a. n^{100},
b. 2^n,
c. $(\ln n)^n$,
d. $n^{\sqrt{n}}$,
e. $n!$.

Problem 5 (5%) $\sum_{i=0}^{n} \binom{n}{i}(-1)^i 2^i 3^{n-i}$ simplifies to _____.

Problem 6 (5%) $\sum_{i=1}^{n} i\binom{n}{i}$ simplifies to _____.

Problem 7 (5%) The number of *integer* solutions of

$$x_1 + x_2 + \cdots + x_n \leq k,$$

where $x_i \geq 0$ for $1 \leq i \leq n$, is _____.

Problem 8 (5%) ___ Which of the following is logically equivalent to $p \to q$:

a. $\neg p \vee q$,
b. $p \vee \neg q$,
c. $p \vee q$,
d. $\neg p \wedge q$.

接背面

Problem 9 (5%) How many ways are there to distribute $m > 0$ distinct objects into 2 identical containers with no container left empty? _____

Problem 10 (5%) Let $G = (V, E)$ be an undirected graph. Then $\sum_{v \in V} \text{degree}(v) =$ _____.

Problem 11 (5%) ___ The number of partitions of positive integer m into *distinct* positive integers where the order of summands is irrelevant equals the coefficient of x^m of

a. $\frac{1}{1-x}\frac{1}{1-x^2}\cdots\frac{1}{1-x^m}$,
b. $\frac{1}{1+x}\frac{1}{1+x^2}\cdots\frac{1}{1+x^m}$,
c. $(1+x)(1+x^2)(1+x^3)\cdots(1+x^m)$,
d. $\frac{1}{1+x+x^2+\cdots+x^m}$.

Problem 12 (5%) Solve the recurrence equation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 0$ and $a_1 = 1$. _____

Problem 13 (5%) The inverse of 4 modulo 7 is _____