

1. Find the number of different binary trees with 5 nodes. (16 points)

2. Given the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Draw a graph that has A as its adjacency matrix. Be sure to label the vertices of the graph. (16 points)

3. Demonstrate the insertion of the keys 15, 18, 29, 13, 21, 35, 11, 12, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots (0..8), and let the hash function be $h(k) = k \bmod 9$. (18 points)

4. Suppose that $\sum_{k=1}^n k^5 = \Theta(n^a)$, $\sum_{k=1}^n (\log k)^2 = \Theta(n^b(\log n)^c)$, and

$$\sum_{k=1}^n k^2(\log k)^3 = \Theta(n^d(\log n)^e). \text{ Find the value of the constants } a, b, c, d, e.$$

(15 points)

5. Given a set $\{6.0, 1.1, 2.3, 3.4, 2.5, 1.2, 2.0, 3.3, 5.2, 3.6, 4.7, 4.5\}$ of points on the real line. Find a smallest set of unit-length closed intervals that contains all of the given points. (15 points)

6. Describe how to use depth first search to determine whether the input directed graph $G = (V, E)$ is acyclic.

(20 points)