國立台灣大學九十三學年度碩士班招生考試試題

科目：數學

題號：452

※ 注意：請於答案卷上依序作答，並應註明作答之題號。

Problem 1 (4%) ___ The number of independent row vectors in a matrix is the same as the number of independent column vectors.

T. true,
F. false.

Problem 2 (4%) ___ If H is a row-echelon form of a matrix A, then the nonzero column vectors in H form a basis for the column space of A.

T. true,
F. false.

Problem 3 (4%) ___ $\text{rank}(AC) \leq \text{rank}(A)$, where A and C are matrices such that the product AC is defined.

T. true,
F. false.

Problem 4 (4%) ___ Let V and W be vector spaces of dimensions n and m, respectively. A linear transformation $T : V \to W$ is invertible if and only if $m = n$.

T. true,
F. false.

Problem 5 (4%) ___ Let $\boldsymbol{v}$ and $\boldsymbol{w}$ be independent vectors in V, and let $T : V \to W$ be a one-to-one linear transformation of V into W, then $T(\boldsymbol{v})$ and $T(\boldsymbol{w})$ are independent vectors in W.

T. true,
F. false.

Problem 6 (10%) Let $V \subset \mathbb{R}^n$ be a subspace. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ and $S : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformations defined respectively by projecting onto and reflecting across V. What are the eigenvalues and eigenvectors of T and S, respectively?

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Problem 7 (20%) Suppose A is an $n \times n$ matrix with the property that $A^2 = A$. Let $a_1, \dots, a_n \in \mathbb{R}^n$ be the column vectors of A and $A_1, \dots, A_n \in \mathbb{R}^n$ be the row vectors of A. Let $C(A) = \text{span}(a_1, \dots, a_n)$ and $R(A) = \text{span}(A_1, \dots, A_n)$ be the column space and the row space of A, respectively. Define

$$\begin{aligned} E(A) &= \{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{x} = A\boldsymbol{x}\}, \\ F(A) &= \{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{x} = \boldsymbol{u} - A\boldsymbol{u} \text{ for some } \boldsymbol{u} \in \mathbb{R}^n\}. \end{aligned}$$

Find the following four sets: $C(A) \cap E(A)$, $N(A) \cap F(A)$, $C(A) \cap N(A)$, $C(A) + N(A)$.

Problem 8 (5%) $7! =$ __________________.

Problem 9 (5%) Consider n distinct objects. Two circular arrangements are equivalent if one can be obtained from the other by rotation. The number of circular arrangements is __________________.

Problem 10 (5%) For $m, n \geq 0$, $\sum_{k=m}^{n} \binom{k}{m} =$ __________________.

Problem 11 (5%) $\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} =$ __________________.

Problem 12 (5%) If $|A| = m$ and $|B| = n$, then there are __________________ relations from A to B.

Problem 13 (10%) There are __________________ ways to distribute m distinct objects into $m-1$ identical containers with no container left empty.

Problem 14 (5%) A relation is called an equivalence relation if it is reflexive, symmetric, and __________________.

Problem 15 (5%) ____ For $2^n - 1$ to be a prime, the necessary condition is that n be a prime.

T. true,
F. false.

Problem 16 (5%) The inverse of 5 modulo 11 is __________________

試題隨卷繳回