※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the general solution of the given differential equation on the interval (0, ∞)

$$x^2 y'' + 2xy' + \alpha^2 x^2 y = 0$$, Hint: change of variable via $y = x^{-1/2} u(x)$ (15%)

2. Find the particular solution of the given differential equation

$y'' - 10y' + 25y = e^{5x}, y(0) = 0, y'(0) = 0$ (10%)

3. Find the Laplace transform $\mathcal{L}\{\cos(t)\sinh(t)\}$ and $\mathcal{L}\{\cos(kt)\sinh(kt)\}$ where k is a positive constant. (10%)

4. $y'' + 2ty' - 4y = 6, y(0) = 0, y'(0) = 0,$ please use the series solution method to find $y(t) = ?$ (10%)

5. $P = \begin{bmatrix} 0 & 0 & a \\ a & 0 & 0 \\ 0 & a & 0 \end{bmatrix}$, Let $P^{-1} = AB$, in which A≠I, B≠I, and the summation of all elements in A and B should be

equal to $6a^{-1/2}$, please find the proper matrix A and B. (10%)

6. List all possible subspaces in R^3. (10%)

7. Please prove that the null space of A and the null space of $A^T A$ are the same. (A is a m by n matrix) (15%)

8. Find a vector x with ‖x‖= 10 such that ‖Ax‖ is maximized, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. (10%)

9. Which of the following attributes are the same between the similar matrices? (a) Eigenvalues, (b) Eigenvectors, (c) Trace and determinant, (d) Rank, (e) Null space and Column space, (f) Number of independent eigenvectors. Please write the answer in terms of (a) to (f), instead of the description. (10%)