1-10 題為填充題,請依題號,將答案填寫於答案卷上。

1. If $A = \begin{bmatrix} 5 & a \\ 1 & 4 \end{bmatrix}$ and $A^T = A$, then $a =$_________. (5%)

2. If $\begin{bmatrix} 1 & -1 & -1 \\ 2 & 1 & -3 \\ -1 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, then $x =$_________. (5%)

3. If $B = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $B^3 = \alpha B + \beta I_2$, then $(\alpha, \beta) =$________. (5%)

4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 1 & 1 & 2 \\ -2 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 4 & 1 \\ 5 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then $\det(A) =$________. (5%)

5. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & a \\ 3 & 1 & 0 \end{bmatrix}$. If $\mathrm{trace}(A^2) = 5$, then $a =$______________. (5%)

6. Suppose $w^T = [2 \quad -1 \quad 0 \quad 2 \quad 1]$ and the matrix $A = I_5 + \alpha w w^T$ is singular, then $(\alpha, \mathrm{rank}(A)) =$___________. (5%)

7. Let the diagonal matrix $D =$_____________. Then for any

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \in R^{3\times3}, \text{ we have } D^{-1}AD = \begin{bmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33} \end{bmatrix}. \text{ (5\%)}$$

8. If $u = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$, $v = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$, and $A = I_5 + [u \quad v] \begin{bmatrix} v^T \\ u^T \end{bmatrix}$,

then all the eigenvalues of A are___________. (5%)

9. If $P_n = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n+1} \\ \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n+2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n+1} & \frac{1}{n+2} & \cdots & \frac{1}{2n} \end{bmatrix}$, then $\frac{\det(P_{n+1})}{\det(P_n)} =$___________. (5%)

10. Given two complex vectors $u, v \in C^n$ with $u^H u = v^H v \neq 0$ and $u \neq v$. If the complex matrix $A(u,v) =$_________, then $A(u,v)^H A(u,v) = I_n$, and $A(u,v)u = v$. (5%)

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11 Find the possibly maximal number of edges contained in

(a) a bipartite graph with 12 vertices and (5%)

(b) a planar graph with 5 vertices. (5%)

12 Suppose that $c_1 2^n + c_2 3^n + n - 7$ is the general solution to $a_{n+2} + p_1 a_{n+1} + p_2 a_n = q_1 n + q_2$, where $n \geq 0$ and p_1, p_2, q_1, q_2 are constants. Find

(a) p_1 and p_2; (5%)

(b) q_1 and q_2. (5%)

13 Suppose $A = \{w, x, y, z\}$. Find the number of relations on A that are

(a) reflexive; (5%)

(b) symmetric and contain (x, y). (5%)

14 Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$. (10%)

15 Suppose that f: $G \to H$ is a group homomorphism and f is onto. Prove that if G is abelian, then H is abelian. (10%)

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