編號： 212

國立成功大學 106 學年度碩士班招生考試試題

系　　所：電機資訊學院-資訊聯招

考試科目：計算機數學

考試日期：0213，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。只有答案，沒有計算過程或說明原因，也不予計分。考卷含線性代數及離散數學

一、線性代數 (50%)

1. (15%) Given A and p, if $\lim_{n\to\infty} A^n p = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$. Please find a,b,c,d.

$$A = \begin{bmatrix} 1 & 2/5 & 1/10 & 0 \\ 0 & 3/10 & 1/2 & 0 \\ 0 & 0 & 1/5 & 0 \\ 0 & 3/10 & 1/5 & 1 \end{bmatrix}, p = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

2. (10%) Let V be the vector space R^3. Determine if U is a subspace of V if U contains the following vectors.
   (a) U={(x,y,z):x<0}
   (b) U={(x,y,z):x+y+z=3}
   (c) U={(x,y,z):x=2z}

3. (10%) Let $U=R^n$ and $V=\{(x_1,x_2,...,x_n)\in U \mid x_1+x_2+...+x_n=0\}$, please find a basis of V over R

4. (15%) Let $T:P_2(R)\rightarrow P_2(R)$ be a linear operator that is defined according to

   $T(g(x)) = g(x) + x\frac{dg(x)}{dx} + \frac{dg(x)}{dx}$, in which $P_2(R)$ is a set of all polynomials with real–value coefficients and degree n, n=0,1,2. Please find the eigenvalues and the associated eigenvectors of the operator $5T+2T^2+T^{10}$

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二. Discrete Mathematics (total: 50%)

5. In how many ways can 36 identical robots be assigned to five assembly lines with

   (a) at least four robots assigned to each line? (5%)

   (b) at least four, but no more than ten, assigned to each line? (5%)

   (no score if you give no details.)

6. Let $D = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & 0 \\ & -1 & 2 & \ddots & \\ & & -1 & \ddots & -1 \\ & 0 & & \ddots & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$, i.e., $D(i,j) = \begin{cases} 2, & \text{if } i = j. \\ -1, & \text{if } |i-j| = 1. \\ 0, & \text{elsewhere.} \end{cases}$

   (a) Use recurrence relations to express determinant of D, i.e., $|D|$. (5%)

   (b) Find the general solution for $|D_n|$. (5%)

   (c) $|D_1| = 2, |D_2| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$. Find $|D_{100}|$. (5%)

   (no score if you give no details.)

7. Solve the following recurrence relations: $6a_n - 5a_{n-1} + a_{n-2} = \sin(n\pi)$ with $a_0 = 1, a_{-1} = a_{-2} = 0$. (10%)(no score if you give no details.)

8. Let $(Q, \oplus, \otimes)$ denote the **field**, where $\oplus$ and $\otimes$ are defined by $a \oplus b = a + b - k$, $a \otimes b = a + b - (ab/m)$, for fixed elements k, m $(\neq 0)$ of Q. Determine the values for k and m in each of the following:

   (a) The zero element for the field is 5. (5%)

   (b) The additive inverse of the element 8 is -7. (5%)

   (c) The multiplicative inverse of 3 is $1/6$. (5%)

   (no score if you give no details.)