國立臺灣大學101學年度碩士班招生考試試題

請於答案卷上標明題號後依序作答
請勿使用選擇題作答區。

1. (30 points) Assume that the elements are pairwise distinct. Answer the following questions on sorting algorithms.

   (1) (5 points) What is the number of permutations for n elements?

   (2) (5 points) A single comparison between two elements can distinguish up to 2 permutations. How many permutations can be distinguished using k comparisons?

   (3) (10 points) Prove that a sorting algorithm using only pairwise comparisons requires $\Omega(n \log n)$ comparisons to sort n elements in the worst case. You may use the following formula

   $$n! \geq \sqrt{2\pi} \cdot n^{n+\frac{1}{2}} e^{-n} \text{ for } n \geq 1.$$

   (4) (10 points) Sorting algorithms using operations other than pairwise comparisons can sort elements in time faster than $O(n \log n)$ under certain assumptions. Three representative algorithms are bucket, counting and radix sorting. Choose one of those algorithms, discuss the assumptions behind it, and analyze its computational complexity.

2. (15 points) Assume that $f(n) = O(g(n))$ with $g(n) \geq 2$ for all n. Are the following claims true or false?

   (1) (3 points) $f(n) + g(n) = O(g(n))$.

   (2) (3 points) $f(n) \cdot g(n) = O(g(n) \log_2 g(n))$.

   (3) (3 points) $\sqrt{f(n)} = O(\sqrt{g(n)})$.

   (4) (3 points) $\log_2 f(n) = O(\log_2 g(n))$.

   (5) (3 points) $2^{f(n)} = O(2^{g(n)})$.

3. (15 points) For each of the following, compute how large a problem instance do you need before algorithm A is faster than or equal to algorithm B. On that particular instance, how much time do the algorithms take?

   (1) (5 points) Algorithm A takes n^2 days to solve a problem of size $n \in N$. Algorithm B takes n^3 seconds on the same problem.

   (2) (10 points) Algorithm A takes n^2 days to solve a problem of size $n \in N$. Algorithm B takes 2^n seconds on the same problem. For your reference, $\log_2 86400 \approx 16.4$, and you may need the following log table for your calculation.

n	21	22	23	24	25	26	27	28	29	30
$\log_2 n$	4.39	4.46	4.52	4.58	4.64	4.70	4.75	4.80	4.86	4.90

4. (20 points) Which of the following is not true.

   (1) (5 points) In a red-black tree: (a) Every node is either red or black. (b) The shortest possible path has all red nodes. (c) The longest possible path alternates between red and black nodes. (d) A red-black tree is a binary search tree.

   (2) (5 points) Chaining, overflow areas, re-hashing, etc. are all techniques applied to resolve collisions in a hash table. (a) Re-hashing is the slowest method for resolving collisions. (b) Chaining allows an unlimited number of collisions to be handled and does not require prior knowledge of how many elements are contained in the collection. (c) To use re-hashing, one must know the maximum number of elements first.

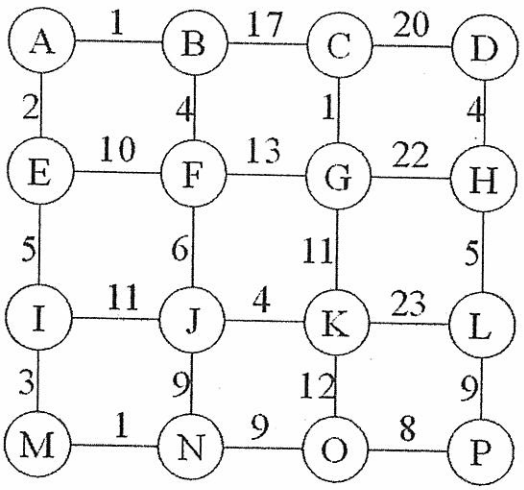
見背面

(3) (5 points) Dynamic algorithm solves problems by solving smaller versions of the problem, saving the solutions to the small problems and then combining them to solve the larger problem. (a) The tradeoff is space for increased speed. (b) Dynamic programming can be effectively applied to solve the longest common subsequence (LCS) problem, for an example, to compare common DNA sequences between two gene sequences. (c) Dynamic programming partitions the problem into independent subproblems, solve the subproblems recursively, and then combine their solutions to solve the original problem.

(4) (5 points) Construct a Huffman encoding for the five characters E, T, A, S, N with the relative frequencies {E: 40, T: 16, A: 8, S: 8, N: 28}. Which of the following encoding does not match with the string? (a) ETA 0 100 1010 (b) SEA 1011 0 1010 (c) TEN 100 0 11 (d) EAS 0 1010 1001

5. (5 points) The Fourier Transform provides the means of transforming a signal defined in the time domain into one defined in the frequency domain. The Discrete Fourier Transform (DFT) can be viewed as a special case of the Fourier Transform that requires the signal to be discrete and of finite duration. DFT can be implemented with the Fast Fourier Transform (FFT) algorithm, a special algorithm with considerable savings in computational time. Given the efficiency of FFT, it is also commonly used to approximate the Fourier Transform of a continuous signal. There are some limitations inherent in this approach, though. Which of the following phenomena is not the result in errors between the FFT-computed and the desired transform: (a) aliasing, (b) leakage, (c) the picket-fence effect, (d) linkage.

6. (15 points) For the following graph G, answer the following questions. Break all ties by picking the vertices in alphabetical order (i.e., A before Z)



(1) (2 points) Report the order of the vertices encountered on a breadth-first search of G starting from vertex A.

(2) (2 points) Report the order of the vertices encountered on a depth-first search of G starting from vertex A.

(3) (5 points) Find the minimum spanning tree of G. Write the answer in DFS order starting from vertex A.

(4) (6 points) Find the shortest path spanning tree of G rooted in vertex A. Write the answer in DFS order starting from vertex A.