

- (1) Given the following three tree traversal combinations, tell which combination results in a unique binary tree. If it corresponds to a unique binary tree, show the tree and list the level order traversal. Otherwise, explain why more than one binary tree is produced, or there does not exist such a binary tree.

(a) preorder: J C B A D E F I G H (5%)

inorder: A B C E D F J G I H

(b) preorder: A B D G C E H I F (5%)

postorder: G D B H I E F C A

(c) postorder: F E C H G D B A (5%)

inorder: F C E A B H D G

- (2) Given the following data structures with  $n$  elements, please specify how to delete the largest and the smallest elements, and list the complexity.

(a) max heap (10%)

(b) 2-3-4 tree (10%)

- (3) XML (eXtensible Markup Language) is a universal format for representing documents. The following shows part of a large-scale XML document collection:

<article>

<author> M.F. Porter </author>

<title> An Algorithm for Suffix Stripping </title>

<abstract> The automatic removal of suffixes from words ... </abstract>

<pages> 313-316 </pages>

</article>

<article>

<author> D.K. Harman </author>

<title> User-Friendly Systems Instead of User-Friendly Front-Ends </title>

<abstract> Most systems are not designed to service end user ... </abstract>

<pages> 413-423 </pages>

</article>

...

Users usually submit queries like "find papers written by a specific author", "find papers about a specific topic", "find papers consisting of some keywords", and so on.

- (a) Propose data structures to support the above queries efficiently. (8 %)
- (b) Explain how to find papers written by some specific author (e.g., M.F. Porter) and concerning some specific topic (e.g., suffix removal) based on the data structures in (a). (7 %)

- (4) Give an  $O(n^2)$ -time algorithm to find the longest monotonically increasing subsequence of a sequence of  $n$  numbers. (20%)
- (5) Argue that the solution to the recurrence  $T(n) = T(\frac{n}{4}) + T(\frac{3n}{4}) + n$  is  $\Omega(n \log n)$  by appealing to a recursion tree. (15%)
- (6) The *square* of a directed graph  $G = (V, E)$  is the graph  $G^2 = (V, E^2)$  such that  $(u, w) \in E^2$  if and only if for some  $v \in V$ , both  $(u, v) \in E$  and  $(v, w) \in E$ . That is,  $G^2$  contains an edge between  $u$  and  $w$  whenever  $G$  contains a path with exactly two edges between  $u$  and  $w$ . Describe an efficient algorithm for computing  $G^2$  from  $G$  for the adjacency-matrix representation of  $G$ . Analyze the running time of your algorithm. (15%)