系　　　所：電機資訊學院-資訊聯招

考試科目：工程數學　　　　　　　　　　　　　　　　　考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。只有答案，沒有計算過程或說明原因，也不予計分。

1. (10%) Solve the initial-value problem $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1-x^2)}, y(0) = 3$

2. (10%) Solve $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 3 + \ln x^3$

3. (10%) $y'' + P(x)y' + Q(x)y = 0$, where P(x) and Q(x) are continuous on some interval I. Let us suppose that y1(x) is a known solution on I and that y1(x)!=0 for every x in the interval I. Please find another solution in terms of y1(x).

4. (10%) Please find $\mathscr{L}^{-1}\{\dfrac{(s+k)^2 - 2k^2}{(s^2+k^2)^2}\} = ?$

5. (10%) Let $p(x)y'' + q(x)y' + r(x)y = 0$, where p(x),q(x), and r(x) can not be reduced. Please summarize the cases when we want to find the series solution at x=a.

6. (15%) Let A=$\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 2 & 4 \end{bmatrix}$,b=$\begin{bmatrix} 113 \\ 226 \\ 791 \end{bmatrix}$

   (a) (5%) Find the projection of b onto the column space of A.

   (b) (5%) Split b into p+q, with p in the column space and q perpendicular to that space. Find q.

   (c) (5%) Which space contains q?

7. (10%) Let A=$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix}$

   (a) (5%) Find Jordan form of A.

   (b) (5%) Find P such that $P^{-1}AP$ is a Jordan form.

8. (25%) Consider the nxn matrix $M = aI_n + bx_nx_n^T$ where a and b are scalars, I_n is an identity matrix with dimension n and x_n is a n-dimension column vector with all elements 1.

   (a) (5%) Find the eigenvalues and eigenspace of M.

   (b) (5%) If M is called singular, find the values of a and b.

   (c) (5%) If M is called idempotent, find the values of a and b.

   (d) (5%) When M is nonsingular, find the inverse of M.

   (e) (5%) When M is nonsingular, find the determinant of M.