

1. (15 points) Given a number sequence  $S[1..n]$ , the MaxSum procedure outputs

$$\max\{0, \max_{1 \leq i \leq j \leq n} \sum_{k=i}^j S[k]\}$$
. Complete the following pseudocode.

```

MaxSum ( $S, n$ )
begin
     $x \leftarrow 0; y \leftarrow 0$ 
    for  $i \leftarrow 1$  to  $n$ 
        do if  $S[i] + y > x$  then  $y \leftarrow$  [1A];  $x \leftarrow$  [1B]
            elseif  $S[i] + y > 0$  then  $y \leftarrow$  [1C]
            else  $y \leftarrow 0$ 
        print  $x$ 
    end

```

2. (15 points) Complete the following pseudocode so that it prints “123 123 132 123 123 213 213 231 213 123 321 321 312 321 123” in order when it is invoked by  $X(S, 1, 3)$ , where  $S[i]=i$  for  $1 \leq i \leq 3$ .

```

X( $S, k, n$ )
begin
    if  $k=n$  then print  $S[1..n]$ 
    else
        for  $i \leftarrow k$  to  $n$ 
            do  $temp \leftarrow S[k]$ 
                 $S[k] \leftarrow S[i]$ 
                 $S[i] \leftarrow temp$ 
                X( $S, [2A], [2B]$ )
                 $temp \leftarrow S[k]$ 
                 $S[k] \leftarrow S[i]$ 
                 $S[i] \leftarrow temp$ 
            X( $S, n, n$ )
    end

```

3. (20 points) The binary-search-tree property: Let  $x$  be a node in a binary search tree. If  $y$  is a node in the left subtree of  $x$ , then  $\text{key}[y] \leq \text{key}[x]$ . If  $y$  is node in the right subtree of  $x$ , then  $\text{key}[x] \leq \text{key}[y]$ . Suppose that we have integer numbers between 1 and 10000 in a binary search tree and want to search for the number 2006. The following sequence is the sequence of nodes examined. Give all the feasible ranges of the variable  $k$  in the sequence.

1000, 5566, 5203,  $k$ , 1314, 1510, 2381, 2006

4. (20 points) Give tight asymptotic bounds for the following recurrences.

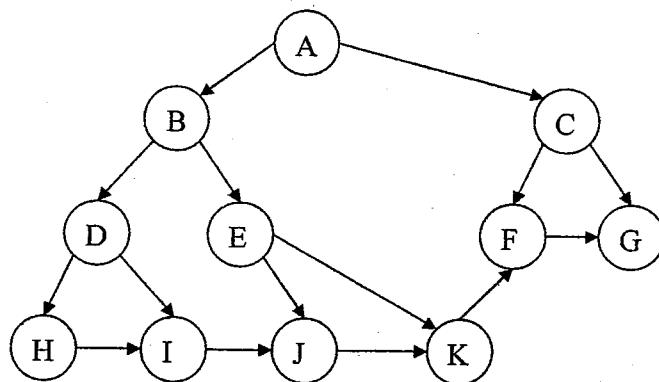
a.  $T(n) = 2T\left(\frac{n}{2}\right) + n$

b.  $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + n$

c.  $T(n) = T\left(\frac{n}{7}\right) + T\left(\frac{3n}{4}\right) + n$

d.  $T(n) = 7T\left(\frac{n}{4}\right) + n^2$

5. (15 points) Add one edge to the following directed graph to make it strongly connected.



6. (15 points) Complete the following Depth-First Search pseudocode, which is used to classify the edges of the input graph  $G=(V, E)$ :

DFS( $G$ )

1. for each vertex  $u \in V[G]$
2. do  $\text{color}[u] \leftarrow \text{WHITE}$
3.  $\pi[u] \leftarrow \text{NIL}$
4. time  $\leftarrow 0$
5. for each vertex  $u \in V[G]$
6. do if  $\text{color}[u]=\text{WHITE}$
7. then DFS-VISIT( $u$ )

DFS-VISIT( $u$ )

1.  $\text{color}[u] \leftarrow \text{GRAY}$
2. time  $\leftarrow \text{time} + 1$
3.  $d[u] \leftarrow \text{time}$
4. for each  $v \in \text{Adj}[u]$
5. do if  $\text{color}[v]=\text{WHITE}$
6. then  $\pi[v] \leftarrow u$
7. DFS-VISIT( $v$ )
8.  $(u, v)$  is a tree edge
9. elseif  $\text{color}[v]=\text{GRAY}$
10. then  $(u, v)$  is a [6A] edge
11. elseif  $\text{color}[v]=\text{BLACK}$
12. then if [6B]
13.  $(u, v)$  is a cross edge
14. else  $(u, v)$  is a [6C] edge
15.  $\text{color}[u] \leftarrow \text{BLACK}$
16. time  $\leftarrow \text{time} + 1$
17.  $f[u] \leftarrow \text{time}$