

This entrance examination includes two parts:

Part I: Linear Algebra (50%); Part II: Discrete Math. (50%).

Part I: Linear Algebra

1. Find the LDU decomposition of

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

where L is a lower triangular matrix with 1's along the main diagonal, D is a diagonal matrix, and U is an upper triangular matrix with 1's along the main diagonal. (14%)

2. Let $T: M_{22} \rightarrow M_{22}$ be the linear operator (or linear mapping) defined by

$$T(X) = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} X + X \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

where $X \in M_{22}$, M_{22} is the vector space of all 2×2 matrices. Calculate the Rank and Nullity of T . (12%)

3. Let A and B be $n \times n$ matrices. Prove that if A is nonsingular, then $A + B$ and $I + BA^{-1}$ are both nonsingular or both singular. (8%)

4. The matrix A is nonsingular. How will A^{-1} be affected if c times the i th row of A is added to the j th row? (8%)

5. Prove that a symmetric matrix A is positive definite if and only if all the eigenvalues of A are positive. (8%)

离散数学 (Part II)

1. Let $G = (V, E)$ be bipartite with V partitioned as $X \cup Y$. For all $x \in X$, $\deg(x) \geq 3$, and for all $y \in Y$, $\deg(y) \leq 7$. If $|X| \leq 50$, find an upper bound on $\delta(G)$. (10%)
2. Determine whether each of the following statements is true or false. (10%)
 - a) A group with only three elements is commutative.
 - b) An additive group can be isomorphic to a multiplication group.
 - c) The empty set \emptyset can be considered a group.
 - d) A group is a cancellative semigroup.
 - e) A group may have more than one identity.
3. Let A be a set with $|A|=n$. A k -ary operation for A is called commutative if $f(a_1, a_2, \dots, a_k) = f(\pi(a_1), \pi(a_2), \dots, \pi(a_k))$, where $a_1, a_2, \dots, a_k \in A$ (repetitions allowed), and $\pi(a_1), \pi(a_2), \dots, \pi(a_k)$ is any rearrangement of a_1, a_2, \dots, a_k . How many of the k -ary operations on A are commutative? (10%)
4. Consider the following Pascal program segment, where i, j, k , and h are integer variables (10%)

```
For i:=1 to 30 do
  For j:=1 to i do
    For k:=1 to j do
      For h:=1 to k do
        writeln (i+j+k+h);
```

How many times is the `writeln` statement executed in this program segment?
5. Find the coefficient of x^{83} in $f(x) = (x^5 + x^6 + x^7 + x^{14} + x^{17})^{10}$. (10%)