Homework - 3
COEN 240-Machine Learning

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1. **Find the derivative of the function $f(x)$=5(x+47)^2**

Sol: $f(x)=5(x+47)^2$
The derivative of $f(x)$ is as below:

$$\boxed{f'(x)=10(x+47)}$$

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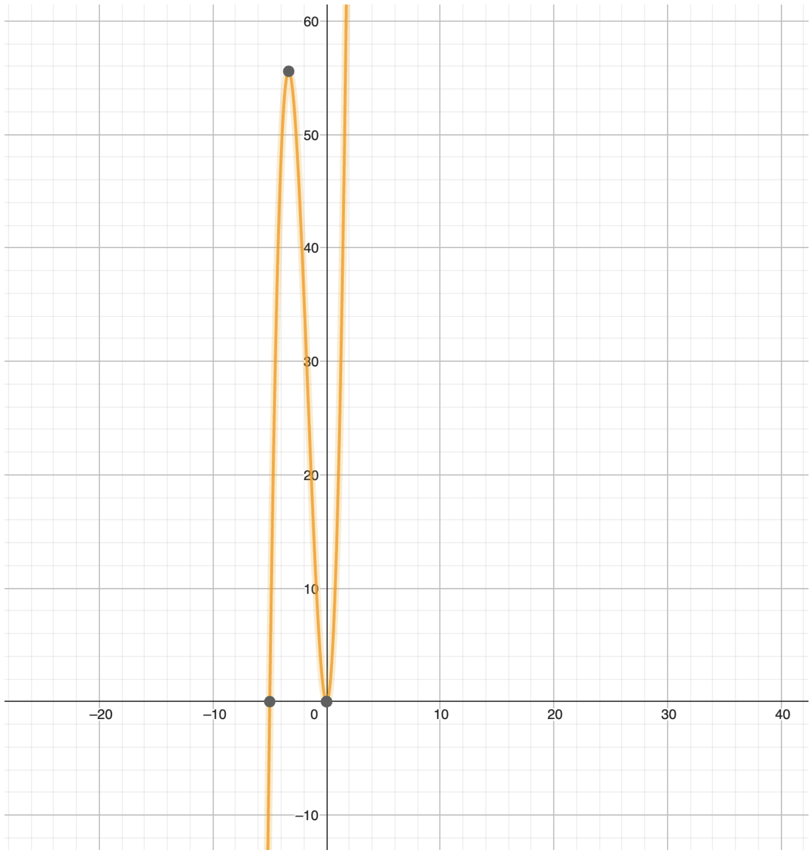
2. **Determine the minimum and maximum of the function $f(x)=3x^3+15x^2$. Then sketch it.**

Sol: $f(x)=3x^3+15x^2$
1st derivative : $f'(x)=9x^2+30x=0$
by solving we get : $x(9x+30)=0$
from above → $x_1=0$; $x_2= -3.33$

2nd derivative $f'(x)=9x^2+30x$
$f'(x)= 18x+30$
→ $f''(0) > 0$ => Minimum.
→ $f''(-3.33) < 0$ => Maximum

$$\boxed{\begin{array}{l} f(0) = 0 \Rightarrow \text{Min: } (0,0) \\ f(-3.33) = 55.56 \Rightarrow \text{Max: } (-3.33, 55.56) \end{array}}$$

Below is the representation on graph:



Find the partial derivative of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

3. ***f(x, y)* = 3x + 4y**

Sol: By deriving as mentioned in the question we get:

$$\boxed{\frac{\partial f}{\partial x} = 3 \text{ and } \frac{\partial f}{\partial y} = 4}$$

4. ***f(x, y)* = $xy^3 + x^2y^2$**

$$\boxed{\frac{\partial f}{\partial x} = y^3+2xy^2 \quad \text{and} \quad \frac{\partial f}{\partial y} = 3xy^2+2x^2y}$$

5. ***f(x, y)* = $x^3y + e^x$**

Sol:

$$\frac{\partial f}{\partial x} = 3x^2y + e^x \text{ and } \frac{\partial f}{\partial y} = x^3$$

6. ***f(x, y)* = xe^{2x+3y}**

Sol:

$$\frac{\partial f}{\partial x} = e^{2x+3y} + 2xe^{2x+3y} \quad \text{and } \frac{\partial f}{\partial y} = 3xe^{2x+3y}$$

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7. **Given a function *J*(w):**

Sol:

$$J(w_0, w_1) = \frac{1}{2m} \sum_{i=1}^{m} x(w_0 + w_1 x^{(i)} - y_i)^2$$

Deriving $\frac{\partial J(w)}{\partial w0}$ and $\frac{\partial J(w)}{\partial w1}$ are as below:

$$\frac{\partial J(w)}{\partial w0} = \frac{1}{m} \sum_{i=1}^{m} (w_0 + w_1 x^{(i)} - y_i)$$

$$\frac{\partial J(w)}{\partial w1} = \frac{1}{m} \sum_{i=1}^{m} (w_0 + w_1 x^{(i)} - y_i) * x^{(i)}$$

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8. **Find the derivative of the function:** *f(x)* = $\frac{1}{1+e^{-x}}$

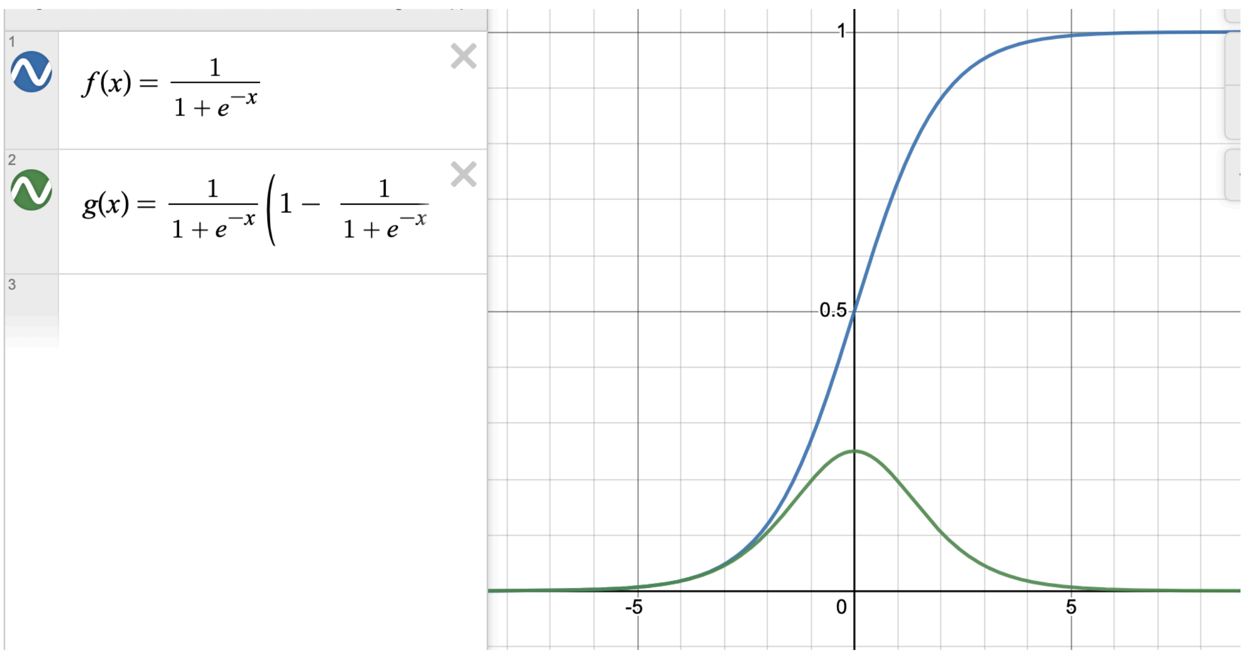
Sol: Using Quotient rule: $h(x) = \frac{f(x)}{g(x)}$ *then,*

$$h'(x) = \frac{f'(x)\, g(x) - f(x) g'(x)}{g(x)^2}$$

$$f'(x) = \frac{0 - -e^{-x}}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})} \frac{1}{(1+e^{-x})}$$

$$f'(x) = \frac{1}{\left(1+e^{-x}\right)} * \frac{\left(1+e^{-x}\right)-1}{\left(1+e^{-x}\right)} = \frac{1}{\left(1+e^{-x}\right)} * \frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}$$

$$\boxed{f'(x) = \left(\frac{1}{\left(1+e^{-x}\right)}\right) * \left(1 - \frac{1}{1+e^{-x}}\right) = f(x)*(1 - f(x))}$$



f(x) = function given to us - represented in blue

g(x) = derivative of f(x) - represented in green

References:

1. Derivatives for machine learning

https://towardsdatascience.com/a-quick-introduction-to-derivatives-for-machine-learning-people-3cd913c5cf33

2. Quotient rule https://www.geeksforgeeks.org/quotient-rule/

3. https://towardsai.net/p/machine-learning/mastering-derivatives-for-machine-learning