Homework - 8
COEN 240-Machine Learning

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Consider N i.i.d. samples drawn from a Poisson distribution. The PMF is defined as follows:

$$Poisson(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!} \text{ for } x \in \{0,1,2,\dots\} \text{ where } \lambda > 0 \text{ is the rate parameter.}$$

Find λ_{MLE}.

Show your work.

Solution is below

Sol:

Niid. samples $\sim$ Poisson (x / λ)

$$= \prod_{i=1}^{N} \frac{\lambda^{x^{(i)}}}{x^{(i)}!}$$

Log function :

$$L(\lambda) = \log \prod_{i=1}^{N} e^{-\lambda} \frac{\lambda^{x^{(i)}}}{x^{(i)}!}$$

$$L(\lambda) = \prod_{i=1}^{N} \left(\log e^{-\lambda} + \log \lambda^{x^{(i)}} - \log x^{(i)}! \right)$$

$$L(\lambda) = \prod_{i=1}^{N} \left(-\lambda + x^{(i)} \log \lambda - \log x^{(i)}! \right)$$

$$= N\lambda + \prod_{i=1}^{N} x^{(i)} \log \lambda - \prod_{i=1}^{N} \log x^{(i)}!$$

$$\frac{\partial L(\lambda)}{\partial \lambda} = -N + \frac{1}{\lambda} \prod_{i=0}^{N} x^{(i)} = 0$$

$$\Rightarrow \lambda_{MLE} = \frac{1}{N} \sum_{i=1}^{N} x^{(i)}$$

Reference:

1. **https://www.investopedia.com/terms/p/poisson-distribution.asp**
2. **https://www.geeksforgeeks.org/poisson-distribution/**