

Problem 1:

- a) 3 [dimensionality of dataset]
- b) [8.2 2.5 1.7] (x^5)
- c) 6.9 (x_1^3)
- d) 0.6 (x_3^1)
- e) blue (y^4)

Problem 2:

- a) 1,2 [values of i]
- b) 3 [value of d]
- c) 3×1 [dimension of μ_i]
- d) 3×3 [dimension of Σ_i]
- e) $1/2$ [value for $P(w_i)$] for equal prior probabilities.

Problem 3:

- A) d Logistic regression
- B) e Elastic regression
- C) c High generalization error
- D) c Logistic regression
- E) a stop training when validation error reaches minimum
- F) c ?
- G) c softmax Regression.
- H) C It is susceptible to outliers

Problem 4:

Sol: $TP = 5$ [True positive]

~~FN~~ = 4 [False Negatives].

FP = 3 [False positives]

$$\text{Precision} = \frac{TP}{(TP+FP)} = \frac{5}{(5+3)} = \frac{5}{8} = 62.5\%$$

$$\text{Recall} = \frac{TP}{(TP+FN)} = \frac{5}{(5+4)} = \frac{5}{9} = 55.6\%$$

$$\boxed{\text{Precision} = 62.5\%}$$

$$\boxed{\text{Recall} = 55.6\%}$$

Problem 5:

a) Given $w = [1, -2, 3, -4]$

$$x = [10, 20, 30, 40]$$

$$\text{Predicted value } \hat{y} = w \cdot x$$

$$\hat{y} = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$$

$$\hat{y} = 1 \cdot 10 + (-2)(20) + (3)(30) + (-4)(40)$$

$$\hat{y} = 10 - 40 + 90 - 160$$

$$\boxed{\hat{y} = -100}$$

b) Ridge Regression :

$$J(\omega) = \text{MSE}(\omega) + \frac{1}{2} \alpha \sum_{i=1}^n w_i^2$$

Penalty term of Ridge Regression when parameter set to 0.6 is :

$$\begin{aligned} &= \frac{1}{2} \times 0.6 (1^2 + (-2)^2 + (3)^2 + (-4)^2) \\ &= 0.3 (1+4+9+16) \\ &= 0.3(30) = 9 \end{aligned}$$

\therefore Penalty term of Ridge Regression is 9

Problem 6 :

Given data set $x = [50, 60, 70, 80]$

$y = [c, a, c, b]$

x	y	x	$y=a$	$y=b$	$y=c$
50	c	50			1
60	a	60	1		
70	c	70			1
80	b	80		1	

Maximum depth = 1 [root node & its 2 children].

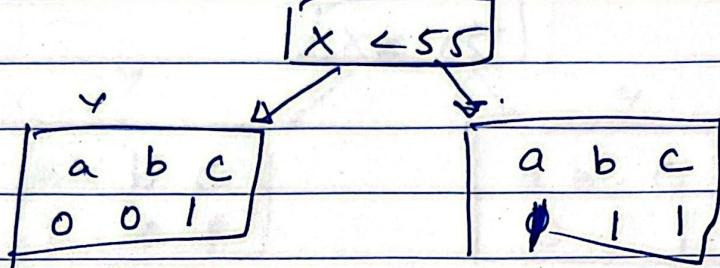
$$\text{Gini} = 1 - \sum_{i=1}^m p_i^2$$

$$= 1 - \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \left(\frac{2}{4}\right)^2$$

$$\text{Gini} = 1 - 0.0625 - 0.0625 - 0.25 = 1 - 0.375$$

$$\text{Gini} = 0.625$$

a) Threshold = 55



Gini Impurity for Left Node =

$$\text{Gini}_{\text{left}} = 1 - (\text{Prob of } a)^2 - (\text{Prob of } b)^2 - (\text{Prob of } c)^2$$

$$\text{Gini}_{\text{left}} = 1 - \left(\frac{1}{3}\right)^2 = 0$$

$$\underline{\text{Gini}_{\text{left}} = 0}$$

Gini Impurity for Right Node =

$$\text{Gini}_{\text{right}} = 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2$$

$$= 1 - 0.1111 - 0.1111 - 0.1111$$

$$= 1 - 0.4444$$

$$\underline{\text{Gini}_{\text{right}} = 0.6667}$$

Minimum CART cost function [combined Gini Impurity]

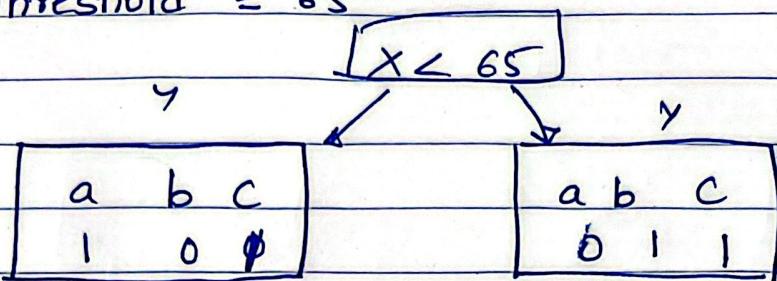
$$J(k, T_L) = \frac{m_{\text{left}}}{m} \text{Gini}_{\text{left}} + \frac{m_{\text{right}}}{m} \text{Gini}_{\text{right}}$$

$$= \left(\frac{1}{4}\right)(0) + \left(\frac{3}{4}\right)(0.6667)$$

$$\boxed{J(k, T_L) = 0.500025}$$

for $\text{Gini}_{\text{right}} \circled{0.667} \geq J(k, T_L) = 0.50025$

Threshold = 65



$$\text{Gini Left} = 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 1 - 0.25 - 0.25 = 1 - 0.5$$

$$\text{Gini Left} = 0.5$$

$$\text{Gini Right} = 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 1 - 0.25 - 0.25 = 1 - 0.5$$

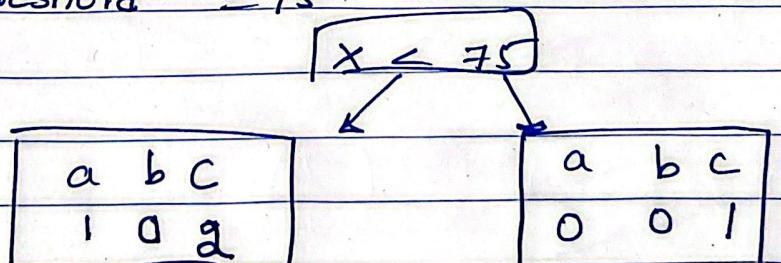
$$\text{Gini Left} = 0.5$$

Minimum CART function:

$$J(K, T_E) = \left(\frac{2}{4}\right)(0.5) + \left(\frac{2}{4}\right)(0.5)$$

$$J(K, T_E) = 0.5$$

Threshold = 75



$$\text{Gini Left} = 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2$$

$$= 1 - 1/9 - 4/9 = 0.444$$

$$\boxed{\text{Gini Left} = 0.444}$$

$$\text{Gini}_{\text{right}} = 1 - \left(\frac{1}{4}\right)^2 = 0$$

$$\text{Gini}_{\text{right}} = 0$$

Minimum CART function

$$J(k, T_L) = \left(\frac{3}{4}\right)(0.444) + \left(\frac{1}{4}\right)0$$

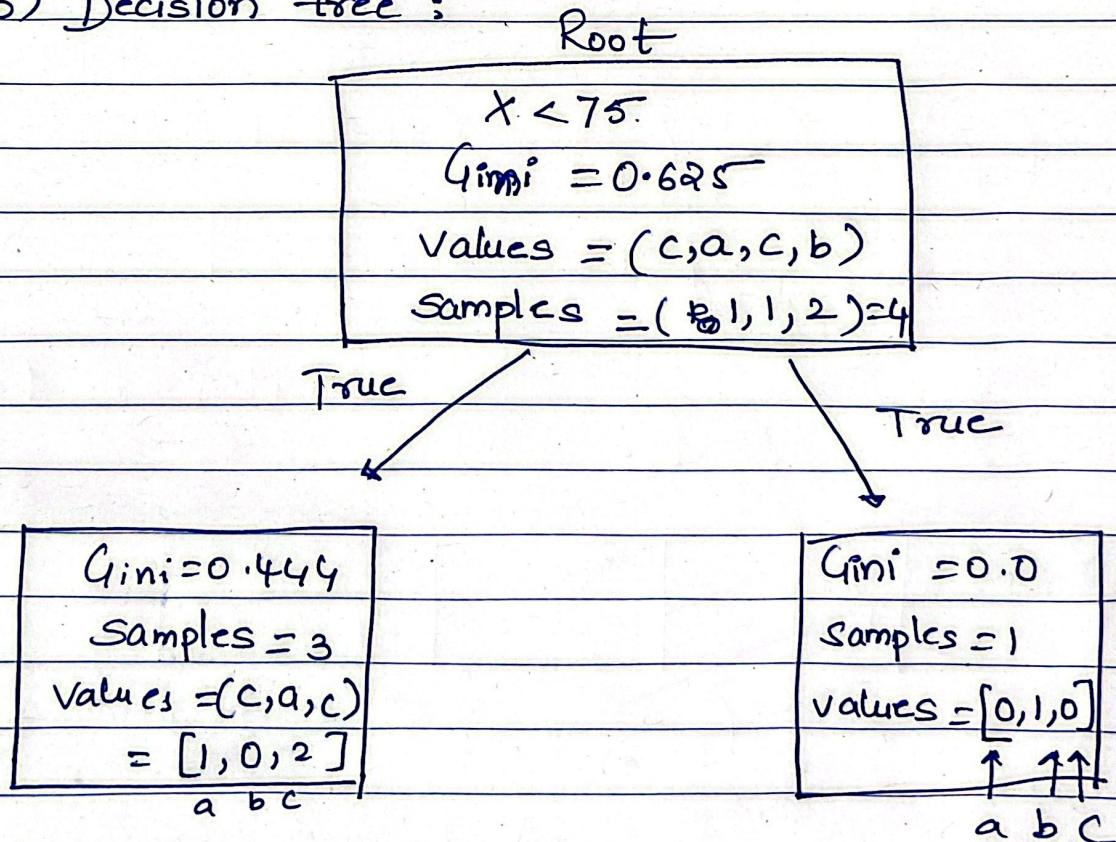
$$J(k, T_L) = 0.333$$

From the above threshold we get

Minimum threshold for $x < 75$

Cost value = 0.333.

b) Decision tree :



$$\text{Gini score of root} = 1 - \sum_{i=1}^m p_i^2$$
$$= 1 - \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \left(\frac{2}{4}\right)^2$$
$$= 1 - \left(\frac{1}{16}\right) - \left(\frac{1}{16}\right) - \left(\frac{1}{4}\right)$$

Gini score of root = 0.625