Quantum computation and information - Indranil Chakravarty

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Contents

Chapter 1

Lecture 1: Introduction

Taught in collaboration with MSR Redmond for the Q# bits.

Topics:

- Intro: Transition from Classical to Quantum: Stern Gerlash, Sequential Stern Gerlash, Rise of randomness.
- Foundations of Quantum Theory: States, Ensembles, Qubits, Pure and Mixed states, Multi qubit states, Tensor products, Unitary transforms, Spectral decomposition, SVD, Generalized measurements, Projective measurements, POVM, Evolution of quantum state, Krauss Representation.
- Quantum Entropy: Subadditivity of Entropy, Avani-Licb(?) Inequality, Quantum channel, Quantum channel capacity, Data compression, Benjamin Schumahur(?) theorem.
- Quantum Entanglement: EPR paradox, Schmidt decomposition, Purification of entanglement, Entanglement separability problem, Pure and mixed entangled states, Measures of Entanglement.
- Quantum information processing protocols: Teleportation, Superdense coding, Entanglement swapping.
- Impossible operations in quantum information theory: No cloning, No deleting, No partial erasure.
- Quantum Computation: Introduction to Quantum Computating, Pauli gates, Hadamard gates, Universal gates, Quantum algorithms (Shor, Grover search, machine learning and optimisation).
- Quantum programming: Programming quantum algorithms, Q# progtramming language, quantum subroutines.

Books:

- Quantum computation and Quantum information — Nielsen and Chuang.

- Preskill lecture notes.

Grading:

- Possibility of open book take-home open ended exam for the finals.
- Mid 1: 15%
- Mid 2: 15%
- End sem (open book?) : 30%
- Assignments: 15%
- Projects: 25%

1.1 Stern-Gerlach: A brief, morally correct construction of qubits

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light rays ---> [z] ---> (z+, z-) --block (z-) --> [x] --- (x+, x-) -- block (x-) --> [z] ---> (z+, z-?!)
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[z] represents a polarizer along that axis.

- Since we first polarized along z, how did we manage to get out light rays in the x direction? The polarization should have killed everything.
- Since we blocked $z-$, How did we get back $z-$ after passing stuff through $[x]$? Something has changed drastically from our classical picture.

We can consider $|z+\rangle$ to be something like:

$$|z+\rangle \equiv_? \frac{1}{2}|x+\rangle + \frac{1}{2}|x-\rangle$$

Where $|x+\rangle$ and $|x-\rangle$ are basis vectors for some vector space over $\mathbb{R}$.

If we were to pass the $z+$ light rays through $[y]$, then we would get $|y+\rangle, |y-\rangle$. So, $|z+\rangle$ is also:

$$|z+\rangle \equiv_? \frac{1}{2}|y+\rangle + \frac{1}{2}|y-\rangle$$

1.1.1 Analogy with polarization of light

Consider a monochromatic light wave in the z direction. A linearly polarized light with polarization in the x direction which we call x polarized light is given by:

$$E_x = E_0 \hat{x} \cos(kz - \omega t)$$

$\omega \equiv$ frequency $\equiv ck$, $c \equiv$ speed of light, $k \equiv$ wave number.

Similarly, y polarized light is given by:

$$E_y = E_0 \hat{y} \cos(kz - \omega t)$$

Consider the case where we have x filters along direction $-$, x' filter along direction $/$, y filters along direction $|$. In this case, we can have x, x', y filters arranged sequentially give us non-zero output (contrast with just having x, y).

We can express the x' polarization as:

$$E_0\hat{x}'\cos(kz-\omega t) = \frac{E_0}{\sqrt{2}}\hat{x}\cos(kz-\omega t) + \frac{E_0}{\sqrt{2}}\hat{y}\cos(kz-\omega t)$$

By analogy, we write:

$$|z_+\rangle \equiv \frac{1}{\sqrt{2}}|x_+\rangle + \frac{1}{\sqrt{2}}|x_-\rangle$$

However, we now have probability $\frac{1}{\sqrt{2}}$, but we want $\frac{1}{2}$. So, we define the probability as:

$$\langle x+|x_-\rangle^2 = \frac{1}{2}$$

$$\begin{aligned}
z_+ &\equiv x \text{ polarization} \\
z_- &\equiv y \text{ polarization} \\
x_+ &\equiv x' \text{ polarization} \\
x_- &\equiv y' \text{ polarization}
\end{aligned}$$

This problem can be solved again by polarization of light. This time, we consider circularly polarized light which can be obtained by letting linearl polarized light passing through a quarter wave plate (?)

When we pass such circularly polarized light through an x or y filter, we again obtain either an x polarized beam, or a y polarized beam of equal intensity. Yet, everybody knows that circularly polarized light is totally different from 45° linearly polarized light.

A right circularly polarized light is a linear combination of x polarized light and y polarized light, where the oscillation of the electric field for the y component is 90° out of phase with the x polarized component.

$$\begin{aligned}
E &= \frac{E_0}{\sqrt{2}}\hat{x}\cos(kz-\omega t) + \frac{E_0}{\sqrt{2}}\hat{y}\cos\left(kz-\omega t+\frac{n}{2}\right) \\
\frac{E}{E_0} &= \frac{1}{\sqrt{2}}\hat{x}e^{i(kz-\omega t)} + \frac{i}{\sqrt{2}}\hat{y}e^{i(kz-\omega t)}
\end{aligned}$$

Similarly, left circularly polarized light is:

$$E = \frac{E_0}{\sqrt{2}}\hat{x}\cos(kz-\omega t) - \frac{E_0}{\sqrt{2}}\hat{y}\cos\left(kz-\omega t+\frac{n}{2}\right)$$

1.2 Observable

An observable is something that we observe.

$$Z\left|z+\right\rangle = \frac{\mathrm{hbar}}{\sqrt{2}}\left|z+\right\rangle \qquad Z\left|z-\right\rangle = \frac{\mathrm{hbar}}{\sqrt{2}}\left|z-\right\rangle$$

TODO: try to construct an operator that takes a vector $\left|v\right\rangle$ to a vector that is orthogonal to it.

1.3 Operators

1.3.1 Projectors — P

Suppose W is a k-dimensional vector subspace of the d-dimensional vector space V.

Using Gram-Schmidt, it is possible to construct an orthonormal basis $|1\rangle, |2\rangle, \ldots |d\rangle$ for V such that $|1\rangle \ldots |k\rangle$ is an orthonormal basis for W. Then the projector P is defined as:

$$P_W \equiv \sum_{i=1}^{k} |i\rangle\langle i|$$