Software Foundations

Siddharth Bhat

Spring 2020

Contents

Chapter 1

Introduction

Let $S = \langle X_s, X_s^0, U_s, \xrightarrow{s}, Y_s, h_s \rangle$ be a transition system.

A *finite run* originating from $x_0 \in X^0$ is a finite sequence $(x_0, u_0, x_1, u_1, \ldots u_{n-1}, x_n)$ such that $\forall i \in \{0, \ldots, n-1\}$, $x_i \xrightarrow{s}_{u_i} x_{i+1}$. This is sometimes denoted as $x_0 \xrightarrow{s}_{u_0} x_1 \xrightarrow{s}_{u_1} \ldots \xrightarrow{s}_{u_{n-1}} x_n$. In the notation of Tabuada, this is called as the *finite internal behaviour*. A run has information about both states and transitions.

$\langle x_0, x_1, \ldots, x_n \rangle$ is a *finite trajectory* iff $\exists u_0, \ldots u_{n-1}$ such that $x_0 \xrightarrow{u_0} x_1 \xrightarrow{u_1} \ldots \xrightarrow{u_{n-1}} x_n$ is a finite run. The trajectory has information only about states.

$\langle y_0, y_1, \ldots, y_n \rangle$ is a *finite trace* iff there exists a finite trajectory $\langle x_0 \ldots x_n \rangle$ and $\forall i \in \{0, \ldots, n\}$, $y_i = h_s(x_i)$. The finite trace has information only about projections of a state.

The *finite behaviour* of a system S is defined as the union of all finite traces of S. This is notated as $\mathcal{B}(S)$.

The infinite behaviour of a system S is the union of all infinite traces of S, notated as $\mathcal{B}^{\omega}(S)$.