1) Consider the harmonic oscillator $dx/dt = v$, $dv/dt = -\omega^2 x$.

Show that the orbits are given by ellipses $-\omega^2x^2 + v^2 = C$, where C is any non-negative constant.

2) For the following systems, find the fixed points, sketch the nullclines, the vector field, and a plausible phase potrait.

a) $dx/dt = x - y$, $dy/dt = 1 - e^x$
b) $dx/dt = x - x^3$, $dy/dt = -y$

3) For each of the following systems, find the fixed points, classify them, sketch the neighbouring trajectories, and try to fill in the rest of the phase potrait.

a) $dx/dt = x - y$, $dy/dt = x^2 - 4$
b) $dx/dt = \sin y$, $dy/dt = x - x^3$

4) Consider the following system :

$$dx/dt = xy,\quad dy/dt = x^2 - y$$

a) Show that the linearization predicts that the origin is a non-isolated fixed point.
b) Show that the origin is in fact an isolated fixed point.
c) Sketch the potrait for the above system.