# Topics in Physics - C. Mukku

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# Chapter 1

# Lagrangian, Hamiltonian mechanics

Mechanics in terms of generalized coords.

## 1.1 Lagrangian

Define a functional. $L$ over the config. space of partibles $q^i$, $qdot^i$. $L = L(q^i, qdot^i)$. We have an explicit dependence on $t$.

$L = KE - PE$

Assuming a 1-particle system of unit mass,

$$L = \frac{1}{2}\dot{q}^2 - V(q)$$

Assuming an n-particle system of unit mass,

$$L = \sum_i \frac{1}{2} qdot^{i2} - V(q^i)$$

## 1.2 Variational principle

Take a minimum path from $A$ to $B$. Now notice that the path that is slightly different from this path will have some delta from the minimum.

Action

$$S(t0, t1) = \int L \mathrm{d}t = \int_{t0}^{t1} L(q^i, qdot^i)\mathrm{d}t$$

. Least action: $\delta S = 0$

# Chapter 2

# Functional calculus

this chapter develops a completely handwavy physics version of functional analysis.

**Definition 1** *A* ***functional*** *$F$ is a function: $F : (\mathbb{R} \to \mathbb{R}) \to \mathbb{R}$*

**Notation 1** *Evaluation of a functional $F$ with respect to $f$ is denoted by $F[f]$.*

## 2.1 Functional Derivative - take 1

Consider a functional $F : (\mathbb{R} \to \mathbb{R}) \to \mathbb{R}$, a function $f : \mathbb{R} \to \mathbb{R}$, and a "test function" $\phi : \mathbb{R} \to \mathbb{R}$.

Consider a functional $F$. We only define the derivative of a functional $F$ with respect to a function $f$ by what happens under an integral sign as follows:

$$\int \frac{\delta F}{\delta f}(x)\phi(x)\mathrm{d}x = \lim_{\epsilon \to 0} \frac{F[f + \epsilon\phi] - F[f]}{\epsilon}$$

Now, we can define a small variation in $F$ as:

$$\begin{aligned} \delta F &: (\mathbb{R} \to \mathbb{R}) \times (\mathbb{R} \to \mathbb{R}) \to \mathbb{R} \\ \delta F(f, \phi) &\equiv \int \frac{\delta F}{\delta f}(x)\phi(x)\mathrm{d}x \end{aligned}$$

Intuitively, $\delta F$ tells us the variation of the function $f$ along a test function $\phi$. So, it encapsulates some kind of "directional derivative".

So, we can look at $\frac{\delta F}{\delta f}$ as a functional as follows:

$$\begin{aligned} \frac{\delta F}{\delta f} &: (\mathbb{R} \to \mathbb{R}) \to \mathbb{R} \\ \frac{\delta F}{\delta f}(\phi) &= \delta F(f, \phi) \end{aligned}$$

Wehre $\frac{\delta F}{\delta f}$ allows us to "test" the change of $F$ with respect to $f$ along a given "direction" $\phi$.

## 2.2 Functional Derivative as taught in class

Substitute $\phi = \delta(x-p)$. Now, the quantity:

$$\frac{\delta F}{\delta f}\phi(x) = \delta F(f, \delta(x-p))$$

Rewriting $\delta F$ by sticking it under an integral:

$$\int \frac{\delta F}{\delta f}(x)\delta(x-p)\mathrm{d}x = \lim_{\epsilon \to 0} \frac{F[f + \epsilon\delta(x-p)] - F[f]}{\epsilon}$$

$$\left.\frac{\delta F}{\delta f}\right|_p = \lim_{\epsilon \to 0} \frac{F[f + \epsilon\delta(x-p)] - F[f]}{\epsilon}$$

That is, we can start talking about "derivative of the functional $F$ with respect to a function $f$ at a point $p$" as long as we only test the functional $F$ against $\delta$-functions.

So, we can alternatively define this quantity as:

$$\left.\frac{\delta F}{\delta f}\right|_p \equiv \lim_{\epsilon \to 0} \frac{F[f + \epsilon\delta(x-p)] - F[f]}{\epsilon}$$

While this does not "look like a functional", it actually is, if we mentally replace:

$$p \to \int \_ \; \delta(x-p)\mathrm{d}x$$

This is how mukku got that expression.