Assignment 1

1. Suppose $|v_i\rangle$ is an orthonormal basis for the inner product space V. What is the matrix representation for the operator $|\phi_j\rangle\langle\phi_k|$ with respect to the $|v_i\rangle$ basis.
2. Show that a positive operator is necessarily Hermitian.
3. Show that for any operator A, $A^\dagger A$ is positive.
4. Show that the eigenvalues of a projector P are all either 0 or 1.
5. Show that the tensor product of two unitary operators is unitary.
6. Show that the tensor product of two projectors is a projector.
7. Find the square root and logarithm of the matrix

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$

8. Show
   - tr(BA) = tr(AB)
   - tr(A+B) = tr(A) + tr(B)
   - tr(2A) = 2 tr(A)
9. Show that
   - $[A, B] = -[B, A]$
   - $\frac{[A,B]+\{A,B\}}{2} = AB$
10. Express the polar decomposition of a normal matrix in the outer product representation.
11. Find the left and the right polar decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$