Nonlinear dynamics and chaos

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Contents

Chapter 1

Useful links

- Course taught at Cornell is `MAE5790`. Video lectures available on YouTube.
- Textbook: nonlinear dyanmics and chaos
- MIT course number: `18.385J / 2.036J` Contains problem sets and some lecture notes which prove theorems.

Chapter 2

1D flows

2.1 Logistic eqn

$$\dot{N} = rN(1 - \frac{N}{K})$$

2.2 Types of bifurcations

- Saddle node bifurcation/ Blue-sky: Fixed points appear. $\dot{X} = r + X^2$
- Transcritical bifurcation: Fixed points switch stability. $\dot{X} = RX - X^2$
- Pitchfork bifurcation: Appear in problems that have a symmetry.
  - Supercritical Pitchfork bifurcation: $\dot{x} = rx - x^3$. Cubic term is stabilizing, pulls system back towards $x = 0$.
  - Subcritical Pitchfork bifurcation: $\dot{x} = rx + x^3$. We will have a blow up since x^3 pushes.
  - Subcritical Pitchfork bifurcation with stability: $\dot{x} = rx + x^3 - x^5$. (Solve `3.4.14`, `3.4.15`)
- Imperfect bifurcation, catastrophe

2.3 Flows on circle

2.3.1 Uniform oscillator

2.3.2 Nonuniform oscillator

2.3.3 Overdamped pendulum

2.3.4 Fireflies

2.4 2D Linear flows

2.5 2D Nonlinear flows

2.5.1 Conservative systems

2.5.2 Rabbit and Sheep

2.5.3 Reversible systems

2.5.4 Pendulum

2.5.5 Index theory

TODO: has links to alternate homology theories (Morse homology and singular homolory. Read Morse homology sometime).