1 Q1. Deterministic TMs

1.1 $f(n) = 2^n$

the idea is that we encode both the input and the output in unary. We strike out inputs' digits one-by-one. When we do, we copy the output digits (that is, we double the current output) that is currently on the tape. We then go back to the input and repeat this process.

1.2 $f(v, b) = \log_b(v)$

One simple way to encode the log is to implement $g(i) = b^i$. We then compute $g(i)$, $i \geq 0$ till $g(i) \geq v$. We then return $i - 1$

To implement $g(i)$, we copy the string b i times, similar to how we copy the string once in the implementation of 2^n.

2 Q2. `NP`$\subseteq$ `PSPACE`

since `3-SAT` is NP-complete, we show that `3-SAT` can be computed in `PSPACE`. Hence, `NP`$\subseteq$ `PSPACE`.

We can simply enumerate all possible 2^n assignments of the given `3-SAT` problem in $\log(2^n) = n = poly(n)$ space, and verify whether they satisfy the given problem.

3 Q3. Nodes not in path between two nodes

Note that a node v is in the path $s \to t$, if there exists paths $s \to v$, $v \to t$.

We can first compute all pairs shortest path using floyd-warshall, which is $O(V^3)$, after which for each v, we can look for the existence of paths $s \to v \to t$, in $O(1)$ time. The total time works out to be $O(V^3) + O(1)O(V) = O(V^3)$

4 Q4. Tshirts

We first compute probabilites which are used to compute expectation.

$$P(\text{k people not receiving t shirts}) =$$
$$P(\text{picking } (n-k) \text{ people from } n \text{ people, } n \text{ times (with reptition)}) =$$
$$\left(\frac{n-k}{n}\right)^n$$

Now that we know the probability of k people not receiving t-shirts, we can compute the expectation of this:

$$\mathbb{E}[\text{number of participants with no tshirt}] =$$

$$\sum_{i=0}^{n} iP(\text{i people not receiving t-shirts}) =$$

$$\sum_{i=0}^{n} i\left(\frac{n-i}{n}\right)^{n}$$