CSE 411: Complexity and Advanced Algorithms
Monsoon 2018
IIIT Hyderabad

Homework 4, Due: October 1, 2018

Questions 1 to 3 is for 5 points each. Question 4 is for 20 points. The program for Question 4 can be submitted in before the second mid exam.

Problem 1. Prove the following statements about the doubly logarithmic tree.

1. It has a depth of $O(\log \log n)$.

2. The number of nodes at level i is $2^{2^k - 2^{k-i}}$ for $0 \leq i < k$.

Problem 2. Suppose we have two algorithms A and B to solve a problem P of size n. Algorithm A takes $O(\log n)$ time on the PRAM using $O(n \log n)$ operations. Algorithm B reduces the size of P by a constant factor in $O(\log n / \log \log n)$ time using $O(n)$ operations. Devise an $O(\log n)$ time algorithm for P that uses only $O(n)$ operations.

Problem 3. Given n numbers in an array A, the All-Nearest-Smaller-Values problem is to find for each element of A, the largest index of the element of A that is smaller than the given element. Imagine you are given a solution to the ANSV problem that runs in time $O(t(n))$ time using $W(n)$ work. Use the solution to the ANSV problem to merge two sorted arrays of size $n/2$ each. What is the time and work complexity of your algorithm for merging.

Problem 4. (Programming Problem) Implement the parallel search and the parallel merge algorithms and study their performance. You can use OpenMP for your programming.