Optimization assignment – Linear algebra 2

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Prove that $(a+b)^T(a-b)^T = \|a\|^2 + \|b\|^2$. This question is wrong. Let us assume the dimension of a, b is $1 \times n$. Hence, $a+b, a-b$ have dimensions $n \times 1$, and can consequently not be matrix multiplied, let alone produce a scalar as output.

The question should probably be: prove that $(a+b)(a-b)^T = \|a\|^2 - \|b\|^2$.

$$
\begin{aligned}
(a+b)(a-b)^T &= \begin{bmatrix} a_1+b_1 & a_2+b_2 & \dots & a_n+b_n \end{bmatrix} \begin{bmatrix} a_1-b_1 \\ a_2-b_2 \\ \dots \\ a_n-b_n \end{bmatrix} \\
&= (a_1+b_1)(a_1-b_1) + (a_2+b_2)(a_2-b_2) + \cdots + (a_n+b_n)(a_n-b_n) \\
&= \sum_i a_i^2 - b_i^2 = \sum_i a_i^2 - \sum_i b_i^2 = \|a\|^2 - \|b\|^2
\end{aligned}
$$