

Principle of Information & Security

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Chapter 1

Lagrangian, Hamiltonian mechanics

Mechanics in terms of generalized coords.

1.1 Lagrangian

Define a functional. L over the config. space of partibles q^i, \dot{q}^i . $L = L(q^i, \dot{q}^i)$. We have an explicit dependence on t .

$$L = KE - PE$$

Assuming a 1-particle system of unit mass,

$$L = \frac{1}{2}\dot{q}^2 - V(q)$$

Assuming an n-particle system of unit mass,

$$L = \sum_i \frac{1}{2}\dot{q}^{i2} - V(q^i)$$

1.2 Variational principle

Take a minimum path from A to B . Now notice that the path that is slightly different from this path will have some delta from the minimum.

Action

$$S(t_0, t_1) = \int_{t_0}^{t_1} L dt = \int_{t_0}^{t_1} L(q^i, \dot{q}^i) dt$$

. Least action: $\delta S = 0$

Chapter 2

Functional calculus

this chapter develops a completely handwavy physics version of functional analysis.

Definition 1 A ***functional*** F is a function: $F : (\mathbb{R} \rightarrow \mathbb{R}) \rightarrow \mathbb{R}$

Notation 1 Evaluation of a functional F with respect to f is denoted by $F[f]$.

We define the derivative of a function F with respect to a function f at a point p as:

$$\left. \frac{\delta F}{\delta f} \right|_p \equiv \lim_{\epsilon \rightarrow 0} \frac{F[f + \epsilon \delta(x - p)] - F[f]}{\epsilon}$$