

Nonlinear dynamics and chaos

Siddharth Bhat

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Chapter 1

Useful links

Course taught at Cornell is MAE5790. [Video lectures available on YouTube](#)

Chapter 2

1D flows

2.1 Logistic eqn

$$\dot{N} = rN(1 - \frac{N}{K})$$

2.2 Types of bifurcations

- Saddle node bifurcation/ Blue-sky: Fixed points appear. $\dot{X} = r + X^2$
- Transcritical bifurcation: Fixed points switch stability. $\dot{X} = RX - X^2$
- Pitchfork bifurcation: Appear in problems that have a symmetry.
 - Supercritical Pitchfork bifurcation: $\dot{x} = rx - x^3$. Cubic term is stabilizing, pulls system back towards $x = 0$.
 - Subcritical Pitchfork bifurcation: $\dot{x} = rx + x^3$. We will have a blow up since x^3 pushes.
 - Subcritical Pitchfork bifurcation with stability: $\dot{x} = rx + x^3 - x^5$. (Solve 3.4.14, 3.4.15)
- Imperfect bifurcation, catastrophe

2.3 Flows on circle

2.3.1 Uniform oscillator

2.3.2 Nonuniform oscillator

2.3.3 Overdamped pendulum

2.3.4 Fireflies

2.4 2D Linear flows

2.5 2D Nonlinear flows

2.5.1 Conservative systems

2.5.2 Rabbit and Sheep

2.5.3 Reversible systems

2.5.4 Pendulum

2.5.5 Index theory

TODO: has links to alternate homology theories (Morse homology and singular homology. Read Morse homology sometime).