**Tag the question** Edit

Other Math | Differential Equa... | Separable Equations

**Step-by-step** Edit

Step 1 of 4

We will use our knowledge and understanding of differential equations to solve this question.

We are given that
$\frac{dy}{dx} = -y(y-1)(y+5)$ (i)

Step 2 of 4

Differential equations have an equilibrium solution at $y = y_{eq}$ if $\frac{dy}{dx}|_{y=y_{eq}} = 0$

By putting $y = 0, 1$ or $-5$ in (i), we can see $\frac{dy}{dx} = 0$
So, the third equilibrium in that list can be $y = -5$, because the other two are given in the question.
Step 3 of 4

We can solve the given differential equation by using the separation of variable method,

$\frac{dy}{dx} = -y(y-1)(y+5)$
$\Rightarrow \frac{dy}{y(y-1)(y+5)} = -dx$
$\Rightarrow \frac{y+(y+5)-2(y-1)}{y(y-1)(y+5)}dy = -7dx$
$\Rightarrow \frac{dy}{(y-1)(y+5)} + \frac{dy}{y(y-1)} - \frac{2dy}{y(y+5)} = -7dx$
$\Rightarrow \left(\frac{1}{6}\frac{y+5-(y-1)}{(y-1)(y+5)} + \frac{y-(y-1)}{y(y-1)} - \frac{2}{5}\frac{(y+5)-y}{y(y+5)}\right)dy = -7dx$
$\Rightarrow \left(\frac{1}{6(y-1)} - \frac{1}{6(y+5)} + \frac{1}{y-1} - \frac{1}{y} - \frac{2}{5y} + \frac{2}{5(y+5)}\right)dy = -7dx$
$\Rightarrow \frac{7dy}{6(y-1)} - \frac{7dy}{5y} + \frac{7dy}{30(y+5)} = -7dx$
$\Rightarrow \frac{dy}{6(y-1)} - \frac{dy}{5y} + \frac{dy}{30(y+5)} = -dx$
Now integrating both side,

$\Rightarrow \frac{1}{6}\ln(y-1) - \frac{1}{5}\ln(y) + \frac{1}{30}\ln(y+5) = -x + c$

Step 4 of 4

The general solution to the given differential equation is,

$\Rightarrow \frac{1}{6}\ln(y-1) - \frac{1}{5}\ln(y) + \frac{1}{30}\ln(y+5) = -x + c$
$\Rightarrow \ln\left[(y-1)^{\frac{1}{6}}(y+5)^{\frac{1}{30}}y^{-\frac{1}{5}}\right] = -x + c$ ...(i)

for the initial value problem, $y(-5) = 2$

$\Rightarrow 5\ln(2-1) - 6\ln(2) + \ln(2+5) = 30(5+c)$
$\Rightarrow \ln\left(\frac{7}{2^6}\right) = 30(5+c)$
$\Rightarrow c = -5 + \frac{\ln\left(\frac{7}{64}\right)}{30} = -5 - 0.073 \approx -5$

from solution (i),
let, $Y(x) = (y-1)^{\frac{1}{6}}(y+5)^{\frac{1}{30}}y^{-\frac{1}{5}}$
$\lim_{x\to\infty}(Y(x)) = \lim_{x\to\infty} e^{-x-5} = 0$
$\Rightarrow Y(x) = 0$

so, $y(x) \to 1$ as $x \to \infty$

You should be able to solve the later part.

Explanation

Ignoring the very small power of $y(x)$, $\lim_{x\to\infty}(y(x)-1)^{\frac{1}{6}}y(x)^{\frac{1}{5}} = 0$

**Final answer** Edit

Third equilibrium in the list is $y = -5$ and $\lim_{x\to\infty} y(x) = 1$

Confirm