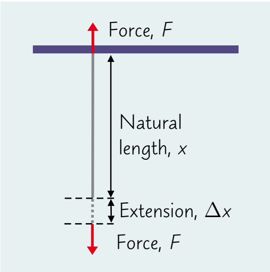
Hooke's Law

Hooke's law applies to all materials, but only up to a point. For some materials that point is so tiny you wouldn't notice...

Hooke's Law Says that Extension is Proportional to Force

If a **metal wire** is supported at the top and then a weight attached to the bottom, it **stretches**. The weight pulls down with force ***F***, producing an equal and opposite force at the support.

The material will only deform (stretch, bend, twist etc.) if there's a pair of opposite forces acting on it.



1) **Robert Hooke** discovered in the 17th century that the extension of a stretched wire, Δx, is proportional to the change in load or force, ΔF. This relationship is now called **Hooke's law**.
2) Hooke's law can be written:

$$\boxed{\Delta F = k\Delta x}$$

Where ***k*** is the stiffness of the object that is being stretched. ***k*** is called the **force constant** (or **stiffness constant**) and has units Nm^{-1}.

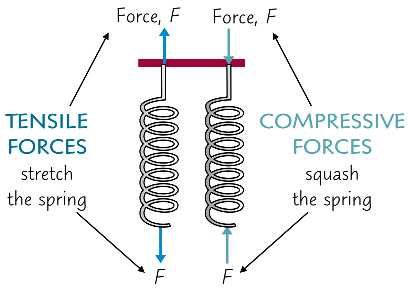
Hooke's Law Also Applies to Springs

A metal spring also changes length when you apply a **pair of opposite forces**.

1) The **extension** or **compression** of a spring is **proportional** to the **force** applied — so Hooke's law applies.
2) For springs, ***k*** in the formula $\Delta F = k\Delta x$ can also be called the **spring stiffness** or **spring constant**.

Hooke's law works just as well for **compressive** forces as **tensile** forces. For a spring, ***k*** has the **same value** whether the forces are tensile or compressive (that's not true for all materials).

3) **Hooke's Law** doesn't just apply to metal **springs** and **wires** — most **other materials** obey it up to a point.

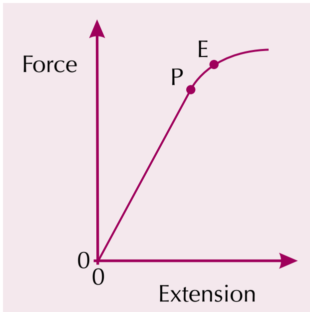


Tensile forces create tension in a stretched spring. Compressive forces create compression in a squashed spring. Tensile or compressive forces in the spring act in the opposite direction to the tensile or compressive forces stretching or squashing it.

Hooke's Law Stops Working when the Load is Great Enough

There's a **limit** to the force you can apply for Hooke's law to stay true.

1) The graph shows force against extension for a **typical metal wire** or **spring**.
2) The first part of the graph (up to point P) shows Hooke's law being obeyed — there's a **straight-line relationship** between **force** and **extension**.
3) When the force becomes great enough, the graph starts to **curve**. **Metals** generally obey Hooke's law up to the **limit of proportionality**, **P**.
4) The point marked **E** on the graph is called the **elastic limit**. If you exceed the elastic limit, the material will be **permanently stretched**. When all the force is removed, the material will be **longer** than at the start.
5) Be careful — there are some materials, like **rubber**, that only obey Hooke's law for **really small** extensions.



A Deformation can be Elastic or Plastic

A material will show elastic deformation **up to** its **elastic limit**, and plastic deformation **beyond** it.
If a **deformation** is **elastic**, the material returns to its **original shape** once the forces are removed.

1) When the material is put under **tension**, the **atoms** of the material are **pulled apart** from one another.
2) Atoms can **move** slightly relative to their **equilibrium positions**, without changing position in the material.
3) Once the **load** is **removed**, the atoms **return** to their **equilibrium** distance apart.

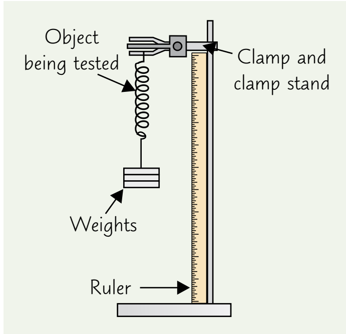
If a deformation is **plastic**, the material is **permanently stretched**.

1) Some atoms in the material move position relative to one another.
2) When the load is removed, the **atoms don't return** to their original positions.

Extension and compression are sometimes called tensile deformation and compressive deformation.

Hooke's Law

Investigating Extension

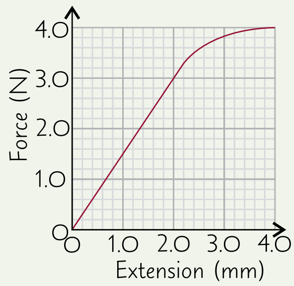


1) Set up the experiment shown in the diagram. Support the object being tested at the top (e.g. with a clamp) and measure its original (natural) length with a ruler.
2) Add weights one at a time to the bottom of the object.
3) After each weight is added, measure the new length of the object, then **calculate the extension**:

   extension = new length – original length

4) To measure an extension as **accurately** as possible, make sure you fix a ruler close to the extending object and take readings with your eye close to the ruler. It's a good idea to use a '**fiducial marker**' (a thin tag on the object that marks where you're measuring) and a set square to ensure the ruler is vertical. Make sure the marker and ruler are both at eye level when you take readings.
5) Plot a graph of **force** (weight) against **extension** for your results. Where the line of best fit is **straight**, then the object obeys Hooke's law and the gradient = $\boldsymbol{k}$ (as $\Delta F = k\Delta x$). If you've loaded the object beyond its limit of proportionality, the graph will start to curve.
6) Make sure you carry out the experiment **safely**. You should be **standing up** so you can get out of the way quickly if the weights fall, and wearing **safety goggles** to protect your eyes in case the object snaps.

Example: A student investigates extension using the set-up shown above, and plots their results on the graph below. Find the stiffness of the object being stretched.



The stiffness of the object, k, is the gradient of the graph up to the limit of proportionality.

For this graph, this is shortly after the point where the load is 3.0 N and the extension is 2.0 mm.

Convert 2.0 mm into m, to get 0.0020 m.

Then $k = \frac{\Delta F}{\Delta x} = \frac{3.0}{0.0020} = \mathbf{1500\ Nm^{-1}}$.

If the markings on your measuring equipment are quite far apart, you can often interpolate between them (e.g. if the length is halfway between the markings for 24 mm and 25 mm you could record it as 24.5 mm). But it's better to use something with a finer scale if you can.

Practice Questions

Q1 State Hooke's law and explain what is meant by the elastic limit of a material.

Q2 Define tensile forces and compressive forces.

Q3 From studying the force-extension graph for a material as it is loaded and unloaded, how can you tell:
   a) if Hooke's law is being obeyed, b) if the elastic limit has been reached?

Q4 What is meant by plastic deformation of a material?

Q5 Describe how you could determine the stiffness of an object being stretched, k, experimentally.

Exam Questions

Q1 A metal guitar string stretches 4.0 mm when a 10.0 N force is applied.

a) If the string obeys Hooke's law, calculate how far the string will stretch when a 15 N force is applied. [1 mark]

b) Calculate the force constant for this string in Nm^{-1}. [1 mark]

c) The string is then stretched beyond its elastic limit. Describe the effect this will have on the string. [1 mark]

Q2 A rubber band is 6.0 cm long. When it is loaded with 2.5 N, its length becomes 10.4 cm. Further loading increases the length to 16.2 cm when the force is 5.0 N.

State whether the rubber band obeys Hooke's law when the force on it is 5.0 N. Explain your answer. [2 marks]

Sod's Law — if you don't learn it, it'll be in the exam...

Three things you didn't know about Robert Hooke — he was the first person to use the word 'cell' (as in biology, not prisons), he helped Christopher Wren with his designs for St. Paul's Cathedral and no-one's sure what he looked like. I'd like to think that if I did all that stuff, then someone would at least remember what I looked like — poor old Hooke.

Stress, Strain and Elastic Strain Energy

How much a material stretches for a particular applied force depends on its dimensions. If you want to compare it to another material, you need to use stress and strain instead. A stress-strain graph is the same for any sample of a particular material — the size of the sample doesn't matter.

A Stress Causes a Strain

A material subjected to a pair of **opposite forces** might **deform** (i.e. **change shape**).
If the forces **stretch** the material, they're **tensile**. If the forces **squash** the material, they're **compressive**.

1) **Tensile stress**, σ, is defined as the **force applied**, F, divided by the **cross-sectional area**, A:

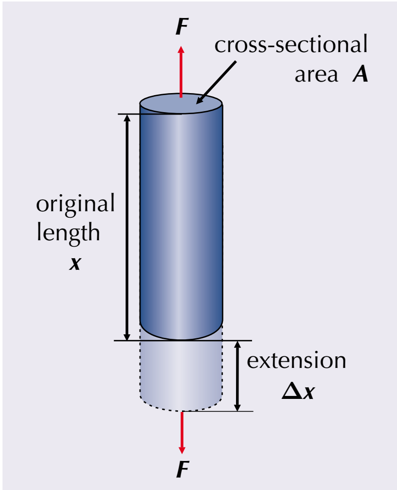
$$\boxed{\sigma = \frac{F}{A}}$$

The **units** of stress are $\mathbf{Nm^{-2}}$ or pascals, **Pa**.

2) **Tensile strain**, ε, is defined as the **change in length** (i.e. the **extension**), divided by the **original length** of the material:

$$\boxed{\varepsilon = \frac{\Delta x}{x}}$$

Strain has **no units**, it's just a **ratio** and is usually written as a **number**. It can also be written as a **percentage**, e.g. extending a 0.5 m wire by 0.02 m would produce a strain of (0.02 ÷ 0.5) × 100 = 4%.

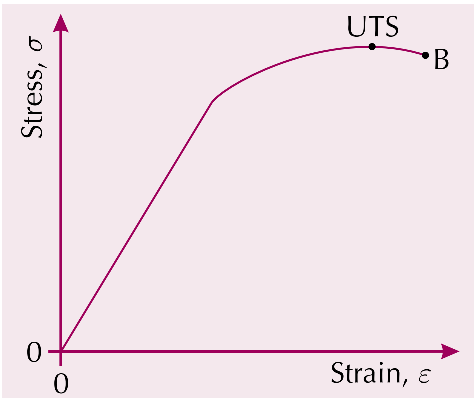


3) It doesn't matter whether the forces producing the **stress** and **strain** are **tensile** or **compressive** — the **same equations** apply. The only difference is that you tend to think of **tensile** forces as **positive**, and **compressive** forces as **negative**.

A Stress Big Enough to Break the Material is Called the Breaking Stress

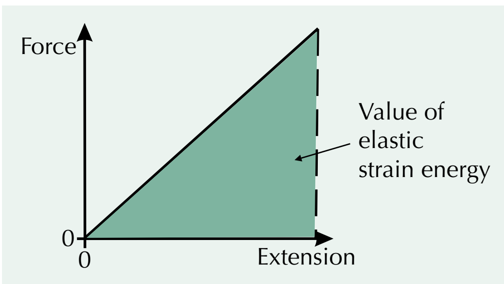
As a greater and greater tensile **force** is applied to a material, the **stress** on it **increases**.

1) The effect of the **stress** is to start to **pull** the **atoms apart** from one another.
2) Eventually the stress becomes **so great** that atoms **separate completely**, and the **material breaks**. This is shown by point **B** on the graph. The stress at which this occurs is called the **breaking stress**.
3) The point marked **UTS** on the graph is called the **ultimate tensile stress**. This is the **maximum stress** that the material can withstand.
4) Both **UTS** and **B** depend on conditions e.g. **temperature**.
5) **Engineers** have to consider the **UTS** and **breaking stress** of materials when designing a **structure** — e.g. they need to make sure the stress on a material won't reach the **UTS** when the **conditions change**.



Elastic Strain Energy is the Area under a Force-Extension Graph

1) **Work** has to be done to **stretch** a material.
2) **Before** the **elastic limit** is reached, **all** this **work done** in stretching is **stored** as **elastic strain energy** in the material.
3) On a **graph** of **force against extension**, the elastic strain energy is given by the **area under the graph**.
4) If the force-extension graph is **non-linear**, you'll need to **estimate** the area by **counting squares** or dividing the curve into **trapeziums** (see page 25).



Stress, Strain and Elastic Strain Energy

You can Calculate the **Energy Stored** in a **Stretched Material**

Provided a material obeys Hooke's law, the **potential energy** stored inside it can be **calculated** quite easily.

1) The work done on the material in stretching it is equal to the energy stored.
2) **Work done** equals **force × displacement**.
3) However, the **force** on the material **isn't constant**. It rises from zero up to force F. To calculate the **work done**, use the **average force** between zero and F, i.e. $\frac{1}{2}F$: $\boxed{\text{work done} = \frac{1}{2}F\Delta x}$
4) So the **elastic strain energy** stored, E_{el}, is: $\boxed{\Delta E_{el} = \frac{1}{2}F\Delta x}$

This is the triangular area under the force-extension graph — see previous page.

5) Because Hooke's law is being obeyed, $\Delta F = k\Delta x$, which means F can be replaced in the equation to give: $\boxed{\Delta E_{el} = \frac{1}{2}k\Delta x^2}$

Example: A metal wire is 55.0 cm long. A force of 550 N is applied to the wire, and the wire stretches. The length of the stretched wire is 56.5 cm. Calculate the elastic strain energy stored in the wire.

The extension of the wire is Δx = 56.5 cm – 55.0 cm = 1.5 cm = 0.015 m

So the elastic strain energy $\Delta E_{el} = \frac{1}{2} \times F \times \Delta x$
$= \frac{1}{2} \times 550 \times 0.015 = 4.125$ J = **4.1 J (to 2 s.f.)**

Practice Questions

Q1 Write a definition for tensile stress.

Q2 Explain what is meant by the tensile strain on a material.

Q3 What is meant by the breaking stress of a material?

Q4 How can the elastic strain energy in a material under load be found from its force-extension graph?

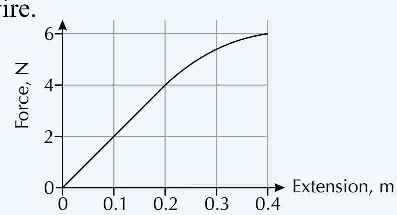
Q5 The work done is usually calculated as force multiplied by displacement.
Explain why the work done in stretching a wire is $\frac{1}{2}F\Delta x$.

Exam Questions

Q1 A steel wire is 2.00 m long. When a 300 N (to 3 s.f.) force is applied to the wire, it stretches by 4.0 mm. The wire has a circular cross-section with a diameter of 1.0 mm.
a) Calculate the tensile stress in the wire. [2 marks]
b) Calculate the tensile strain of the wire. [1 mark]

Q2 A copper wire (which obeys Hooke's law) is stretched by 3.0 mm when a force of 50.0 N is applied.
a) Calculate the stiffness constant for this wire in Nm^{-1}. [2 marks]
b) Calculate the value of the elastic strain energy in the stretched wire. [1 mark]

Q3 A force is applied to stretch an unknown material. The graph shows the force-extension graph for this stretch. Estimate the elastic strain energy stored in the material as a result of this stretch. [2 marks]



Q4 A pinball machine contains a spring which is used to fire a small, light metal ball to start the game. The spring has a stiffness constant of 40.8 Nm^{-1}. It is compressed by 5.00 cm and then released to fire the ball. Calculate the maximum possible kinetic energy of the ball. [3 marks]

UTS a laugh a minute, this stuff...

...or it would be, if there were more jokes. But you know what they say — a stress causes a strain, and nobody likes strained jokes. So you'd better just get on with learning this stuff proper good like. And before you ask, no — grabbing a slingshot and shooting the neighbours' begonias doesn't count as 'revising the energy stored in a stretched material'.

The Young Modulus

Busy chap, Thomas Young. He did this work on tensile stress as something of a sideline. Light was his main thing. He proved that light behaved like a wave, explained how we see in colour and worked out what causes astigmatism.

The **Young Modulus** is Stress ÷ Strain

When you apply a **load** to stretch a material, it experiences a **tensile stress** and a **tensile strain**.

1) Up to the **limit of proportionality** (see p.56), the stress and strain of a material are proportional to each other.
2) So below this limit, for a particular material, stress divided by strain is a **constant**. This constant is called the **Young modulus**, ***E***.

$$\textbf{Young modulus} = E = \frac{\textbf{stress}}{\textbf{strain}} = \frac{\sigma}{\varepsilon}$$

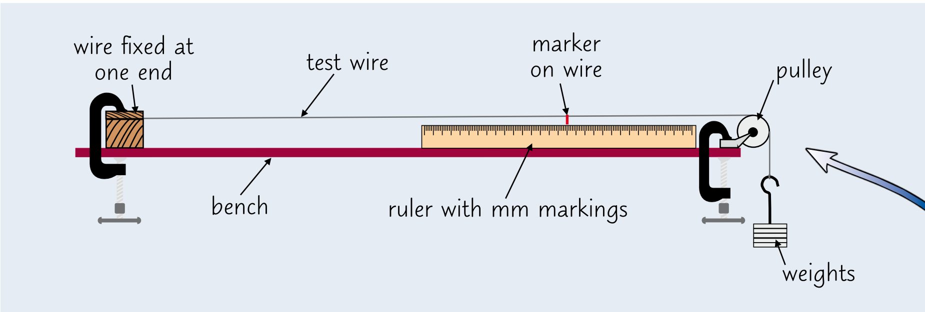
Where σ = stress and ε = strain

Remember that $\sigma = \frac{F}{A}$ and $\varepsilon = \frac{\Delta x}{x}$ — see p.58 for more.

3) The **units** of the Young modulus are the same as stress (**Nm⁻²** or pascals), since strain has no units.
4) The Young modulus is a measure of the **stiffness** of a material. It is used by **engineers** to make sure the materials they are using can withstand sufficient forces.

To **Find** the Young Modulus, You need a **Very Long Wire**

This is the experiment you're most likely to do in class:



"Okay, found one. Now what?"

Mum moment: if you're doing this experiment, wear safety goggles — if the wire snaps, it could get very messy...

1) The test wire should be thin, and as long as possible. The **longer and thinner** the wire, the more it **extends** for the same force — this reduces the percentage uncertainty in your measurements.
2) First you need to find the **cross-sectional area** of the wire. Use a **micrometer** to measure the **diameter** of the wire in several places and take an **average** of your measurements. By assuming that the cross-section is **circular**, you can use the formula for the area of a circle:

$$\textbf{area of a circle} = \pi r^2$$

The thickness of wires is sometimes given in swg (standard wire gauge) — a 36 swg wire has a radius of about 0.1 mm.

3) **Clamp** the wire to the bench (as shown in the diagram above) so you can hang **weights** off one end of it. Start with the **smallest weight** necessary to **straighten** the wire. (**Don't** include this weight in your final calculations.)
4) Measure the **distance** between the **fixed end of the wire** and the **marker** — this is your unstretched length.
5) Then if you increase the weight, the **wire stretches** and the **marker moves**.
6) **Increase** the **weight** in equal steps (e.g. 100 g intervals), recording the marker reading each time — the **extension** is the **difference** between this reading and the **unstretched length**. Because you can't take repeat readings (the wire might snap or be permanently stretched), you should take more readings than usual.
7) You can use your results from this experiment to calculate the **stress** and **strain** of the wire and plot a stress-strain curve (see next page).

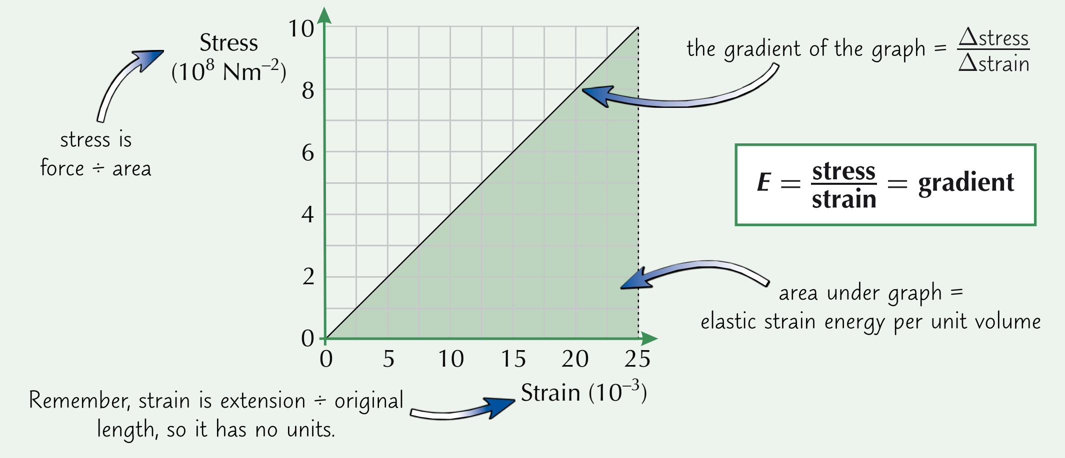
To reduce random errors you should use a thin marker on the wire, and always look from directly above the marker and ruler when measuring the extension.

(The other standard way of measuring the Young modulus in the lab is using **Searle's apparatus**. This is a bit more accurate, but it's harder to do and the equipment's more complicated.)

The Young Modulus

Use a *Stress-Strain Graph* to Find *E*

You can plot a **graph** of **stress against strain** from your results.



1) The **gradient** of the graph gives the Young modulus, ***E***.
2) The **area under the graph** gives the **strain energy** (or energy stored) per unit volume, i.e. the energy stored per 1 m^3 of wire.
3) The stress-strain graph is a **straight line** provided that Hooke's law is obeyed, so you can also calculate the energy per unit volume (Jm^{-3}) as:

energy per unit vol = ½ × stress × strain

Example: The stress-strain graph above is for a thin metal wire. Find the Young modulus of the wire from the graph.

E = stress ÷ strain = gradient

The gradient of the graph $= \frac{\Delta \text{stress}}{\Delta \text{strain}} = \frac{10 \times 10^8}{25 \times 10^{-3}}$

$= \mathbf{4 \times 10^{10}\ Nm^{-2}}$

Practice Questions

Q1 Define the Young modulus for a material. What are the units for the Young modulus?

Q2 Describe an experiment to find the Young modulus of a test wire. Explain why a thin test wire should be used.

Q3 What is given by the area contained under a stress-strain graph?

Exam Questions

Q1 A steel wire is stretched elastically. For a load of 80.0 N, the wire extends by 3.6 mm. The original length of the wire was 2.50 m and its average diameter is 0.60 mm. Calculate the value of the Young modulus for steel. [4 marks]

Q2 Two wires, A and B, are stretched elastically under a load of 50.0 N. The original length and the extension of both wires under this load are the same. The Young modulus of wire A is found to be $7.0 \times 10^{10}\ Nm^{-2}$. The cross-sectional area of wire B is half that of wire A. Calculate the Young modulus of wire B. [2 marks]

Q3 The Young modulus for copper is $1.3 \times 10^{11}\ Nm^{-2}$.

a) The stress on a copper wire is $2.6 \times 10^8\ Nm^{-2}$. Calculate the strain on the wire. [2 marks]

b) The load applied to the copper wire is 100 N (to 3 s.f.). Calculate the cross-sectional area of the wire. [2 marks]

c) Calculate the strain energy per unit volume for this loaded wire. [2 marks]

Learn that experiment — it's important...

Getting back to the good Dr Young... As if ground-breaking work in light, the physics of vision and materials science wasn't enough, he was also a well-respected physician, a linguist and an Egyptologist. He was one of the first to try to decipher the Rosetta stone (he didn't get it right, but nobody's perfect). Makes you feel kind of inferior, doesn't it...

Stress-Strain and Force-Extension Graphs

I hope the stresses and strains of this section aren't getting to you too much.

*There are **Three Important Points** on a **Stress-Strain Graph***

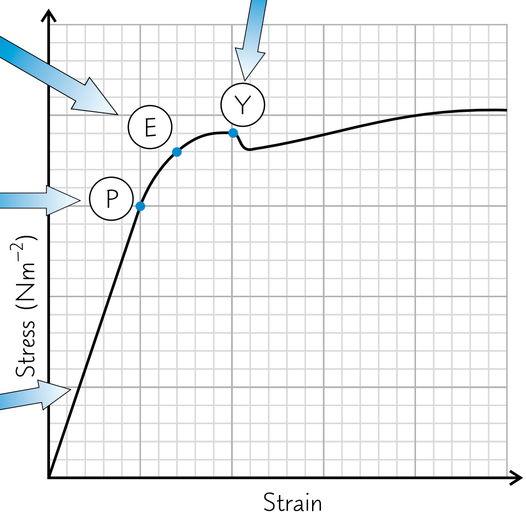
In the exam you could be given a **stress-strain graph** and asked to **interpret** it. Luckily, most stress-strain graphs share **three** important points — as shown in the **diagram**.

Point **Y** is the **yield point** — here the material suddenly starts to **stretch** without any extra load. The **yield point** (or yield stress) is the **stress** at which a large amount of **plastic deformation** takes place with a **constant** or **reduced load**.

Point **E** is the **elastic limit** — at this point the material starts to behave **plastically**. From point E onwards, the material would **no longer** return to its **original shape** once the stress was removed.

Point **P** is the **limit of proportionality** — after this, the graph is no longer a straight line but starts to **bend**. At this point, the material **stops** obeying **Hooke's law**, but would still **return** to its **original shape** if the stress was removed.

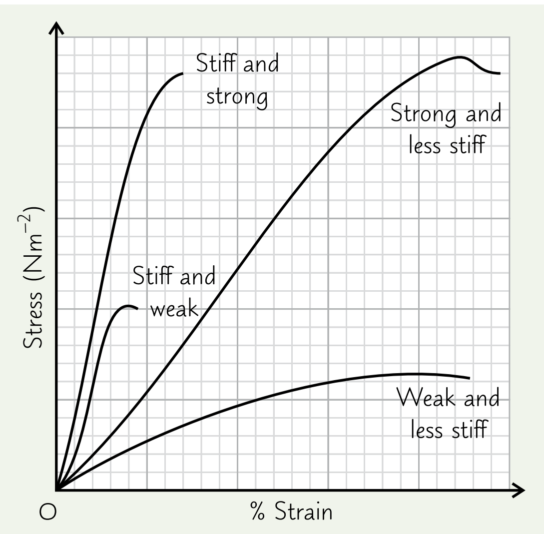
Before point **P**, the graph is a **straight line** through the **origin**. This shows that the material is obeying **Hooke's law** (page 56). The **gradient** of the line is constant — it's the **Young modulus** (see pages 60-61).



You might see the x-axis labelled as percentage strain instead — this is just the strain (a ratio) expressed as a percentage.

Stronger** Materials can Withstand **More Stress** Before they **Break

The graph below shows the stress-strain curves for materials of different strengths and stiffnesses.



1) Different materials have different **breaking stresses** (p.58).
2) The **stronger** the material, the higher the breaking stress.
3) **Stiff** materials are difficult to stretch or compress. They have a **large** Young's modulus.
4) For a given stress, a stiff material will have a lower strain (i.e. a smaller extension) than a less stiff material.
5) A stiff material **doesn't** have to be strong (and vice versa). Some stiff materials **break** under a low stress, and some strong materials **aren't** very stiff.

When a line on a stress-strain graph just stops, you can assume the material has reached its breaking stress and fractured (unless the question says otherwise).

Steve took a break after a stressful day at the office.

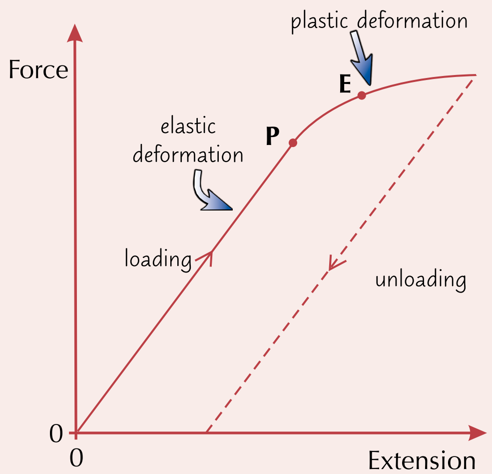
Stress-Strain and Force-Extension Graphs

Force-Extension Graphs Are Similar to *Stress-Strain* Graphs

You first met force-extension graphs on page 56.

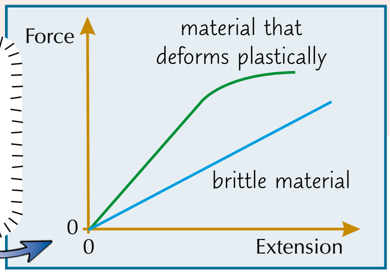
1) Force-extension graphs look a lot like **stress-strain** graphs, but they show slightly different things.
2) Force-extension graphs are **specific** for the tested **object** and **depend on its dimensions**. Stress-strain graphs describe the **general behaviour** of a **material**, they are **independent of the dimensions**.
3) You can plot a force-extension graph of what happens when you gradually **remove** a **force** from an object. The **unloading** line doesn't always match up with the **loading** line though.

A force-extension graph for a metal wire



1) This graph is for a metal wire that has been stretched beyond its **limit of proportionality** (**P**) so the graph starts to **curve**.
2) When the load is **removed**, the **extension decreases**.
3) The unloading line is **parallel** to the loading line because the stiffness constant ***k*** is still the same (since the forces between the atoms are the same as they were during the loading).
4) But because the wire was stretched beyond its **elastic limit** (**E**) and deformed **plastically**, it has been **permanently stretched**. This means the unloading line doesn't go through the origin.
5) The **area** between the two lines is the **work done** to permanently deform the wire.

Of course, if you apply a big enough load to fracture the object, you can't draw an unloading line — you just get a force-extension graph like one of these graphs.



Practice Questions

Q1 What is the difference between the limit of proportionality and the elastic limit?

Q2 Describe what happens at the yield point.

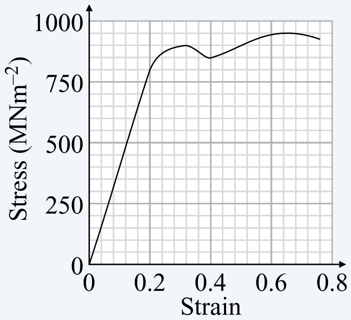
Q3 A metal wire is stretched beyond its elastic limit. Why does the unloading line on the force-extension graph for the wire not go through the origin?

Exam Questions

Q1 Sketch a stress-strain graph to show (use the same axes):

a) A stiff and weak material being stretched to its breaking point. [1 mark]

b) A strong and less stiff material being stretched to its breaking point. [1 mark]



Q2 The diagram shows a stress-strain graph for a nylon thread.

a) State the yield stress for nylon. [1 mark]

b) Calculate how much energy per unit volume is stored in the thread when the limit of proportionality is reached. [2 marks]

I don't want to stress you, but this page can strain even the strongest brains...

The trick to revising is realising when you've really reached your yield point. That's when you need to take a break and get up and go for a walk. (It's not when you get the urge to have a quick check of your FaceTwittFeed or whatnot...)

Density, Upthrust and Viscosity

Yikes, three whole things in one go. Grab a coffee, take a deep breath and put on your snazziest learning hat...

Density *is* ***Mass*** *per* ***Unit Volume***

Density is a measure of the '**compactness**' (for want of a better word) of a substance. It relates the **mass** of a substance to how much **space** it takes up.

1) The density of an object depends on what it's made of. Density **doesn't vary** with **size or shape**.
2) The **average density** of an object determines whether it **floats** or **sinks**. A solid object will **float** on a fluid if it has a **lower density** than the **fluid**.

The symbol for density is a Greek letter rho (ρ) — it looks like a p but it isn't.

$$\textbf{density} = \frac{\textbf{mass}}{\textbf{volume}} \qquad \rho = \frac{m}{V}$$

The units of density are g cm^{-3} or kg m^{-3}
1 g cm^{-3} = 1000 kg m^{-3}

Bodies *in* ***Fluids*** *Experience* ***Pressure*** *and* ***Upthrust***

1) Pressure is the **amount of force** applied per **unit area**. It is measured in **pascals (Pa)**, which are equivalent to newtons per square metre (Nm^{-2}).

$$\textbf{pressure} = \frac{\textbf{force}}{\textbf{area}} \qquad p = \frac{F}{A}$$

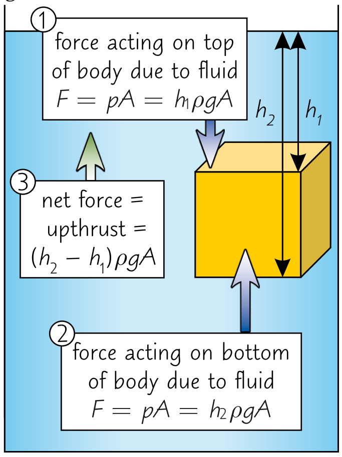
2) The extra pressure acting on an object due to a fluid depends on the **depth** of the object in the fluid (h), the **density** of the fluid (ρ) and the **gravitational field strength** (g).

$$p = h\rho g$$

3) Upthrust is an **upward force** that fluids exert on objects that are **completely** or **partially submerged** in the fluid. It's caused because the top and bottom of a submerged object are at different depths. Since $p = h\rho g$, there is a difference in pressure which causes an overall upwards force known as upthrust.
4) **Archimedes' principle** says that when a body is completely or partially immersed in a fluid, it experiences an **upthrust** equal to the **weight** of the fluid it has **displaced**.

upthrust = weight of fluid displaced

This is just because $(h_2 - h_1)\rho g A = V\rho g = mg = W$



Example: **Submarines** make use of upthrust to dive underwater and return to the surface. To sink, large tanks are filled with water to **increase** the weight of the submarine so that it **exceeds** the upthrust. To rise to the surface, the tanks are filled with compressed air to **reduce** the weight so that it's **less** than the upthrust.

Viscous Drag *Acts on* ***Objects Moving Through Fluids***

When an object moves through a fluid, or a fluid moves past an object, you get **friction** between the surface of the **object** and the **fluid**. This is **viscous drag.** You can calculate the **force due to viscous drag** on a **spherical** object moving through a fluid using **Stokes's law**.

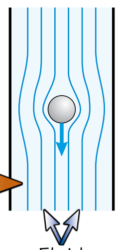
$$F = 6\pi\eta rv$$

F is the viscous drag (N), η (eta) is the viscosity of the fluid (Nsm^{-2} or Pa s), r is the radius of the object (m) and v is the speed the object is moving at (ms^{-1}).

Viscous drag depends on the **viscosity** (or "thickness") of a fluid, η. Viscosity is **temperature-dependent** — **liquids** get **less viscous** as temperature **increases**, but **gases** (like **dry air**) get **more viscous** as temperature **increases.**

Stokes's Law ***Only*** *Applies to* ***Small*** *Objects* ***Moving Slowly*** *with* ***Laminar Flow***

1) **Laminar flow** is a flow **pattern** where all the parts of the fluid are flowing in the **same direction** — the layers in the fluid **do not mix**.
2) Laminar flow usually occurs when a fluid is **flowing slowly**, or when an object is **moving slowly** through a fluid. The diagram shows a ball falling slowly with laminar flow.



Fluid 'layers'

3) **Turbulent flow** is a different flow **pattern**. You don't get nice layers like you do with laminar flow, all the parts of the **fluid** get **mixed up**.
4) Turbulent flow usually occurs when a **fluid** is flowing **quickly**, or an object is **moving quickly** through a fluid.
5) **Stokes's law only** applies to **small**, **spherical** objects moving **slowly** with **laminar flow**. It **doesn't** apply to **turbulent flow**.

What do you get if you cross a baby sheep with a river? A lamb in a flow.

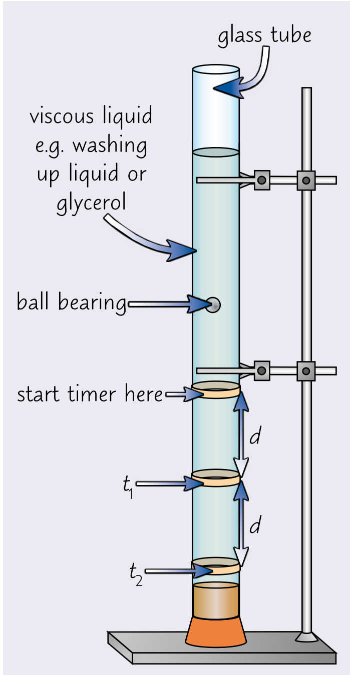
Density, Upthrust and Viscosity

Measure** the **Viscosity** of a **Liquid

You don't have to use elastic bands — you could also use insulation tape or another marker for your intervals.

You can calculate the **viscosity** of a liquid by timing the **fall** of different sizes of **ball bearings** (little steel balls) of **known density**, ρ. using an experiment like this:

1) Fill a wide, clear tube with the liquid you want to investigate (e.g. washing up liquid). Make sure you know the **density** of the liquid, σ.
2) Put one rubber band about halfway down the tube at a position such that the ball bearings will have achieved **terminal velocity** (see page 30) when they reach it.
3) Place two more elastic bands **below** the first so that the distance between each band is **equal** and the lowest band is near the **bottom** of the tube. Record the distance between them. These are the points at which you will record t_1 and t_2.
4) Measure the diameter of your ball bearing and halve it to get the **radius**.
5) **Drop** the ball bearing into the tube. Start a **stopwatch** when the ball reaches the first band, and record the time at which it reaches each of the other bands. Record your results in a **table** (see below).
6) **Repeat** this at least three times for each ball bearing to reduce the effect of **random errors** on your results, then repeat the whole thing for different sizes of ball bearing. You can use a **strong magnet** to remove the ball bearings from the tube.
7) If the ball falls **close** to the **wall**, **re-do the reading** — the flow will no longer be laminar and Stokes's law will not apply.
8) **Calculate** the **average time taken** for each size of ball bearing to fall between the elastic bands. Use the **average time** and the **distance between bands** to calculate the **average (terminal) velocity** of the ball bearing between the elastic bands.
9) You can then calculate the **viscosity**, η, of the liquid. The ball bearings are falling at terminal velocity, so the **sum** of the forces acting on the ball is **zero**:



weight – drag – upthrust = 0

Substituting the equation for each force and rearranging gives $\eta = \frac{2r^2 g(\rho - \sigma)}{9v}$, where r is the radius of the ball bearing, g is the gravitational field strength due to gravity, ρ is the density of the ball bearing, σ is the density of the liquid and v is the average terminal velocity between the elastic bands.

10) Your table could look like this:

radius of ball bearing /mm	density of ball bearing / kg m^{-3}	time to t_1 / s	velocity at t_1 / ms^{-1}	time between t_1 and t_2 / s	velocity at t_2 / ms^{-1}	average velocity / ms^{-1}	viscosity / Nsm^{-2}	average viscosity / Nsm^{-2}
2.0		1		1				
		2		2				
		3		3				
3.0		1						
		2						

If the velocities at t_2 are consistently larger than those at t_1, the ball bearing might not have reached terminal velocity when you started timing — move the elastic bands down the tube a bit and try again.

Practice Questions

Q1 Show that the mass of 3.8 m^3 of a liquid with density 1.5 kg m^{-3} is 5.7 kg.

Q2 State Archimedes's principle and explain how it is used by submarines.

Q3 What is meant by laminar flow?

Q4 Describe an experiment you could carry out to determine the viscosity of a liquid using ball bearings.

Exam Question

Q1 a) State Stokes's law and explain what each term represents. [1 mark]

b) State three conditions that must be true for Stokes's law to be apply. [3 marks]

c) A ball bearing of radius 3.0 mm is falling at a constant speed of 0.040 ms^{-1} through a liquid of viscosity 3.0 Pa s. The density of the liquid is 1400 kg m^{-3}. Calculate the weight of the ball bearing. ($V_{sphere} = \frac{4}{3}\pi r^3$) [5 marks]

My discosity increases with temperature — warm me up, I'll dance for hours...

Sorry about the 'lamb-in-a-flow' joke — it really is awful. My boss Joe thought it up, I'm just not that funny. If you're struggling to remember this page, try explaining it to someone else (little brothers are useful for this) — it really helps.