Scalars and Vectors

And now time to draw some lovely triangles. Please, don't all thank me at once...

Scalars *Only Have* ***Size****, but* ***Vectors*** *Have* ***Size*** *and* ***Direction***

1) A **scalar** has **no direction** — it's **just an amount** of something, like the **mass** of a **sack of meaty dog food**.
2) A **vector** has magnitude (**size**) and **direction** — like the **speed and direction** of next door's **cat** running away.
3) **Force**, **velocity and momentum** are all **vectors** — you need to know **which way** they're going as well as **how big** they are. Here are some of the common scalars and vectors that you'll come across in your exams:

Scalars	Vectors
mass, time, energy temperature, length, speed	displacement, force, velocity, acceleration, momentum

4) Vectors are drawn as **arrows** (to show their direction) with their **size** written next to them (see Example 1 below). In the exam, you might see quantities written with **arrows** above them, e.g. $\vec{v}$, to show that they're vectors.

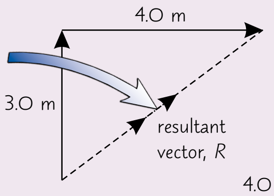
Sometimes vectors are printed in bold, e.g. ***v***, but it's quite hard to handwrite in bold, so the arrow is used too.

You can ***Add*** *Vectors to Find the* ***Resultant***

1) Adding two or more vectors is called finding the **resultant** of them.
2) You should always start by drawing a **diagram**. Draw the vectors '**tip to tail**'. If you're doing a **vector subtraction**, draw the vector you're subtracting with the same magnitude but pointing in the **opposite direction**.
3) If the vectors are at **right angles** to each other, then you can use **Pythagoras** and **trigonometry** to find the resultant.
4) If the vectors aren't at right angles, you may need to draw a **scale diagram**.

Example 1: Jemima goes for a walk. She walks 3.0 m north and 4.0 m east. She has walked 7.0 m but she isn't 7.0 m from her starting point. Find the magnitude and direction of her displacement.

First, draw the vectors **tip-to-tail**. Then draw a line from the **tail** of the first vector to the **tip** of the last vector to give the **resultant**:



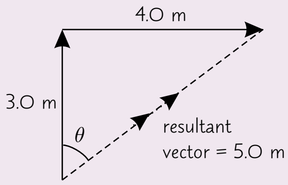
Jemima's 'displacement' gives her position relative to her starting point, see p.16.

Because the vectors are at right angles, you get the **magnitude** of the resultant using Pythagoras:

$R^2 = 3.0^2 + 4.0^2 = 25.0$ So $R =$ **5.0 m**

Jemima

Now find the **bearing** of Jemima's new position from her original position.



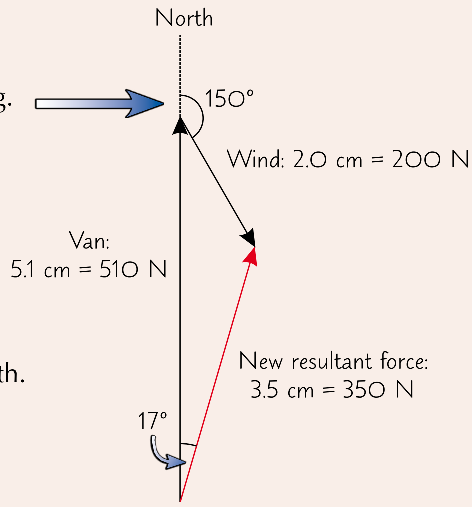
You use the triangle again, but this time you need to use trigonometry. You know the opposite and the adjacent sides, so you can use:

$\tan\theta = 4.0 / 3.0$ So $\theta =$ **053° (to 2 s.f.)**

A bearing is just an angle measured clockwise from the north line, represented by three digits, e.g. 10° = 010°.

Example 2: A van is accelerating north, with a resultant force of 510 N. A wind begins to blow on a bearing of 150°. It exerts a force of 200 N (to 2 s.f.) on the van. What is the new resultant force acting on the van?

The vectors **aren't** at right angles, so you need to do a scale drawing. Pick a sensible scale. Here, 1 cm = 100 N seems good.



Using a really sharp pencil, draw the initial resultant force on the van. As the van is going north, this should be a 5.1 cm long line going straight up.

The force of the wind acts on a bearing of 150°, so add this to your diagram. Using the same scale, this vector has a length of 2.0 cm.

Then you can draw on the new resultant force and measure its length. Measure the angle carefully to get the bearing.

The resultant force has a magnitude of 350 N (to 2 s.f.), acting on a bearing of 017° (to 2 s.f.).

Scalars and Vectors

It's Useful to Split a **Vector** into **Horizontal** and **Vertical Components**

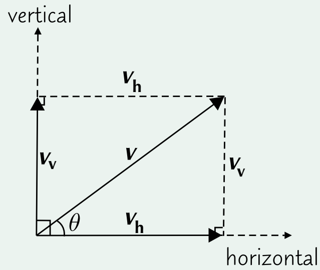
This is the opposite of finding the resultant — it's called **resolving**. You start from the resultant vector and split it into two **components** at right angles to each other. You're basically **working backwards** from Example 1 on the last page.

Resolving a vector *v* into horizontal and vertical components:

You get the **horizontal** component v_h like this:

$\cos\theta = v_h / v$

$$\boxed{v_h = v\cos\theta}$$



...and the **vertical** component v_v like this:

$\sin\theta = v_v / v$

$$\boxed{v_v = v\sin\theta}$$

Where θ is the angle from the horizontal.

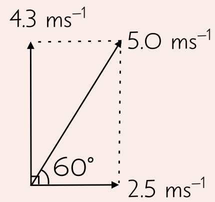
Example: Charley's amazing floating home is travelling at a speed of 5.0 ms^{-1} at an angle of 60° (to 2 s.f.) up from the horizontal. Find the vertical and horizontal components.

The **horizontal** component v_h is:

$v_h = v\cos\theta = 5.0\cos 60° = \mathbf{2.5\ ms^{-1}}$

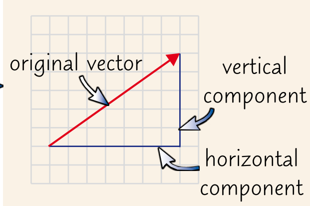
The **vertical** component v_v is:

$v_v = v\sin\theta = 5.0\sin 60° = \mathbf{4.3\ ms^{-1}}$ **(to 2 s.f.)**



Charley's mobile home was the envy of all his friends.

You can also resolve vectors using **scale drawings**. It's easiest to do this on **squared paper**. Just draw the vector to **scale** in the **correct direction**, then draw a **horizontal** line and **vertical** line from the tip and tail of the vector to form a **right-angled triangle**. Then measure the **lengths** of the lines and use the **scale** to convert them — this gives you the **horizontal** and **vertical** components of the vector.



Resolving is dead useful because the two components of a vector **don't affect each other**. This means you can deal with the two directions **completely separately** (there's more on this on page 20).

Practice Questions

Q1 What is the difference between a vector and a scalar? Give three vector quantities and three scalar quantities.

Q2 Describe how to draw a vector, and how to use vector notation.

Q3 Describe how to use a scale diagram to: a) find a resultant vector
b) resolve a vector into components at right angles to each other.

Exam Questions

Q1 The wind applies a horizontal force of 20.0 N on a falling rock of weight 75 N.
Calculate the magnitude and direction of the resultant force. [2 marks]

Q2 A glider is travelling at a velocity of 20.0 ms^{-1} at an angle of 15.0° below the horizontal.
Calculate the horizontal and vertical components of the glider's velocity. [2 marks]

Q3 A remote controlled boat is placed in a river. The boat produces a driving speed of 1.54 ms^{-1} at an angle of 60.0° to the current (travelling with the current). The river is flowing at 0.20 ms^{-1}.
By resolving the vectors into their horizontal and vertical components, show that the resultant velocity of the boat is 1.6 ms^{-1} at an angle of 54° to the current. [4 marks]

I think I'm a scalar quantity, my Mum says I'm completely direction-less...

*Lots of different ways to solve vector problems on these pages, it must be your lucky day. Trigonometry comes up all over the shop in physics, so make sure you're completely okay with it. Remember: **S**in θ = **O**pposite ÷ **H**ypotenuse, **C**os θ = **A**djacent ÷ **H**ypotenuse, and **T**an θ = **O**pposite ÷ **A**djacent. Or SOH CAH TOA.*

Motion with Uniform Acceleration

All the equations on this page are for motion with constant acceleration. It makes life a whole lot easier, trust me.

Learn the **Definitions** of **Speed**, **Displacement**, **Velocity** and **Acceleration**

Displacement, velocity and acceleration are all **vector** quantities (page 14), so the **direction** matters.

Speed — How fast something is moving, regardless of direction.
Displacement (s) — How far an object's travelled from its starting point in a given direction.
Velocity (v) — The rate of change of an object's displacement (its speed in a given direction).
Acceleration (a) — The rate of change of an object's velocity.

During a journey, the **average speed** is just the **total distance** covered over the **total time** elapsed.
The speed of an object at any given point in time is known as its **instantaneous** speed.

Uniform Acceleration *is* ***Constant*** *Acceleration*

Acceleration could mean a change in speed or direction or both.

Uniform means constant here. It's nothing to do with what you wear.
There are **four main equations** that you use to solve problems involving **uniform acceleration**. You need to be able to **use them**, but you don't have to know how they're **derived** — we've just put it in to help you learn them.

1) **Acceleration is the rate of change of velocity.**
From this definition you get:

$$a = \frac{(v - u)}{t} \quad \text{so} \quad \boxed{v = u + at}$$

where:
u = initial velocity a = acceleration
v = final velocity t = time taken

2) **s = average velocity × time**
If acceleration is constant, the average velocity is just the average of the initial and final velocities, so:

$$\boxed{s = \frac{(u + v)t}{2}} \quad s = \text{displacement}$$

3) Substitute the expression for v from equation 1 into equation 2 to give:

$$s = \frac{(u + u + at) \times t}{2} = \frac{2ut + at^2}{2}$$

$$\boxed{s = ut + \tfrac{1}{2}at^2}$$

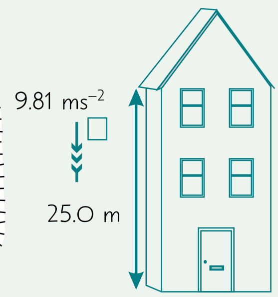
4) You can **derive** the fourth equation from equations **1** and **2**:
Use equation **1** in the form: $a = \dfrac{v - u}{t}$ Multiply both sides by s, where: $s = \dfrac{(u + v)}{2} \times t$

This gives us: $as = \dfrac{(v - u)}{t} \times \dfrac{(u + v)t}{2}$

The t's on the right cancel, so:
$2as = (v - u)(v + u)$
$2as = v^2 - uv + uv - u^2$

so: $\boxed{v^2 = u^2 + 2as}$

Example: A tile falls from a roof 25.0 m high. Calculate its speed when it hits the ground and how long it takes to fall. Take $g = 9.81\ \text{ms}^{-2}$.



First of all, write out what you know:
$s = 25.0$ m
$u = 0\ \text{ms}^{-1}$ since the tile's stationary to start with
$a = 9.81\ \text{ms}^{-2}$ due to gravity
$v = ?$ $t = ?$

Usually you take upwards as the positive direction. In this question it's probably easier to take downwards as positive, so you get $g = +9.81\ \text{ms}^{-2}$ instead of $g = -9.81\ \text{ms}^{-2}$.

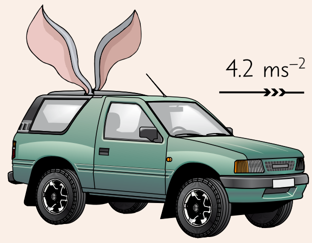
Then, choose an equation with only **one unknown quantity**.
So start with $v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 9.81 \times 25.0$
$v^2 = 490.5$ v = **22.1 ms⁻¹ (to 3 s.f.)**

Now, find t using:
$s = ut + \frac{1}{2}at^2$
$25.0 = 0 + \frac{1}{2} \times 9.81 \times t^2$
$t^2 = \dfrac{25.0}{4.905}$

Final answers:
t = **2.26 s (to 3 s.f.)**
v = **22.1 ms^{-1} (to 3 s.f.)**

Motion with Uniform Acceleration

Example: A car accelerates steadily from rest at a rate of 4.2 ms^{-2} for 6.5 seconds.
a) Calculate the final speed.
b) Calculate the distance travelled in 6.5 seconds.



Remember — always start by writing down what you know.

a) $a = 4.2$ ms^{-2}
$u = 0$ ms^{-1}
$t = 6.5$ s
$v = ?$

choose the right equation... $v = u + at$

$v = 0 + 4.2 \times 6.5$

Final answer: $v = 27.3$ ms^{-1}
= **27 ms^{-1} (to 2 s.f.)**

b) $s = ?$
$t = 6.5$ s
$u = 0$ ms^{-1}
$a = 4.2$ ms^{-2}
$v = 27.3$ ms^{-1}

you can use: $s = \frac{(u+v)t}{2}$ **or:** $s = ut + \frac{1}{2}at^2$

$s = \frac{(0+27.3) \times 6.5}{2}$ $\qquad$ $s = 0 + \frac{1}{2} \times 4.2 \times (6.5)^2$

Final answer: s = **89 m (to 2 s.f.)** $\qquad$ s = **89 m (to 2 s.f.)**

Practice Questions

Q1 Write down definitions for speed, displacement, average velocity, instantaneous velocity and acceleration.

Q2 Write out the four constant acceleration equations.

Mona's experiments into uniform acceleration had gone a bit far.

Exam Questions

Q1 A skydiver jumps from an aeroplane when it is flying horizontally. She accelerates due to gravity for 5.0 s.

a) Calculate her maximum vertical velocity. (Assume no air resistance.) [2 marks]

b) Calculate how far she falls in this time. [2 marks]

Q2 A motorcyclist slows down uniformly as he approaches a red light.
He takes 3.2 seconds to come to a halt and travels 40 m (to 2 s.f.) in this time.

a) Calculate how fast he was travelling initially. [2 marks]

b) Calculate his acceleration. (N.B. a negative value shows a deceleration.) [2 marks]

Q3 A stream provides a constant acceleration of 6 ms^{-2}. A toy boat is pushed directly against the current and then released from a point 1.2 m upstream from a small waterfall. Just before it reaches the waterfall, it is travelling at a speed of 5 ms^{-1}.

a) Calculate the initial velocity of the boat. [2 marks]

b) Calculate the maximum distance upstream from the waterfall the boat reaches. [2 marks]

Constant acceleration — it'll end in tears...

If a question talks about "uniform" or "constant" acceleration, it's a dead giveaway they want you to use one of these equations. The tricky bit is working out which one to use — start every question by writing out what you know and what you need to know. That makes it much easier to see which equation you need. To be sure. Arrr.

Free Fall

Free fall is all about objects falling — no really, that's all that the next two pages are about. Have fun.

Free Fall *is when there's Only* ***Gravity*** *and Nothing Else*

1) All objects on Earth experience a **force** due to the Earth's **gravitational field** that depends on their **mass**.
2) The **gravitational field strength** near the surface of the Earth is called g and is equal to **9.81 Nkg⁻¹**. g is given by the equation: $\boxed{g = \frac{F}{m}}$

Where m is the mass of the object and F is the force due to gravity.

3) The force on an object due to gravity is called its **weight** (see page 38).
4) When the only force acting on an object is its weight, it will undergo **free fall**, and will **accelerate towards the ground**.
5) Acceleration and force are related by $F = ma$. So, for an object in free fall, $a = \frac{F}{m} = g$.

Force and acceleration are both vector quantities (p.14). In free fall, they act vertically downwards.

> Objects undergoing free fall on Earth have an acceleration of g = **9.81 ms⁻²**.

6) Objects can have an initial velocity in any direction and still undergo **free fall** as long as the **force** providing the initial velocity is **no longer acting**.

It seems weird that g is both a gravitational field strength, in Nkg⁻¹, and an acceleration, in ms⁻². However, if you break down Nkg⁻¹ into SI units, they are actually the same as ms⁻².

You can ***Replace a*** *with* ***g*** *in the* ***Equations of Motion***

You need to be able to work out **speeds**, **distances** and **times** for objects in **free fall**. Since g is a **constant acceleration** you can use the **constant acceleration equations**.

$$v = u + gt \qquad s = ut + \tfrac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs \qquad s = \frac{(u + v)t}{2}$$

g acts **downwards**, so you need to be careful about directions. To make it clear, there's a sign convention: **upwards is positive**, **downwards is negative**.

> g is always **downwards** so it's **usually negative** $\quad$ t is **always positive**
> u and v can be **positive or negative** $\quad$ s can be **positive or negative**

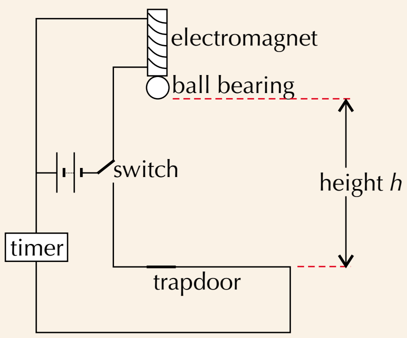
If an object is dropped, not thrown, u is zero.

You Can Calculate ***g*** *By Performing an* ***Experiment***...

See pages 6-13 for more on doing experiments.

This is just one way of **measuring g**, there are loads of different experiments you could do — just make sure you know **one** method for your exams.

1) Set up the equipment shown in the diagram on the right.
2) Measure the height h from the **bottom** of the ball bearing to the **trapdoor**.
3) Flick the switch to simultaneously **start the timer** and **disconnect the electromagnet**, releasing the ball bearing.
4) The ball bearing falls, knocking the trapdoor down and **breaking the circuit** — which **stops the timer**. Record the time t shown on the timer.
5) **Repeat** this experiment three times and **average** the time taken to fall from this height. Repeat this experiment but drop the ball from several **different heights**.
6) You can then use these results to find g using a **graph** (see the next page).

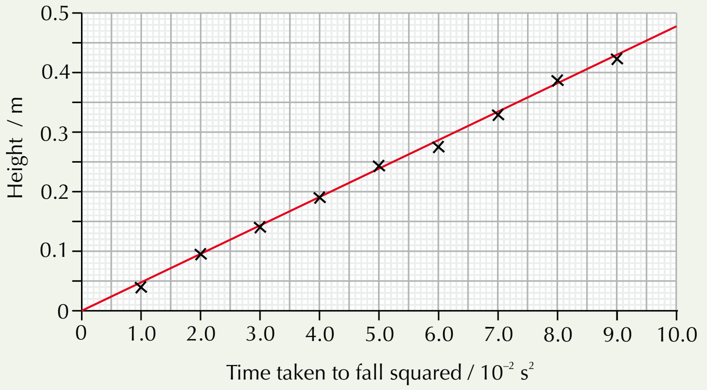


- Using a **small** and **heavy** ball bearing means you can assume air resistance is so small you can **ignore it**.
- Having a computer **automatically release** and **time** the ball-bearing's fall can measure times with a **smaller uncertainty** than if you tried to drop the ball and time the fall using a stopwatch.
- The most significant source of **error** in this experiment will be in the measurement of h. Using a ruler, you'll have an uncertainty of about ±1 mm. This dwarfs any error from switch delay or air resistance. By making the values of h as **large** as possible, you can reduce the **percentage uncertainty** in your measurement of the height, (see p.10) as well as the percentage uncertainty caused by the **resolution** of the **timer** (as the bigger h is, the longer the ball-bearing will take to fall).

Free Fall

...and Plotting a **Graph** of Your **Results**

1) Use your data from the experiment on the last page to plot a graph of **height** (s) against the **time** it takes the ball to fall, **squared** (t^2). Then draw a **line of best fit**.



You could plot error bars on your data graph to find the error in your final value for g. See page 11 for more on error bars.

In the exam you might be asked to find g from a displacement-time graph (see p.22-23). g is an acceleration and the gradient of the graph will be velocity, so you can find g by finding the change in gradient between two points on the graph (as $a = \Delta v \div \Delta t$).

2) You know that with constant acceleration, $s = ut + ½at^2$. If you drop the ball, initial speed $u = 0$, so $s = ½at^2$.

3) Rearranging this gives $½a = \frac{s}{t^2}$, or $½g = \frac{s}{t^2}$ (remember the acceleration is all due to gravity).

4) So the gradient of the line of best fit, $\frac{\Delta s}{\Delta t^2}$, is equal to $½g$:

$$g = 2 \times \frac{\Delta s}{\Delta t^2} = 2 \times \frac{0.43}{0.090} = 9.555...$$
$$= \mathbf{9.6\ ms^{-2}\ (to\ 2\ s.f.)}$$

5) As you know g (9.8 ms^{-2} to 2 s.f.) you can calculate the **percentage difference** (see p.12) between your value of g and the true value, and use this to evaluate the **accuracy** of your results.

$$\%\ \text{difference} = \frac{9.6 - 9.8}{9.8} \times 100\ \% = -2.040...$$
$$= \mathbf{-2.0\%\ (to\ 2\ s.f.)}$$

Practice Questions

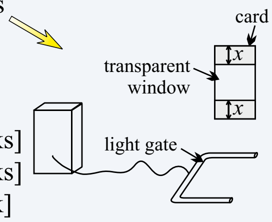
Q1 State the equation linking gravitational field strength, force due to gravity, and mass.

Q2 What is the value of the acceleration of a free-falling object on Earth?

Q3 Describe how you would find a value for g by using a trapdoor and electromagnet set-up.

Exam Questions

Q1 A student has designed a device to estimate the value of 'g'. It consists of two narrow strips of card joined by a piece of transparent plastic. The student measures the widths of the strips of card then drops the device through a light gate connected to a computer. As the device falls, the strips of card break the light beam.



a) Give three pieces of data that the student will need from the computer to estimate g. [3 marks]

b) Explain how these measurements can be used to estimate 'g'. [3 marks]

c) Give one reason why the student's value of 'g' will not be entirely accurate. [1 mark]

Q2 Jan bounces on a trampoline, reaching a point 5.0 m above the trampoline's surface. Assume there is no air resistance.

a) Calculate the speed with which she leaves the trampoline surface. [2 marks]

b) Calculate how long it takes Jan to reach her highest point. [2 marks]

c) State her velocity as she lands back on the trampoline. [1 mark]

It's not the falling that hurts — it's the being pelted with rotten vegetables... okay, okay...

The hardest bit with free fall questions is getting your signs right. Draw yourself a little diagram before you start doing any calculations, and label it with what you know and what you want to know.

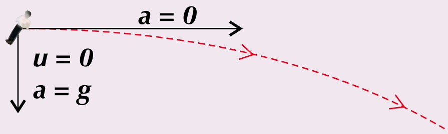
Projectile Motion

Any object given an initial velocity then left to move freely under gravity is a projectile. Time to resolve some vectors...

*You have to think of **Horizontal** and **Vertical** Motion **Separately***

In projectiles, the **horizontal** and **vertical** components of the object's motion are **completely independent**. Projectiles follow a **curved path** because the horizontal velocity remains **constant**, while the vertical velocity is affected by the **acceleration due to gravity**, *g*.

Example: Jane fires a scale model of a TV talent show presenter horizontally from 1.5 m above the ground with a velocity of 100 ms^{-1} (to 2 s.f.). How long does it take to hit the ground, and how far does it travel horizontally? Assume the model acts as a particle, the ground is horizontal and there's no air resistance.



Think about vertical motion first:

1) It's **constant acceleration** under gravity...
2) You know $u = 0$ (no vertical velocity at first), $s = -1.5$ m and $a = g = -9.81$ ms^{-2}. You need to find t.
3) Use $s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times -1.5}{-9.81}} = 0.553...$ s. So the model hits the ground after **0.55 (to 2 s.f.)** seconds.

Then do the horizontal motion:

1) The horizontal motion isn't affected by gravity or any other force, so it moves at a **constant speed**. That means you can just use good old **speed = distance / time**.
2) Now $v_h = 100$ ms^{-1}, $t = 0.553...$ s and $a = 0$. You need to find s_h.
3) $s_h = v_h t = 100 \times 0.553... =$ **55 m (to 2 s.f.)**

Where v_h is the horizontal velocity, and s_h is the horizontal distance travelled (rather than the height fallen).

*It's **Slightly Trickier** if it **Starts Off** at an **Angle***

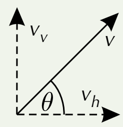
If something's projected at an **angle** (e.g. a javelin) you'll start with **horizontal** and **vertical velocity**. Here's what to do:

1) **Resolve** the initial velocity into **horizontal** and **vertical** components:
2) Often you'll use the vertical component to work out **how long** it's in the air and/or **how high** it goes, and the horizontal component to work out **how far** it goes while it's in the air.

If an object has velocity v, at an angle of θ to the horizontal:

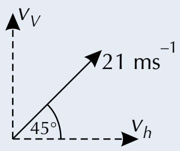
The horizontal component of its velocity is: $v_h = v \cos\theta$

The vertical component of its velocity is: $v_v = v \sin\theta$

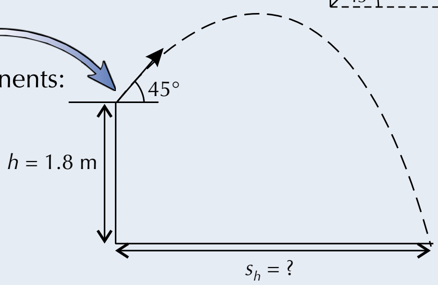


(see page 15)

Example: An athlete throws a javelin from a height of 1.8 m with a velocity of 21 ms^{-1} at an upward angle of 45° to the ground. How far is the javelin thrown? Assume the javelin acts as a particle, the ground is horizontal and there is no air resistance.



1) Draw a quick sketch of the information given in the question.



2) Start by resolving the velocity into horizontal and vertical components:
   $u_h = \cos 45° \times 21 = 14.84...$ ms^{-1}
   $u_v = \sin 45° \times 21 = 14.84...$ ms^{-1}
3) Then find how long it's in the air for — start by finding v_v. The javelin starts from a height of 1.8 m and finishes at ground level, so its final vertical distance $s_V = -1.8$ m:
   $v_v^2 = u_v^2 + 2gs$
   $v_v = \sqrt{14.84...^2 + 2 \times (-9.81) \times (-1.8)} = -15.99...$ ms^{-1}

   You need the negative square root, as this is a velocity towards the ground.

   Now you can use this v_v value and $s = \frac{(u+v)t}{2}$ to find the time it stays in the air:
   $s_v = \frac{(u_V + v_V)t}{2} \Rightarrow t = \frac{s_V}{(u_V + v_V)} \times 2 = \frac{-1.8}{14.84... - 15.99...} \times 2 = 3.144...$ s
4) Finally, as $a_h = 0$, you can use **speed = distance / time** to work out how far it travels horizontally in this time. The horizontal velocity is just u_h, so: $s_h = u_h t = 14.84... \times 3.144... = 46.68... =$ **47 m (to 2 s.f.)**

Projectile Motion

You can Investigate Projectile Motion Using a **Video Camera**...

If you **video** a projectile moving, you can use **video analysis software** to investigate its motion:

1) You can **plot the course** taken by an object by recording its **position** in **each frame**.
2) If you know the **frame rate**, and your video includes a metre ruler or grid lines that you can use as a scale, you can calculate the **velocity** of the projectile between **different points** in its motion, by looking at how far it travels **between frames**.

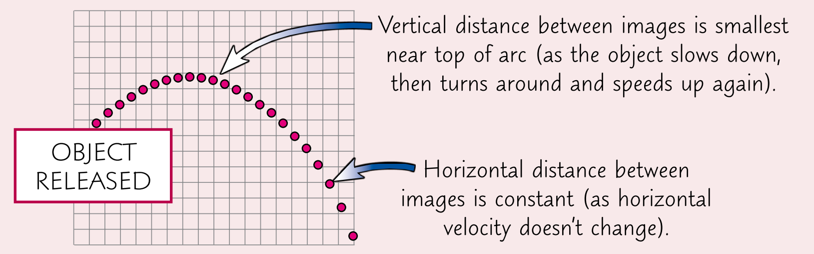
A video camera records a series of pictures, or frames (typically around 25 frames per second). Video analysis software lets you view videos frame by frame.

... or **Strobe Photography**

In strobe photography, a camera is set to take a **long exposure**. While the camera is taking the photo, a **strobe light** flashes repeatedly and the projectile is released. The strobe light **lights up** the projectile at regular intervals. This means that the projectile appears **multiple times** in the same photograph, in a **different position** each time.

Again, if you've got a **reference object** in the photo (for example, you might throw an object in front of a **screen** with a **grid** drawn on it), you can calculate **how far** the object travels **between flashes** of the strobe, and use the **time** between flashes to calculate the **velocity** of the projectile between the flashes.

The motion of a typical projectile captured with strobe photography is shown below.



Strobe photography and video cameras give you more information than using light-gates to study an object's projectile motion. They can be used whatever the size of the object, unlike a light-gate.

Practice Questions

Q1 What is the initial vertical velocity for an object projected horizontally with a velocity of 5 ms^{-1}?

Q2 How does the horizontal velocity of a projectile change with time?

Q3 What is the horizontal component of the velocity of a stone, hurled at 30 ms^{-1} at 35° to the horizontal?

Q4 Explain how you might use video analysis software to analyse the motion of a projectile.

Exam Questions

Q1 Jason stands on a vertical cliff edge throwing stones into the sea below. He throws a stone horizontally with a velocity of 20 ms^{-1} (to 2 s.f.), 560 m above sea level.

a) Calculate how long it takes for the stone to hit the water from leaving Jason's hand. Use $g = 9.81$ ms^{-2} and ignore air resistance. [2 marks]

b) Calculate the distance of the stone from the base of the cliff when it hits the water. [2 marks]

Q2 Robin fires an arrow into the air with a vertical velocity of 30.0 ms^{-1}, and a horizontal velocity of 20.0 ms^{-1}, from 1.0 m above horizontal ground. Choose the correct option which shows the maximum height from the ground (to 2 significant figures) reached by his arrow. Use $g = 9.81$ ms^{-2} and ignore air resistance.

A	45 m	C	47 m
B	46 m	D	48 m

[1 mark]

All this physics makes me want to create projectile motions...

...by throwing my revision books out of the window. The maths on these two pages can be tricky, but take it step by step and all will be fine. On the plus side, the next page is full of lovely graphs. Who doesn't love a good graph?

Displacement-Time Graphs

Drawing graphs by hand — oh joy. You'd think examiners had never heard of the graphical calculator. Ah well, until they manage to drag themselves out of the Dark Ages, you'll just have to grit your teeth and get on with it.

Acceleration** Means a **Curved Displacement-Time Graph

A graph of displacement against time for an **accelerating object** always produces a **curve**.
If the object is accelerating at a **uniform rate**, then the **rate of change** of the **gradient** will be constant.

Example: Plot a displacement-time graph for a lion who accelerates constantly from rest at 2 ms^{-2} for 5 seconds.

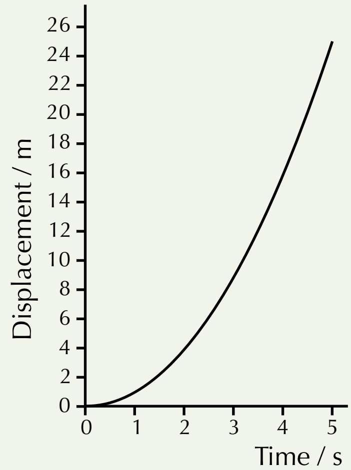
You want to find ***s***, and you know that:
$a = 2\ ms^{-2}$
$u = 0\ ms^{-1}$

Use $s = ut + \frac{1}{2}at^2$
If you substitute in u and a, this simplifies to:
$s = 0 \times t + \frac{1}{2} \times 2t^2$
$s = t^2$

Do a **table of values**:

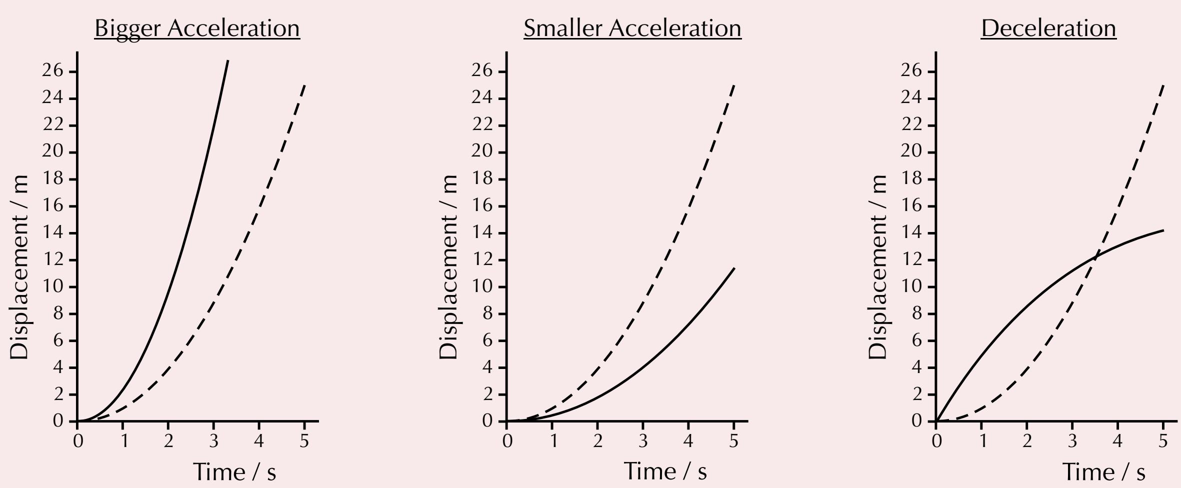
t / s	s / m
0	**0**
1	**1**
2	**4**
3	**9**
4	**16**
5	**25**

...then plot the **graph**:



*Different Accelerations Have **Different Gradients***

In the example above, if the lion has a **different acceleration** it'll change the **gradient** of the curve like this:



deceleration — the line has a decreasing gradient and curves the other way.

Norman (the lion).
Ooo, he's mean...

Displacement-Time Graphs

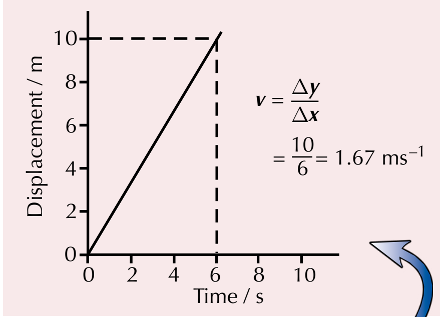
The Gradient of a Displacement-Time Graph Tells You the Velocity

When the velocity is constant, the graph's a **straight line**.
Velocity is defined as...

$$\textbf{velocity} = \frac{\textbf{change in displacement}}{\textbf{change in time}}$$

On the graph, this is $\dfrac{\text{change in } y\ (\Delta y)}{\text{change in } x\ (\Delta x)}$, i.e. the gradient.

So to get the velocity from a displacement-time graph, just find the gradient.



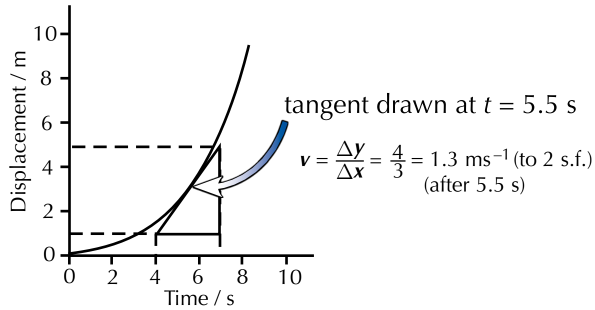
Acceleration is $\dfrac{\text{change in velocity } (\Delta v)}{\text{change in time } (\Delta t)}$, so it is the rate of change of this gradient. If the gradient is constant (straight line) then there is no acceleration, and if it's changing (curved line) then there's acceleration or deceleration.

It's the Same with Curved Graphs

If the gradient **isn't constant** (i.e. if it's a curved line), it means the object is **accelerating**.

> To find the **instantaneous velocity** at a certain point you need to draw a **tangent** to the curve at that point and find its gradient.

To find the **average velocity** over a period of time, just divide the final (change in) displacement by the final (change in) time — it doesn't matter if the graph is curved or not.



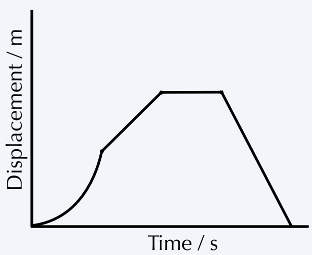
Practice Questions

Q1 What is given by the slope of a displacement-time graph?

Q2 Sketch a displacement-time graph to show: a) constant velocity, b) acceleration, c) deceleration

Exam Questions

Q1 Describe the motion of the cyclist as shown by the graph below. [4 marks]



Q2 A baby crawls 5 m in 8 seconds at a constant velocity. She then rests for 5 seconds before crawling a further 3 m in 5 seconds at a constant velocity. Finally, she makes her way back to her starting point in 10 seconds, travelling at a constant speed all the way.

a) Draw a displacement-time graph to show the baby's journey. [4 marks]

b) Calculate her velocity at all the different stages of her journey. [2 marks]

Be ahead of the curve, get to grips with this stuff now...

Whether it's a straight line or a curve, the steeper it is, the greater the velocity. There's nothing difficult about these graphs — the problem is that it's easy to confuse them with velocity-time graphs (next page). If in doubt, think about the gradient — is it velocity or acceleration, is it changing (curve), is it constant (straight line), is it 0 (horizontal line)...

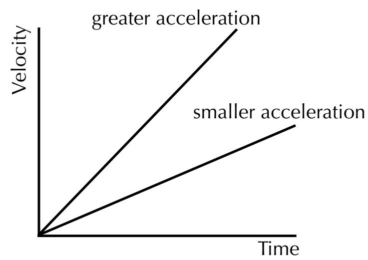
Velocity-Time and Acceleration-Time Graphs

Speed-time graphs and velocity-time graphs are pretty similar. The big difference is that velocity-time graphs can have a negative part to show something travelling in the opposite direction:

*The **Gradient** of a **Velocity-Time Graph** tells you the **Acceleration***

$$\textbf{acceleration} = \frac{\textbf{change in velocity}}{\textbf{time taken}}$$

likewise for a speed-time graph



So the acceleration is just the **gradient** of a **velocity-time graph**.

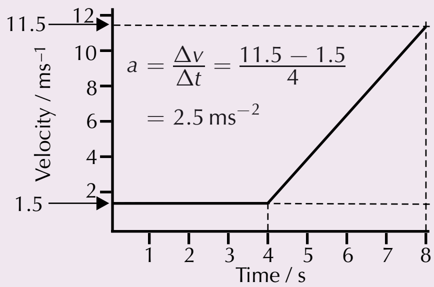
1) **Uniform** acceleration is always a **straight line**.
2) The **steeper** the **gradient**, the **greater** the **acceleration**.

When the **acceleration** is **constant**, you get a **straight-line** *v-t* graph. The equation for a straight line is ***y* = *mx* + *c***. You can rearrange the acceleration equation into the same form, getting ***v* = *u* + *at***.
So on a linear *v-t* graph, **acceleration**, *a*, is the **gradient** (*m*) and the **initial speed**, *u*, is the ***y*-intercept** (*c*).

Example: A lion strolls along at 1.5 ms^{-1} for 4 s and then accelerates uniformly at a rate of 2.5 ms^{-2} for 4 s. Plot this information on a velocity-time graph.

So, for the first four seconds, the velocity is 1.5 ms^{-1}, then it increases by **2.5 ms^{-1} every second**:

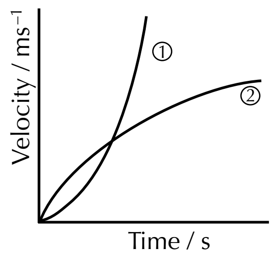
time / s	0-4	5	6	7	8
velocity / ms^{-1}	1.5	4.0	6.5	9.0	11.5



You can see that the **gradient of the line** is **constant** between 4 s and 8 s and has a value of 2.5 ms^{-2}, representing the **acceleration of the lion**.

*Acceleration isn't Always **Uniform***

1) If the acceleration is changing, the gradient of the velocity-time graph will also be changing — so you **won't** get a **straight line**.
2) **Increasing acceleration** is shown by an **increasing gradient** — like in curve ①.
3) **Decreasing acceleration** is shown by a **decreasing gradient** — like in curve ②.



Displacement** = **Area** under **Velocity-Time Graph

Similarly, the area under a speed-time graph is the total distance travelled.

You know that: **displacement = velocity × time**

The **area** under a velocity-time graph tells you the **displacement** of an object. Areas under any **negative** parts of the graph count as negative areas, as they show the object moving **back** to its **start point**.

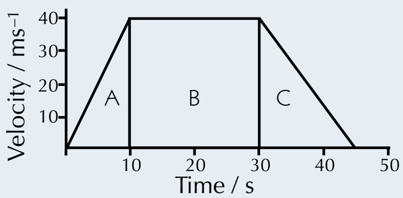
Example: A racing car on a straight track accelerates uniformly from rest to 40 ms^{-1} in 10 s. It maintains this speed for a further 20 s before coming to rest by decelerating at a constant rate over the next 15 s. Draw a velocity-time graph for this journey and use it to calculate the total displacement of the racing car.

Split the **graph** up into **sections**: A, B and C. Calculate the **area** of each and **add** the three results together.

A: Area = ½ base × height = ½ × 10 × 40 = 200 m

B: Area = *b* × *h* = 20 × 40 = 800 m

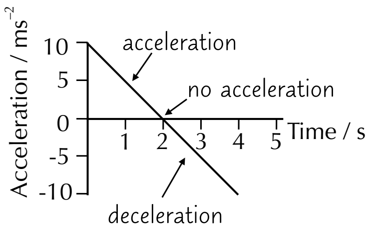
C: Area = ½ *b* × *h* = ½ × 15 × 40 = 300 m



Total displacement = 1300 m

Velocity-Time and Acceleration-Time Graphs

Acceleration-Time (a-t) Graphs are Useful Too



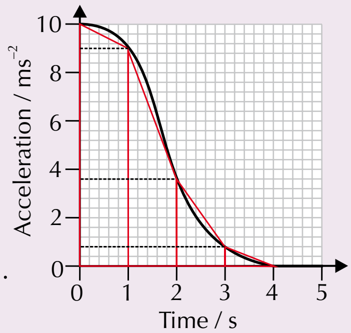
An **acceleration-time graph** shows how an object's **acceleration** changes over time.

1) The **height** of the graph gives the object's **acceleration** at that time.
2) The **area** under the graph gives the object's **change in velocity**.
3) A negative acceleration is a **deceleration**.
4) If $\boldsymbol{a = 0}$, then the object is moving with **constant velocity**.

You Have to **Estimate** the Area Under a **Curved** Graph

If an object's acceleration **isn't constant**, you won't get a straight line *a*-*t* graph. You need to know how to **estimate** the area under a curved graph. If the graph is on **squared paper**, you can work out the value represented by the **area** of **one** square and multiply by the approximate **number of squares** under the curve. Another way is to split the area approximately into simple shapes, calculate the value of the **area** of each of them, and then **add** them all up.

Example: The acceleration of a car in a drag race is shown in this acceleration-time graph. Calculate its change in velocity.



Change in velocity = area under graph

Split the area under the curve up into trapeziums and a triangle.

0-1 s — estimate the area using a trapezium. Area = $\frac{1}{2}(a + b) \times h$
a is the length of the first side, $a = 10$
b is the length of the second side, $b = 9$
h is the width of each strip, so $h = 1$. Area = $\frac{1}{2}(10 + 9) \times 1 = 9.5$ ms^{-1}

1-2 s — this can also be estimated with another trapezium. $a = 9$, $b = 3.6$, $h = 1$.
So area = $\frac{1}{2}(9 + 3.6) \times 1 = 6.3$ ms^{-1}

2-3 s — estimated with another trapezium. $a = 3.6$, $b = 0.8$, $h = 1$. So area = $\frac{1}{2}(3.6 + 0.8) \times 1 = 2.2$ ms^{-1}

3-4 s — this estimation uses a triangle. Area = $\frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(0.8) \times 1 = 0.4$ ms^{-1}

Now add the areas together — Total area = $9.5 + 6.3 + 2.2 + 0.4 = 18.4$ ms^{-1}
The estimated change in velocity of the car is 18.4 ms^{-1} = **20 ms^{-1} (to 1 s.f.)**

You can use the same method to find the area under any non-linear graph.

Practice Questions

Q1 How do you calculate acceleration from a velocity-time graph?
Q2 How do you calculate the displacement travelled from a velocity-time graph?
Q3 Sketch velocity-time graphs for constant velocity and constant acceleration.
Q4 Sketch velocity-time and acceleration-time graphs for a boy bouncing on a trampoline.
Q5 What does the area under an acceleration-time graph tell you?

Exam Question

Q1 A skier accelerates uniformly from rest at 2 ms^{-2} down a straight slope for 5 seconds. He then reaches the bottom of the slope and continues along the flat ground, decelerating at 1 ms^{-2} until he stops.

a) Sketch the velocity-time and acceleration-time graphs for his journey. [4 marks]

b) Use your *v*-*t* graph from part a) to find the distance travelled by the skier during the first 5 seconds. [2 marks]

Still awake — I'll give you five more minutes...

There's a really nice sunset outside my window. It's one of those ones that makes the whole landscape go pinky-yellowish. And that's about as much interest as I can muster on this topic. Normal service will be resumed on page 27.

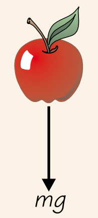
Forces

Remember the vector stuff from the beginning of the section? Good, you're going to need it...

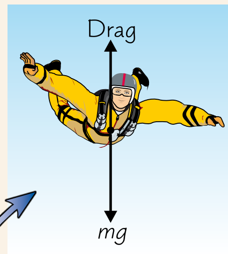
Free-Body Force Diagrams show All Forces on a Single Body

1) **Free-body force** diagrams show a **single body** on its own.
2) The diagram should include all the **forces** that **act on** the body, but **not** the **forces it exerts** on the rest of the world.
3) Remember **forces** are **vector quantities** and so the **arrow labels** should show the **size** and **direction** of the forces.
4) If a body is in **equilibrium** (i.e. not accelerating) the **forces** acting on it will be **balanced**.

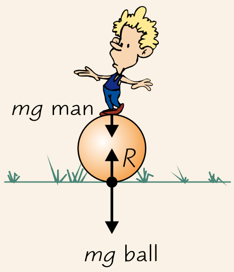
Drawing free-body force diagrams isn't too hard — you just need to practise them. Here are a few **examples**:



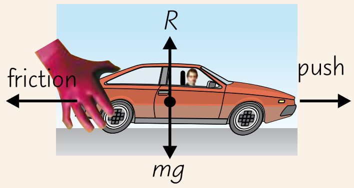
Gravity pulls apple downwards (weight = $m \times g$, see page 38).



Gravity pulls man down. Air resistance acts to push up on man.



Gravity pulls down on ball. Man pushes down on ball. Earth pushes up on ball.



Hand pushes toy car forward. Friction acts against motion. Gravity pulls down on car. Earth pushes up on car.

If the man is falling at a constant speed, then drag = *mg* (see p.31).

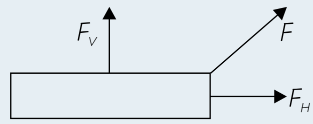
Resolving a Force means Splitting it into Components

See p.15 for a reminder on resolving vectors.

1) Forces can be in **any direction**, so they're not always at right angles to each other. This is sometimes a bit **awkward** for **calculations**.
2) To make an 'awkward' force easier to deal with, you can think of it as two **separate**, **independent** forces, acting at **right angles** to each other.
3) These two forces have **no effect** on each other as they are **perpendicular**. E.g. a horizontal force will have no vertical effect, and vice-versa.

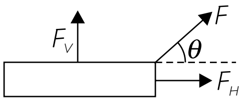
The force F has exactly the same effect as the horizontal and vertical forces, F_H and F_V.

Replacing F with F_H and F_V is called **resolving the force F**.

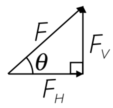


4) To find the **size** of a **component** force in a particular **direction**, you need to use trigonometry. Forces are vectors, so you treat them in the same way as velocity or displacement — put them end to end.

So this...



...could be drawn like this:



Using trigonometry you get:

$\boxed{\frac{F_H}{F} = \cos\theta}$ or $\boxed{F_H = F\cos\theta}$

And:

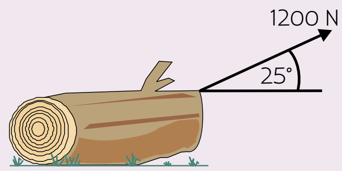
$\boxed{\frac{F_V}{F} = \sin\theta}$ or $\boxed{F_V = F\sin\theta}$

Remember that $\cos 90° = 0$, so forces which act at an angle of **90°** to each other are **independent** (i.e. they have **no effect** on each other).

Example: A tree trunk is pulled along the ground by an elephant exerting a force of 1200 N at an angle of 25° to the horizontal. Calculate the components of this force in the horizontal and vertical directions.

Horizontal force:
$1200 \times \cos 25° = 1087.5...$
= **1100 N (to 2 s.f.)**

Vertical force:
$1200 \times \sin 25° = 507.1...$
= **510 N (to 2 s.f.)**

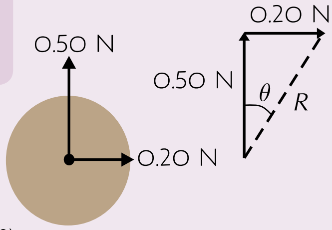


Forces

You Add the Components Back Together to get the Resultant Force

1) If **two forces** act on an object, you can find the **resultant** (total) **force** by adding the **vectors** together and creating a **closed triangle**, with the resultant force represented by the **third side**.
2) Forces are vectors (as you know), so use **vector addition** — draw the forces as vector arrows 'tip-to-tail'.
3) Then it's yet more trigonometry and Pythagoras to find the **angle** and the **length** of the third side.

Example: Two dung beetles roll a dung ball along the ground at a constant velocity. Beetle A applies a force of 0.50 N northwards while beetle B exerts a force of 0.20 N eastwards. What is the resultant force on the dung ball?



By Pythagoras, $R^2 = 0.50^2 + 0.20^2 = 0.29$

$R = \sqrt{0.29} = 0.538... =$ **0.54 N (to 2 s.f.)**

$\tan\theta = \frac{0.20}{0.50}$ so $\theta = \tan^{-1}\left(\frac{0.20}{0.50}\right) = 21.8°... =$ **22° (to 2 s.f.)**

So the resultant force is **0.54 N** at an angle of **22° to the vertical** (i.e. a bearing of 022°).

Choose Sensible Axes for Resolving

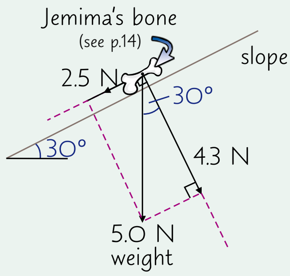
Use directions that **make sense** for the situation you're dealing with. If you've got an object on a slope, choose your directions **along the slope** and **at right angles to it**. You can turn the paper to an angle if that helps.

Always choose sensible axes.

Examiners like to call a slope an "inclined plane".

Example:

The component of the bone's weight down the slope is 2.5 N so you'd need 2.5 N of friction (see p.30) to stop it sliding down.



Practice Questions

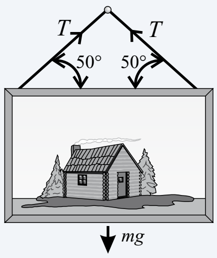
Q1 Sketch a free-body force diagram for an ice hockey puck moving horizontally across a flat sheet of ice at a constant speed (assuming it is not being pushed and there is no friction).

Q2 What are the horizontal and vertical components of a force F if it is applied at an angle of θ to the horizontal?

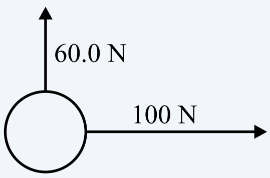
Q3 Use trigonometry to explain why perpendicular components of a force are independent of each other.

Exam Questions

Q1 An 8 kg picture is suspended from a hook as shown in the diagram. Calculate the tension force, T, in the string. [2 marks]



Q2 Two dogs pull a frisbee as shown in the diagram. Both values given are correct to 3 significant figures. Calculate the resultant force on the frisbee. [2 marks]



Free-body force diagram — sounds like a dance competition...

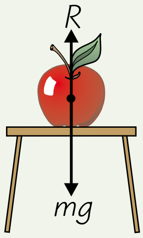
Remember those $F\cos\theta$ and $F\sin\theta$ bits. Write them on bits of paper and stick them to your wall. Scrawl them on your pillow. Tattoo them on your brain. Whatever it takes — you just have to learn them.

Newton's Laws of Motion

You did most of this at GCSE, but that doesn't mean you can just skip over it now. You'll be kicking yourself if you forget this stuff in the exam — it's easy marks...

Newton's **1st Law** Says That a **Force** is Needed to Change Velocity

1) **Newton's 1st law of motion** states that the **velocity** of an object will **not change** unless a **resultant force** acts on it.
2) In plain English this means a body will stay still ('at rest') or move in a **straight line** at a **constant speed**, unless there's a **resultant force** acting on it.
3) An example of constant velocity is when an object reaches its **terminal velocity** (see p.30). This occurs when the **weight** of a **falling** object is exactly **balanced** by **drag** (e.g. air resistance). Since there is **no resultant force**, there is **no acceleration**, and the object falls at a **constant velocity**.
4) If the forces **aren't balanced**, the **overall resultant force** will make the body **accelerate**. This could be a change in **direction**, or **speed**, or both. (See Newton's 2nd law, below.)



An apple sitting on a table won't go anywhere because the **forces** on it are **balanced**.

reaction (R) (force of table pushing apple up) = **weight** (mg) (force of gravity pulling apple down, see p.38)

Newton's **2nd Law** Says That **Acceleration** is **Proportional** to the Force

...which can be written as the well-known equation:

resultant force (N) = mass (kg) × acceleration (ms^{-2})

$$\boxed{\Sigma F = ma}$$

The resultant force is the vector sum of all the forces on an object. Σ is the symbol for 'sum of', but you'll often see resultant force as just F.

Learn this — it crops up all over the place in Physics. And learn what it means too:

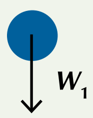
1) It says that the **more force** you have acting on a certain mass, the **more acceleration** you get.
2) It says that for a given force, the **more mass** you have, the **less acceleration** you get.
3) If you have **no resultant force**, then $\boldsymbol{\Sigma F = 0}$. You can see from $\Sigma F = ma$ that this happens when $\boldsymbol{a = 0}$. And as if by magic (or physics), this **matches** what was said above in Newton's 1st law — **no resultant force** means **no acceleration**.
4) There's more on this most excellent law on p.33.

REMEMBER:
1) The **resultant force** is the **vector sum** of all the forces.
2) The force is **always** measured in **newtons**.
3) The **mass** is always measured in **kilograms** and is a **constant**.
4) The **acceleration** is always in the **same direction** as the **resultant force** and is measured in **ms^{-2}**.

Galileo said: **All Objects Fall** at the **Same Rate** (if You **Ignore Air Resistance**)

You need to understand **why** this is true. Newton's 2nd law explains it neatly — consider two balls dropped at the same time — ball **1** being heavy, and ball **2** being light. Then use Newton's 2nd law to find their acceleration.

mass = m_1
resultant force = F_1
acceleration = a_1



By Newton's Second Law:

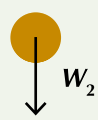
$$F_1 = m_1 a_1$$

Ignoring air resistance, the only force acting on the ball is weight, given by $W_1 = m_1 g$ (where g = gravitational field strength = 9.81 Nkg^{-1}).

So: $F_1 = m_1 a_1 = W_1 = m_1 g$

So: $m_1 a_1 = m_1 g$, then m_1 cancels out to give: $a_1 = g$

mass = m_2
resultant force = F_2
acceleration = a_2



By Newton's Second Law:

$$F_2 = m_2 a_2$$

Ignoring air resistance, the only force acting on the ball is weight, given by $W_2 = m_2 g$ (where g = gravitational field strength = 9.81 Nkg^{-1}).

So: $F_2 = m_2 a_2 = W_2 = m_2 g$

So: $m_2 a_2 = m_2 g$, then m_2 cancels out to give: $a_2 = g$

...in other words, the **acceleration** is **independent of the mass**. It makes **no difference** whether the ball is **heavy or light**. And I've kindly **hammered home the point** by showing you two almost identical examples.

Newton's Laws of Motion

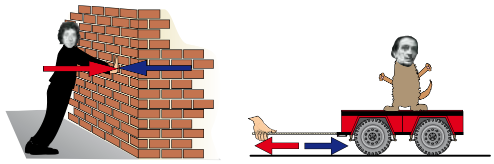
Newton's **3rd Law** Says Each Force has an **Equal**, **Opposite Reaction Force**

There are a few different ways of stating Newton's 3rd law, but the clearest way is:

If an object A exerts a FORCE on object B, then object B exerts AN EQUAL BUT OPPOSITE FORCE on object A.

You'll also hear the law as "every action has an equal and opposite reaction". But this confuses people who wrongly think the forces are both applied to the same object. (If that were the case, you'd get a resultant force of zero and nothing would ever move anywhere...)

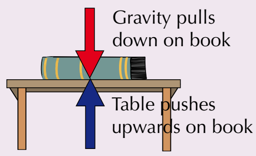
The two forces actually represent the **same interaction**, just seen from two **different perspectives**:



1) If you **push against a wall**, the wall will **push back** against you, **just as hard**. As soon as you stop pushing, so does the wall. Amazing...
2) If you **pull a cart**, whatever force **you exert** on the rope, the rope exerts the **exact opposite** pull on you (unless the rope's stretching).
3) When you go **swimming**, you push **back** against the water with your arms and legs, and the water pushes you **forwards** with an equal-sized force.

This looks like Newton's 3rd law...

But it's <u>NOT</u>.



...because both forces are acting on the book, and they're not of the same type. They are two separate interactions. The forces are equal and opposite, resulting in zero acceleration, so this is showing Newton's 1st law.

Newton's 3rd law applies in **all situations** and to all **types of force**. But the pairs of forces are always the **same type**, e.g. both gravitational or both electrical, and they act along the same line.

Practice Questions

Q1 State Newton's 1st, 2nd and 3rd laws of motion, and explain what they mean.

Q2 Explain how you can demonstrate Newton's 1st law using Newton's 2nd law.

Q3 What are the two equal and opposite forces acting between an orbiting satellite and the Earth?

Exam Questions

Q1 A boat is moving across a river. The engines provide a force of 500 N at right angles to the flow of the river and the boat experiences a drag of 100 N in the opposite direction. The force on the boat due to the flow of the river is 300 N. The mass of the boat is 250 kg.

a) Calculate the magnitude of the resultant force acting on the boat. [2 marks]

b) Calculate the magnitude of the acceleration of the boat. [1 mark]

Q2 John's bike, which has a mass of m, breaks and he has to push it home. The bike has a constant acceleration a and a frictional force F opposes the motion. What force is John using to push his bike?

A	ma
B	$ma + F$
C	$m(a - F)$
D	$ma - F$

[1 mark]

Q3 Michael and Tom are both keen on diving. They notice that they seem to take the same time to drop from the diving board to the water. Use Newton's second law to explain why this is the case. (Assume no air resistance.) [3 marks]

Newton's three incredibly important laws of motion...

These laws may not really fill you with a huge amount of excitement (and I could hardly blame you if they don't)... but it was pretty fantastic at the time — suddenly people actually understood how forces work, and how they affect motion. I mean arguably it was one of the most important scientific discoveries ever...

Drag and Terminal Velocity

If you jump out of a plane at 1500 m, you want to know that you're not going to be accelerating all the way.

Friction *is a* ***Force*** *that* ***Opposes Motion***

There are two main types of friction — **friction** between **solid surfaces** and **friction** in a **fluid**. Friction in a fluid is known as **drag** or fluid resistance. **Air resistance** is a type of fluid resistance.

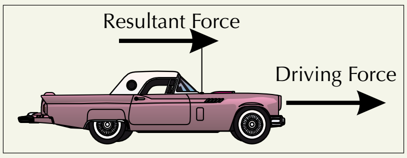
Fluid Friction or Drag:
1) 'Fluid' is a word that means either a **liquid or a gas** — something that can **flow**.
2) The force depends on the thickness (or **viscosity**) of the fluid (see p.64).
3) It **increases** as the **speed increases** (for simple situations it's directly proportional, but you don't need to worry about the mathematical relationship).
4) It also depends on the **shape** of the object moving through it — the larger the **area** pushing against the fluid, the greater the resistance force.
5) A **projectile** (see p.20) is **slowed down** by air resistance. If you calculate how far a projectile will travel without thinking about air resistance, your answer will be **too large**.

Things you need to remember about frictional forces:
1) They **always** act in the **opposite direction** to the **motion** of the object.
2) They can **never** speed things up or start something moving.
3) They convert **kinetic energy** into **heat** and **sound**.

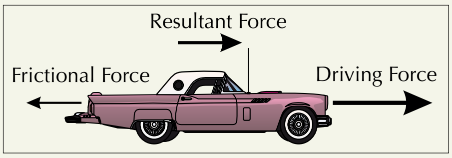
Terminal Velocity *— When the* ***Friction*** *Force* ***Equals*** *the* ***Driving Force***

You will reach a **terminal (maximum) velocity** at some point, if you have:
1) a **driving force** that stays the **same** all the time
2) a **frictional** or **drag force** (or collection of forces) that increases with velocity

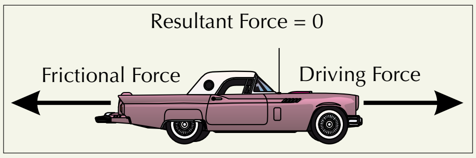
There are **three main stages** to reaching terminal velocity:



The car **accelerates** from **rest** using a constant driving force.



As the **velocity increases**, the **frictional forces increase** (because of things like turbulence — you don't need the details). This **reduces the resultant force** on the car and hence **reduces its acceleration**.



Eventually the car reaches a velocity at which the **frictional forces are equal to the driving force**. There is now **no resultant force** and **no acceleration**, so the car carries on at **constant velocity**.

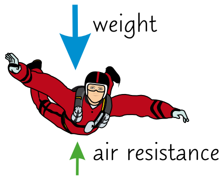
Different factors affect a vehicle's maximum velocity
As you just saw, a vehicle reaches maximum velocity when the driving force is equal to the frictional force. So there are two main ways of increasing a vehicle's maximum velocity:
1) **Increasing the driving force**, e.g. by increasing the engine size.
2) **Reducing the frictional force**, e.g. making the body more streamlined.

Drag and Terminal Velocity

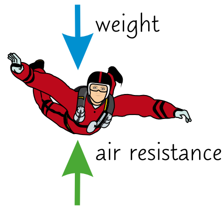
*Things **Falling** through **Air** or **Water** Reach a **Terminal Velocity** too*

When something's falling through air, the weight of the object (p.38) is a constant force accelerating the object downwards. Air resistance is a frictional force opposing this motion, which increases with speed.
So before a parachutist opens the parachute, exactly the same thing happens as with the car example:

1) A skydiver leaves a plane and will **accelerate** until the **air resistance** equals his **weight**.

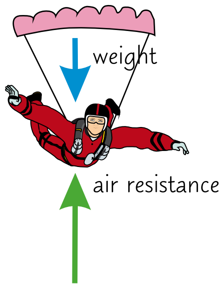


2) He will then be travelling at a **terminal velocity**.

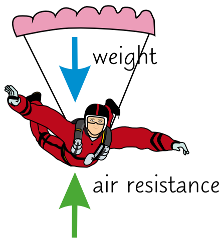


But... the terminal velocity of a person in free fall is too great to land without dying a horrible death.
The **parachute increases** the **air resistance massively**, which slows him down to a lower terminal velocity:

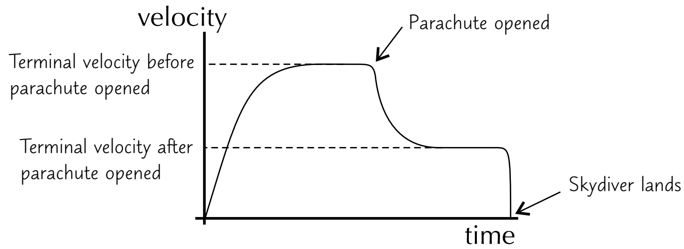
3) Before reaching the ground he will **open his parachute**, which immediately **increases the air resistance** so it is now **bigger** than his **weight**.



4) This **slows him down** until his speed has dropped enough for the **air resistance** to be **equal to his weight** again. This new terminal velocity is small enough for him to land safely.



A *v-t* graph of the skydiver looks like this. He reaches terminal velocity twice during his fall — the second one is much slower than the first.



Practice Questions

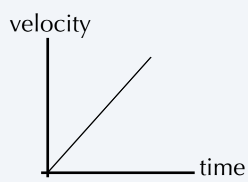
Q1 What forces limit the velocity of a skier going down a slope?

Q2 Suggest two ways in which the maximum velocity of a car can be increased.

Q3 What conditions cause a terminal velocity to be reached?

Exam Question

Q1 A space probe free-falls towards the surface of a planet.
The graph on the right shows the velocity of the probe as it falls.



a) The planet does not have an atmosphere. Explain how you can tell this from the graph. [2 marks]

b) Sketch the velocity-time graph you would expect to see if the planet did have an atmosphere. [2 marks]

c) Explain the shape of the graph you have drawn. [3 marks]

*You'll never understand this without going parachuting...**

When you're doing questions about terminal velocity, remember the frictional forces reduce acceleration, not speed. They usually don't slow an object down, apart from in the parachute example, where the skydiver is travelling faster just before the parachute opens than the terminal velocity for the open parachute-skydiver combination.

* No. 37 in a series of the 100 least convincing excuses for an interesting holiday.

Momentum

These pages are about linear momentum — that's momentum in a straight line (not a circle).

Understanding **Momentum** Helps You Do **Calculations** for **Collisions**

The **momentum** of an object depends on two things — its **mass** and **velocity**.
The **product** of these two values is the momentum of the object:

$$\boxed{p = mv}$$

where p is the momentum in kg ms^{-1},
m is the mass in kg, and v is the velocity in ms^{-1}.

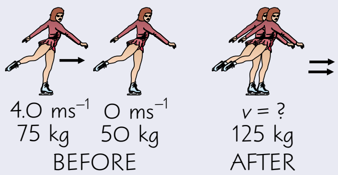
Remember, momentum is a vector quantity, so it has size **and** direction.

Momentum is Always **Conserved**

1) Assuming **no external forces** act, momentum is always **conserved**.
2) This means the **total momentum** of two objects **before** they collide **equals** the total momentum **after** the collision.
3) This is really handy for working out the **velocity** of objects after a collision (as you do...):

You might see momentum referred to as 'linear momentum'. The other kind is 'angular momentum', but you don't need to know about that for now.

Example: A skater of mass 75 kg and velocity 4.0 ms^{-1} collides with a stationary skater of mass 50 kg (to 2 s.f.). The two skaters join together and move off in the same direction. Calculate their velocity after impact.

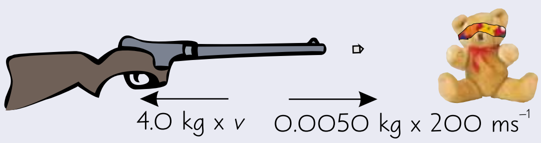


Before you start a momentum calculation, always draw a quick sketch.

$$\begin{aligned}\text{Momentum of skaters before} &= \text{Momentum of skaters after}\\ (75 \times 4.0) + (50 \times 0) &= 125v\\ 300 &= 125v\\ \text{So } \boldsymbol{v} &= \mathbf{2.4\ ms^{-1}}\end{aligned}$$

4) The same principle can be applied in **explosions**. E.g. if you fire an **air rifle**, the **forward momentum** gained by the pellet **equals** the **backward momentum** of the rifle, and you feel the rifle recoiling into your shoulder.

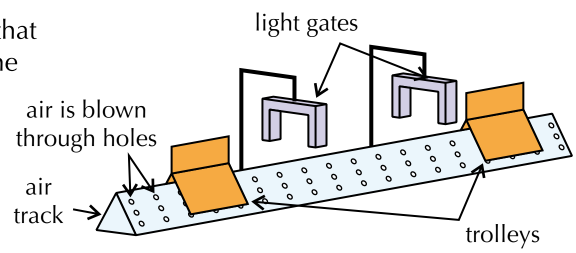
Example: A bullet of mass 0.0050 kg is shot from a rifle at a speed of 200 ms^{-1} (to 2 s.f.). The rifle has a mass of 4.0 kg. Calculate the velocity at which the rifle recoils.



$$\begin{aligned}\text{Momentum before explosion} &= \text{Momentum after explosion}\\ 0 &= (0.0050 \times 200) + (4.0 \times v)\\ 0 &= 1 + 4v\\ \text{So } v &= \mathbf{-0.25\ ms^{-1}}\end{aligned}$$

An **Air Track** and **Light Gates** are Used to **Investigate** Momentum

1) An **air track** is a long piece of metal with a series of **small holes** along its surfaces. Air is blown through the holes.
2) This air **reduces friction** between the **track** and the **trolleys** that move on top of it.
3) Momentum is only conserved if **no external forces act**, so air tracks are useful for studying conservation of momentum as you can assume there is **no friction**.
4) **Two** trolleys are pushed towards each other on an air track, so that they **collide** between the **light gates**. The light gates measure the **speed** of each trolley as they pass through them.
5) If the speed of each trolley is measured **before** and **after** the collision, the **initial** and **final** momentum of each trolley can be calculated using $p = mv$.



Momentum

Newton's 2nd Law Says That Force is the *Rate of Change in Momentum*

Newton's second law (p.28) can also be expressed in terms of momentum. The **rate of change of momentum** of an object is **directly proportional** to the **resultant force** which acts on the object.

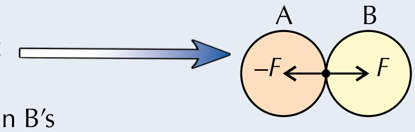
So: $\boxed{F = \dfrac{\Delta(mv)}{\Delta t}}$ or $\boxed{F\Delta t = \Delta(mv)}$

where F is the resultant force in N,
m is the mass in kg, v is the velocity in ms^{-1},
and t is the time taken for the velocity to change in s.

If the force is changing, F gives the average force over the time interval Δt.

Remember that acceleration is equal to the rate of change of velocity (page 16), so if mass is constant then this formula gives you that mechanics favourite, $F = ma$ (or $\Sigma F = ma$) (p.28).

1) You can also relate this formula to **conservation of momentum** by considering **Newton's 3rd law**.
2) If object A **collides** with object B and **exerts** a force F on B for a time Δt, Newton's 3rd law says that object B will also exert a force $-F$ on A for a time Δt.



3) The **change** in A's **momentum** is equal to $-F\Delta t$, and the change in B's momentum is $F\Delta t$, so the **overall** change in momentum is $(-F\Delta t) + F\Delta t = 0$. So momentum is **conserved**.

Example: A snooker ball that is initially at rest is hit with a cue. The cue is in contact with the ball for 0.0040 s and the speed of the ball immediately after being hit is 0.80 ms^{-1}. The mass of the snooker ball is 0.16 kg. Calculate the average force exerted on the snooker ball by the cue.

Use the equation $F = \dfrac{\Delta(mv)}{\Delta t}$ and substitute in the values:

$$F = \frac{\Delta(mv)}{\Delta t} = \frac{(0.16 \times 0.80) - (0.16 \times 0)}{0.0040} = \mathbf{32\ N}$$

Practice Questions

Q1 Give two examples of conservation of momentum in practice.

Q2 Explain how light gates and an air track can be used to investigate conservation of momentum.

Q3 Explain how Newton's 2nd law and Newton's 3rd law can be used to demonstrate conservation of momentum.

Exam Questions

Q1 A ball of mass 0.60 kg moving at 5.0 ms^{-1} collides with a larger stationary ball of mass 2.0 kg. The smaller ball rebounds in the opposite direction at 2.4 ms^{-1}.

a) Calculate the velocity of the larger ball immediately after the collision. [3 marks]

b) The collision lasts for 0.0055 s. Calculate the average force acting on the smaller ball in this time. [2 marks]

Q2 A toy train of mass 0.7 kg, travelling at 0.3 ms^{-1}, collides with a stationary toy carriage of mass 0.4 kg. The two toys couple together. Calculate their new velocity. [3 marks]

Momentum'll never be an endangered species — it's always conserved...

...unlike record stores, which are quite the opposite and could probably do with your help. You can't match the excitement of running home with a shiny new CD, spending 20 minutes trying to pick through the shrink wrap then sitting back and enjoying 12 glorious new tracks... Speaking of (air) tracks, there's a lot of important stuff here — make sure you know how Newton's laws relate to conservation of momentum.

Work and Power

As everyone knows, work in Physics isn't like normal work. It's harder. Work also has a specific meaning that's to do with movement and forces. You'll have seen this at GCSE — it just comes up in more detail here.

Work *is Done Whenever* ***Energy*** *is* ***Transferred***

This table gives you some examples of **work being done** and the **energy changes** that happen.

1) Usually you need a force to move something because you're having to **overcome another force**.
2) The thing being moved has **kinetic energy** while it's **moving**.
3) The kinetic energy is transferred to **another form of energy** when the movement stops.

ACTIVITY	WORK DONE AGAINST	FINAL ENERGY FORM
Lifting up a box.	gravity	gravitational potential energy
Pushing a chair across a level floor.	friction	heat
Pushing two magnetic north poles together.	magnetic force	magnetic energy
Stretching a spring.	stiffness of spring	elastic potential energy

The word **'work'** in Physics means the **amount of energy transferred** from one form to another when a force causes a movement of some sort. It's measured in **joules** (J).

Work = Force × Distance

When a car tows a caravan, it applies a force to the caravan to move it to where it's wanted.
To **find out** how much **work** has been **done**, you need to use the **equation**:

work done (ΔW) = force causing motion (F) × distance moved (Δs), or $\Delta W = F\Delta s$
...where ΔW is measured in joules (J), F is measured in newtons (N) and Δs is measured in metres (m).

Points to remember:

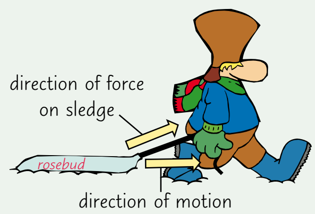
1) **Work** is the **energy** that's been **changed** from one form to another — it's not necessarily the **total** energy. E.g. moving a book from a low shelf to a higher one will increase its gravitational potential energy, but it had some potential energy to start with. Here, the **work done** would be the **increase** in potential energy, **not the total** potential energy.
2) Remember the distance needs to be measured in metres — if you have a **distance in centimetres or kilometres**, you need to **convert** it to metres first.
3) The force F will be a **fixed** value in any calculations, either because it's **constant** or because it's the **average** force.
4) The equation assumes that the **direction of the force** is the **same** as the **direction of movement**.
5) The equation gives you the **definition** of the joule (symbol J): 'One joule is the work done when a force of 1 newton moves an object through a distance of 1 metre'.

The ***Force*** *isn't always in the* ***Same Direction*** *as the* ***Movement***

Sometimes the **direction of movement** is **different** from the **direction of the** force.

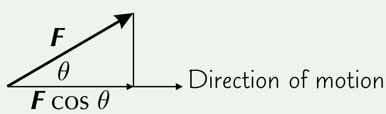
Example:

1) To **calculate the work done** in a situation like the one on the right, you need to consider the **horizontal** and **vertical components** of the **force**.
2) The only **movement** is in the **horizontal** direction. This means the **vertical force** is not causing any motion (and hence not doing any work) — it's just **balancing** out some of the **weight**, meaning there's a **smaller reaction force**.



3) The horizontal force is causing the motion — so to **calculate** the **work done**, this is the **only force** you need to consider. Which means we get: $\boxed{\Delta W = F\Delta s\cos\theta}$

Where θ is the **angle** between the **direction of the force** and the **direction of motion**. See page 26 for more on resolving forces.



Work and Power

Power = Work Done per Second

Power means many things in everyday speech, but in physics (of course!) it has a special meaning. Power is the **rate of doing work** — in other words it is the **amount of energy transferred** from one form to another **per second**. You **calculate power** from the equation:

> **Power (P) = energy transferred (E) ÷ time (t)**, or $P = \frac{E}{t}$ ...where P is measured in watts (W), E is measured in joules (J) and t is measured in seconds (s).

Since **work done** is **equal** to the **energy transferred**, you can also write this as:

> **Power (P) = work done (W) ÷ time (t)**, or $P = \frac{W}{t}$ ...where P is measured in watts (W), W is measured in joules (J) and t is measured in seconds (s).

The **watt** (symbol W) is defined as a **rate of energy transfer** equal to **1 joule per second** (Js^{-1}).
Yep, that's more **equations and definitions** for you to **learn**.

In the equations on this page, W stands for watts (the unit of power) — don't get it confused with W, work done.

Example: A light bulb transfers 230 kJ of electrical energy into light and heat in one hour. Calculate the power of the bulb.

Energy transferred = 230 kJ = 230 000 J
Time taken to transfer the energy = 1 hour = 3600 s
So $P = \frac{E}{t} = \frac{230\,000}{3600} = 63.888... =$ **64 W (to 2 s.f.)**

Remember — 1 kJ = 1000 J

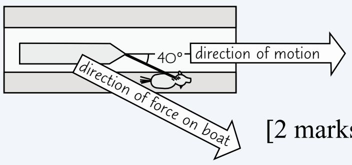
Alice always took a long time to transfer any energy.

Practice Questions

Q1 Write down the equation used to calculate work if the force and motion are in the same direction.
Q2 Write down the equation for work if the force is at an angle to the direction of motion.
Q3 Write down the equations relating: a) power and work done b) power and energy transferred.

Exam Questions

Q1 A traditional narrowboat is drawn by a horse walking along a canal towpath. The horse pulls the boat at a constant speed between two locks which are 1500 m apart. The tension in the rope is 100 N at 40° to the direction of motion.



a) Calculate the work done on the boat. [2 marks]

b) It take 31 minutes for the horse to pull the boat between the two locks. Calculate the power supplied to the boat. [2 marks]

Q2 A motor is used to lift a 20.0 kg load a height of 3.0 m. ($g = 9.81\ ms^{-2}$)

a) Calculate the work done in lifting the load. [2 marks]

b) The speed of the load as it is lifted is $0.25\ ms^{-1}$. Calculate the power delivered by the motor. [3 marks]

Work — there's just no getting away from it...

Loads of equations to learn. Well, that's what you came here for, after all. Can't beat a good bit of equation-learning, as I've heard you say quietly to yourself when you think no one's listening. Aha, can't fool me. Ahahahahahahahahaha.

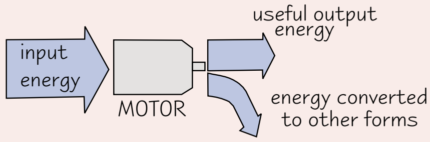
Conservation of Energy and Efficiency

Energy cannot be created or destroyed (don't forget that). Which is basically what I'm about to take up two whole pages saying. But that's, of course, because you need to do exam questions on this as well as understand the principle.

Learn the *Principle* of *Conservation* of *Energy*

The **principle of conservation of energy** says that:

Energy **cannot be created** or **destroyed**. Energy **can be transferred** from one form to another but the total amount of energy in a closed system will not change.



Total energy in = Total energy out

Greg started panic-buying when he heard that more energy couldn't be created.

You need it for *Questions* about *Kinetic* and *Potential Energy*

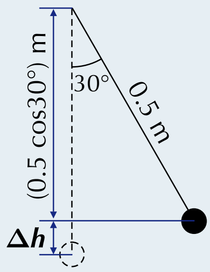
The principle of conservation of energy nearly always comes up when you're doing questions about **changes** between **kinetic** and **potential energy**. A quick reminder:

1) **Kinetic energy** is the energy of anything due to its **motion**, which you work out from: $\boxed{E_K = \frac{1}{2}mv^2}$ — m is the mass of the object (kg) and v is its velocity (ms^{-1})
2) There are **different types of potential energy** — e.g. gravitational potential energy and elastic strain (see page 58).
3) **Gravitational potential energy** is the energy something gains if you lift it up. The **equation** for the change in gravitational potential energy close to the **Earth's surface** is:

$$\boxed{\textbf{change in gravitational potential energy} = \Delta E_{grav} = mg\Delta h}$$

g is the acceleration due to gravity, $g = 9.81\ ms^{-2}$ and Δh is the change in height (m)

Example: A pendulum has a mass of 700 g and a length of 50 cm. It is pulled out to an angle of 30° from the vertical.
a) Find the gravitational potential energy stored in the pendulum bob.



Start by drawing a diagram.
You can work out the increase in height, Δh, of the end of the pendulum using trig.

$\Delta E_{grav} = mg\Delta h = 0.7 \times 9.81 \times (0.5 - 0.5\cos30°) = 0.460... =$ **0.5 J (to 1 s.f.)**

b) The pendulum is released. Find the maximum speed of the pendulum bob as it passes the vertical position. Assume there is no air resistance.

When travelling at its maximum speed, all of the gravitational potential energy has been transferred to kinetic energy, so $mg\Delta h = \frac{1}{2}mv^2 = 0.460...$

$\frac{1}{2}mv^2 = 0.460...$, so $v = \sqrt{\frac{2 \times 0.460...}{0.7}} = 1.146... =$ **1 ms^{-1} (to 1 s.f.)**

Or, you could cancel the 'm's and rearrange to give: $v = \sqrt{2g\Delta h}$
$= \sqrt{2 \times 9.81 \times (0.5 - 0.5\cos30°)}$
$=$ **1 ms^{-1} (to 1 s.f.)**

Conservation of Energy and Efficiency

All Energy Transfers Involve Losses

You saw on the last page that **energy can never be created or destroyed**. But whenever **energy** is **converted** from one form to another, some is always **'lost'**. It's still there (i.e. it's **not destroyed**) — it's just not in a form you can **use**.

Most often, **energy** is lost as **heat** — e.g. **computers** and **TVs** are always **warm** when they've been on for a while. In fact, **no device** (except possibly a heater) is ever **100% efficient** (see below) because some energy is **always** lost as **heat**. (You want heaters to give out heat, but in other devices the heat loss isn't useful.) Energy can be **lost** as other forms too (e.g. **sound**) — the important thing is the lost energy **isn't** in a **useful** form and you **can't** get it back.

Energy is often lost as **heat** due to **friction**. For example, imagine pushing a **box** along a **table**. **Frictional forces** between the box and the table surface act in the **opposite direction** to the **motion** of the box (see p.30). Work has to be done to **overcome** this friction — energy is transferred to **heat**. Luckily you can usually assume that **friction** is **zero** in exams.

Efficiency is the Ratio of Useful Energy Output to Total Energy Input

Efficiency is one of those words we use all the time, but it has a **specific meaning** in Physics. It's a measure of how well a **device** converts the **energy** you put **in** into the energy you **want** it to give **out**. So, a device that **wastes** loads of **energy** as heat and sound has a really **low efficiency**.

$$\text{Efficiency} = \frac{\textbf{useful energy output}}{\textbf{total energy input}}$$

Or

$$\text{Efficiency} = \frac{\textbf{useful power output}}{\textbf{total power input}}$$

You can multiply either of these equations by 100 to find the efficiency as a percentage.

Some questions will be kind and **give you** the **useful output energy** — others will tell you how much is **wasted**. You just have to **subtract** the **wasted energy** from the **total input energy** to find the **useful output energy**, so it's not too tricky if you keep your wits about you.

Energy, as always, is measured in joules (J). Efficiency has no units because it's a fraction (or a percentage).

Practice Questions

Q1 State the principle of conservation of energy.

Q2 Show that, if there's no air resistance and the mass of the string is negligible, the speed of a pendulum is independent of the mass of the bob.

Q3 Why can a device never be 100% efficient?

Q4 Give an equation for efficiency in terms of a) energy and b) power.

Exam Questions

acceleration of free fall, g = 9.81 ms^{-2}

Q1 A skateboarder is skating on a half-pipe. He lets the board run down one side of the ramp and up the other. The height of the ramp is 2.0 m.

a) Calculate his speed at the lowest point of the ramp. Assume friction is negligible. [3 marks]

b) State how high he will rise up the other side of the half-pipe. [1 mark]

c) Real ramps are not frictionless. Describe what the skater must do to reach the top on the other side. [1 mark]

Q2 A 20.0 g rubber ball is released from a height of 8.0 m. Assume that the effect of air resistance is negligible.

a) Calculate the kinetic energy of the ball just before it hits the ground. [2 marks]

b) The ball strikes the ground and rebounds to a height of 6.5 m.
Calculate the amount of energy that is converted to heat and sound in the impact with the ground. [2 marks]

Q3 Calculate the efficiency (as a fraction) of a device that wastes 65 J for every 140 J of input energy. [1 mark]

Eat, sleep, state the principle of conservation of energy, repeat...

Remember to check your answers — I can't count the number of times I've forgotten to square the velocities or to multiply by the ½... I reckon it's definitely worth the extra minute to check. You never know what you might find.

Mass, Weight and Centre of Gravity

I'm sure you know all this 'mass', 'weight' and 'centre of gravity' stuff from GCSE. But let's just make sure...

The Mass** of a **Body** makes it **Resist Changes** in **Motion

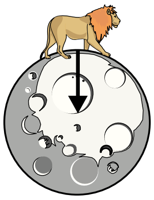
1) The **mass** of an object is the **amount of 'stuff'** (or **matter**) in it. It's measured in **kg**.
2) The greater an object's mass, the greater its **resistance** to a **change in velocity**.
3) The **mass** of an object **doesn't change** if the strength of the **gravitational field** changes.
4) As you've already seen in this topic, weight is a **force**. It's measured in **newtons** (N), like all forces.
5) Weight is the **force experienced by a mass** due to a **gravitational field**.
6) The weight of an object **does vary** according to the size of the **gravitational field** acting on it.

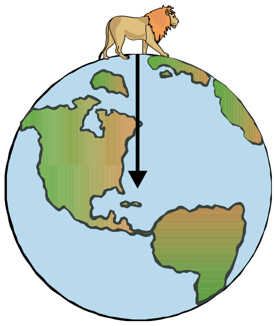
weight = mass × gravitational field strength ($W = mg$) where g = 9.81 Nkg^{-1} on Earth.

The value of g = 9.81 Nkg^{-1} is only true close to the Earth's surface (p.18).

This table shows a lion's mass and weight on the Earth and the Moon.

Name	Quantity	Earth (g = 9.81 Nkg^{-1})	Moon (g = 1.6 Nkg^{-1})
Mass	Mass (scalar)	150 kg	150 kg
Weight	Force (vector)	1470 N (to 3 s.f.)	240 N (to 2 s.f.)

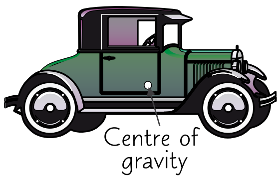


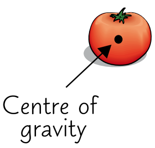


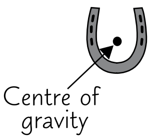
Centre of Gravity** — Assume **All the Mass** is in **One Place

1) The **centre of gravity** of an object is the **single point** that you can consider its **whole weight** to **act through** (whatever its orientation).
2) The object will always **balance** around this **point**, although in some cases the **centre of gravity** will **fall outside** the object.

You might come across the phrase 'extended body' when dealing with the centre of gravity. This is just another name for an object that isn't a single point.

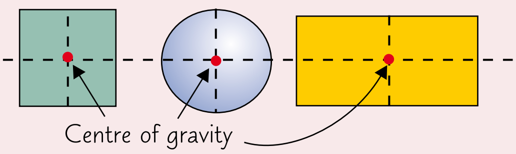






***Find the Centre of Gravity** either by **Symmetry**...*

1) To find the centre of gravity for a **regular** object you can just use **symmetry**.
2) The centre of gravity of any regular shape is at its **centre** — where the lines of symmetry will cross.
3) The centre of gravity is **halfway** through the **thickness** of the object at the point the lines meet.



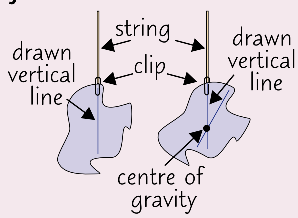
The symmetry in this picture shows the centre of cuteness.

Mass, Weight and Centre of Gravity

... Or By **Experiment**

Experiment to find the Centre of Gravity of an Irregular Object

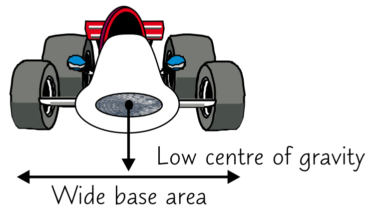
1) **Hang** the object freely from a point (e.g. one corner).
2) Draw a **vertical line** downwards from the point of suspension — use a plumb bob to get your line exactly vertical.
3) Hang the object from a different point.
4) Draw another vertical line down.
5) The centre of gravity is where the two lines **cross**.



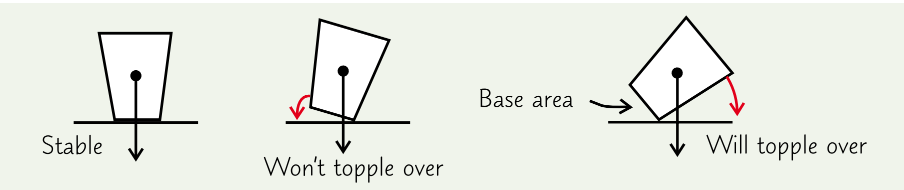
A plumb bob is just a weight on a string — when suspended, the string will be exactly vertical.

How High *the* ***Centre of Gravity*** *is tells you* ***How Stable*** *the Object is*

1) An object will be nice and **stable** if it has a **low centre** of **gravity** and a **wide base area**. This idea is used a lot in design, e.g. racing cars.
2) The **higher** the **centre of gravity**, and the **smaller** the **base area**, the **less stable** the object will be. Think of unicyclists...



3) An object will topple over if a **vertical line** drawn **downwards** from its **centre of gravity** falls **outside** its **base area**. This is because the object's weight is causing a turning force (a moment — see p.40) around a pivot.



The vertical line is the line of action of the weight — see next page.

Practice Questions

Q1 A lioness has a mass of 200 kg. What would be her mass and weight on Earth and on the Moon (where g = 1.6 Nkg^{-1})?

Q2 Define centre of gravity.

Exam Questions

Q1 Joanne weighs X N on Earth. Which of the following statements are correct?

A She will weigh the same on the Moon as on Earth.

B Her mass is equal to $\frac{X}{g}$ kg.

C Her mass depends on the gravitational field strength.

D Her acceleration due to gravity will be the same on Earth as the Moon. [1 mark]

Q2 a) Describe an experiment to find the centre of gravity of an object of uniform density with a constant thickness and irregular cross-section. [3 marks]

b) Identify one major source of uncertainty for this method and suggest a way to reduce its effect on the accuracy of the result. [2 marks]

The centre of gravity of this book should be round about page 50...

This is a really useful area of physics. To would-be nuclear physicists it might seem a little dull, but if you want to be an engineer — then things like centre of gravity and weight are dead important things to understand. You know, for designing things like cars and submarines... yep, pretty useful I'd say.

Moments

*This is not a time for jokes. There is not a moment to lose. Oh ho ho ho *bang*. (Ow.)*

A *Moment* is the *Turning Effect* of a *Force*

The **moment** of a **force** depends on the **size** of the force and **how far** the force is applied from the **turning point** (also called the **axis of rotation**):

The line of action of a force is a line along which it acts.

moment of a force (in Nm) **= force** (in N) **× perpendicular distance from the line of action of the force to the axis of rotation** (in m)

$$M = Fx$$

Moments Must be *Balanced* or the *Object* will *Turn*

The **principle of moments** states that for a body to be in **equilibrium**, the **sum of the clockwise moments** about any point **equals** the **sum of the anticlockwise moments** about the same point.

Remember $\sum$ means "the sum of".

Example:

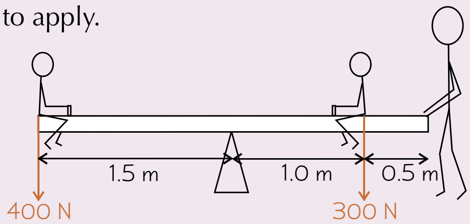
Two children sit on a seesaw as shown in the diagram. An adult balances the seesaw at one end. Find the size and direction of the force that the adult needs to apply.

In equilibrium, $\sum$ anticlockwise moments = $\sum$ clockwise moments

$$400 \times 1.5 = (300 \times 1.0) + 1.5F$$

$$600 = 300 + 1.5F$$

Final answer: F = **200 N downwards**



Muscles, *Bones* and *Joints* Act as *Levers*

1) In a lever, an **effort force** (in this case from a muscle) acts against a **load force** (e.g. the weight of your arm) by means of a **rigid object** (the bone) rotating around a **pivot** (the joint).
2) You can use the **principle of moments** to answer lever questions:

Example:

Find the force, E, exerted by the biceps in holding a bag of gold still. The bag of gold weighs 100 N and the forearm weighs 20 N.

Take moments about **A**.

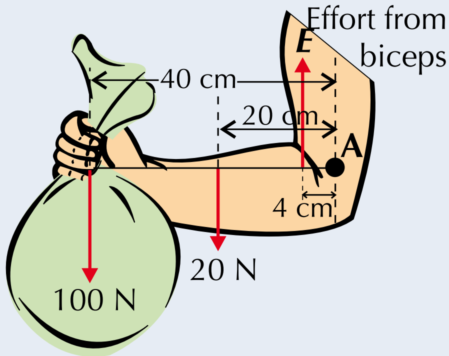
In equilibrium:

$\sum$ anticlockwise moments = $\sum$ clockwise moments

$$(100 \times 0.4) + (20 \times 0.2) = 0.04E$$

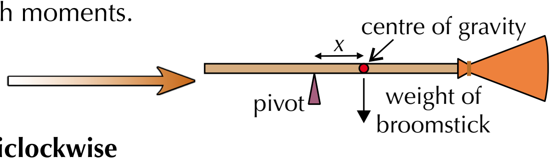
$$40 + 4 = 0.04E$$

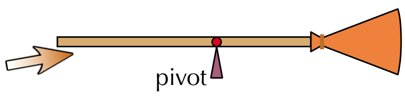
Final answer: E = 1100 = **1000 N (to 1 s.f.)**



The Principle of Moments can *Explain* why Things *Fall Over*

1) As you saw on page 39, you can assume that **all** the weight of an object **acts through** its **centre of gravity**. This is important when dealing with moments.
2) Imagine you are trying to balance a broomstick on a pivot. If the **centre of gravity** is to one side of the **pivot** (as shown here) then there will be a **clockwise** moment due to the **weight** of the broomstick acting at a **distance x** from the pivot. There is **no anticlockwise** moment, so the broomstick will **rotate** clockwise (fall off the pivot).
3) However, if the centre of gravity is **directly above** the pivot, then there are no clockwise or anticlockwise **moments** and so the broomstick is in **equilibrium**.





Moments

Sometimes you Need to ***Resolve Forces*** to Calculate ***Moments***

Example: Jamie Band is trying to save some top secret documents falling from a plank which is hanging precariously over a cliff. The diagram shows the plank pivoting at the edge of the cliff, labelled point P. He has tied a rope to one end of the plank and attached the other end of the rope to the ground. Jamie edges towards the top secret documents, but gets scared and stops at the point shown in the diagram below. Jamie has a mass of 75 kg and the top secret documents have a mass of 12 kg.

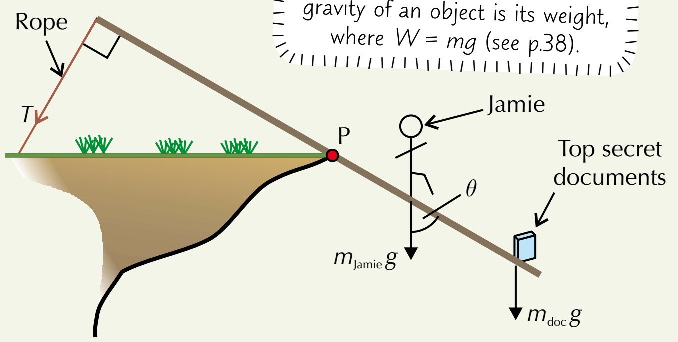
The distances and angles have been drawn to scale on the diagram below. Assuming the mass of the plank is negligible, calculate the tension, T, in the rope.

Remember — the force due to gravity of an object is its weight, where $W = mg$ (see p.38).

First you need to resolve all the forces (see p.26) acting perpendicular to the plank by measuring the angle between the line of action of Jamie's weight and the plank.

Angle $\theta = 60°$, so the force exerted by Jamie's weight acting perpendicular to the plank = $m_{Jamie}g \times \sin 60°$ and the force exerted by the documents acting perpendicular to the plank = $m_{doc}g \times \sin 60°$.

The angle between the rope and plank is 90°, so the tension acting perpendicular to the plank is just T.



You then need to measure all the distances: from P to rope = 3.6 cm, from P to Jamie = 1.2 cm, from P to documents = 2.8 cm. Using these values and the values for the forces, you can use the principle of moments to give:

Σ anticlockwise moments = Σ clockwise moments

$T \times 3.6 = (m_{Jamie}g \times \sin 60° \times 1.2) + (m_{doc}g \times \sin 60° \times 2.8)$

Which gives: $T = \dfrac{(75 \times 9.81 \times \sin 60° \times 1.2) + (12 \times 9.81 \times \sin 60° \times 2.8)}{3.6} = 291.68... = $ **290 N (to 2 s.f.)**

Practice Questions

Q1 A force of 54 N acts at a perpendicular distance of 84 cm from a pivot. Calculate the moment of the force.

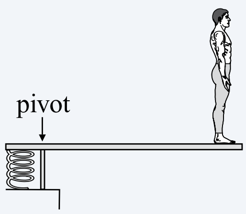
Q2 A girl of mass 40 kg sits 1.5 m from the middle of a seesaw.
Show that her brother, mass 50 kg, must sit 1.2 m from the middle if the seesaw is to balance.

Q3 Explain why a ruler will only balance on a narrow pivot if it is positioned so the pivot is exactly halfway along the ruler.

Exam Questions

Q1 A driver is changing his flat tyre. The moment required to undo the nut is 60 Nm (to 2 s.f.).
He uses a 0.40 m long spanner. Calculate the force that he must apply at the end of the spanner. [2 marks]

Q2 A diver of mass 60 kg stands on the end of a diving board 2.0 m from the pivot point.
Calculate the downward force exerted on the board by the retaining spring 30 cm from the pivot.



[2 marks]

It's all about balancing — just ask a tightrope walker...

They're always boring questions aren't they — seesaws or bicycles. It'd be nice if just once, they'd have a question on... I don't know, rotating knives or something. Just something unexpected. It'd make physics a lot more fun, I'm sure.