Charge, Current and Potential Difference

Electricity's brilliant, I love it. It's what gets me out of bed in the morning. (Not literally of course, that'd be quite painful.)

Current is the Rate of Flow of Charge

1) The **current** in a **wire** is like **water** flowing in a **pipe**. The **amount** of water that flows depends on the **flow rate** and the **time**. It's the same with electricity — **current is the rate of flow of charge**.

$$I = \frac{\Delta Q}{\Delta t}$$

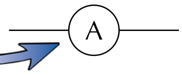
Where I is the current in amperes, ΔQ is the charge in coulombs, and Δt is the time taken in seconds.

Remember that conventional current flows from + to −, the opposite way from electron flow.

2) The **coulomb** is the **unit of charge**.

One coulomb (C) is defined as the **amount of charge** that passes in **1 second** when the **current** is **1 ampere**.

3) You can measure the current flowing through part of a circuit using an **ammeter**. This is the circuit symbol for an ammeter:



Attach an ammeter in series with the component you're investigating.

Potential Difference is the Work Done per Unit Charge

1) To make electric charge flow through a conductor, you need to do **work** on it.
2) **Potential difference** (p.d.), or **voltage**, is defined as the **work done per unit charge moved**:

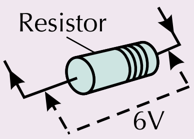
$$V = \frac{W}{Q}$$

W is the work done in joules (see p.34). It's the energy transferred in moving the charge.

The **potential difference** across a component is **1 volt** (V) when you do **1 joule** of work moving **1 coulomb** of charge through the component. This **defines** the volt.

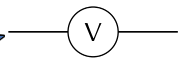
$$1\ \mathbf{V} = 1\ \mathbf{J\,C^{-1}}$$

Back to the 'water analogy' again. The p.d. is like the pressure that's forcing water along the pipe.



Here you do 6 J of work moving each coulomb of charge through the resistor, so the p.d. across it is 6 V. The energy gets converted to heat.

3) You can measure the potential difference across a component using a **voltmeter**. This is the circuit symbol for a voltmeter:



4) Remember, the potential difference across components in parallel is **the same**, so the **voltmeter** should be connected in **parallel** with the component you're investigating.

The maximum value that a voltmeter or ammeter can measure is called the full scale deflection.

Charge Carriers in Liquids and Gases are Ions

1) A current is the **rate of flow of charged particles** (called **charge carriers**). In a circuit, the charge carriers are free electrons (sometimes called **conduction electrons**), but there are **other types** of charge carrier.
2) A flow of **positively-charged** particles produces **exactly** the **same current** as an **equal** flow of negatively-charged particles in the **opposite direction**. This is why we use **conventional current**, defined as 'in the same direction as a flow of positive charges'.
3) **Ionic crystals** like sodium chloride are **insulators**. Once **molten**, though, the liquid **conducts**. Positive and negative **ions** are the **charge carriers**. The **same thing** happens in an **ionic solution** like copper sulfate solution.
4) **Gases** are **insulators**, but if you apply a **high enough voltage** electrons get **ripped out** of **atoms**, giving you **ions** along a path. You get a **spark**.

Uses of ions in air include creating a dramatic backdrop to a Gothic horror story, and bringing the creations of mad scientists to life.

Charge, Current and Potential Difference

*The **Mean Drift Velocity** is the **Average** Velocity of the **Charge Carriers***

When **current** flows through a wire, you might imagine the **electrons** all moving uniformly in the **same direction**. In fact, they move **randomly** in **all directions**, but tend to **drift** one way. The **mean drift velocity** is just the **average velocity** and it's **much, much less** than the electrons' **actual speed**. (Their actual speed is about 10^6 ms^{-1}.)

The Current Depends on the Mean Drift Velocity:

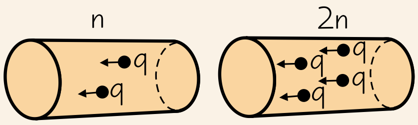
The charge on an electron, e, is -1.60×10^{-19} C.

The **current** is given by the equation:

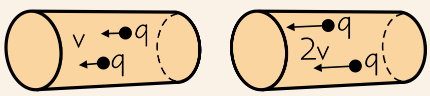
$$\boxed{I = nqvA}$$

where: I = electrical current (A)
n = number density of charge carriers (m^{-3}) (number per unit volume)
q = charge on each charge carrier (C)
v = mean drift velocity (ms^{-1})
A = cross-sectional area (m^2)

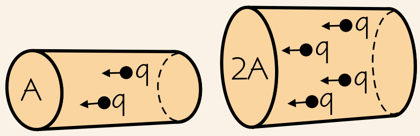
So...



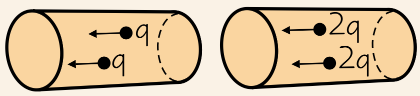
1) Double the number of charge carriers and the current doubles.



2) If the carriers move twice as fast you get twice the charge in the same time — twice the current.



3) Doubling the area also doubles the current.



4) Doubling the charge carried by each carrier gives you twice the charge in the same time — twice the current.

Different Materials have Different Numbers of Charge Carriers

1) In a **metal**, the **charge carriers** are **free electrons** — they're the ones from the **outer shell** of each atom. Thinking about the formula ***I = nqvA***, there are **loads** of charge carriers per unit volume, making ***n*** **big**. The **drift velocity** is **small**, even for a **high current**.
2) **Semiconductors** have **fewer charge carriers**, so the **drift velocity** needs to be **higher** to give the **same current**.
3) A **perfect insulator** wouldn't have **any charge carriers**, so $n = 0$ in the formula and you'd get **no current**. **Real** insulators have a **very small *n***.

Practice Questions

Q1 Describe in words and symbols how current and charge are related.

Q2 Define potential difference.

Q3 What happens to the current in a wire if the mean drift velocity of the electrons is halved?

Q4 Describe how metals, semiconductors and insulators differ in terms of n.

Exam Questions

Q1 A battery delivers 4500 C of electric charge to a circuit in 10 minutes. Calculate the average current. [1 mark]

Q2 A kettle runs off the mains supply (230 V) and has an overall efficiency of 88%. Calculate how much electric charge will pass through the kettle if it transfers 308 J of energy to the water it contains. [2 marks]

Q3 Copper has 1.0×10^{29} free electrons per m^3. Calculate the mean drift velocity of the electrons in a copper wire of cross-sectional area 5.0×10^{-6} m^2 when it is carrying a current of 13 A. [2 marks]

I can't even be bothered to make the current joke...

Talking of currant jokes, I saw this bottle of wine the other day called 'raisin d'être' — 'raison d'être' meaning 'reason for living', but spelled slightly differently to make 'raisin', meaning 'grape'. Ho ho. Chuckled all the way home.

Resistance and Resistivity

Resistance and resistivity. Not quite the same word, not quite the same thing. Make sure you know which is which...

Everything has Resistance

1) If you put a **potential difference** (p.d.) across an **electrical component**, a **current** will flow.
2) **How much** current you get for a particular **p.d.** depends on the **resistance** of the component.
3) You can think of a component's **resistance** as a **measure** of how **difficult** it is to get a **current** to **flow** through it.

Mathematically, **resistance** is: $\boxed{R = \frac{V}{I}}$
This equation really **defines** what is meant by resistance.

This is the **circuit symbol** for a resistor: 

4) **Resistance** is measured in **ohms** (Ω).

A component has a resistance of **1 Ω** if a **potential difference** of **1 V** makes a **current** of **1 A** flow through it.

Three Things Determine Resistance

If you think about a nice, **simple electrical component**, like a **length of wire**, its **resistance** depends on:

1) **Length** (***l***). The **longer** the wire the **more difficult** it is to make a **current flow**.
2) **Area** (***A***). The **wider** the wire the **easier** it is to make a current flow.
3) **Resistivity** (ρ). This **depends** on the **material** the wire's made from, as the **structure** of the material may make it easy or difficult for charge to flow. In general, resistivity depends on **environmental factors** as well, like **temperature**.

ρ is the Greek letter rho, the symbol for resistivity.

The **resistivity** of a material is defined as the **resistance** of a **1 m length** with a **1 m² cross-sectional area**, so $\rho = \frac{RA}{l}$. Resistivity is measured in **ohm metres** (Ωm).

In your exams, you'll be given this equation in the **form**: $\boxed{R = \frac{\rho l}{A}}$ where A = cross-sectional area in m², and l = length in m

Typical values for the **resistivity** of **conductors** are **really small**. However, if you **calculate** a **resistance** for a **conductor** and end up with something **really small** (e.g. $1 \times 10^{-7}\ \Omega$), go back and **check** that you've **converted** your **area** into **m²**.

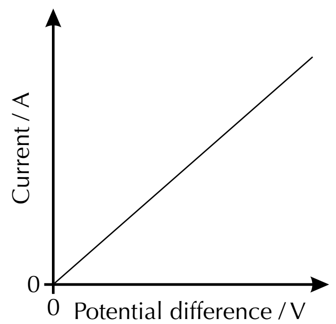
The resistivity of a material is related to the **number density of charge carriers**, (and their **mean drift velocity**, which often varies with temperature, p.46). The higher the number of charge carriers, (and the higher their mean drift velocity), the higher the current at a given p.d. (as $I = nqvA$, see p.43), and so the lower the resistance and therefore the lower the material's resistivity.
The number density of charge carriers **varies greatly** between different materials (see p.43), which means there can be a **huge variation** in their resistivities.

For an *Ohmic Conductor*, *R* is a *Constant*

A chap called **Ohm** did most of the early work on resistance. He developed a rule to **predict** how the **current** would **change** as the applied **potential difference increased**, for **certain types** of conductor.
The rule is now called **Ohm's law** and the conductors that **obey** it (mostly metals) are called **ohmic conductors**.

Provided the **temperature** is **constant**, the **current** through an ohmic conductor is **directly proportional** to the **potential difference** across it (that's $I \propto V$).

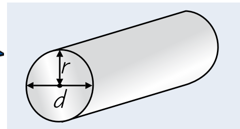
1) As you can see from the graph, **doubling** the **p.d. doubles** the **current**.
2) What this means is that the **resistance** is **constant**.
3) Often **external factors**, such as **temperature** will have a **significant effect** on resistance, so you need to remember that Ohm's law is **only** true for **ohmic conductors** at **constant temperature**.



Resistance and Resistivity

To Find the **Resistivity** of a **Wire** You Need to Find its **Resistance**

Before you start, you need to know the **cross-sectional area** of your test wire.
Assume that the wire is **cylindrical**, and so the cross-section is **circular**.

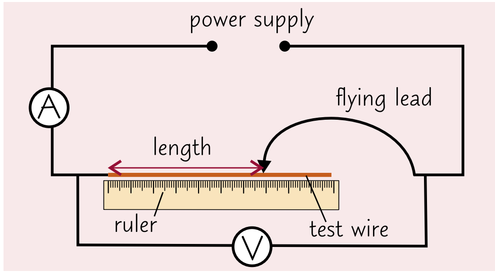


Then you can find its **cross-sectional area** using: $\boxed{\text{area of a circle} = \pi r^2}$

Use a **micrometer** to measure the **diameter** of the test wire for at least **three** different points along the wire. Take an **average** value of the diameter and divide by **2** to get the **radius** (make sure this is in m). Plug it into the equation for cross-sectional area and... **ta da**. Now you can get your teeth into the electricity bit...

A micrometer, sometimes called a micrometer caliper, is used to precisely measure very small distances.

1) The **test wire** should be **clamped** to a ruler and connected to the rest of the circuit at the point where the ruler reads zero.
2) Attach the **flying lead** to the test wire — the lead is just a wire with a crocodile clip at the end to allow connection to any point along the test wire.
3) Record the **length** of the test wire **connected** in the circuit, the **voltmeter reading** and the **ammeter reading**.
4) Use your readings to calculate the **resistance** of the length of wire, using: $\boxed{R = \frac{V}{I}}$
5) Repeat for several **different** lengths within a sensible range, e.g. at 0.10 m intervals from 0.10 m to 1.00 m.
6) Plot your results on a graph of **resistance** against **length**, and draw a **line of best fit** (see page 8).



You could also use a digital multimeter to measure the resistance of the wire directly — you'd connect it in parallel with the length of wire you're investigating and set it to measure resistance.

The **gradient** of the line of best fit is equal to $\frac{R}{l} = \frac{\rho}{A}$. So **multiply** the **gradient** of the line of best fit by the **cross-sectional area** of the wire to find the resistivity of the wire material.

7) The **resistivity** of a material depends on its **temperature**, so you can only find the resistivity of a material **at a certain temperature**. Current flowing in the test wire can cause its temperature to increase, so failing to keep the wire at a **constant temperature** could invalidate your results (see p.12). Try to keep the temperature of the test wire constant by e.g. only having small currents flow through the wire.

Practice Questions

Q1 State the equation that links the resistance of a wire to its resistivity.

Q2 The resistivity of a piece of glass is 1×10^{11} Ωm at 20 °C. The resistivity of aluminium at 20 °C is around 3×10^{-8} Ωm. Explain this difference in terms of charge carriers.

Q3 What is Ohm's law?

Q4 Describe an experiment to find the resistivity of a metal.

Exam Questions

Q1 Aluminium has a resistivity of 2.8×10^{-8} Ω m at 20 °C.
Calculate the resistance of a pure aluminium wire of length 4.0 m and diameter 1.0 mm, at 20 °C. [3 marks]

Q2 The table on the right shows some measurements taken by a student during an experiment investigating an unknown electrical component. The temperature of the circuit is held constant throughout the experiment.

P.d. / V	3.00	7.00	11.00
Current /mA	4.00	9.33	14.67

a) Calculate the resistance of the component when a p.d. of 7.00 V is applied. [1 mark]

b) State whether the component is an ohmic conductor. Explain your answer. [3 marks]

I find the resistivity to my chat-up lines is very high...

Examiners love to ask questions about this experiment, so make sure you learn it well. Make sure you can think of some ways to reduce random errors too — e.g. by repeating measurements and by using more sensitive equipment.

I-V Characteristics

Woohoo — real physics. This stuff's actually kind of interesting.

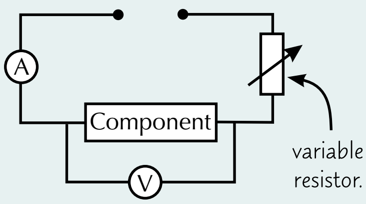
Make sure you learn all the circuit symbols that come up in this section, and know how to design and use circuits using them.

I-V Graphs** Show How **Resistance Varies

The term '***I-V* characteristic**' refers to a **graph** which shows how the **current** (***I***) flowing through a **component changes** as the **potential difference** (***V***) across it is increased.

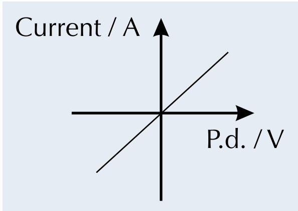
You can investigate the *I-V* characteristic of a component using a **test circuit** like this one:

1) Use the **variable resistor** to alter the **potential difference** across the component and the **current** flowing through it, and record *V* and *I*.
2) **Repeat** your measurements and take **averages** to reduce the effect of random errors on your results.
3) **Plot a graph** of current against potential difference from your results. This graph is the ***I-V* characteristic** of the component.



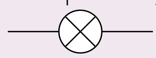
*The **I-V Characteristic** for a **Metallic Conductor** is a **Straight Line***

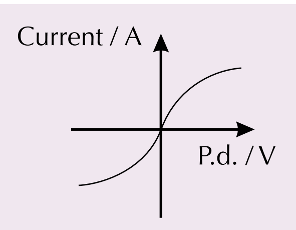
1) At **constant temperature**, the **current** through a **metallic conductor**, e.g. a **wire** or a **resistor**, is **directly proportional** to the **potential difference**.
2) The fact that the characteristic graph is a **straight line through the origin** tells you that the **resistance doesn't change** — it's equal to 1 / gradient.
3) The **shallower** the **gradient** of the characteristic ***I-V*** graph, the **greater** the **resistance** of the conductor.
4) **Metallic conductors** are **ohmic** — they have **constant resistance provided** their temperature doesn't change (see below).



*The **I-V Characteristic** for a **Filament Lamp** is **Curved***

Filament lamp circuit symbol:

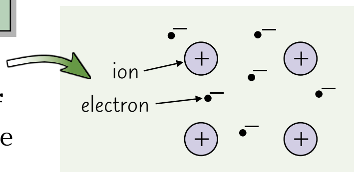




1) The characteristic graph for a **filament lamp** is a **curve**, which starts **steep** but gets **shallower** as the **potential difference rises**.
2) The **filament** in a lamp is just a **coiled up** length of **metal wire**, so you might think it should have the **same characteristic graph** as a **metallic conductor**.
3) However, **current** flowing through the lamp **increases** its **temperature**, so its **resistance increases** (see below).

*The **Resistivity** of a Metal **Increases** with **Temperature***

1) **Charge** is carried through **metals** by **free electrons** in a **lattice** of **positive ions**.
2) Heating up a metal makes it **harder** for electrons to **move about**. The **lattice of ions vibrates more** when heated, meaning the electrons **collide** with them more frequently, **transferring** some of their **kinetic energy** into other forms.
3) When kinetic energy is **lost** by the individual electrons, their speed and therefore the **mean drift velocity** (see page 43) decreases. As current is proportional to drift velocity, ($I = nqvA$) this means the **current** in the wire **decreases** so its **resistance** (and its resistivity, as it's dimensions haven't changed) **increases**.



Semiconductors** are Used in **Sensors

1) **Semiconductors** have a **higher resistivity** than **metals** because there are fewer **charge carriers** available (see page 43).
2) However, if **energy** is supplied to some types of semiconductor (e.g. by increasing their temperature), **more charge carriers** are **released**, so the current increases (as $I = nqvA$) and their resistance and resistivity **decrease**.
3) This means that they can make **excellent sensors** for detecting **changes** in their **environment**. You need to know about **three** semiconductor components — **thermistors**, **LDRs** and **diodes** (see the next page).

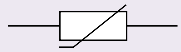
Like metals, increasing the temperature of semiconductors increases their lattice vibrations, reducing the mean drift velocity of electrons. But this effect is dwarfed by the effect of releasing more charge carriers with increasing temperature.

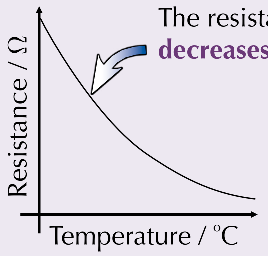
I-V Characteristics

*The **Resistance** of a **Thermistor** Depends on **Temperature***

A **thermistor** is a **resistor** with a **resistance** that depends on its **temperature**. You only need to know about **NTC** thermistors — NTC stands for 'Negative Temperature Coefficient'. This means that the **resistance decreases** as the **temperature goes up**.

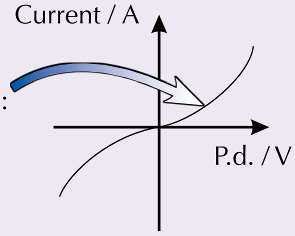
Thermistor circuit symbol:





The resistance of an NTC thermistor **decreases** with **temperature**.

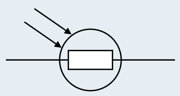
The characteristic ***I-V* graph** for an NTC thermistor looks like this: As the voltage **increases**, the current **increases**. More current leads to an **increase in temperature** and so a **decrease in resistance**. This in turn means more current can flow, so the graph **curves upwards**.



Warming the thermistor gives more **electrons** enough **energy** to **escape** from their atoms. This means that there are **more charge carriers** available, so the **current increases** and the **resistance decreases** ($R = V/I$).

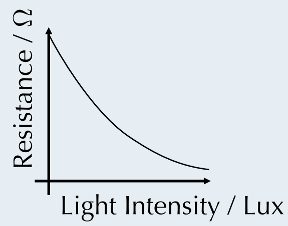
*The **Resistance** of an **LDR** Depends on **Light Intensity***

LDR circuit symbol:

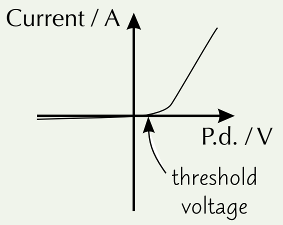


LDR stands for **Light-Dependent Resistor**. The **greater** the intensity of **light** shining on an LDR, the **lower** its **resistance**.

The explanation for this is similar to that for the thermistor. In this case, **light** provides the **energy** that releases more electrons. This means more charge carriers, which means a higher current and a lower resistance.



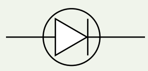
Diodes** Only Let **Current Flow** in **One Direction



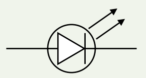
Diodes (including light-emitting diodes (LEDs)) are designed to let **current flow** in **one direction** only. You don't need to be able to explain how they work, just what they do.

1) **Forward bias** is the **direction** in which the **current** is **allowed to flow** — it's the direction the triangle points in the circuit symbols on the right.
2) **Most** diodes require a **threshold voltage** of about **0.6 V** in the **forward direction** before they will conduct.
3) In **reverse bias**, the **resistance** of the diode is **very high** and the current that flows is **very tiny**.

Diode circuit symbol:



LED circuit symbol:



Practice Questions

Q1 If an *I-V* graph is a straight line through the origin, what does this tell you about the resistance?

Q2 Draw an *I-V* characteristic graph for a resistor.

Q3 What is an LDR?

Q4 Draw an *I-V* characteristic graph for a diode. Label the areas of forward bias and reverse bias.

Exam Question

Q1 a) Sketch a characteristic *I-V* graph for a filament lamp. [1 mark]

b) Explain how increasing the voltage across a filament lamp affects its resistance, with reference to the mean drift velocity of electrons in the lamp. [4 marks]

c) Explain how the resistance of an NTC thermistor changes with temperature. [3 marks]

You light up my world like an LED — with One-Directional current...

Make sure you learn all these graphs and can explain them all. It's all about energy — for metals, more energy means more heat and a higher resistance. For semiconductors, more energy means more charge carriers and lower resistance.

Electrical Energy and Power

Power and energy are pretty familiar concepts — and here they are again. Same principles, just different equations.

Power is the Rate of Transfer of Energy

Power (***P***) is **defined** as the **rate** of **doing work**. It's measured in **watts** (**W**), where **1 watt** is equivalent to **1 joule of work done per second**.

in symbols: $$P = \frac{W}{t}$$

In an electrical circuit, W is the work done moving a charge.

There's a really simple formula for **power** in **electrical circuits**:

$$P = VI$$

This makes sense, since:

1) **Potential difference** (***V***) is defined as the **work done** per **coulomb**.
2) **Current** (***I***) is defined as the **number** of **coulombs** transferred per **second**.
3) So **p.d.** × **current** is **work done per second**, i.e. **power**.


Arnold had a pretty high resistance to doing work.

You also know (from the definition of **resistance**) that $V = IR$ (see p.44). **Combining** this with the equation above gives you loads of **different ways** to **calculate power**.

$$P = VI \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

Obviously, which equation you should use depends on what **quantities** you're given in the **question**.

Example: A robotic mutant Santa from the future converts 750 J of electrical energy into heat every second.
a) What is the operating power of the robotic mutant Santa?
b) All of the robotic mutant Santa's components are connected in series, with a total resistance of 30 Ω. What current flows through his wire veins?

a) Power = $W \div t = 750 \div 1 =$ **750 W**

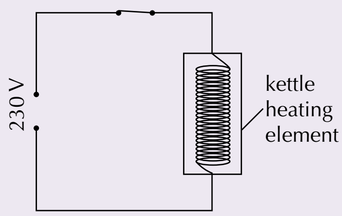
b) $P = I^2R$ so $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{750}{30}} = \sqrt{25} =$ **5.0 A**

Energy is Easy to Calculate if you Know the Power

Sometimes it's the **total energy** transferred that you're interested in. In this case you simply need to **multiply** the **power** by the **time**. So: $$W = VIt$$ (or $W = \frac{V^2}{R}t$ or $W = I^2Rt$)

Make sure that the time is in seconds before you use these equations.

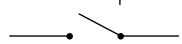
Example: The circuit diagram on the right is part of an electric kettle. A current of 4.0 A flows through the kettle's heating element once it is connected to the mains (230 V).
The kettle takes 4.5 minutes to boil the water it contains. How much energy does the kettle's heating element transfer to the water in the time it takes to boil?



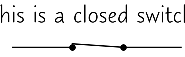
Time the kettle takes to boil in seconds = 4.5 × 60 = 270 seconds.

Use the equation $W = VIt = 230 \times 4.0 \times 270 = 248\,400$ J = **250 kJ (to 2 s.f.)**

Remember, this is the circuit symbol for an open switch:



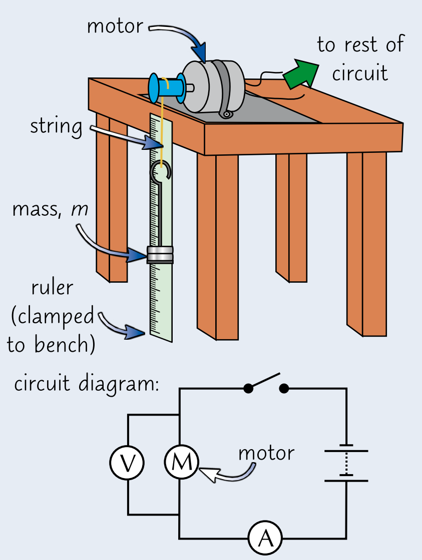
This is a closed switch:



Electrical Energy and Power

You can *Investigate* the *Power Output* of an *Electric Motor*

1) Attach a mass, m, to a motor, as shown on the right, and connect the motor to the **circuit** shown. When you close the switch, the motor will **turn on**, winding the string around the axle and **raising the mass**.
2) Record the **time taken**, t, for the motor to raise the mass a **set distance**, Δh, e.g. 50 cm, using a **stopwatch** and the metre ruler.
3) Calculate the **work done against gravity** by the motor to raise the mass using the equation: $\Delta E_{grav} = mg\Delta h$ (see p.36).
4) You can then calculate the **power output** of the motor, using the equation $P = W/t$ (p.48).
5) You could also investigate the **efficiency** of the motor when lifting a given mass using this equipment. You'd need to measure the **potential difference** across the motor, and the **current** through it, as it lifts the load (see the circuit diagram on the right). You could then calculate the input power using $P = VI$.
6) You could then calculate the efficiency of the motor using the equation: **Efficiency = (useful power output ÷ total power input)** (see page 37). The motor won't be **100% efficient** as some energy will be lost, e.g. as heat to **resistance** in the circuit components.
7) You could also use this apparatus to investigate whether the power input and power output of the motor **varies** with the **load** attached to it by **varying the mass** attached to it.



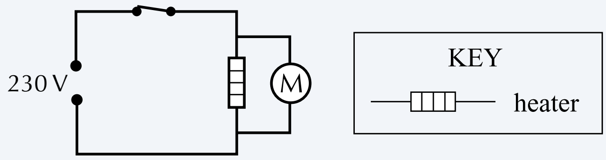
(Voltmeter and ammeter only required for efficiency experiment.)

Practice Questions

Q1 Power is measured in watts. What is 1 watt equivalent to?
Q2 What equation links power, voltage and resistance?
Q3 Write down the equation linking power, current and resistance.
Q4 Describe how you could investigate the power output of an electric motor when raising different loads.

Exam Questions

Q1 The circuit diagram for a mains-powered hairdryer is shown below.



a) The heater has a power of 920 W in normal operation. Calculate the current in the heater. [1 mark]

b) The motor's resistance is 190 Ω. Calculate the current through the motor when the hairdryer is used. [1 mark]

c) Show that the total power of the hairdryer in normal operation is just under 1.2 kW. [2 marks]

Q2 A 12 V car battery supplies a current of 48 A for 2.0 seconds to the car's starter motor.
The total resistance of the connecting wires is 0.010 Ω.

a) Calculate the energy transferred by the battery in this time. [1 mark]

b) Calculate the energy wasted as heat in the wires. [1 mark]

Ultimate cosmic powers...

Another load of equations on these pages to add to your collection, oh joy. Make sure you learn the circuit symbol for a heater in exam question 1 — you won't get a key in the exam (I gave you one because I'm nice).

E.m.f and Internal Resistance

There's resistance everywhere — inside batteries, in all the wires (although it's very small) and in the circuit components themselves. Who said current had it easy?

Batteries have an **Internal Resistance**

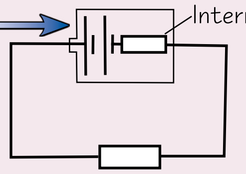
Resistance in metals comes from **electrons colliding** with **atoms** (or ions) and **losing energy** (see p.46).

In a **battery**, **chemical energy** is used to make **electrons move**. As they move, they collide with atoms inside the battery — so batteries **must** have resistance. This is called **internal resistance**.

Internal resistance is what makes **batteries** and **cells warm up** when they're used.

Because the resistance of wires is so small, on these two pages I'm assuming they have zero resistance.

Chemical reactions in the battery produce electrical energy.



In general, *r* is typically less than 1 ohm for a battery. This means a high current can flow.

Load resistance is the total resistance of all the components in the external circuit. You might see it called 'external resistance'.

1) The total amount of **work** the battery does on each **coulomb** of charge is called its **electromotive force** or **e.m.f.** (ε). Be careful — e.m.f. **isn't** actually a force. It's measured in **volts**.

$$W = \varepsilon Q \quad \text{or} \quad \varepsilon = \frac{W}{Q}$$

W is the work done on the charge (i.e. the energy transferred to the charge) in joules.

2) The **potential difference** across the **load resistance** (***R***) is the **work done** when **one coulomb** of charge flows through the **load resistance**. This potential difference is called the **terminal p.d.** (***V***).
3) If there was **no internal resistance**, the **terminal p.d**. would be the **same** as the **e.m.f.** However, in **real** power supplies, there's **always some energy lost** overcoming the internal resistance.
4) The **energy wasted per coulomb** overcoming the internal resistance is called the **lost volts** (***v***).

Conservation of energy tells us:

energy per coulomb supplied by the source = **energy per coulomb used in load resistance** + **energy per coulomb wasted in internal resistance**

There are Loads of **Calculations** with **E.m.f.** and **Internal Resistance**

Examiners can ask you to do **calculations** with **e.m.f.** and **internal resistance** in loads of **different** ways. You've got to be ready for whatever they throw at you.

$$\varepsilon = V + v \qquad \varepsilon = I(R + r) \qquad V = \varepsilon - v \qquad \varepsilon = V + Ir$$

These are all basically the **same equation**, just written differently. If you're given enough information you can calculate the e.m.f. (ε), terminal p.d. (V), lost volts (v), current (I), load resistance (R) or internal resistance (r). Which equation you should use depends on what information you've got, and what you need to calculate.

You Can Work Out the **E.m.f.** of **Multiple** Cells in **Series** or **Parallel**

For cells **in series**, you can calculate the **total e.m.f.** of the cells by **adding** their individual e.m.f.s.

$$\varepsilon_{total} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots$$

This makes sense if you think about it, because each charge goes through each of the cells and so gains e.m.f. from each one.

For identical cells **in parallel**, the **total e.m.f.** of the combination of cells is the **same size** as the e.m.f. of each of the individual cells.

See p.52 for all the rules for parallel and series circuits.

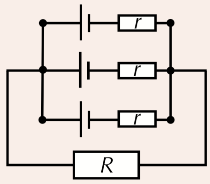
$$\varepsilon_{total} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots$$

This is because the current will split equally between identical cells. The charge only gains e.m.f. from the cells it travels through — so the overall e.m.f. in the circuit doesn't increase.

E.m.f and Internal Resistance

Time for an Example ***E.m.f. Calculation Question...***

Example Three identical cells each with an e.m.f. of 2.0 V and an internal resistance of 0.20 Ω are connected in parallel in the circuit shown to the right. A current of 0.90 A is flowing through the circuit. Calculate the total p.d. across the cells.



First calculate the lost volts, v, for 1 cell using $v = Ir$.

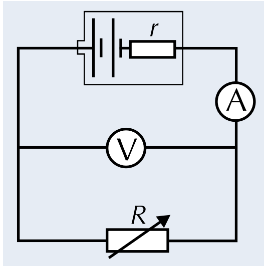
Since the current flowing through the circuit is split equally between each of the three cells, the current through one cell is $I/3$. So for 1 cell: $v = I/3 \times r = 0.90/3 \times 0.20 = 0.30 \times 0.20 = 0.06$ V

Then find the terminal p.d. across 1 cell using the equation: $V = \varepsilon - v = 2 - 0.06 = 1.94$

So the total p.d. across the cells combined = **1.9 V (to 2 s.f.)**

Investigate ***Internal Resistance*** *and* ***E.m.f.*** *With This* ***Circuit***

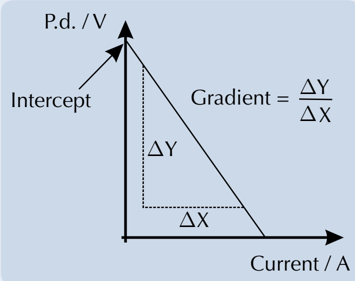
1) **Vary** the **current** in the circuit by changing the value of the **load resistance** (***R***) using the variable resistor. **Measure** the **p.d.** (***V***) for several different values of **current** (***I***).
2) Record your data for V and I in a table, and **plot the results** in a graph of V against I.



To find the **e.m.f.** and **internal resistance** of the cell, start with the equation: $\boxed{\varepsilon = V + Ir}$

1) Rearrange to give $V = -rI + \varepsilon$
2) Since ε and r are constants, that's just the equation of a **straight line**:

Equation of a straight line
$y = mx + c$
gradient y-intercept



3) So the intercept on the vertical axis is ε.
4) And the gradient is $-r$.

You probably don't need to take repeated readings in an experiment like this one, but make sure you get plenty of data points to draw your line.

An **easier** way to **measure** the **e.m.f.** of a **power source** to just connect a **voltmeter** across its **terminals**. Voltmeters have a very **high resistance**, but a **small current** will still flow through them. This means there must be some **lost volts**, which means you measure a value **very slightly less** than the **e.m.f.** (Although in practice the difference isn't usually significant.)

Practice Questions

Q1 What causes internal resistance? Write down the equation linking work, e.m.f. and charge.

Q2 What is the difference between e.m.f. and terminal p.d.?

Q3 What is meant by 'lost volts'?

Q4 Give an example of a source of e.m.f and describe an experiment to find the value of its e.m.f.

Exam Questions

Q1 A battery with an internal resistance of 0.8 Ω and an e.m.f. of 24 V powers a dentist's drill with resistance 4.0 Ω.

a) Calculate the current in the circuit when the drill is connected to the power supply. [1 mark]

b) Calculate the voltage across the drill while it is being used. [1 mark]

Q2 A bulb of resistance R is powered by two cells connected in series, each with internal resistance r and e.m.f. ε. Which expression represents the current flowing through each cell? [1 mark]

A $\frac{\varepsilon}{R + r}$ **B** $\frac{\varepsilon}{2(R + 2r)}$ **C** $\frac{2\varepsilon}{R + 2r}$ **D** $\frac{\varepsilon}{R + 2r}$

Why'd the physicist swallow a multimeter? To find his internal resistance...

Thank you, thank you, I'm here all week. A jam-packed pair of pages here, but it's all stuff you need to know. Make sure you know the difference between terminal p.d. and e.m.f., and that you've got a handle on all those equations.

Conservation of Charge & Energy in Circuits

There are some things in Physics that are so fundamental that you just have to accept them. Like the fact that there's loads of Maths in it. And that energy is conserved. And that Physicists get more homework than everyone else.

Charge Doesn't 'Leak Away' *Anywhere — it's* ***Conserved***

1) As **charge flows** through a circuit, it **doesn't** get **used up** or **lost**.
2) This means that whatever charge flows **into** a junction will flow **out** again.
3) Since **current** is **rate of flow of charge**, it follows that whatever current flows **into** a junction is the same as the current flowing **out** of it.

Example: 6 coulombs of charge flow into a junction in 1 second, and split in the ratio 1:2.

$Q_1 = 6\text{ C} \Rightarrow I_1 = 6\text{ A}$ → at junction, current branches in two → $Q_2 = 2\text{ C} \Rightarrow I_2 = 2\text{ A}$ and $Q_3 = 4\text{ C} \Rightarrow I_3 = 4\text{ A}$

$I_1 = I_2 + I_3$

Kirchhoff's first law says:

> The total **current entering a junction** = the total **current leaving it.**

You'll probably see these laws called "conservation laws" rather than "Kirchhoff's laws" in the exam.

Energy *is* ***Conserved*** *too*

1) **Energy is conserved**. You already know that. In **electrical circuits**, **energy** is **transferred round** the circuit. Energy **transferred to** a unit charge is **e.m.f.**, and energy **transferred from** a unit charge is **potential difference**.
2) In a **closed loop**, these two quantities must be **equal** if energy is conserved (which it is).

Kirchhoff's second law says:

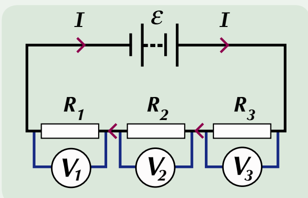
> The **total e.m.f.** around a **series circuit** = the **sum** of the **p.d.**s across each component.

(or $\varepsilon = \Sigma IR$ in symbols)

The ***Conservation Laws*** *Apply to* ***Different Combinations*** *of Resistors*

A **typical exam question** will give you a **circuit** with bits of information missing, leaving you to fill in the gaps. Not the most fun... but on the plus side you get to ignore any internal resistance stuff (unless the question tells you otherwise)... hurrah. You need to remember the **following rules**:

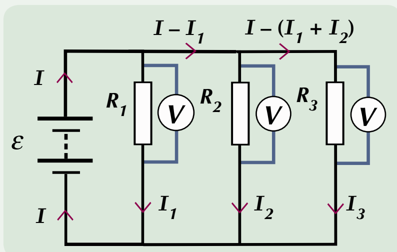
Series Circuits:



1) **same current** at **all points** of the circuit (since there are no junctions)
2) **e.m.f. split** between **components** (by Kirchhoff's 2nd law), so: $\varepsilon = V_1 + V_2 + V_3$
3) $V = IR$, so if I is constant: $IR_{total} = IR_1 + IR_2 + IR_3$
4) cancelling the Is gives:

$$\boxed{R_{total} = R_1 + R_2 + R_3}$$

Parallel Circuits:



1) **current** is **split** at each **junction**, so: $I = I_1 + I_2 + I_3$
2) **same p.d.** across **all components** (remember that within a loop the e.m.f. equals the sum of individual p.d.s)
3) so, $V/R_{total} = V/R_1 + V/R_2 + V/R_3$
4) cancelling the Vs gives:

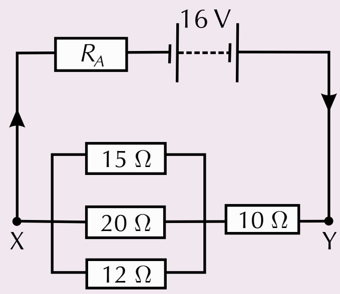
$$\boxed{\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

...and there's an example on the next page to make sure you know what to do with all that...

Conservation of Charge & Energy in Circuits

Worked Exam Question

Example:



A battery with an e.m.f. of 16 V and negligible internal resistance is connected in a circuit as shown on the right. The resistances are all correct to 2 s.f..

a) Show that the group of resistors between X and Y could be replaced by a single resistor of resistance 15 Ω.

You can find the combined resistance of the 15 Ω, 20 Ω and 12 Ω resistors using:

$1/R = 1/R_1 + 1/R_2 + 1/R_3 = 1/15 + 1/20 + 1/12 = 1/5 \quad \Rightarrow R = 5\ \Omega$

So overall resistance between X and Y can be found by: $R = R_1 + R_2 = 5 + 10 =$ **15 Ω**

b) If $R_A = 20\ \Omega$ (to 2 s.f.):

i) calculate the potential difference across R_A,

Careful — there are a few steps here. You need the p.d. across R_A, but you don't know the current through it. So start there: total resistance in circuit = 20 + 15 = 35 Ω,
so current through R_A can be found using $I = V_{total}/R_{total} = 16/35$ A
then you can use $V = IR_A$ to find the p.d. across R_A: $V = 16/35 \times 20 =$ **9.1 V (to 2 s.f.)**

ii) calculate the current in the 15 Ω resistor.

You know the current flowing into the group of three resistors and out of it, but not through the individual branches. But you know that their combined resistance is 5 Ω from part a), so you can work out the p.d. across the group:

$V = IR = 16/35 \times 5 = 16/7$ V

The p.d. across the whole group is the same as the p.d. across each individual resistor, so you can use this to find the current through the 15 Ω resistor:

$I = V/R = (16/7) / 15 =$ **0.15 A (to 2 s.f.)**

Practice Questions

Q1 State Kirchhoff's first and second laws and state what quantity is conserved in each case.

Q2 Show how conservation of charge leads to the rule for combining resistors in series: $R_{total} = R_1 + R_2 + R_3$.

Q3 Show how conservation of energy leads to the rule for combining resistors in parallel: $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

Q4 Find the current through and potential difference across each of two 5 Ω resistors when they are placed in a circuit containing a 5 V battery, and are wired: a) in series, b) in parallel.

Exam Question

Q1 For the circuit on the right:

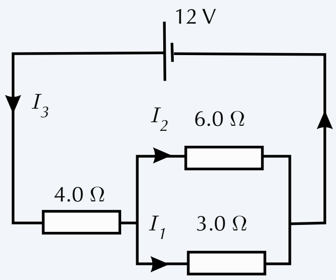
a) Calculate the total effective resistance of the three resistors in this combination. [2 marks]

b) Calculate the main current, I_3. [1 mark]

c) Calculate the potential difference across the 4.0 Ω resistor. [1 mark]

d) Calculate the potential difference across the parallel pair of resistors. [1 mark]

e) Calculate the currents I_1 and I_2. [2 marks]



Conservation of energy is really important — time for a nap I think...

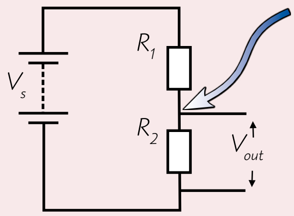
$V = IR$ is the formula you'll use most often in these questions. Make sure you know whether you're using it on the overall circuit, or just one specific component. It's amazingly easy to get muddled up — you've been warned.

The Potential Divider

I remember the days when potential dividers were pretty much the hardest thing they could throw at you. Then along came A level Physics. Hey ho. Anyway, in context this doesn't seem too hard now, so get stuck in.

Use a **Potential Divider** to get a **Fraction** of a **Source Voltage**

1) At its simplest, a **potential divider** is a circuit with a **voltage source** and a couple of **resistors** in series.
2) The **potential difference** across the voltage source (e.g. a battery) is **split** in the **ratio** of the **resistances** (p.52).
3) So, if you had a **2 Ω** resistor and a **3 Ω** resistor, you'd get **2/5** of the p.d. across the **2 Ω** resistor and **3/5** across the **3 Ω**.
4) You can use potential dividers to supply a potential difference, V_{out}, between **zero** and the potential difference across the voltage source. This can be useful, e.g. if you need a **varying** p.d. supply or one that is at a **lower p.d.** than the voltage source.



The voltage has **dropped** by V_1 (the voltage across R_1) by the time it gets to here. The **remaining voltage** that can be supplied, e.g. to another component, is V_{out}.

In the circuit shown, R_2 has $\frac{R_2}{R_1 + R_2}$ of the total resistance. So: $\boxed{V_{out} = \frac{R_2}{R_1 + R_2} V_s}$

E.g. if V_s = 9 V and you want V_{out} to be 6 V, then you need:

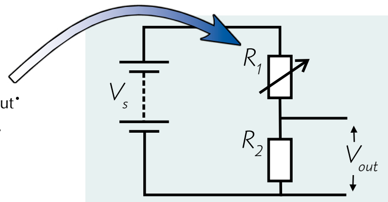
$$\boxed{\frac{R_2}{R_1 + R_2} = \frac{2}{3} \text{ which gives } R_2 = 2R_1}$$

So you could have, say, $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$

5) This circuit is mainly used for **calibrating voltmeters**, which have a **very high resistance**.
6) If you put something with a **relatively low resistance** across R_2 though, you start to run into **problems**. You've **effectively** got **two resistors** in **parallel**, which will **always** have a **total** resistance **less** than R_2. That means that V_{out} will be **less** than you've calculated, and will depend on what's connected across R_2. Hrrumph.

Use a **Variable Resistor** to Vary the **Voltage**

If you replace R_1 with a **variable resistor**, you can change V_{out}. When $R_1 = 0$, $V_{out} = V_s$. As you increase R_1, V_{out} gets smaller.

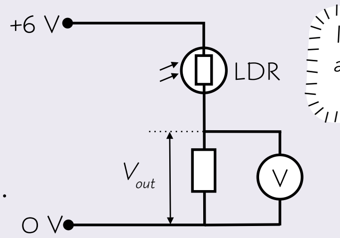


Add an **LDR** or **Thermistor** for a **Light** or **Temperature Sensor**

1) A **light-dependent resistor** (LDR) has a very **high resistance** in the **dark**, but a **lower resistance** in the **light**.
2) An **NTC thermistor** has a **high resistance** at **low temperatures**, but a much **lower resistance** at **high temperatures** (it varies in the opposite way to a normal resistor, only much more so).
3) Either of these can be used as one of the **resistors** in a **potential divider**, giving an **output voltage** that **varies** with the **light level** or **temperature**.

See page 47 for why the resistances of LDRs and NTC thermistors change like this.

The diagram shows a **sensor** used to detect **light levels**.
When light shines on the LDR its **resistance decreases**, so V_{out} increases.
You can include LDRs and thermistors in circuits that control **switches**, e.g. to turn on a light or a heating system.

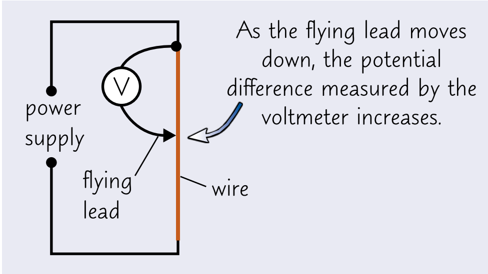


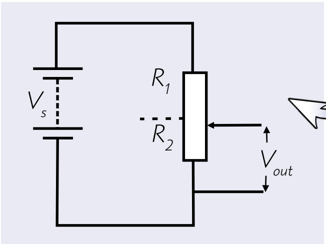
If you replace the LDR with an NTC thermistor, V_{out} will increase with temperature.

The Potential Divider

A **Potentiometer** uses a **Variable Resistor** to give a **Variable Voltage**

1) Imagine you have a long length of wire connected to a power supply. If the wire is **uniform** (i.e. same cross-sectional area and material throughout), then its **resistance** is **proportional** to its **length**.
2) This means that if you were to connect a voltmeter across different lengths of the wire, the **potential difference** you'd record would be **proportional** to the **length** you'd connected it over — you're measuring across a bigger share of the total resistance so you get a bigger potential difference.

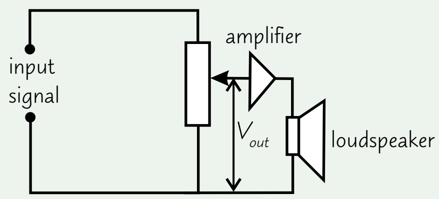




3) This is the basis of how a **potentiometer** works. A **potentiometer** is basically a potential divider, with a variable resistor replacing $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ (it's even sometimes **called** a potential divider just to confuse things).
4) You move a **slider** or turn a knob to **adjust** the **relative sizes** of $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$. That way you can vary $\boldsymbol{V_{out}}$ from **0 V** up to the source voltage.
5) This is dead handy when you want to be able to **change** a **voltage continuously**, like in the **volume control** of a stereo:

I've often wished bagpipes had a volume control. Or just an off switch.

Example:
Here, $\boldsymbol{V_s}$ is replaced by the input signal (e.g. from a CD player) and $\boldsymbol{V_{out}}$ is the output to the amplifier and loudspeaker.



Practice Questions

Q1 Look at the light sensor circuit on page 54.
How could you change the circuit so that it could be used to detect temperature changes?

Q2 The LDR in the circuit on page 54 has a resistance of 300 Ω when in light conditions, and 900 Ω in dark conditions. The fixed resistor has a value of 100 Ω. Show that V_{out} (light) = 1.5 V and V_{out} (dark) = 0.6 V.

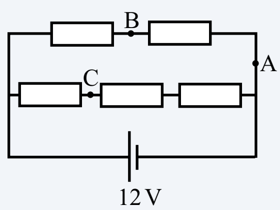
Exam Questions

Q1 In the circuit on the right, all the resistors have the same value. Calculate the p.d. between:

a) A and B. [1 mark]

b) A and C. [1 mark]

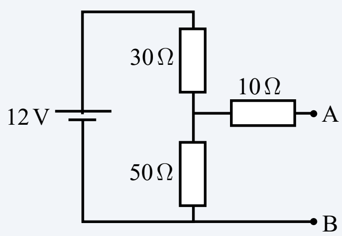
c) B and C. [1 mark]



Q2 Look at the circuit on the right. All the resistances are given to 2 significant figures.

a) Calculate the p.d. between A and B as shown by a high resistance voltmeter placed between the two points. [1 mark]

b) A 40.0 Ω resistor is now placed between points A and B. Calculate the p.d. across AB and the current flowing through the 40.0 Ω resistor. [4 marks]



OI...YOU... [bang bang bang]... turn that potentiometer down...

You'll probably have to use a potentiometer in every experiment you do with electricity from now on in, so you'd better get used to them. I can't stand the things myself, but then lab and me don't mix — far too technical.