OPTICS (EP204)
PROJECT REPORT

THE SPOT IN THE DARK

POISSON'S SPOT

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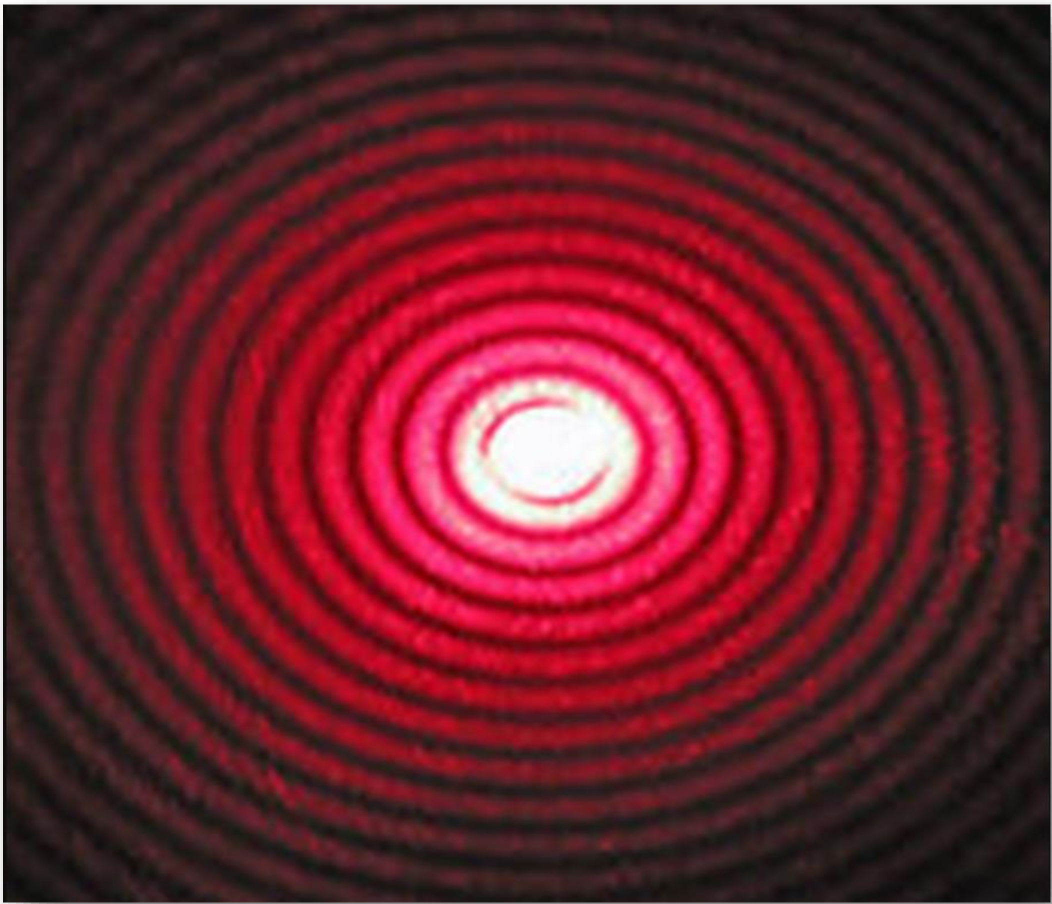

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CONTENTS

I. PREFACE

ABSTRACT~ The Fresnel diffraction phenomenon referred to as Poisson's spot or spot of Arago has, beside its historical significance, become relevant in a number of fields. Among them are for example fundamental tests of the super-position principle in the transition from quantum to classical physics and the search for extra-solar planets using star shades. Poisson's spot refers to the positive on-axis wave interference in the shadow of any spherical or circular obstacle. While the spot's intensity is equal to the undisturbed field in the plane wave picture, its intensity in general depends on a number of factors, namely the size and wavelength of the source, the size and surface corrugation of the diffraction obstacle, and the distances between source, obstacle and detector. The intensity can be calculated by solving the Fresnel–Kirchhoff diffraction integral numerically, which however tends to be computationally expensive. We have therefore devised an analytical model for the on-axis intensity of Poisson's spot relative to the intensity of the undisturbed wave field and successfully validated it both using a simple light diffraction setup and numerical methods. The model will be useful for optimizing future Poisson-spot matter-wave diffraction experiments and determining under what experimental conditions the spot can be observed.

POINT SPREAD FUCNTION~ The point spread function (PSF) describes the response of an imaging system to a point source or point object. The point spread function is based on an infinitely small **point source** of light originating in the specimen (object) space. Because the microscope imaging system collects only a fraction of the light emitted by this point, it cannot focus the light into a perfect three-dimensional image of the point. Instead, the point appears widened and spread into a three-dimensional diffraction pattern. Thus, the point spread function is formally defined as the three-dimensional diffraction pattern generated by an ideal point source of light. The PSF in many contexts can be thought of as the extended blob in an image that represents a single point object. It is a useful concept in Fourier optics, astronomical imaging, medical imaging, electron microscopy and other imaging techniques such as 3D microscopy (like in confocal laser scanning microscopy) and fluorescence microscopy.

CONVULSION~ In mathematics (in particular, functional analysis), convolution is a mathematical operation on two functions (f and g) that produces a third function (f*g) that expresses how the shape of one is modified by the other. The term convolution refers to both the result function and to the process of computing it. It is defined as the integral of the product of the two functions after one is reversed and shifted. Convolution has applications that include probability statistics, acoustics, spectroscopy, signal, engineering, physics, computer vision and differential equations

HUYGENS' PRINCIPLE~ The Huygens–Fresnel principle (named after Dutch physicist Christian Huygyen and French physicist Augustin Jean Fresnel) is a method of analysis applied to problems of wave propogation both in the far field limit and in near-field diffraction and also reflection. It states that every point on a wavefront is itself the source of spherical wavelets, and the secondary wavelets emanating from different points mutually interfere.[1] The sum of these spherical wavelets forms the wavefront.

II - DISCOVERY OF ARGO'S SPOT

BACKGROUND THEORY

❖ CORPUSCULAR THEORY OF LIGHT

The corpuscular theory states that light is made up of tiny particles called 'corpuscles' (little particles) that always travel in a straight line. Light is a form of energy that travels from one place to another place at high velocity. Various scientists have attempted to explain the nature of light. However, the first scientific attempt to explain the nature of light was made by Sir Isaac Newton.

In the 17^{th} century, two different theories about the nature of light were proposed; these theories were the 'wave theory' and 'corpuscular theory'.

The 'Corpuscular theory of light was proposed by Newton in 1704. In this theory, he successfully explained the nature of light.

The corpuscular theory is the simplest theory of light in which light is assumed as the tiny particles called 'corpuscles'. The corpuscular theory is often referred to as particle theory or Newton's theory of light

➢ Newton's Corpuscular Theory Statement

According to the Newton's corpuscular theory:

- Light is made up of tiny particles called 'corpuscles' having negligible mass.
- These particles (corpuscles) are perfectly elastic.
- The corpuscles are emitted from the luminous sources such as Sun, candle, electric lamp etc.
- The tiny particles (corpuscles) always travel in a straight line in all directions.
- Each particle (corpuscle) carries kinetic energy with it while moving.
- The corpuscles travel at high velocity.
- The corpuscles (light) would travel faster in the denser medium than in rarer medium. But later this is proved wrong. We know that light travels faster in the rarer medium than in denser medium.
- When the particles (corpuscles) fall on the retina of the eye, they produce an image of the object or sensation of vision.
- The corpuscles can be of different sizes. The different colors of light are due to the different sizes of the corpuscles.

➢ Corpuscular theory about reflection of light

The corpuscular theory explains the reflection of light in exactly the same way as the reflection of a perfectly elastic ball from a rigid plane.

When the corpuscles (particles) hit the reflecting surface, they are reflected from it in such a way that the angle of incidence is equal to the angle of reflection. This is due to the repulsion between the corpuscles and the reflecting surface.

➢ Corpuscular theory about refraction of light

According to Newton, when corpuscles (light particles) approach the refracting surface, they are attracted near the surface. When they enter the denser medium from a rarer medium, their speed increases and hence change their direction.

❖ YOUNG’S DOUBLE SLIT EXPERIMENT

Young’s Double-Slit Experiment is one of the most successful demonstrations that light and matter can display characteristics of both waves and particles. The scientific inquiry into wave characteristics of light began in the 17th - 18th Century. In 1803 Thomas Young described his famous Double-Slit Experiment which is known as Young’s Double-Slit Experiment. The Double Slit Experiment was first conducted by Thomas Young back in 1803, although Sir Isaac Newton is said to have performed a similar experiment in his own time. Newton shone a light on hair but Thomas Young did it on the slit.

The Double-Slit Experiment uses two coherent sources of light placed at a small distance apart. There is a screen at some distance. As the light sources turn on the interference pattern appears on the screen. The Original Double-Slit Experiment was used two-slit and one source of light. The light source was placed behind the slit. As light passes through the slit, both of them behave like point sources.

At the start of the 19th century London ophthalmologist and physicist Thomas Young was obsessed with light rays. For years he had been working to unravel the mysteries of light. Since antiquity, two contradictory theories have been argued, whether light is a wave or the sum of tiny particles.

Isaac Newton defined light as corpuscular, that is, made up of tiny particles. But Young dared to contradict him. For Young, light had too many characteristics of a classic wave form, like diffraction and refraction. These two basic physical phenomena could not be explained by Newton's theory.

Young chose the difficult path of trying to disprove Newton's theory of light. In 1802 his investigations lead him to devise an experiment known as the double-slit experiment, which has become part of scientific history. Using a mirror Young directed a beam of light from a narrow slit in a windowpane of his lab onto a simple apparatus. The experiment could only work if light exists as waves.

The window slit allows just enough light to enter that it remains constant enough for the experiment. A card just 20 millimeters wide with two slits divides the incoming light beam into two overlapping beams of light. This results in a pattern that Young knows well, an interference pattern that only waves can produce. The waves of the light ray meet a barrier. Part of the wave front is blocked, the rest is allowed through. Diffraction occurs because the waves are steered around the barrier, creating twin light sources whose rays, when they overlap, alternately add and subtract from each other, behavior only possible of a wave. Young was familiar with this effect, but had never seen it happen with light. It was a discovery that confirmed his notions of the nature of light. But it took a great deal of courage for Young to publish his findings. For a humble ophtrician to contradict the theories of the great Isaac Newton.
The result of the experiment is shocking. In the screen, the light was creating bright and dark bands (interference pattern)

Now the new law stated that light could consist of particles but, particles or not, it does behave as though it were a wave. The concept of wave-particle duality was born - a wavicle. Light is both wave and particle, or particles that travel through space as a wave.

Before knowing the reason, we need to learn what is an interference pattern. It is a phenomenon When two waves superpose to form a resultant wave of greater, lower, or the same amplitude. Using this phenomenon we can easily identify the characteristics of the suspect.
So in this case The wave nature of light causes the light waves passing through the two slits to interfere, producing bright and dark bands on the screen.

❖ DIFFRACTION

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. It is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings. Similar effects are observed when light waves travel through a medium with a varying refractive index or a sound wave through one with varying acoustic impedance. Diffraction occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, x-rays and radio waves. As physical objects have wave-like properties (at the atomic level), diffraction also occurs with matter and can be studied according to the principles of quantum mechanics.

Diffraction was first observed by Francesco Grimaldi in 1665. He noticed that light waves spread out when made to pass through a slit. Later it was observed that diffraction not only occurs in small slits or holes but in every case where light waves bend round a corner.

One of the most common examples of diffraction in nature is the tiny specks or hair-like transparent structures, known as "floaters" that we can see when we look up at the sky. This illusion is produced within the eye-ball, when light passes through tiny bits in the vitreous humour. They are more prominently observed when one half-closes his eyes and peeps through them.

The phenomenon of diffraction can be readily explained using Huygens' principle: When the wavefront of a light ray is partially obstructed, only those wavelets which belong to the exposed parts superpose, in such a way that the resulting wavefront has a different shape. This permits bending of light around the edges. Colourful fringe patterns are observed on a screen due to diffraction.

In the early 1800s, most of the people who wrote and submitted papers on diffraction of light were believers of the wave-theory of light. However, their views contradicted those of Newton's supporters' and their would be regular discussions between these two sides. One such person, who believed in the wave theory was Augustin Fresnel, who in 1819, handed a paper to the French Academy of Sciences, about the phenomenon of diffraction. However, the Academy mainly consisting of Newton's supporters, tried to challenge Fresnel's point of view by saying that if light was indeed a wave, these waves, which were diffracted from the edges of a sphere, would cause a bright area to occur within the shadow of the sphere. This was indeed oberved later, and the area is today known as the Fresnel Bright Spot.

Diffraction is the slight bending of light as it passes around the edge of an object. The amount of bending depends on the relative size of the wavelength of light to the size of the opening. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. However, if the two are closer in size or equal, the amount of bending is considerable, and easily seen with the naked eye.

Diffracted light can produce fringes of light, dark or colored bands. An optical effect that results from the diffraction of light is the silver lining sometimes found around the edges of clouds or coronas surrounding the sun or moon.

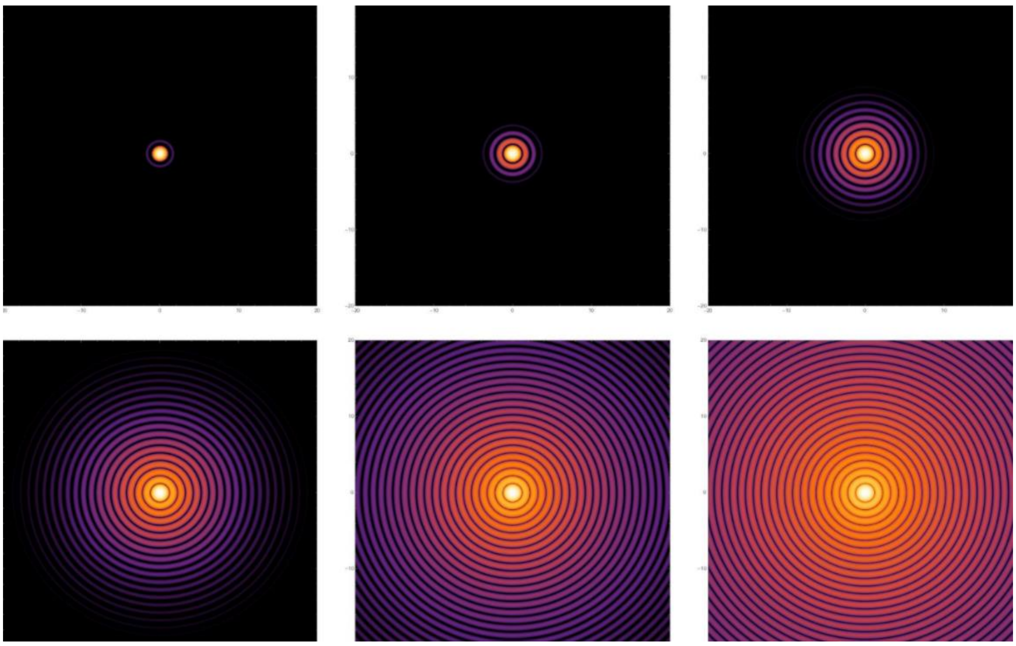
Optical effects resulting from diffraction are produced through the interference of light waves. To visualize this, imagine light waves as water waves. If water waves were incident upon a float residing on the water surface, the float would bounce up and down in response to the incident waves, producing waves of its own. As these waves spread outward in all directions from the float, they interact with other water waves. If the crests of two waves combine, an amplified wave is produced (constructive interference). However, if a crest of one wave and a trough of another wave combine, they cancel each other out to produce no vertical displacement (destructive interference).

This concept also applies to light waves. When sunlight (or moonlight) encounters a cloud droplet, light waves are altered and interact with one another in a similar manner as the water waves described above. If there is constructive interference, (the crests of two light waves combining), the light will appear brighter. If there is destructive interference, (the trough of one light wave meeting the crest of another), the light will either appear darker or disappear entirely.

In optics, Fresnel diffraction or near-field diffraction is a process of diffraction which occurs when a wave passes through an aperture and diffracts in the near field, causing any diffraction pattern observed to differ in size and shape, relative to the distance. It occurs due to the short distance in which the diffracted waves propagate, which results in a fresnel number greater than 1. When the distance is increased, outgoing diffracted waves become planar and Fraunhofer diffraction occurs. The multiple Fresnel diffraction at nearly placed periodical ridges (ridged mirror) causes the specular reflection; this effect can be used for atomic mirrors.

Fresnel diffraction refers to the general case where those restrictions are relaxed. This makes it much more complex mathematically. Some cases can be treated in a reasonable empirical and graphical manner to explain some observed phenomena.

In optics, Fresnel diffraction or near-field diffraction is a process of diffraction that occurs when a wave passes through an aperture and diffracts in the near field, causing any diffraction pattern observed to differ in size and shape, depending on the distance between the aperture and the projection. It occurs due to the short distance in which the diffracted waves propagate, which results in a Fresnel number greater than 1. When the distance is increased, outgoing diffracted waves become planar and Fraunhofer diffraction occurs. Fresnel diffraction showing centre black spot. The multiple Fresnel diffraction at nearly placed periodical ridges (ridged mirror) causes the specular reflection; this effect can be used for atomic mirrors. Diffraction due to circular aperture.



III - FUNDAMENTALS OF POSSON'S SPOT

This section is structured into four main sections. In the first one we present the analytical model as well as its derivation and limitations. In the second we describe the numerical simulation used to calculate the diffraction images in the detection plane and how we derived from them the relative intensity of Poisson's spot. In the third section, the light diffraction experiment, which we used to further validate the analytical model is reported on. Finally, the results from the analytical model, the numerical simulation. All is followed by a discussion and a conclusion.

❖ INTRODUCTION

One of the most tell-tale properties of waves in general is diffraction: the deviation from rectilinear propagation in the presence of obstacles due to interference. In the history of science the phenomenon has helped to reveal the wave-character of light and material particles such as electrons[3, 4], helium atoms and hydrogen molecules[5], and more recently C60 molecules as well as large bio-molecules exceeding 10 000 amu in mass. These latest diffraction experiments have sparked renewed interest in the particle-wave duality and the role of quantum decoherence in the quantum-to-macroscopic world transition.

Diffraction in the Fresnel-regime, as the Talbot–Lau interferometers used in belong to, is particularly useful in the determination of wave-nature, due to the possibility to observe diffraction from obstacles that are much larger than the wavelength of the incident wave. A most prominent effect in the Fresnel diffraction regime is Poisson's spot, also sometimes referred to as spot of Arago. It refers to the bright interference spot that can be observed in the shadow of an object with a circular rim such as a circular disc or sphere (for brevity we call it a disc from here). Its prediction by SD Poisson and subsequent surprising observation by DFJ Arago established the wave-nature of light at the beginning of the 19th century. The intensity of Poisson's spot as a function of experimental parameters is the subject of this article.

For an ideal point source at infinity (plane wave) the on-axis intensity of Poisson's spot is equal to that of the undisturbed wave front(at equal distance from the source). This is referred to as unit relative intensity $I_{rel} = 1$in this article.

Closer to the obstacle the ideal relative intensity is

$$I_{\mathrm{rel}} = \frac{b^2}{b^2 + R^2}$$

where R is the radius of the disc and b is the distance between disc and detector. In practice the intensity of Poisson's spot is affected by a number of experimental factors. These include beside the distances g and b, the source size, the diameter of the disc, any additional blocking due to support structure and edge corrugation of the disc. Why the last of these parameters influences the spot intensity can be best understood using the Fresnel zone concept. The spot intensity results from the annular Fresnel zone adjacent to the rim of the disc. The radial phase profile of the wave passing through this zone depends on the radius of the rim. A variation of the radius of the order of the width of the adjacent Fresnel zone thus results in destructive interference.

❖ ANALYTICAL MODEL

In this section we derive an analytical formula valid in the Fresnel regime for the relative peak intensity of Poisson's spot in the presence of finite spatial coherence and imperfections of the diffraction obstacle. The starting point for the analytical model is the lateral intensity distribution of Poisson's spot from a source of plane waves (point source at infinite distance) in the Fresnel regime, which can be expressed in terms of a zero-order Bessel function of the first kind J_0.

$$I_{\mathrm{pw}}(r) = \frac{b^2}{b^2 + R^2} J_0^2\left(\frac{r}{w_p/2}\right) = \frac{b^2}{b^2 + R^2} J_0^2\left(r\frac{2R\pi}{\lambda\, b}\right) \approx J_0^2\left(r\frac{2R\pi}{\lambda\, b}\right). \tag{1}$$

Here,

$r = \sqrt{x^2 + y^2}$ is the radial lateral coordinate in the detection plane.

It is assumed to be small. w_p is the full-width at half-maximum (FWHM) of the plane-wave Poisson spot. The latter can be expressed in terms of the radius of the disc R, the wavelength λ at which the source emits, and the length b, which is the distance from diffraction obstacle to detection plane as mentioned before. The Fresnel approximation following.

- $\lambda \ll R$
- $g, b \gg R$

The fraction in front of the Bessel function is thus very close to

$$\frac{b^2}{b^2 + R^2} \approx 1$$

and we will neglect it therefore from here on. Note that we assumed incoming plane waves in this first step and thus the source distance g to be infinite.

The influence of the partial spatial coherence of the source, i.e. the source width, results in a much more significant variation of the relative on-axis intensity. The derivation of an analytical model that reflects this variation and thus gives the correct dependence on g (g larger than a few disc radii), is presented in the following paragraphs. In brief, we first note that off-axis source points result in 2d point-spread function images equal to the plane wave Poisson spot given in equation, but with an offset from the optical axis.

The extended-source Poisson spot corresponds to an incoherent super-position of these point-spread function images, which we express using a 2d convolution integral. For on-axis points the integral has an exact solution. First the lateral offset of the off-axis point-source Poisson-spot images in the detection plane can be explained as follows. The point-source Poisson spot images will be at the exact centre of the shadow cast by each off-axis point-source. It is therefore located at the intersection point of the detection plane and the line going through source point and disc centre, giving the magnification factor b/g. The extended-source Poisson spot therefore results in a magnified or de-magnified image of the source depending on the value of this factor.

The lateral intensity distribution $I_{xs}(r)$ of Poisson's spot from a source of diameter W_s is then given by a convolution of $I_{pw}(r)$with the function $I_i(r)$ defined below. The latter represents the ideal image of the source that would be formed by a delta-function as point-spread-function. We assume that the source emits evenly from a circular area of diameter ws and take into account that the self-image formed in the detection plane is magnified by the factor b/g and would be also the case for a thin lens in geometrical optics:

$$I_i(r) = \begin{cases} \frac{1}{\pi (w_i/2)^2} & \text{if } |r| < w_i/2, \\ 0 & \text{elsewhere,} \end{cases}$$

where W_i= W_s b/g is the FWHM of the ideal source image. The Poisson spot $I_{xs}(r)$ from a spatially incoherent source is thus characterized by the following convolution integral in polar coordinates:

$$I_{xs}(r) = I_{pw}(r) * I_i(r) = \int_0^{2\pi} \int_0^{\infty} J_0^2\left(\frac{r-t}{w_p/2}\right) I_i(t)\, t\, \mathrm{d}\phi \mathrm{d}t.$$

We are interested in the peak intensity atr = 0 and can therefore evaluate the integral as follows:

$$\begin{aligned} I_{\mathrm{rel},xs} = I_{xs}(0) &= \int_0^{2\pi} \mathrm{d}\phi \int_0^{w_i/2} J_0^2\left(-\frac{2t}{w_p}\right) \frac{4}{\pi w_i^2}\, t\, \mathrm{d}t \\ &= J_0^2\left(\frac{w_i}{w_p}\right) + J_1^2\left(\frac{w_i}{w_p}\right) \\ &= J_0^2\left(\frac{w_s R\pi}{g\lambda}\right) + J_1^2\left(\frac{w_s R\pi}{g\lambda}\right). \end{aligned}$$

This surprisingly simple equation forms the centre piece of the analytical model and shows that $I_{rel,xs}$ is independent of b for an extended uniform source and ideal spherical obstacle.

The Bessel-functions in equation, can be evaluated directly in most mathematics software packages to arbitrary precision, but for completeness it's been realised experimentally, the following asymptotic form for large ratios w_i/w_p.

$$I_{\mathrm{rel},xs} \approx \frac{2}{\pi}\frac{w_p}{w_i} = \frac{2}{\pi^2}\frac{g}{w_s}\frac{\lambda}{R}.$$

If we define the transverse coherence length lc of the source of diameter ws at a distance g, in the usual way, namely by

$$l_c = \frac{\lambda g}{2 w_s}$$

it is clear that the intensity of Poisson's spot is directly related to the degree of coherent illumination of the disc:

$$I_{\mathrm{rel},xs} \approx \frac{4}{\pi^2}\frac{l_c}{R}.$$

Thus, to observe the intensity of Poisson's spot to be as large as if the disc was not there, i.e. $I_{rel,xs}$ » 1, one can as a rule of thumb state that the transverse coherence of the beam incident on the disc must be approximately equal to the diameter of the disc.

Furthermore, it should be noted that if the source width exceeds the radius of the disc, i.e $w_s > R$, the geometric width of Poisson's spot w_i may become larger than the geometrical shadow (umbra shadow) width

$$w_u = 2R + \frac{b}{g}(2R - w_s)$$

making the observation of Poisson's spot increasingly difficult. To ensure that $w_i < w_u$ we must have

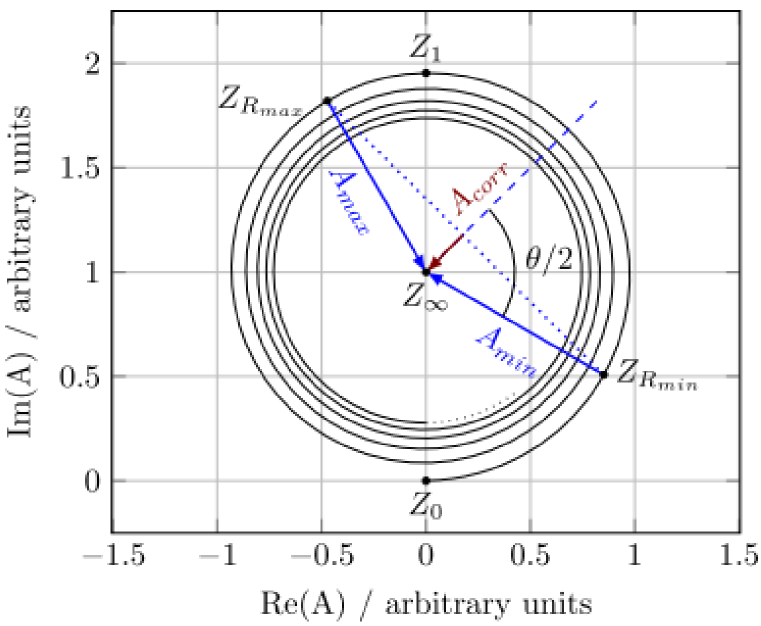
$$\frac{b}{g} < \frac{R}{w_s - R} \quad \text{if} \quad w_s > R.$$

For large wavelengths the width of Poisson's spot will be even larger and the shadow even narrower than assumed for this purely geometrical requirement, restricting its visibility to even smaller b/g.

Beside partial spatial coherence a deviation of the shape of the diffraction obstacle from a circular cross-section reduces the relative intensity of Poisson's spot. The parameter we consider in this respect is the edge corrugation of the disc. Its effect on the on-axis diffraction intensity can be best understood by considering the vibration curve. . It is a visual representation of the Fresnel–Kirchhoff integral.

$$A(P) = -\frac{\mathrm{i}}{\lambda g b}\int_0^{2\pi}\int_0^{\infty} g(\rho, \theta)\, \mathrm{e}^{\frac{\mathrm{i}\pi\rho^2}{\lambda}\left(\frac{1}{g}+\frac{1}{b}\right)}\rho \,\mathrm{d}\rho\, \mathrm{d}\theta.$$

Vibration curve. This spiral-shaped graph is often used in literature to visualize the result of the Fresnel–Kirchhoff integral. Here we use it in particular to explain the corrugation factor defined in equation. Points along the spiral starting from the origin (Z_0) correspond to increasing radii R in the integration plane. For a blocking circular disc the phasor points from the point on the spiral corresponding to the radius of the disc inwards to the centre of the spiral (point Z_∞). Point Z_1 corresponds to the radius of the first Fresnel zone. The phasor resulting from blocking the source with a disc with edge corrugation can be derived by a weighted averaging of the phasors that correspond to the range of disc radii.



The resulting phasor A(P) (point P on the optical axis)is composed of the infinitesimal phasors, each corresponding to a particular radius in the integration plane, that when joined head-to-tail follow the vibration curve. The end of the spiral is located at point Z_∞, resulting in the expected unit amplitude of A(P), if none of the incident field is blocked. However, if part of the integration plane is blocked by a disc with origin at the optical axis, the resulting phasor starts at the point on the vibration curve that corresponds to the radius of the rim of the disc, instead of the point Z_0.

The edge corrugation can be thus accounted for by an averaging of the phasors that correspond to the different disc radii. A rotation of the phasor by q = 180° corresponds to a change in the radius by one Fresnel zone, which results in a near complete cancellation of the amplitude or intensity of Poisson's spot if the two phasors are averaged. The width of a Fresnel zone starting from a particular radius R is approximately given by:

$$w_{\mathrm{fz}} = \sqrt{R^2 + \frac{\lambda\, g\, b}{(g+b)}} - R.$$

We assume from here that the corrugation is less than the width of the adjacent Fresnel zone, and that the intensity is negligible for corrugation of larger amplitude. The latter is not accurate for ideally shaped rectangular corrugation profiles, but we are more interested in finding a worst-case analysis for random edge corrugation. Neglecting the change in length of the phasors due to the spiral shape of the vibration curve, the averaging of the phasors can be accomplished by averaging the projections of the phasors in the direction of the resulting phasor.

The special case of a square wave corrugation profile with a peak-to-peak amplitude

$\sigma_{\mathrm{corr}} = R_{\mathrm{max}} - R_{\mathrm{min}}$

and sufficiently small period is depicted vibrational diagram. The phasors A_{min} and A_{max} correspond to the part of the corrugated edge with radii R_{min} and R_{max}, respectively. A rotation of the phasor by theta(Q) = pi corresponds to a change in the radius by w_{fz}. For simplicity we assume that the phase angle of the phasors is approximately proportional to R(more precisely we have that phase ∝ R^2), and we can thus write

$$|A_{\rm corr}| \approx |A_{\rm min}| \cdot \cos\left(\frac{\pi}{2}\frac{\sigma_{\rm corr}}{w_{\rm fz}}\right).$$

Since the measured intensity at point P corresponds to the square of the length of the resulting phasor, the attenuation due to rectangular-profile corrugation can be accounted for by the following factor(with attenuation we here refer to the ratio between the on-axis intensity behind a corrugated disc and the on-axis intensity behind a disc of perfect circular shape):

$$C_{\rm corr} = \begin{cases} \cos^2\left(\frac{\pi}{2}\frac{\sigma_{\rm corr}}{w_{\rm fz}}\right) & \text{if } \sigma_{\rm corr} < w_{\rm fz}, \\ \sim 0 & \text{otherwise.} \end{cases}$$

Finally, we take into account that in most realizations of the Poisson spot experiment the disc needs to be fixed in space by some type of support structure. In the model we propose, we assume the use of a number nsupp of straight radial support bars each of width w_{supp}. Together they block a distance $n_{supp} \cdot w_{supp}$ along the circumference of the disc. Again we use the vibration curve and the concept of phasor averaging to derive the effect on the on-axis intensity. If we set the phasors corresponding to blocked parts of the disc's circumference to zero, then it is clear that the attenuation is proportional to the ratio of unblocked circumference to the total circumference. We thus include the following factor C_{supp} in the model with a proportionality constant c_{supp}.

$$C_{\rm supp} = 1 - \frac{c_{\rm supp}\, n_{\rm supp}\, w_{\rm supp}}{2\,\pi\, R}.$$

Certain dependence of c_{supp} on the Fresnel number of the particular setup since the Fresnel zones at larger radii are blocked at decreasing proportions.

The complete analytical model for the relative on-axis intensity of Poisson's spot is thus given by:

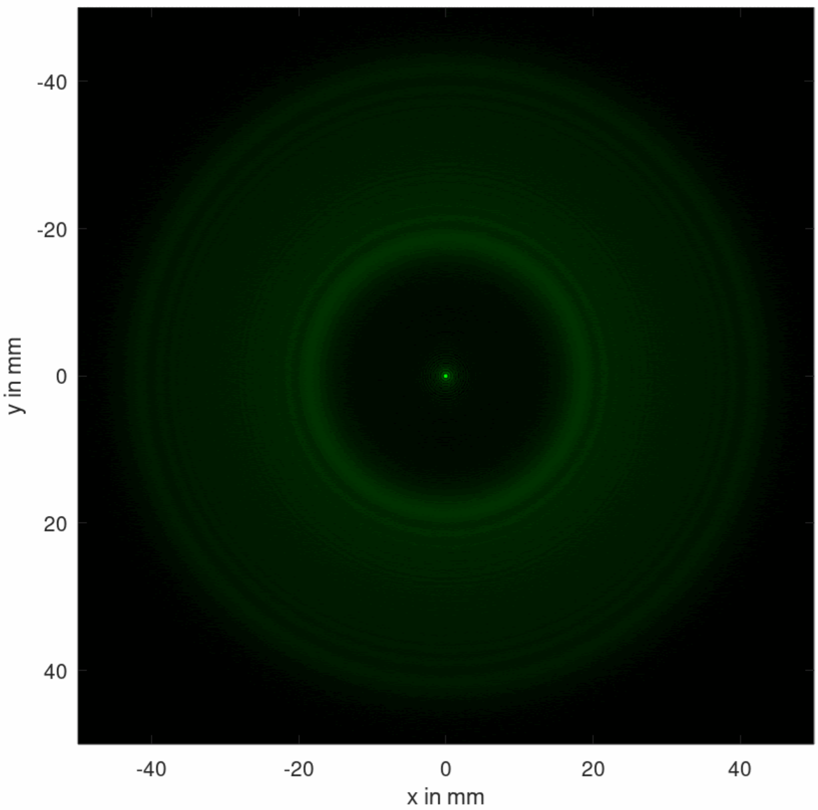
$$I_{\rm rel,model}(g, b, \lambda, w_s, R, \sigma_{\rm corr}, w_{\rm supp}, n_{\rm supp}) = I_{\rm rel,xs} \cdot C_{\rm corr} \cdot C_{\rm supp}.$$

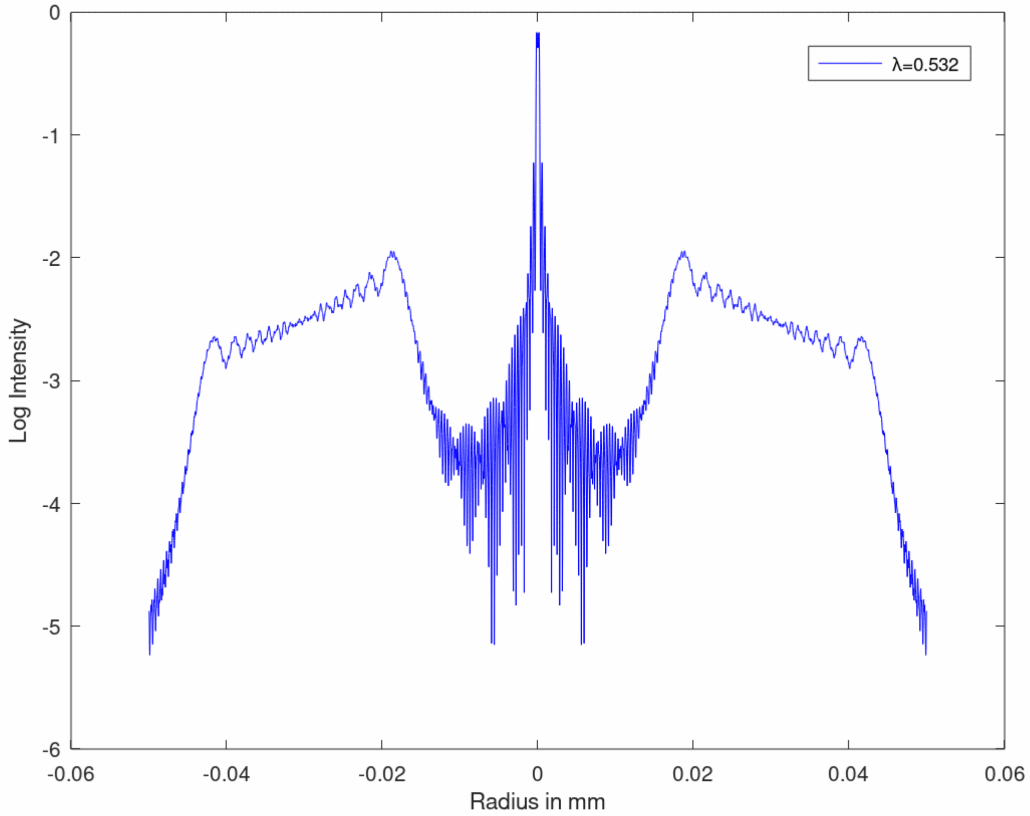
❖ CONCLUSIONS/RESULTS

Thus, we arrived at final equation of on-axis lateral intensity distribution of Poisson's spot. The equation depends on spatial coherence and edge corrugation of obstacle. We analysed the blocked part of obstacle that takes part in process and came up with a factor $C_{supp..}$

The Simulation of project comprises of two sets of code. C code solve the lateral intensity distribution of Poisson's spot. It creates two files corresponding to radial distance values and logarithmic of intensity values. Matlab code then gives the visual representation of Poisson's spot.

The green spot in the middle is Poisson's spot formed diffraction of light rays at the circumference of obstacle which can only be explained by optical wave nature of light.




λ=0.532
Log Intensity
Radius in mm
0
-1
-2
-3
-4
-5
-6
-0.06
-0.04
-0.02
0
0.02
0.04
0.06

IV. APPLICATIONS OF POISSON'S SPOT

- Understanding of this phenomenon helps in the study of the wave-particle duality for objects of increasing size and the measurement of particle–surface interaction potentials.
- The characterization of wave-front aberrations in annular high-energy-laser systems using the transverse intensity distribution of Poisson's spot was proposed by Harvey and Forgham.
- The characterization of surface corrugation and shape of balls, as used for example in ball-bearings.
- Kouznetsov and Lara proposed the use of the distance at which the Poisson spot vanishes behind the obstacle as a measure for its surface corrugation.
- The extinction of Poisson's spot is also of relevance in the search for extra-solar planets, where a petal-shaped star shade has been proposed as part of an external coronagraph aiming at their detection and spectroscopic characterization
- Other applications include the precise and rapid measurement of the position of inertial fusion energy targets and particle velocities in fluids.
- The Poisson spot has been used in lithography to fabricate microtube arrays for which its shaping via incomplete spiral phase modulation has interesting prospects.
- Little particles actually drifting around inside your eyeball. And they can be all sorts of different shapes, but some of them are spheres. And so they cast a shadow on the back of your retina. And right in the middle of that shadow is Poisson's bright spot.

V. BIBLIOGRAPHY

- Sci-Hub | Numerical evaluation of diffraction integrals. Journal of Research of the National Institute of Standards and Technology, 105(4), 581 | 10.6028/jres.105.048 (sci-hub.st)
- Convolution - Wikipedia
- A Beginners Guide to The Point Spread Function (bitesizebio.com)
- https://en.wikipedia.org/wiki/Arago_spot
- https://arxiv.org/pdf/1606.09472.pdf
- https://www.usna.edu/lime/_files/documents/Intro%20to%20Laser%20Research/Arago-Spot-Verification.pdf
- https://www.google.com/search?q=POISSON+SPOT+INTENSITY+VS+RADIUS&rlz=1C1NDCM_enIN872IN872&hl=en&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjqv7Cz7OTvAhV8xTgGHR7fCUUQ_AUoAXoECAEQAw&biw=1280&bih=521#imgrc=uVBZrcXBmbO9jM
- https://hal.archives-ouvertes.fr/hal-00416055v1/document
- https://www.zeiss.com/microscopy/int/solutions/reference/basic-microscopy/the-point-spread-function.html#:~:text=The%20ideal%20point%20spread%20function,numerical%20aperture%20(NA)%20objective.
- https://iopscience.iop.org/article/10.1088/1367-2630/aa5e7f
- http://www.yorku.ca/eye/psf.htm
- http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/opt/mch/diff.rxml
- https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/diffraction/