



PERTURBATION THEORY

PROJECT REPORT

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CONTENTS

1. PREFACE	04
1) Abstract	
2) Wave Function	
3) Dirac-Delta Notation	
4) Expectation Values	
5) Normalization	
6) Hamiltonian Operator	
7) Hermitian Operator	
2. Literature Review.....	08
(a) Perturbation Theory	
(b) Harmonic Oscillator	
(c) Degeneracy Vs Non-Degeneracy	
(d) Degenerate Perturbation Theory	
3. Simulation & Website Details.....	17
(a) Problem Statement : System Considered	
(b) Plan of Action	
4. Results.....	18
5. <i>Bibliography</i>.....	22

I PREFACE

ABSTRACT—

In this era of advancement ,we very well acknowledge the use of Quantum Mechanics in varied Technological Fronts like Atomic clocks, GPS Navigation, Telecommunication, Cryptography and Super Powerful Computers. This Document presents an overview of the Simulation proposed ,that is, Applications of Perturbation Theory and approximating the error between the Theoretical and Experimental Values (as referenced). As we are already aware that perturbation theory is a set of approximation methods directly related to mathematical perturbation for describing a complicated Quantum System such as Harmonic Oscillator , Stark Effect , Infinite Potential Well, Electron-Photon Interaction. Therefore, Perturbation Equations can be very well Computed and Visually represented in the form of Contour graphs through Matlab and can further be compiled using Python or Javascript in order to fabricate an Interactive Simulation as a Final Result.

Wave Function—

A wave function in quantum physics is a mathematical description of the quantum state of an isolated quantum system. The wave function is a complex-valued probability amplitude, and the probabilities for the possible results of measurements made on the system can be derived from it.

Time-dependent equation—

The form of the Schrödinger equation depends on the physical situation (see below for special cases). The most general form is the time-dependent Schrödinger equation (TDSE), which gives a description of a system evolving with time

$$i\hbar \frac{\partial}{\partial t} \Psi = H\Psi$$

where i is the imaginary unit, $\hbar=h/2\pi$ is the reduced Planck constant having the dimension of action, Ψ (the Greek letter psi) is the state vector of the quantum system, t is time, and H is the Hamiltonian operator.

Time-Independent equation—

The time-dependent Schrödinger equation described above predicts that wave functions can form standing waves, called stationary states. These states are particularly important as their individual study later simplifies the task of solving the time-dependent Schrödinger equation for any state. Stationary states can also be described by a simpler form of the Schrödinger equation, the time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m} \frac{\partial^2 u(x)}{\partial x^2} + V(x)u(x) = E u(x)$$

EXPECTATION VALUE—

To relate a quantum mechanical calculation to something you can observe in the laboratory, the "expectation value" of the measurable parameter is calculated. For the position x , the expectation value is defined as

This integral can be interpreted as the average value of x that we would expect to obtain from a large number of measurements. Alternatively it could be viewed as the average value of position for a large number of particles which are described by the same wavefunction. For example, the expectation value of the radius of the electron in the ground state of the hydrogen atom is the average value you expect to obtain from making the measurement for a large number of hydrogen atoms.

While the expectation value of a function of position has the appearance of an average of the function, the expectation value of momentum involves the representation of momentum as a quantum mechanical operator.

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)x\psi(x,t)dx$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\hbar}{i} \frac{\partial}{\partial x} \psi(x,t) dx$$

where

QUANTUM MECHANICS

Submitted: November 2020

$$p_{operator} = \frac{\hbar}{i} \frac{\partial}{\partial x}$$

is the operator for the x component of momentum. Since the energy of a free particle is given by

$$E = \frac{p^2}{2m} \quad \text{then} \quad \langle E \rangle = \frac{\langle p^2 \rangle}{2m}$$

and the expectation value for energy becomes

for a particle in one dimension.

In general, the expectation value for any observable quantity is found by putting the quantum mechanical operator for that observable in the integral of the wave function over space:

$$\langle E \rangle_{free\ particle} = \int_{-\infty}^{\infty} \psi * \frac{(-\hbar^2)}{2m} \frac{\partial^2}{\partial x^2} \psi dx$$
$$\langle Q \rangle = \int_{-\infty}^{\infty} \psi * Q_{operator} \psi dV$$

integral over
all space

NORMALISATION—

Whenever we speak of Quantum Mechanics, one the most fundamental concept that comes to mind is the Schrödinger's equation. This equation is a second order differential equation. Solution of which is the wavefunction that describes an electron.

Max Born (a 20th century physicist), linked this mathematical analysis of an electron to the real world physicality of electron. He interpreted the modulus of the calculated wavefunction as the probability of finding an electron at a specific space and time.

Now since, this mathematical wavefunction (which had a certain form, being a solution of a differential equation) had to be viewed as a probability distribution, the equation had to incorporate the fact, that the maximum probability of finding an electron over the entire space

has to be 1.

This is where a new quantity i.e. “normalization constant” was introduced that took care of this.

A normalized wave function $\phi(x)\phi(x)$ would be said to be normalized if $\int|\phi(x)|^2=1$. If it is not 1 and is instead equal to some other constant, we incorporate that constant into the wave function to normalize it and scale the probability to 1 again.

Hamiltonian Operator—

Associated with each measurable parameter in a physical system is a quantum mechanical operator, and the operator associated with the system energy is called the Hamiltonian. In classical mechanics, the system energy can be expressed as the sum of the kinetic and potential energies. For quantum mechanics, the elements of this energy expression are transformed into the corresponding quantum mechanical operators. The Hamiltonian contains the operations associated with the kinetic and potential energies and for a particle in one dimension can be written:

$$H_{operator} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$$

Operator associated Potential
with kinetic energy energy

Operating on the wave function with the Hamiltonian produces the Schrodinger equation. In the time independent Schrodinger equation, the operation may produce specific values for the energy called energy eigenvalues. This situation can be shown in the form

where the specific values of energy are called energy eigenvalues and the functions Ψ_i are called eigenfunctions.

In addition to its role in determining system energies, the Hamiltonian operator generates the time evolution of the wave function in the form

$$H_{op}\Psi_i = E_i\Psi_i$$

$$H\Psi = i\hbar \frac{\partial}{\partial t}$$

The full role of the Hamiltonian is shown in the time dependent Schrodinger equation where both its spatial and time operations manifest themselves.

II LITERATURE REVIEW

Perturbation Theory

A perturbation is a small disturbance in potential to a system that slightly changes the energy and wave equation solutions to the system. Perturbation theory in general allows us to calculate approximate solutions to problems involving perturbation potentials by using what we already know about very closely related unperturbed problems. We start from what we know about the solutions to the unperturbed problem, and make small corrections that approximate the effects of the perturbation under consideration.

Degeneracy—

Degeneracy occurs when more than one possible state of a system corresponds to the exact same energy. If you took a measurement of the energy of one of these degenerate states, and then took a measurement of the energy of another of the degenerate states, both being FUNDAMENTALLY and PHYSICALLY DIFFERENT states, you would get the same result. By this experiment alone you would not be able to distinguish between the two, and by your reckoning, the two states would appear identical. If these energy measurements were all of the information you had at your disposal, you would not have any knowledge of the degeneracy.

Degeneracy stems from symmetries inherent to a systems geometry. Distortion of the symmetry that allows multiple states to have the same energy spreads the energies of the different states apart, and they become measurably distinguishable. Consider this rudimentary illustration: A straight stick has a joint one third along it's length. Another has a joint at the 2/3 mark. Both appear as straight sticks in the absence of any outside force, *but when you push on them* they start to fold in a different manner from one another, and now *they are distinguishable*.

Consider a Hamiltonian of the form:

$$\hat{H} = \hat{H}_0 + \hat{H}'$$

where \hat{H} is the total Hamiltonian, \hat{H}_0 is the unperturbed Hamiltonian, and \hat{H}' is the relatively small contribution due to the perturbation.

We now suppose that \hat{H}_0 has degenerate eigenstates, and in so doing depart from non-degenerate perturbation theory. Suppose for example that the ground state of \hat{H}_0 has q degenerate states (q-fold degeneracy). If a perturbation potential is applied that destroys

the symmetry permitting this degeneracy, the ground state $E_n^{(0)}$ will separate into q distinct energy levels.

One of the primary goals of Degenerate Perturbation Theory is to allow us to calculate these new energies, which have become distinguishable due to the effects of the perturbation. As in the non-degenerate case, we start out by expanding the first order wavefunctions of \hat{H} in terms of the eigenstates of \hat{H}_0 .

$$\varphi_n^{(1)} = \sum_i c_{ni} \varphi_{ni}$$

Solving for the coefficients $\{c_{ni}\}$ gives the expression:

$$c_{ni} = \frac{H'_{in}}{E_n^{(0)} - E_i^{(0)}}$$

But consider the denominator of c_{ni} when $E_n^{(0)} = E_i^{(0)}$ (our degeneracy condition) . . . it is zero! This causes c_{ni} to blow up to infinity due to the degeneracy of H_0 . In fact c_{ni} blows up for any $(n, i \leq q)$ because all q of those eigenenergies are equal.

$$E_1^{(0)} = E_2^{(0)} = \dots = E_q^{(0)}$$

To avoid this problem, we need to construct a new set of basis functions from the set $\{\psi_n^{(0)}\}$ that will diagonalize the submatrix, H'_{in} for $(n, i \leq q)$. With all off diagonal elements of H'_{in} as zero, the corresponding c_{ni} are also zero, and we may proceed as in the non-degenerate case.

Diagonalization of the Submatrix

Now we know that degeneracy forces us to diagonalize a q by q submatrix of H' , which we label H'_{in} , in order to obtain non-singular coefficients of expansion, but how does this help us find the q new energies of the previously degenerate eigenstates once they are split apart by the perturbation potential? We will find that the diagonal elements of a diagonalized H'_{in} are precisely the incremental energies that when added to $E_1^{(0)}$ separate the q energy levels when in the presence of the perturbation.

Let the q functions that diagonalize H'_{in} for $(i, n \leq q)$ be called $\bar{\varphi}_n$:

$$\bar{\varphi}_n = \sum_{i=1}^q a_{ni} \varphi_i^{(0)}$$

where each $\bar{\varphi}_n$ is a linear combination of the degenerate eigenstates $\{\psi_i^{(0)}\}$. If the set of $\bar{\varphi}_n$ diagonalizes H'_{in} , we may write the following

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_p \rangle = H'_{np} \delta_{np}$$

where the Kronecker Delta reflects the fact that all non-diagonal elements have vanished. We are now prepared to construct a new, complete, and orthogonal basis by joining the new set of states that we have specifically constructed to replace those states that previously were degenerate $\{\bar{\varphi}_n, n \leq q\}$, with the set of all of the states that were previously non-degenerate $\{\varphi_{i(0)}, i > q\}$.

$$\beta = \{\bar{\varphi}_1, \bar{\varphi}_2, \dots, \bar{\varphi}_q, \varphi_{q+1}^{(0)}, \varphi_{q+2}^{(0)}, \dots\}$$

The matrix of \hat{H}' when calculated in this new set of basis functions appears as

$$\hat{H}' = \begin{pmatrix} H'_{11} & 0 & H'_{1,q+1} & \dots \\ & H'_{22} & & \\ 0 & & \ddots & \\ & & & H'_{qq} \\ H'_{q+1,1} & & & \\ \vdots & & & \end{pmatrix}$$

The submatrix of elements $n \leq q, p \leq q$ is diagonalized with all off-diagonal elements as zero.

First Order Energies

Now that we have diagonalized the appropriate submatrix, we must show that the diagonal elements H'_{nn} of this $q \times q$ submatrix are the first-order energy corrections E'_n to $E_n^{(0)}$, $n \leq q$. What we wish to show may be written

$$E'_n = \langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle = H'_{nn} \quad (n \leq q).$$

The eigenvalue equation for our perturbated Hamiltonian appears as

$$\hat{H}\varphi_n = (\hat{H}_0 + \hat{H}')\varphi_n = E_n\varphi_n.$$

Recalling our newly constructed basis functions $\{\bar{\varphi}_n, (n \leq q)\}$, and expressing energy in terms of unperturbated and perturbated components,

$$\left. \begin{array}{l} \varphi_n = \bar{\varphi}_n \\ E_n = E_n^{(0)} + E'_n \end{array} \right\} (n \leq q)$$

and then substituting back into the eigenvalue equation,

$$\hat{H}_0\bar{\varphi}_n + \hat{H}'\bar{\varphi}_n = E_n^{(0)}\bar{\varphi}_n + E'_n\bar{\varphi}_n$$

and cancelling unperturbated components, gives the expression

$$\hat{H}'\bar{\varphi}_n = E'_n\bar{\varphi}_n \quad (n \leq q).$$

Expressing this in Dirac notation,

and multiplying from the left with $\langle \bar{\varphi}_n |$

$$\hat{H}' |\bar{\varphi}_n\rangle = E'_n |\bar{\varphi}_n\rangle$$

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle = \langle \bar{\varphi}_n | E'_n | \bar{\varphi}_n \rangle$$

then pulling the scalar energy out,

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle = E'_n \langle \bar{\varphi}_n | \bar{\varphi}_n \rangle$$

and noting that the projection of any vector upon itself is 1 leaves us with:

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle = E'_n$$

Comparing this with

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_p \rangle = H'_{np} \delta_{np}$$

when $n = p$, gives

$$\langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle = H'_{nn} = E'_n \quad (n \leq q)$$

which is what we set out to show: that the diagonal elements of the submatrix H'_{np} are the first order corrections to the total Hamiltonian \hat{H} (for $n \leq q$).

Now let's construct the new basis functions $\{\bar{\varphi}_n\}$ that diagonalize the submatrix of \hat{H}' . Recall that $\bar{\varphi}_n^{(0)}$ is a linear combination of the degenerate eigenstates,

$$\bar{\varphi}_n = \sum_{i=1}^q a_{ni} \varphi_i^{(0)}$$

and that we can write an eigenvalue equation in terms of the perturbation contributions to the Hamiltonian and the energy

$$\hat{H}' \bar{\varphi}_n = E'_n \bar{\varphi}_n \quad (n \leq q).$$

Substituting $\bar{\varphi}_n$ into the above eigenvalue equation gives

$$\hat{H}' \sum_{i=1}^q a_{ni} |\varphi_i^{(0)}\rangle = E'_n \sum_{i=1}^q a_{ni} |\varphi_i^{(0)}\rangle.$$

Multiplying from the left with $\langle \varphi_p^{(0)} |$ yields

$$\sum_i a_{ni} H'_{pi} = E'_n \sum_i a_{ni} \delta_{pi} = E'_n a_{np} \quad (\text{for fixed } n, p \leq q)$$

which may be rewritten

QUANTUM MECHANICS

Submitted: November 2020

$$\sum_{i=1}^q (H'_{pi} - E'_n \delta_{pi}) a_{ni} = 0 \quad (n, p \leq q).$$

This may be interpreted as follows: the coefficients $\{a_{ni}\}$ for a fixed value of n are the column vector representation of $\bar{\varphi}_n$ in the subbasis $\{\varphi_l^{(0)}, (l \leq q)\}$

$$\bar{\varphi}_n = \begin{pmatrix} a_{n1} \\ a_{n2} \\ \vdots \\ a_{nq} \end{pmatrix} \left(\begin{array}{cccc} \varphi_1^{(0)} & \varphi_2^{(0)} & \dots & \varphi_q^{(0)} \end{array} \right)$$

The matrix elements H'_{pi} in this same basis appear as

$$H'_{pi} = \langle \varphi_p^{(0)} | \hat{H}' | \varphi_i^{(0)} \rangle$$

The q roots of $\det|H'_{pi} - E'_n \delta_{pi}| = 0$
are the eigenvalues of

$$\hat{H}' \bar{\varphi}_n = E'_n \bar{\varphi}_n \quad (n \leq q)$$

and also the diagonal elements of the $q \times q$ submatrix portion of

$$\hat{H}' = \begin{pmatrix} H'_{11} & & 0 & & H'_{1,q+1} & \dots \\ & H'_{22} & & & & \\ 0 & & \ddots & & & \\ & & & H'_{qq} & & \\ H'_{q+1,1} & & & & & \\ \vdots & & & & & \end{pmatrix}$$

When any of these q roots (in this case E'_1) are substituted back into

$$\begin{pmatrix} H'_{11} - E'_1 & H'_{12} & H'_{13} & \dots & H'_{1q} \\ H'_{21} & H'_{22} - E'_1 & H'_{23} & \dots & H'_{2q} \\ \vdots & \vdots & \vdots & & \vdots \\ H'_{q1} & H'_{q2} & H'_{q3} & \dots & H'_{qq} \end{pmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1q} \end{pmatrix} = 0$$

we will be able to solve for the coefficients $\{a_{1i}\}$ (had we substituted E'_2 we would be able to solve for $\{a_{2i}\}$ etc) which in turn allows the calculation of the basis functions $\{\bar{\varphi}_n\}$

$$\bar{\varphi}_n = \begin{pmatrix} a_{n1} \\ a_{n2} \\ \vdots \\ a_{nq} \end{pmatrix} \left(\begin{array}{cccc} \varphi_1^{(0)} & \varphi_2^{(0)} & \dots & \varphi_q^{(0)} \end{array} \right)$$

Using the new basis, the ambiguities due to the degeneracy of \hat{H}_0 are removed and we once again have the ability to use non-degenerate analysis. For $n \leq q$ we use our newly constructed set of q basis functions, and for $n > q$ we use the original and unaltered non-degenerate states.

$$\varphi_n = \bar{\varphi}_n + \lambda \bar{\varphi}_n^{(1)} + \lambda^2 \bar{\varphi}_n^{(2)} + \dots \quad (n \leq q)$$

$$\varphi_n = \varphi_n^{(0)} + \lambda \varphi_n^{(1)} + \lambda^2 \varphi_n^{(2)} + \dots \quad (n > q)$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots \quad (n \leq q) (E_1^{(0)} = \dots = E_q^{(0)})$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots \quad (n > q)$$

$$E'_n = \langle \bar{\varphi}_n | \hat{H}' | \bar{\varphi}_n \rangle \quad (n \leq q)$$

$$E_n^{(1)} = \langle \varphi_n^{(0)} | \hat{H}' | \varphi_n^{(0)} \rangle \quad (n > q)$$

□ 2D HARMONIC OSCILLATOR—

Some basics on the Harmonic Oscillator might come in handy before reading on. Consider the case of a two-dimensional harmonic oscillator with the following Hamiltonian:

$$\hat{H}_0 = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \frac{K}{2}(x^2 + y^2)$$

which may be equivalently expressed in terms of the annihilation and creation operators

$$\hat{H}_0 = \hbar\omega_0(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + 1).$$

For your reference

$$\hat{a} \equiv \frac{\beta}{\sqrt{2}} \left(\hat{x} + \frac{i\hat{p}_x}{m\omega_0} \right) \quad \hat{a}^\dagger \equiv \frac{\beta}{\sqrt{2}} \left(\hat{x} - \frac{i\hat{p}_x}{m\omega_0} \right)$$

$$x = \frac{1}{\sqrt{2}\beta}(\hat{a} + \hat{a}^\dagger) \quad y = \frac{1}{\sqrt{2}\beta}(\hat{b} + \hat{b}^\dagger) \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\hat{b} \equiv \frac{\beta}{\sqrt{2}} \left(\hat{y} + \frac{i\hat{p}_y}{m\omega_0} \right) \quad \hat{b}^\dagger \equiv \frac{\beta}{\sqrt{2}} \left(\hat{y} - \frac{i\hat{p}_y}{m\omega_0} \right)$$

QUANTUM MECHANICS

Submitted: November 2020

The 2-D eigenstates of \hat{H}_0 are each the product of two 1-D eigenstates:

$$\varphi_{np} \equiv \varphi_n(x)\varphi_p(y) \equiv |np\rangle$$

which have the corresponding eigen energies

$$E_{np} = \hbar\omega_0(n + p + 1)$$

which are $(n + p + 1)$ -fold degenerate. Consider the case of $n = 1$ $p = 0$, which is 2-fold degenerate, with corresponding eigenstates $|10\rangle$ and $|01\rangle$.

$$E_{10} = E_{01} = 2\hbar\omega_0$$

Using degenerate perturbation analysis we will determine how this energy separates under the influence of the perturbing potential

$$H' = K' xy$$

In addition, we will find two new wave functions that diagonalize \hat{H}' which are given by

$$\bar{\varphi}_n = \sum_{i=1}^q a_{ni} \varphi_i^{(0)}.$$

Because we have a basis with only 2 states, we will express this directly as

$$\bar{\varphi}_1 = a\varphi_{10} + b\varphi_{01} \quad \bar{\varphi}_2 = a'\varphi_{10} + b'\varphi_{01}$$

The submatrix \hat{H}' that needs to be diagonalized in this case is a full (2×2) matrix due to the fact that each and every state in our example is a degenerate eigenstate. In the basis $\{\varphi_{10}, \varphi_{01}\}$ this appears as

$$\hat{H}' = K' \begin{pmatrix} \langle 10 | xy | 10 \rangle & \langle 10 | xy | 01 \rangle \\ \langle 01 | xy | 10 \rangle & \langle 01 | xy | 01 \rangle \end{pmatrix} = \mathbb{E} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where

$$\mathbb{E} = \frac{K'}{2\beta^2}$$

Consider for example, the calculation of the upper right $(2,1)$ matrix element of \hat{H}' .

$$\langle 10 | \hat{x}\hat{y} | 01 \rangle = \frac{1}{2\beta^2} \langle 10 | \hat{a}\hat{b} + \hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}^\dagger | 01 \rangle$$

Remember that

$$\hat{a}|10\rangle = |00\rangle, \text{ and } \hat{b}|01\rangle = |00\rangle$$

(\hat{a} and \hat{b} subtract 1 from their respective quantum numbers) and

$$\hat{a}^\dagger|00\rangle = |10\rangle, \text{ and } \hat{b}^\dagger|00\rangle = |01\rangle$$

(\hat{a}^\dagger and \hat{b}^\dagger add 1 to their respective quantum numbers). In our case, if you try to subtract from zero, you get zero, and if you try to add to one, you also get zero. In other words you cannot promote above the highest, or demote below the lowest possible states.

Therefore

$$\begin{aligned}\langle 10|\hat{x}\hat{y}|01\rangle &= \frac{1}{2\beta^2}\langle 10|\hat{a}\hat{b} + \hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}^\dagger|01\rangle \\ &= \frac{1}{2\beta^2}\langle 10|\hat{a}^\dagger\hat{b}|01\rangle = \frac{1}{2\beta^2}\end{aligned}$$

After calculating all of the matrix elements of \hat{H}' we are now prepared to solve

$$\det|H'_{pi} - E'_n \delta_{pi}| = 0$$

which for the present example appears as

$$\begin{vmatrix} -E' & \mathbb{E} \\ \mathbb{E} & -E' \end{vmatrix} = 0$$

and has the solutions

$$E' = \pm \mathbb{E}$$

Thus we find that our perturbation has separated the degenerate first excited state by the amount $2\mathbb{E}$.

$$E_+ = E_{10} + \mathbb{E}$$

$$E_- = E_{10} - \mathbb{E}$$

Substituting these values back into

gives

$$\sum_{i=1}^q (H'_{pi} - E'_n \delta_{pi}) a_{ni} = 0 \quad (n, p \leq q)$$

QUANTUM MECHANICS

Submitted: November 2020

$$\begin{pmatrix} -E' & \mathbb{E} \\ \mathbb{E} & -E' \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

into which we can substitute (on at a time) the two values of E' we have found: (E_+ and E_-). These substitutions allow us to solve for the column vectors (a, b) , and (a', b') .

For $E' = E_+$:

$$\begin{pmatrix} -\mathbb{E} & \mathbb{E} \\ \mathbb{E} & -\mathbb{E} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0 \quad \begin{aligned} -a\mathbb{E} + b\mathbb{E} &= 0 & a &= b \\ a\mathbb{E} - b\mathbb{E} &= 0 & a &= b \end{aligned}$$

For $E' = E_-$:

$$\begin{pmatrix} \mathbb{E} & \mathbb{E} \\ \mathbb{E} & \mathbb{E} \end{pmatrix} \begin{pmatrix} a' \\ b' \end{pmatrix} = 0 \quad \begin{aligned} a'\mathbb{E} + b'\mathbb{E} &= 0 & a' &= -b' \\ a'\mathbb{E} + b'\mathbb{E} &= 0 & a' &= -b' \end{aligned}$$

Having solved for the coefficients of expansion, we can now fully construct our new basis states, which diagonalize the perturbation Hamiltonian H' :

$$E' = +\mathbb{E} \rightarrow \bar{\varphi}_1 = \frac{1}{\sqrt{2}}(\varphi_{10} + \varphi_{01})$$

$$E' = -\mathbb{E} \rightarrow \bar{\varphi}_2 = \frac{1}{\sqrt{2}}(\varphi_{10} - \varphi_{01})$$

III SIMULATIONS AND WEBSITE DETAILS

System Considered—

Let us consider the Hamiltonian for our system to be:

$$H = (p_1^2 + p_2^2) / 2 + \epsilon m \omega^2 (x \cos\theta + y \sin\theta)$$

$$H^0 \Phi_a^0 = E^0 \Phi_a^0 \quad H^0 \Phi_b^0 = E^0 \Phi_b^0 \quad \langle \Phi_a^0 | \Phi_b^0 \rangle = 0$$

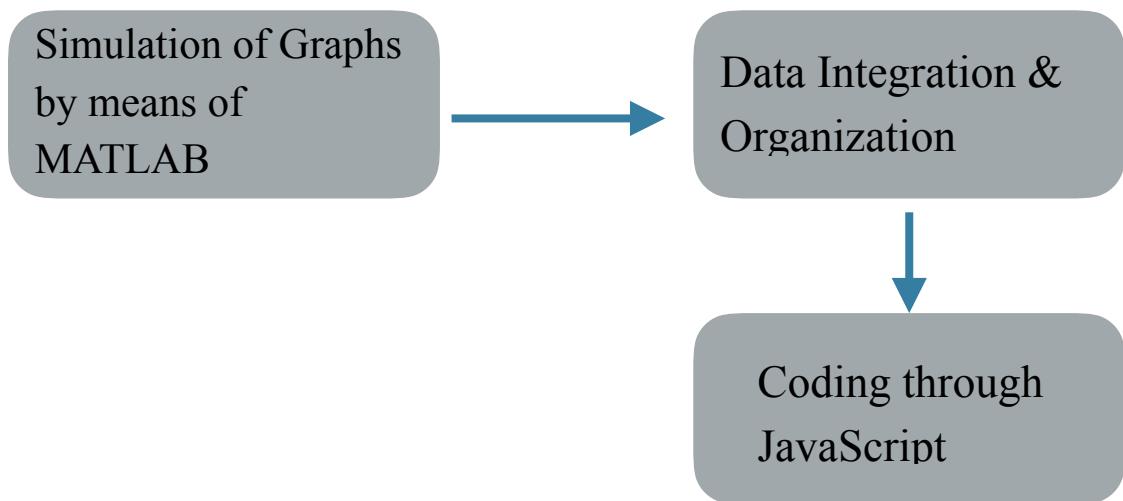
With Φ_a^0 and Φ_b^0 both normalised,
We may express it in terms of linear combination,

$$\Phi^0 = \alpha \Phi_a^0 + \beta \Phi_b^0 \quad \text{where } \alpha = \cos\theta \text{ and } \beta = \sin\theta$$

Now when we go through the Derivation for the approximation of the Applied Perturbation,

$$\begin{matrix} W_{aa} & W_{ab} & \alpha \\ W_{ba} & W_{bb} & \beta \end{matrix} = \begin{matrix} E^1 \\ \alpha \\ \beta \end{matrix} \quad \text{where } W_{ij} = \langle \Phi_i^0 | H^1 | \Phi_j^0 \rangle$$

Plan of Action Executed—

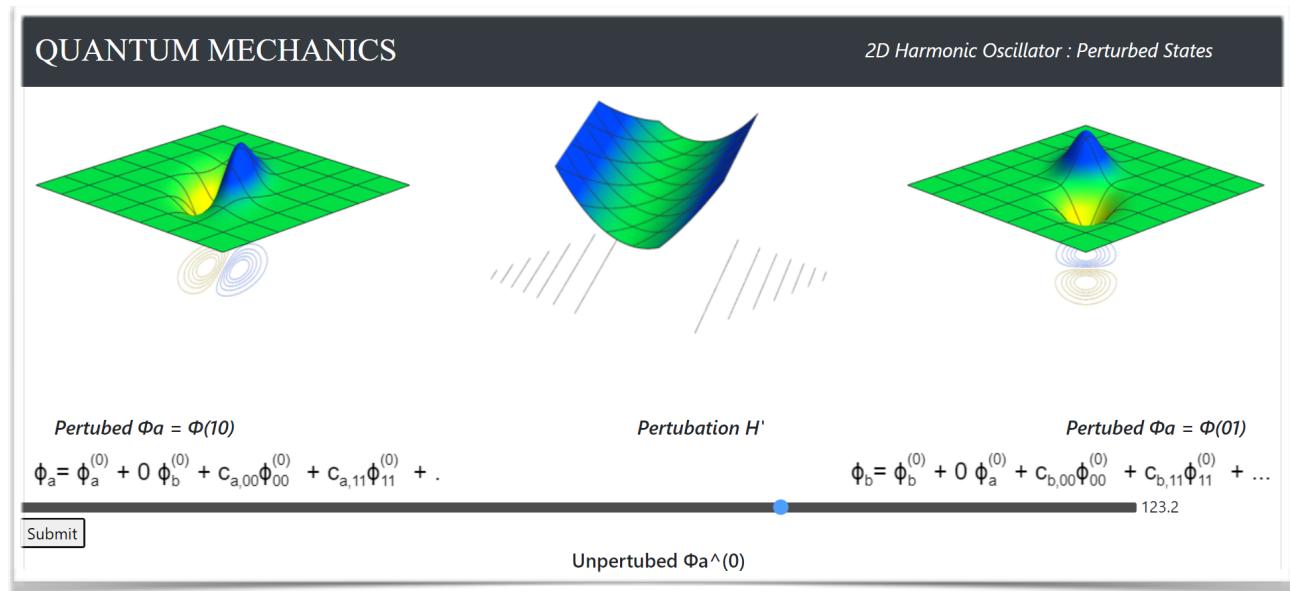
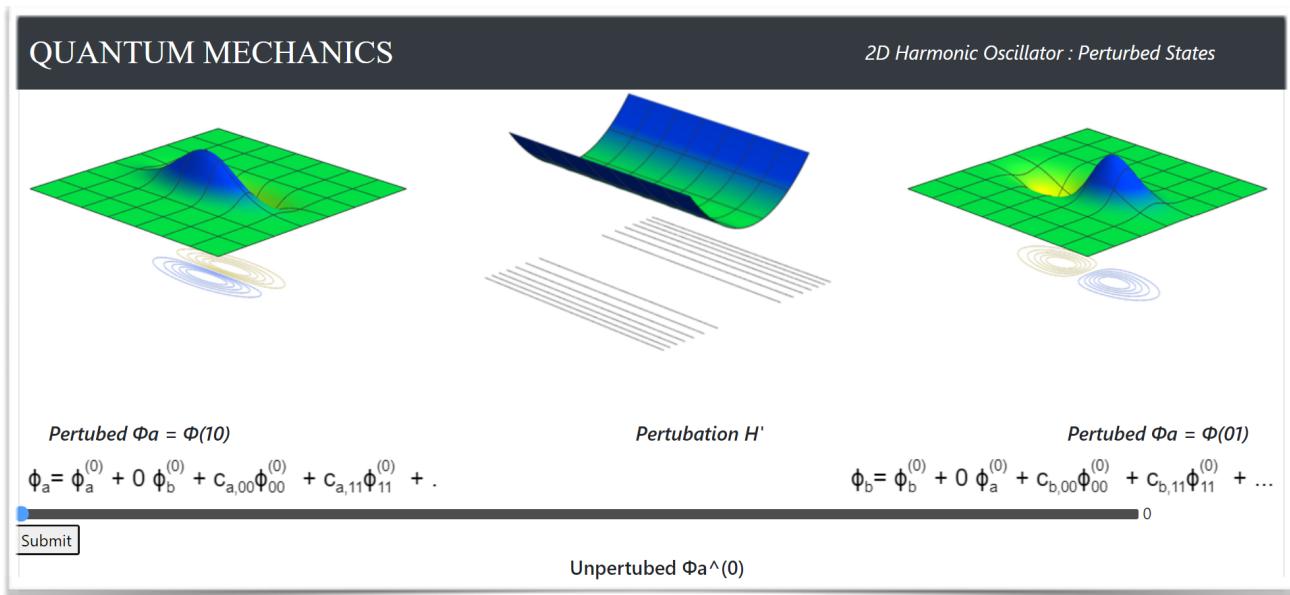


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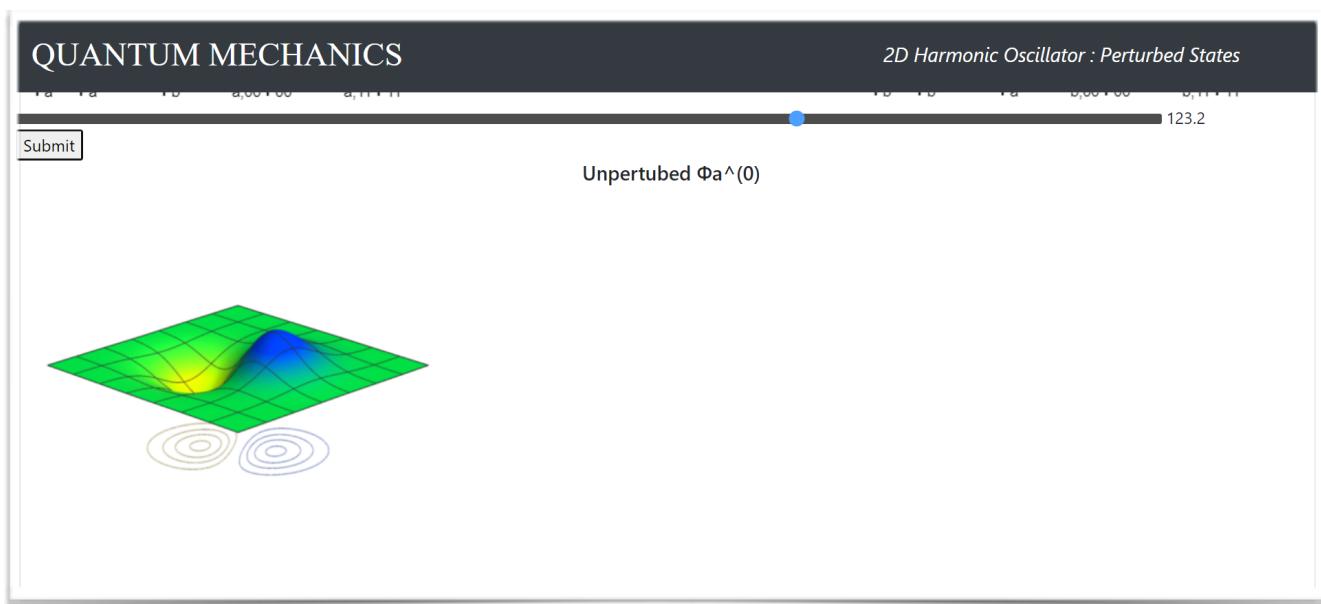
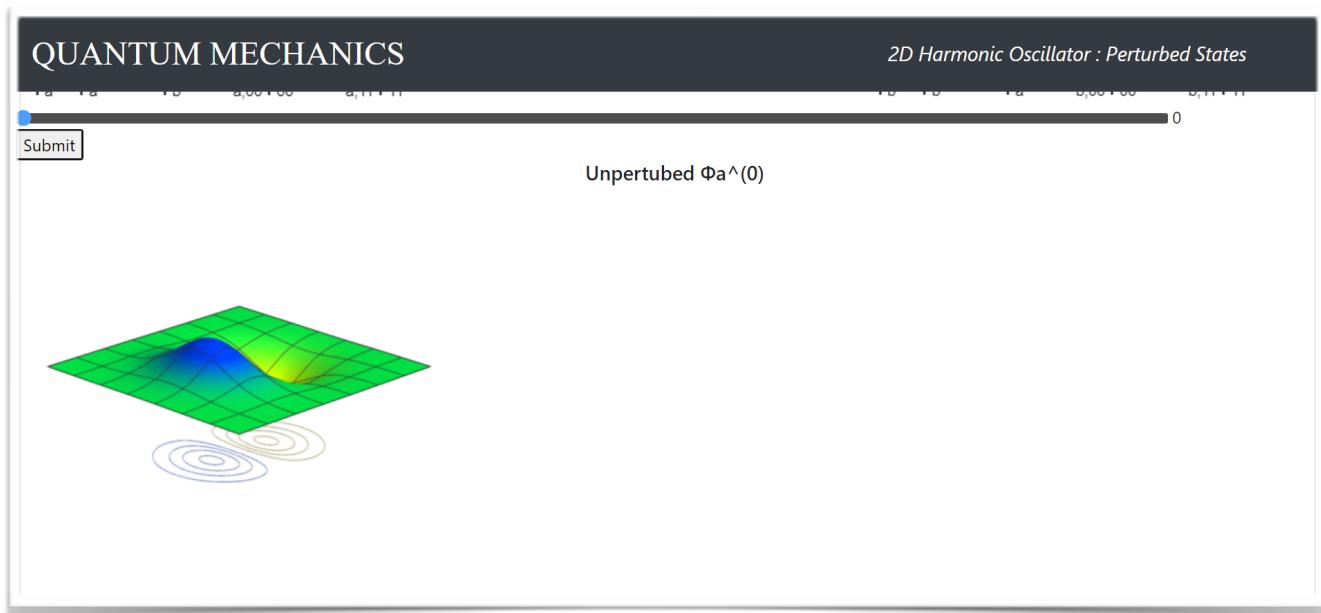
IV RESULTS

The following are the screenshots of the results obtained through Coding in JavaScript



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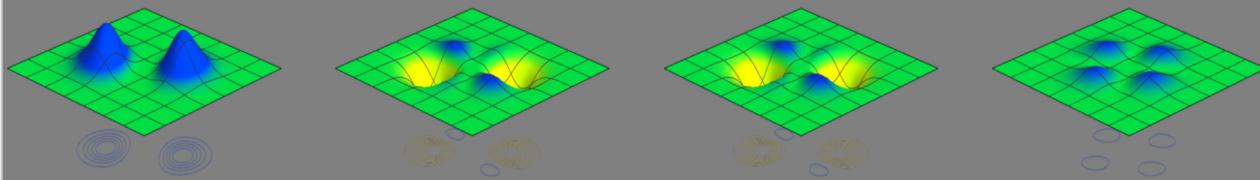
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123.2

Graph of integrand $\Phi a^{\wedge}(0)|H'| \Phi b^{\wedge}(0)$



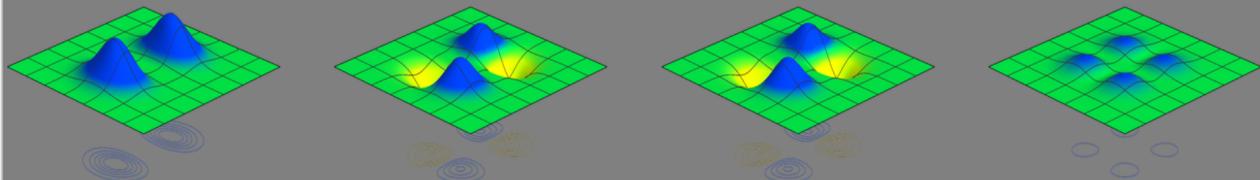
QUANTUM MECHANICS

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Submit

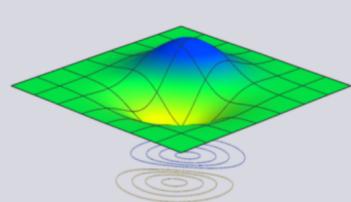
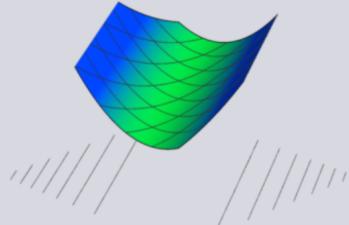
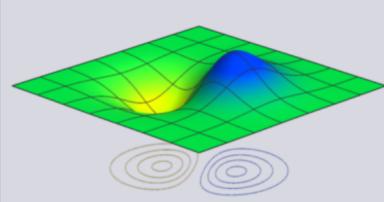
0

Graph of integrand $\Phi a^{\wedge}(0)|H'| \Phi b^{\wedge}(0)$



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Degenerate Perturbation Theory : 2D Harmonic Oscillator



Unperturbed first excited state

$$\Phi a^{\wedge}(0) = \cos(123.2)\Phi 10^{\wedge}(0) + \sin(123.2)\Phi 01^{\wedge}(0)$$

Perturbation H'

Unperturbed first excited state

$$\Phi b^{\wedge}(0) = -\sin(123.2)\Phi 10^{\wedge}(0) + \cos(123.2)\Phi 01^{\wedge}(0)$$

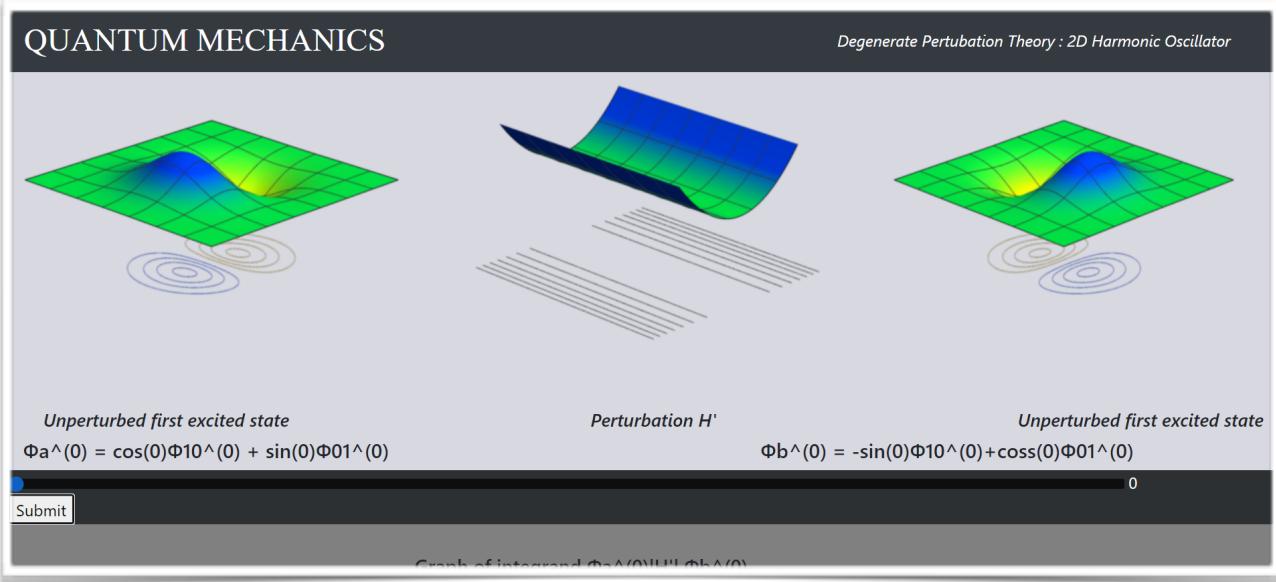
Submit

123.2

Graph of integrand $\Phi a^{\wedge}(0)|H'| \Phi b^{\wedge}(0)$

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