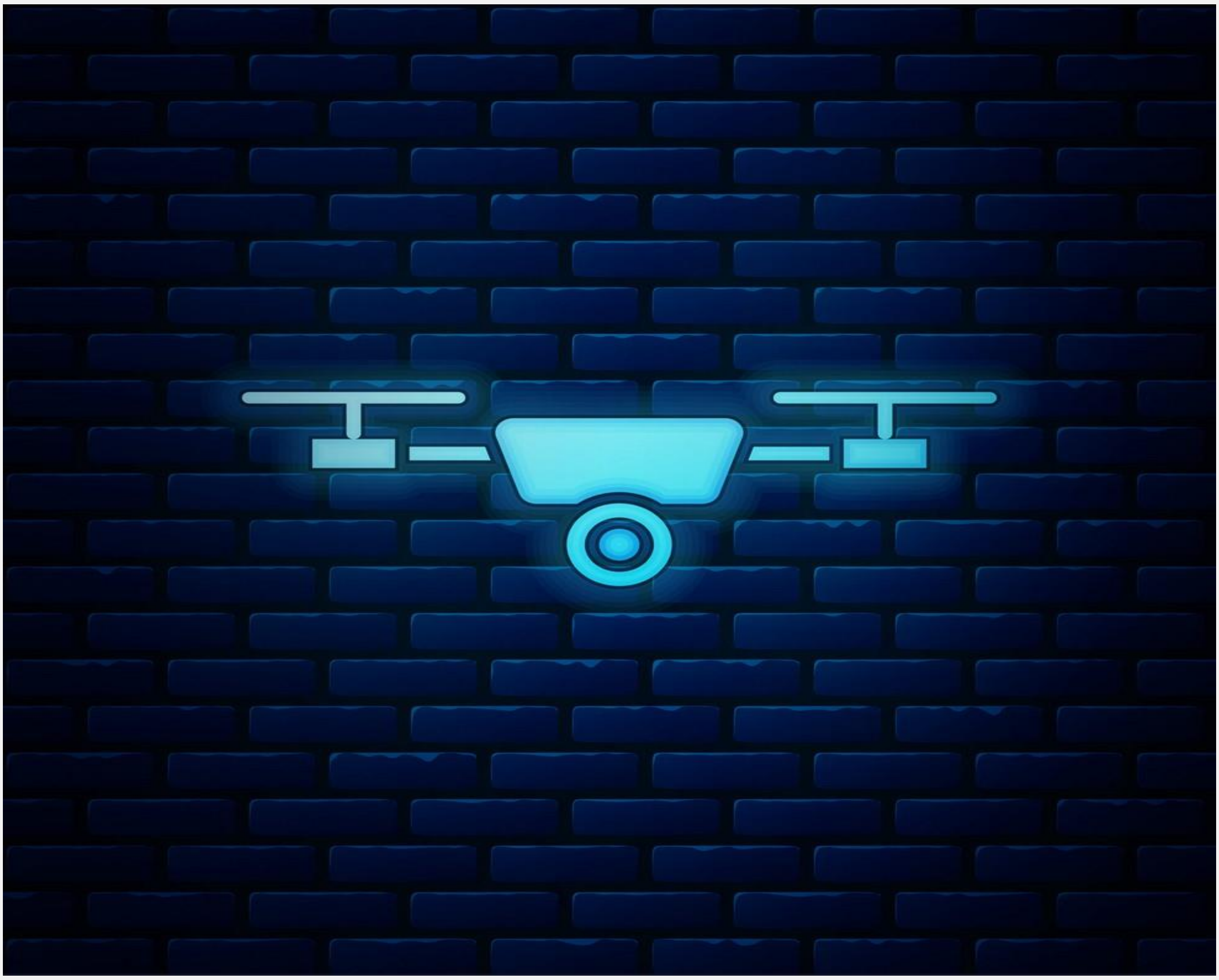
AERIAL SYSTEMS

STABLE DYNAMICS OF QUADCOPTOR

PROJECT REPORT

SUBMITTED BY: PULKIT PANDEY – 2K19/EP/076



DEPARTMENT OF APPLIED PHYSICS

DELHI TECHNOLOGICAL UNIVERSITY, NEW DELHI, 11042, INDIA

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INTRODUCTION

An unmanned aerial vehicle (UAV), commonly known as a drone, is an aerial system without a human pilot on board. Its flight is either controlled autonomously by computers in the vehicle. A common drone consist of four motors, control circuitry in middle and propellers mounted on its rotors. Two rotors rotate in clockwise direction while other two in counter clockwise directions to provide appropriate thrusts.

A quadcopter is an aircraft lifted and propelled by four horizontal rotors; each rotor consists of two or three rotor blades. A four-propeller multi-copter is called a quad-copter, a six propeller multi-copter is called a hexa-copter.

Each rotor produces both lift and torque about its centre of rotation, as well as drag opposite to the vehicle's direction of flight. Quadcopters generally have two rotors spinning clockwise (CW) and two counter-clockwise (CCW). Flight control is provided by independent variation of the speed and hence lift and torque of each rotor. Pitch and roll are controlled by varying the net centre of thrust, with yaw controlled by varying the net torque.

- **Frame**
  It is like the body of the drone which holds each part together and it has to be light and strong. Generally it is recommended to use carbon fibre frames because they are strong and light. We’ve used a glass fibre frame which is a good frame to start.

- **ESC**
  An electronic speed control or ESC is an electronic circuit that controls and regulates the speed of an electric motor.

- **Brushless Motors**
  A brushless motor is a direct current (DC) electric motor that operates without the mechanical brushes and commutator of a traditional brush motor

- **Arduino (Flight Controller)**
  Arduino is an open-source electronics platform based on easy-to-use hardware and software. Arduino boards are able to read inputs.

DYNAMICS OF UAV

UAV (quadcopter) dynamics would be introduced with two frames of reference in which it operates.

The inertial frame is defined by the ground, with gravity pointing in negative z direction.

The body frame is defined by the orientation of the drone, with the rotor axes pointing in the positive z direction and the arms pointing in the x and y directions.

Defining the position and velocity of the quadcopter in the inertial frame as

$$x = (x, y, z)^T$$

$$\dot{x} = (\dot{x}, \dot{y}, \dot{z})^T$$

respectively. Similarly, then under this project the Euler angles were defined, the roll, pitch, and yaw angles in the body frame as

$$\theta = (\phi, \theta, \psi)^T$$

with corresponding angular velocities equal to

$$\dot{\theta} = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$$

However, note that the angular velocity vector $\omega \neq \dot{\theta}$. The angular velocity is a vector pointing along the axis of rotation, while $\dot{\theta}$ is just the time derivative of yaw, pitch, and roll.

In order to properly model the dynamics of the system, the flow of project proceeded to gain an understanding of the physical properties that govern it. Thus, description of the motors being used for our quadcopter, and then use energy considerations to derive the forces and thrusts that these motors produce on the entire quadcopter. All motors on the quadcopter are identical, so we can analyze a single one without loss of generality. Note that adjacent propellers, however, are oriented opposite each other; if a propeller is spinning “clockwise”, then the two adjacent ones will be CLASSICAL MECHANICS PROJECT REPORT spinning “counter-clockwise”, so that torques are balanced if all propellers are spinning at the same rate.

MOTORS

Brushless motors are used for all quadcopter applications. For our electric motors, the torque produced is given by

$$\tau = K_t(I - I_0)$$

where τ is the motor torque, I is the input current, I_0 is the current when there is no load on the motor, and k^t is the torque proportionality constant. The voltage across the motor is the sum of the back-EMF and some resistive loss:

$$V = IR_m + K_v\omega$$

where V is the voltage drop across the motor, R^m is the motor resistance, ω is the angular velocity of the motor, and K_v is a proportionality constant (indicating back-EMF generated per RPM).

I can use this description of our motor to calculate the power it consumes. The power is

$$P = IV = \frac{(\tau + K_t I_0)(K_t I_0 R_m + \tau R_m + K_t K_v \omega)}{{K_t}^2}$$

For the purposes of our simple model, we will assume a negligible motor resistance. Then, the power becomes proportional to the angular velocity:

$$P \approx \frac{(\tau + K_t I_0) K_v \omega}{K_t}$$

Further simplifying our model, we assume that

$K_t\ I_0 \ll \tau$.

This is not altogether unreasonable, since I_0 is the current when there is no load, and is thus rather small. In practice, this approximation holds well enough.

Thus, we obtain our final, simplified equation for power:

$$P \approx \frac{K_v}{K_t}\tau\omega$$

EULER ANGLES

A drone has orientation and position attributes which can be determined using coordinates system (x,y,z coordinates) with certain angles (theta,phi,psi) with respect to world /Inertial frame. To measure the angles, usually drone's onboard IMU is used. This sensor measures how fast drone's body is rotating around it's body frame. This sensor returns angular velocity as it outputs with respect to body frame.

We can represent an orientation in 3D Euclidean space with three numbers. The sequence of rotation about basis vectors are called Euler angle sequence.

Euler angles are a method to determine and represent the rotation of a body as expressed in a given coordinate frame. They are defined as phi for x axis rotation, theta for y axis rotation and psi for z axis rotation. Any rotation can be described through a combination of these angles.

Rotation of any aerial system is expressed using these Tait-Bryan angles. Consider an orthogonal axis system passing through the centre of gravity of drone. Rotation about each axis is given by these angles.

FORCES ACTING ON DRONE

The power is used to keep the quadcopter aloft. By conservation of energy, we know that the energy the motor expends in a given time period is equal to the force generated on the propeller times the distance that the air it displaces moves ($P \cdot dt = F \cdot dx$).

Equivalently, the power is equal to the thrust times the air velocity ($P = F \cdot dx/dt$).

$P = T \cdot V_h$ assume vehicle speeds are low, so V_h is the air velocity when hovering. We also assume that the free stream velocity, v_∞, is zero (the air in the surrounding environment is stationary relative to the quadcopter).

Momentum theory gives us the equation for hover velocity as a function of thrust(T),

$$v_h = \sqrt{\frac{T}{2\rho A}}$$

where ρ is the density of the surrounding air and A is the area swept out by the rotor.

Using our simplified equation for power, we can then write

$$P = \frac{K_v}{K_t}\tau\omega = \frac{K_v K_\tau}{K_t}T\omega = \frac{T^{\frac{3}{2}}}{\sqrt{2\rho A}}$$

Note that in the general case, $\tau = \vec{r} \times \vec{F}$,in this case. Solving for the thrust magnitude T, we obtain that thrust is proportional to the square of angular velocity of the motor:

$$T = \left(\frac{K_v K_\tau \sqrt{2\rho A}}{K_t}\omega\right)^2 = k\omega^2$$

where k is some appropriately dimensioned constant. Summing over all the motors, we find that the total thrust on the quadcopter (in the body frame) is given by

$$T_B = \sum_{i=1}^{4} T_i = k\begin{bmatrix} 0 \\ 0 \\ \sum {\omega_i}^2 \end{bmatrix}$$

In addition to the thrust force, we will model friction as a force proportional to the linear velocity in each direction. This is a highly simplified view of fluid friction, but will be sufficient for our modeling and simulation. Our global drag forces will be modeled by an additional force term

$$F_D = \begin{bmatrix} -k_d \dot{x} \\ -k_d \dot{y} \\ -k_d \dot{z} \end{bmatrix}$$

TORQUES ACTING ON DRONE

Now that we have computed the forces on the quadcopter, we would also like to compute the torques. Each rotor contributes some torque about the body z axis. This torque is the torque required to keep the propeller spinning and providing thrust; it creates the instantaneous angular acceleration and overcomes the frictional drag forces.

The drag equation from fluid dynamics gives us the frictional force:

$$F_D = \frac{1}{2}\rho C_D A v^2$$

where ρ is the surrounding fluid density, A is the reference area (propeller cross-section), and C_D is a dimensionless constant.

This implies that the torque due to drag is given by

$$\tau_D = \frac{1}{2}R\rho C_D A v^2 = \frac{1}{2}R\rho C_D A(\omega R)^2 = b\omega^2$$

where ω is the angular velocity of the propeller, R is the radius of the propeller, and b is some appropriately dimensioned constant. Note that we've assumed that all the force is applied at the tip of the propeller, which is certainly inaccurate; however, the only result that matters for our purposes is that the drag torque is proportional to the square of the angular velocity.

We can then write the complete torque about the z axis for the ith motor:

$$\tau_z = b\omega^2 + I_M\dot{\omega}$$

where I_M is the moment of inertia about the motor z axis, $\dot{\omega}$ is the angular acceleration of the propeller, and b is our drag coefficient. Note that in steady state flight (i.e. not takeoff or landing), $\dot{\omega} \approx 0$, since most of the time the propellers will be maintaining a constant (or almost constant) thrust and won't be accelerating. Thus, we ignore this term, simplifying the entire expression to

$$\tau_z = (-1)^{i+1} b{\omega_i}^2$$

where the $(-1)^{i+1}$ term is positive for the i-th propeller if the propeller is spinning clockwise and negative if it is spinning counter-clockwise. The total torque about the z axis is given by the sum of all the torques from each propeller:

$$\tau_\psi = b\left({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2\right)$$

The roll and pitch torques are derived from standard mechanics. We can arbitrarily choose the i=1i=1 and i=3i=3 motors to be on the roll axis, so

$$\tau_\phi = \sum r \times T = L(k{\omega_1}^2 - k{\omega_3}^2) = Lk({\omega_1}^2 - {\omega_3}^2)$$

Correspondingly, the pitch torque is given by a similar expression

$$\tau_\theta = Lk({\omega_2}^2 - {\omega_4}^2)$$

where L is the distance from the centre of the quadcopter to any of the propellers. All together, we find that the torques in the body frame are

$$\tau_B = \begin{bmatrix} Lk({\omega_1}^2 - {\omega_3}^2) \\ Lk({\omega_2}^2 - {\omega_4}^2) \\ b\left({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2\right) \end{bmatrix}$$

The model we’ve derived so far is highly simplified. We ignore a multitude of advanced effects that contribute to the highly nonlinear dynamics of a quadcopter. We ignore rotational drag forces (our rotational velocities are relatively low), blade flapping (deformation of propeller blades due to high velocities and flexible materials), surrounding fluid velocities (wind), etc. With that said, we now have all the parts necessary to write out the dynamics of our quadcopter.

EQUATIONS OF MOTIONS

In the inertial frame, the acceleration of the quadcopter is due to thrust, gravity, and linear friction. We can obtain the thrust vector in the inertial frame by using our rotation matrix R to map the thrust vector from the body frame to the inertial frame. Thus, the linear motion can be summarized as

$$m\ddot{x} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + RT_B + F_D$$

where $\vec{x}$ is the position of the quadcopter, g is the acceleration due to gravity, F_D is the drag force, and T_b is the thrust vector in the body frame. While it is convenient to have the linear equations of motion in the inertial frame, the rotational equations of motion are useful to us in the body frame, so that we can express rotations about the center of the quadcopter instead of about our inertial center.

We derive the rotational equations of motion from Euler's equations for rigid body dynamics. Expressed in vector form, Euler's equations are written as

$$I\dot{\omega} + \omega \times (I\omega) = \tau$$

where ω is the angular velocity vector, I is the inertia matrix, and τ is a vector of external torques.We can rewrite this as

$$\dot{\omega} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = I^{-1}\left(\tau - \omega \times (I\omega)\right)$$

We can model our quadcopter as two thin uniform rods crossed at the origin with a point mass (motor) at the end of each. With this in mind, it's clear that the symmetries result in a diagonal inertia matrix of the form

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Therefore, we obtain our final result for the body frame rotational equations of motion

$$\dot{\omega} = \begin{bmatrix} \tau_\phi {I_{xx}}^{-1} \\ \tau_\theta {I_{yy}}^{-1} \\ \tau_\psi {I_{zz}}^{-1} \end{bmatrix} - \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}} \omega_y \omega_z \\ \frac{I_{zz}-I_{xx}}{I_{yy}} \omega_x \omega_z \\ \frac{I_{xx}-I_{yy}}{I_{zz}} \omega_x \omega_y \end{bmatrix}$$

INERTIAL MEASUREMENT UNIT

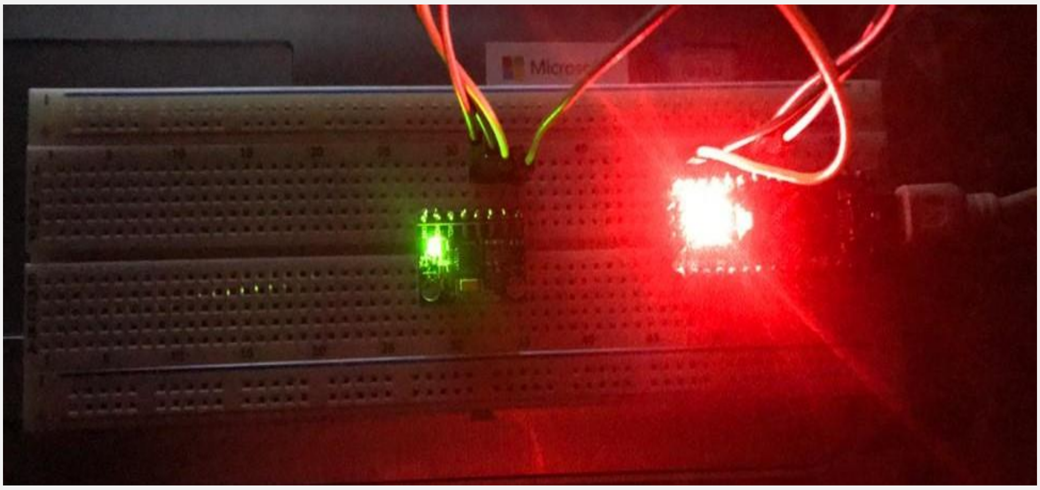
The GY-521 module is a breakout board for MPU-6050 MEMS (Microelectrochemical systems) that features a 3-axis gyroscope, a 3-axis accelerometer, a digital motion processor (DMP), and a temperature sensor. The digital motion processor can be used to process a complex algorithms directly on the board. Inertial sensors are for aerial dynamics typically comes in the form of an Inertial Measurement Unit (IMU). Subsequently, brief summary of main principles of accelerometers and gyroscopes is given below.

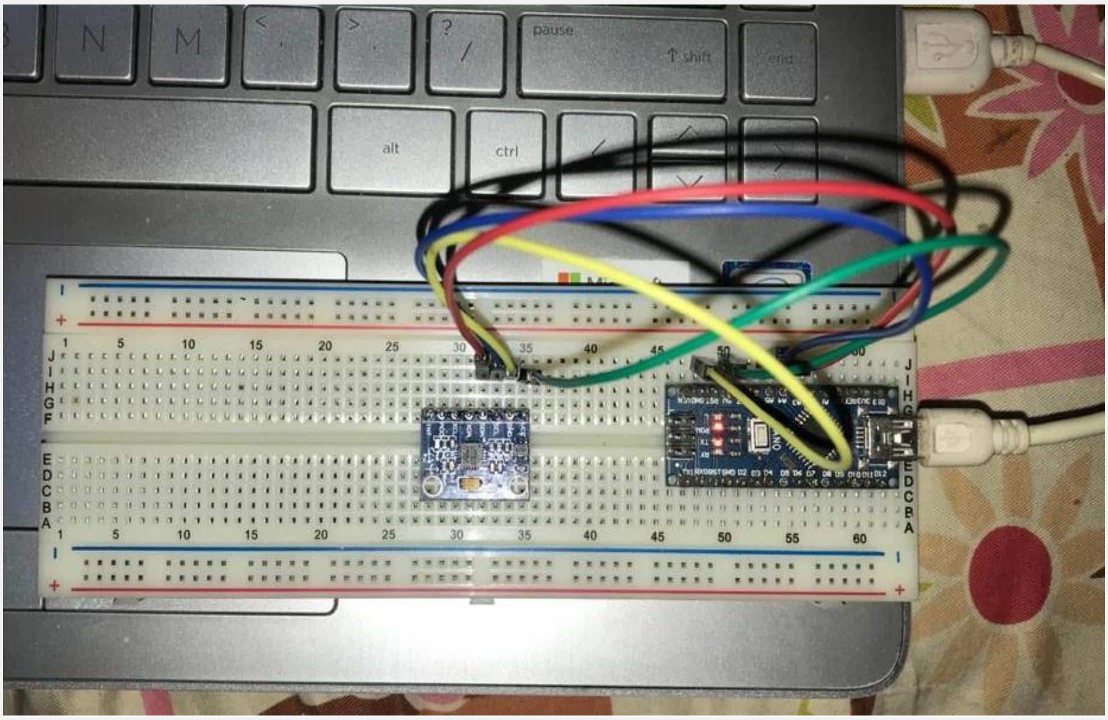
- ACCELEROMTER

  Accelerometers are devices that measure proper acceleration ("g-force"). Proper acceleration is not the same as coordinate acceleration (rate of change of velocity). For example, an accelerometer at rest on the surface of the Earth will measure an acceleration g= 9.81 m/s^2 straight upwards. By contrast, accelerometers in free fall orbiting and accelerating due to the gravity of Earth will measure zero.

- GYROSCOPE

  MEMS Gyroscope is small miniaturized sensors designed possible through integrating MEMS (Micro-Electro-Mechanical-System) technology into it. This allows for the functionality of gyroscopes to be utilized in a smaller package. While accelerometers can measure linear acceleration, they can’t measure twisting or rotational movement. Gyroscopes, however, measure angular velocity about three axes: pitch (x axis), roll (y axis) and yaw (z axis). When integrated with sensor fusion software, a gyro can be used to determine an object’s orientation within 3D space. While a gyroscope has no initial frame of reference (like gravity), you can combine its data with data from an accelerometer to measure angular position.



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AcX = -1724 | AcY = 128 | AcZ = 15752 | Tmp = 30.27 | GyX = -393 | GyY = -127 | GyZ = -145
AcX = -1768 | AcY = 272 | AcZ = 15932 | Tmp = 30.27 | GyX = -388 | GyY = -95 | GyZ = -152
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