SNOWFLAKE GROWTH

PROJECT REPORT

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Presentation, Inspiration and Motivation have always played a key role in success of any project.

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The accomplishment of the project was due to our equal efforts and contribution. We are highly indebted towards our University, that is, Delhi Technological University, New Delhi.

CONTENTS

I OBJECTIVE

The essence of science is figuring out what the universe comprises of, and how things work.This led us to think about our natural environment, which was the case almost everywhere in the world, thanks to the Pandemic!

However, As a result , we started thinking about the very elementary Snow Crystals and the Complex Science behind it.

Therefore, the objective of choosing this concept became to gain the knowledge of fundamental functioning of the very environment we live in, by means of condensed matter physics.

While the core-objective remained the understanding of the fundamental physical processes controlling ice solidification.

The practical Applicative Research perspective of Material sciences concept in our project involves the amalgamation   of Fundamental Physics, Fundamental Chemistry, Mathematical Modeling as well as a peak of Meteorology.

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II PROJECT OVERVIEW

This document encompasses the examination of ice crystallization from liquid water and from water vapor, focusing on the underlying physical processes that determine growth rates and structure formation.

The major focus lies upon molecular attachment kinetics on faceted surfaces and large-scale diffusion processes, yielding a remarkably rich phenomenology of solidification behaviors under different conditions. Layer nucleation plays an especially important role, with nucleation rates determined primarily by step energies on faceted ice/water and ice/vapor interfaces. The measured step energies depend strongly on temperature and other factors. On larger scales, computational techniques have recently demonstrated the ability to accurately model the diffusion-limited growth of complex structures that are both faceted and branched. Together with proper boundary conditions determined by surface attachment kinetics, this opens a path to fully reproducing the variety of complex structures that commonly arise during ice crystal growth.

We conclude the report with simulation of Ice Crystal Growth by means of development on Wolfram .

III BASIC TERMINOLOGIES

1. **ICE SOLIDIFICATION :**

   Solidification is a process in which atoms are converted into an ordered solid state from a liquid disordered state. The conversion rate for the process of solidification can be achieved by following the kinetic laws. The movement of atoms for the conversion of liquid can be observed by these laws. A driving force is the actual cause of conversion rate of process of solidification.When the system is shifted from liquid state to a solid one, then change occur in Gibbs free energy and become equal to driving force of solidification. The process of solidification can be analyzed by finding the liquid's Gibbs free energy and then analyzing temperature and composition of solid.

2. **SUPERCOOLING :**

   Supercooling, also known as undercooling, is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid. It achieves this in the absence of a seed crystal or nucleus around which a crystal structure can form.
   The supercooling of water can be achieved without any special techniques other than chemical demineralization, down to −48.3 °C (−55 °F). Droplets of supercooled water often exist in stratus and cumulus clouds.

3. **WATER VAPOR SUPERSATURATION :**

   The supercooling of water can be achieved without any special techniques other than chemical demineralization, down to −48.3 °C (−55 °F). Droplets of supercooled water often exist in stratus and cumulus clouds.

4. **TRIPLE POINT :**

   The triple point of a substance is the temperature and pressure at which the three phases of that substance coexist in thermodynamic equilibrium.In short, it is that temperature and pressure at which the sublimation curve, fusion curve and the vaporization curve meet.

5. **BASAL FACETING :**

   There is no blueprint or genetic code that guides the growth of a snowflake. The shape of each crystal is not determined by any plan or predetermined design, but by different processes that govern its growth behavior.

One such process is faceting, which causes flat surfaces to appear on the crystal, surfaces that reflect the underlying molecular symmetry.

The simplest snowflakes are called hexagonal prisms, like the one shown on the right. These have two basal facets and six prism facets.

6. SURFACE PRE-MELTING :

Water molecules near an ice/vapor interface are not as tightly bound to the crystal lattice as molecules below the surface, and this leads to a phenomenon called surface pre-melting. As the temperature approaches the melting point for an ice surface in equilibrium, the top molecular layers form a quasi-liquid layer (QLL) that has a disordered structure with characteristics similar to bulk liquid.

The video shown represents, The surface of ice undergoing a pre-melting transition. we can view the fluctuations of this liquid layer on top of a slab of ice. The grey spheres locate the centers of the water molecules and the red surface separates liquid-like molecules from crystal-like molecules.

IV Dynamics of Ice-Crystal Growth

☐ General Overview :

Water ice is one of the most common solid materials on the earth's surface, playing important roles in a broad spectrum of meteorological, biological, environmental, chemical, and physical processes. Although the material properties of ice have been known for many decades, our basic understanding of ice crystal growth remains surprisingly poor. A prime example can be found simply by observing the falling snow. The winter clouds produce a great diversity of snow-crystal forms, from slender columns to thin plates, at times branched, sectored, hollowed, and faceted, as shown in **Figure 1**. Yet extensive laboratory and theoretical investigations have still not been
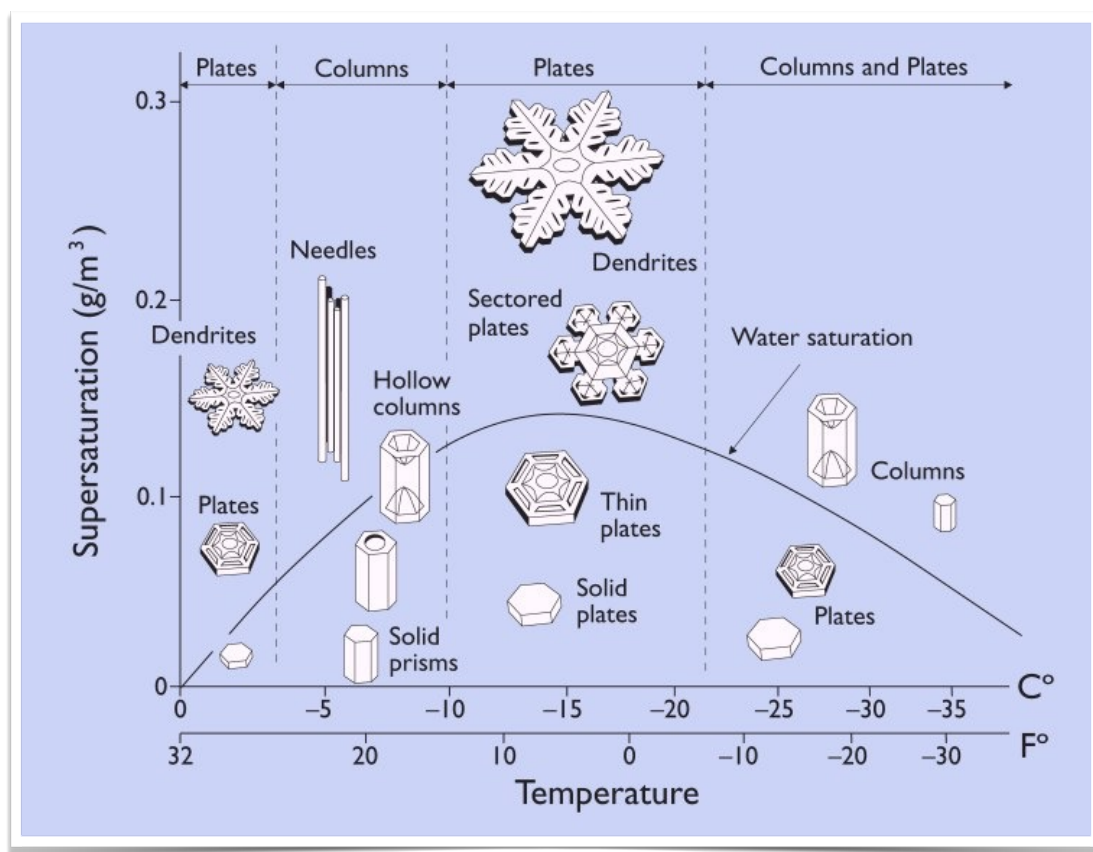

Fig. 1 : The above figure represents the morphology diagram qualitatively describes the growth of snow crystals as a function of temperature and water vapor supersaturation. The water saturation line shows the supersaturation of liquid water relative to ice.

determined why these varied structures appear under different growth conditions. For example, we do not yet possess even a qualitative understanding of why snow crystal growth alternates between plate-like and columnar forms as a function of temperature, as seen in **Figure 1**, although this behavior was first observed more than 75 years ago .

The focus will be on single-crystal ice Ih growing near the triple point, including ice forming from liquid water and from water vapor. Although this confines attention to just a small region of the water phase diagram, the phenomenology we encounter is already quite puzzling, driven by complex non-equilibrium molecular dynamics at the ice/water and ice/vapor interfaces together with large-scale heat- and particle-diffusion effects. We ignore isotopic effects here, as these appear to be relatively minor regarding all aspects of ice growth, at least insofar as they have been measured.

☐ GROWTH MORPHOLOGIES :

In broad brushstrokes, there are two primary physical effects that govern ice growth rates and the formation of complex structures during solidification. One is surface attachment kinetics, and the other is the diffusion of heat and/or particles. The interplay of these two processes, acting in concert over a range of length and time scales, leads to a fascinating phenomenology of observed ice growth behaviors, and we begin with brief descriptions of the ice/water and ice/vapor cases.

▷ Ice Growth from Liquid Water :

A key variable describing the solidification of ice from liquid water is the degree of supercooling, $T_{\text{surf}} = T_{\text{m}} - T_{\text{surf}}$, where $T_{\text{m}} \approx 0°C$ is the temperature of the ice/water interface in equilibrium and T_{surf} is the temperature of the ice/water interface during growth. It is customary to write the growth velocity v_{n} (normal to the surface) in the Wilson-Frenkel form, $v_{\text{n}}(T_{\text{surf}}) = K_{\text{T}} T_{\text{surf}}$, where K_{T} is the kinetic coefficient. This automatically gives the equilibrium condition $v_{\text{n}} = 0$ when $T_{\text{surf}} = 0$, although in general K_{T} is also temperature dependent. Other important variables may include the surface orientation relative to the crystal axes, solute concentrations, overall crystal morphology, the presence of container walls or other foreign materials, or simply the initial conditions and/or boundary conditions describing a particular system. Except where noted, in this review we focus primarily on ice growth in a pure water bath, unconstrained by the presence of solutes or other materials.

Another commonly used measure of supercooling is $T_{\text{bath}} = T_{\text{m}} - T_{\infty}$, where T_{∞} is the temperature of the supercooled water surrounding the crystal, far from the growing interface. Latent heat created at the ice/water interface is usually removed via diffusion, so generally $T_{\text{surf}} > T_{\infty}$. Often T_{surf} and T_{bath} are used interchangeably in the literature, which can lead to some con- fusion. Because T_{surf} is often difficult to determine near a growing interface, T_{bath} is commonly used when describing experimental results.

For a small supercooling with $T_{bath} < 1°C$, a microscopic seed crystal tends to grow into the form of a simple, circular disk. The slow-growing basal surfaces form the two faceted faces of the disk, indicating growth limited by surface attachment kinetics. Meanwhile the outward growth of the edge of the disk is limited primarily by the diffusion of latent heat. As T_{bath} is increased, the disk growth becomes unstable via the Mullins-Sekerka instability , resulting in the formation of dendritic branching, still mainly restricted to two dimensions by basal faceting.

As a function of T_{bath}, the ice growth morphologies progress from circular disks to dendritic plates when $T_{bath} < 2°C$, to unbranched needle-like structures when $T_{bath} \approx 4°C$, to branched needle-like structures when $T_{bath} \approx 8°C$, to platelets when T_{bath} is in the range 10–30°C, as described in (18). For all of these growth morphologies, one can measure the growth velocity v_{tip} of an outermost structural point (for example, the tip of a growing dendrite), and doing this yields approximately a single function $v_{tip}(T_{bath})$. Diffusion-limited dendrite growth theory reproduces this function reasonably well, confirming that ice growth from liquid water is largely limited by heat diffusion.

For dilute solutions, the presence of a solute typically yields a freezing point depression proportional to concentration (Blagden's law), plus solvated molecules are not readily incorporated into the ice lattice. Thus both heat and solute diffusion should be included in a solidification model.

▷ Ice Growth from Water Vapor :

The growth of ice from water vapor is usually described in terms of two primary parameters: the growth temperature T_{surf} and the water vapor supersaturation $\sigma_{surf} = (c_{surf} - c_{sat}) / c_{sat}$, where c_{surf} is the volumetric number density of water vapor molecules just above the growing ice/vapor interface and $c_{sat} = c_{sat} (T_{surf})$ is the saturated number density above a flat ice surface in equilibrium .Unlike the ice/water case, much of the ice/vapor coexistence line is rather easily accessible experimentally, and a good sampling of (T , σ) parameter space can even be found in the earth's atmosphere. Again one must distinguish σ_{surf} and T_{surf} from σ_∞ and T_∞ , the latter referring to the boundary conditions far from the crystal. Typically $\sigma_{surf} < \sigma_\infty$ because of particle diffusion if a background gas (such as air) is present, and $T_{surf} > T_\infty$ because of latent heat considerations. Both particle and heat diffusion factor into ice growth from water vapor, plus the growth rates may be influenced by substrate interactions, surface chemical interactions, initial growth conditions, and other factors.

In the limit of infinitely fast surface attachment kinetics, the ice growth rate is determined simply by the net flux of water vapor molecules striking the ice/vapor interface, which can be calculated from ideal-gas statistical mechanics. This gives the Wilson-Frenkel form $v_n = \alpha v_{kin} \sigma_{surf}$, where $v_{kin} = (c_{sat}/c_{ice})(kT_{surf}/2\pi m_{mol})^{1/2}$ is a temperature - dependent kinetic velocity,the mass of a water molecule is $m_{mol} \approx 3.0 \times 10^{-26}$ kg, and $c_{ice} = \varrho_{ice}/m_{mol} \approx 3.1 \times 10^{28}$ m^{-3} is the number density of ice ($\varrho_{ice} \approx 917$ kg/m^3). The dimensionless factor α, called the

attachment coefficient, describes the deviation from fast kinetics. For an infinite planar surface, we must have $0 \leq \alpha \leq 1$ on average , although nonlocal transport effects on finite crystals may yield violations of this rule .

Attachment kinetics is typically the dominant factor determining growth behavior when the supersaturation is low, or the crystal size is small, or there is little background gas pressure . In any of these circumstances, ice crystals grow as simple hexagonal prisms, defined by faceted basal and prism surfaces. For large crystals growing at low background pressures, heat diffusion becomes an important factor as well . Evacuated voids in solid ice, known as negative crystals, also grow in the shape of hexagonal prisms .
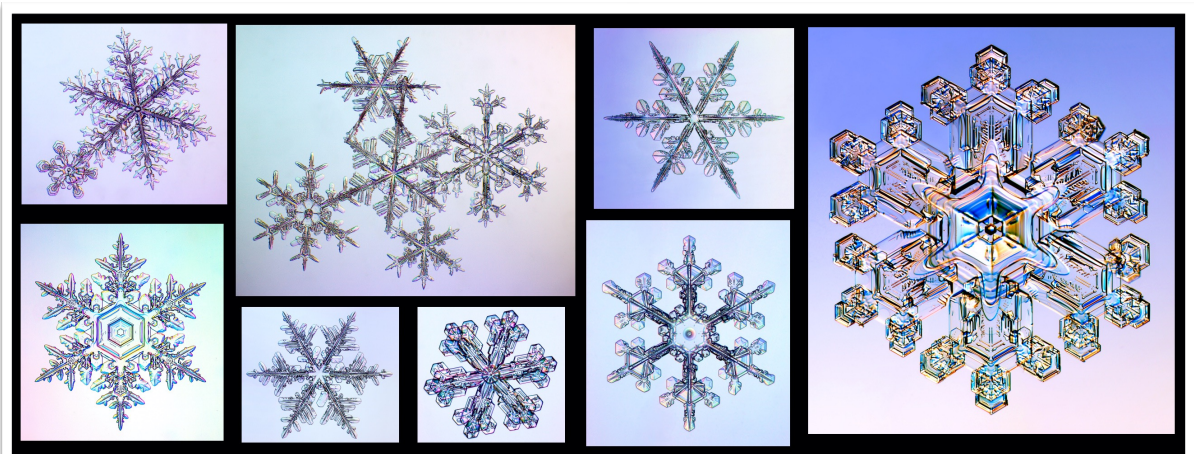

Fig. 3 : *The above figure represents single-crystal dendritic structures like these arise during the diffusion-limited growth of ice from water vapor in air at high supersaturation levels, as seen in the snow-crystal morphology diagram.At temperatures near $-15°C$, slow basal growth yields nearly planar crystals with essentially two-dimensional branching. Each main branch features a roughly parabolic tip that grows outward with a tip velocity proportional to supersaturation, and side branches grow parallel to neighboring main branches. Note that the side branch spacing is both aperiodic and asymmetrical in these fast-growing dendrites.*

As σ_∞ increases around ice crystals growing in air, both particle diffusion and attachment kinetics play large roles in structure formation. The morphology diagram provides only an over- simplified picture of the large menagerie of different types of atmospheric snow crystals that result .Observations down to $T = -70°C$ reveal additional behavioral changes at low temperatures . [However, low-temperature ice growth is sensitive to the presence of chemical contaminants , and this may explain

some of the variability seen when $T < -20^{\circ}$C.] Faceting generally diminishes as σ∞ increases in air, yielding highly branched crystals like those shown in **Figure 3.**

Even with extensive branching, however, attachment kinetics strongly affects the overall structure. Ice growth morphologies generally become more complex with increasing crystal size, increasing σ∞, and increasing background gas pressure.

The detailed formation of an iconic winter snowflake — a stellar-dendrite crystal with six-fold symmetry, for example, the photo shown in sub-panel *i* of. Once a cloud droplet freezes to form a nascent snowflake, the crystal begins growing by absorbing water vapor from its surroundings. The basal and prism surfaces grow slowly (because of their attachment kinetics), so the small crystal initially takes the form of a faceted hexagonal plate near $T = -15^{\circ}$C. As it grows larger, the Mullins - Sekerka instability causes branches to sprout from the corners of the plate, where the near-surface supersaturation is highest. Subsequent development of the branches depends on the temperature and supersaturation surrounding the crystal, as the growth behavior is quite sensitive to these conditions. At times the branches sprout additional side branches, and at other times the branches grow more slowly and become faceted. During a typical 30-min lifetime of a growing snowflake, the growth conditions may change many times, yielding a final crystal with a complex, branched shape. However, the individual branches all experience nearly the same changing conditions as a function of time, so all six grow in synchrony, yielding a structure that is both complex and symmetrical on macroscopic scales. Snowflake photographers tend to seek out specimens with near-perfect symmetry, but these are actually quite rare in nature; the vast majority of natural snow crystals are lopsided or malformed to varying degrees. The repeated application of induced side branching tends to yield the most symmetrical crystals.

The morphology diagram describes the shapes of frost crystals as well, as these also grow from water vapor in the atmosphere. Frost crystals appearing on the ground overnight are typically quite small, but plate-like and needle-like morphologies can often be seen using a low-power magnifier. Larger crystals are called hoarfrost, and centimeter-scale crystals are not uncommon, especially when crystals grow in undisturbed places for days or weeks. Window frost forms from water vapor as well, but its development is substantially influenced by the presence of the glass substrate, including microscopic scratches or chemical residues that often guide the formation of large-scale patterns

V EQUILIBRIUM PROPERTIES

☐ ENERGETICS :

Now we will examine the limiting case in which the growth rate goes to zero, giving us ice/water or ice/vapor interfaces in equilibrium. Of particular importance are the equilibrium energies, including bulk (3D), surface (2D), and step (1D) energies.

The bulk energies are the latent heats released during the formation of condensed phases at the relevant phase boundaries. For ice near the triple point, the latent heat of fusion (solid/liquid) is $L_{sl} \approx 3.3 \times 10^5$ J/kg, the latent heat of deposition (solid/vapor) is $L_{sv} \approx 2.8 \times 10^6$ J/kg, and for completeness we include the latent heat of condensation (liquid/vapor) $L_{lv} \approx 2.5 \times 10^6$ J/kg. These quantities change with position on the phase boundary: L_{sl} drops to $L_{sl} \approx 2.3 \times 10^5$ J/kg at $T \approx -22^{\circ}$C (near the lowest-temperature, highest-pressure point on the ice/water phase boundary), and L_{sv} drops to $L_{sv} \approx 2.6 \times 10^6$ J/kg near absolute zero.

Surface energies are more difficult to measure (or calculate) and are therefore not as well known. The liquid/vapor surface energy has been determined from observations of the oscillations of water droplets, giving $\gamma_{lv} \approx 0.076$ J/m^2 near the triple point, dropping to $\gamma_{lv} \approx 0.059$ J/m^2 at $T = 100^{\circ}$C. The solid/liquid surface energy is derived from the rates of homogeneous nucleation of water droplets, giving $\gamma_{sl} \approx 0.033$ J/m^2 near the triple point, dropping to approximately $\gamma_{sl} \approx 0.020$ J/m^2 at $T = -40^{\circ}$C (44). The solid/vapor surface energy has proven quite difficult to measure, although surface wetting measurements suggest that Antonow's relation is a reasonable approximation near the triple point: $\gamma_{sv} \approx \gamma_{sl} + \gamma_{lv} \approx 0.109$ J/m^2 (2, 3). The ice surface energies are likely isotropic to a few percent, although anisotropies have not yet been measured.

Step free energies play an important role in ice crystal growth, factoring into the 2D nucleation of new terraces on faceted surfaces. The ice/water step energy on the basal facet near the triple point is $\beta_{basal,sl} \approx 5.6 \times 10^{-13}$ J/m (46–48), and $\beta_{prism,sl}$ is essentially zero, as the prism surface is not faceted except at high pressures. Measurements of step energies on the ice/vapor interface give $\beta_{basal,sv} \approx \beta_{basal,sl}$ near T_m (48), and both $\beta_{basal,sv}$ and $\beta_{prism,sv}$ increase sharply at lower temperatures.

Considering a simple geometric picture of a faceted ice/vapor interface, an abrupt terrace step of height a adds an additional surface area equal to al, where l is the length of the step, implying a step energy $\beta_{0,sv} \approx a\gamma_{sv}$. Surface relaxation essentially makes the step less abrupt, yielding $\beta_{sv} < \beta_{sv,0}$. If the terrace step is a gradual one of effective width w, we have $\beta_{sv,w} \approx (a/2w)\beta_{sv,0}$ in the limit $a/w \ll 1$. Measurements of $\beta_{sv}/\beta_{sv,0}$ thus provide some indication of the effective width of a terrace step, which may be relevant for modeling terrace steps and computing step energies using molecular dynamics simulations, as is discussed below.

☐ SURFACE PRE-MELTING :

Water molecules near an ice/vapor interface are not as tightly bound to the crystal lattice as molecules below the surface, and this leads to a phenomenon called surface pre-melting. As the temperature approaches the melting point for an ice surface in equilibrium, the top molecular layers form a quasi-liquid layer (QLL) that has a disordered structure with characteristics similar

to bulk liquid.The thickness of the QLL diverges as $T \rightarrow T_m$, making it impossible to superheat ice above T_m. One physically intuitive way to think about surface melting uses the Lindemann criterion, which states that a solid will melt if thermal fluctuations of the intermolecular distance become larger than approximately 10–15% of the average distance, and these fluctuations are larger near the surface than in the bulk. Surface pre-melting is common in many other materials, and cluster pre-melting is a related phenomenon.

Physical models of ice pre-melting are fairly crude, and the overall thickness of the QLL as a function of temperature $d_{QLL}(T)$ is not well known, with different experiments yielding quite different estimates for $d_{QLL}(T)$. Very roughly, d_{QLL} becomes comparable to the lattice spacing at $T \approx -10°$ C, and increases rapidly as $T \rightarrow T_m$. Molecular dynamics simulations also indicate the presence of surface pre-melting It has long been speculated that surface pre-melting affects the ice/vapor attachment kinetics, but the importance of its role is not yet known.

VI SURFACE ATTACHMENT KINETICS

☐ NUCLEATION THEORY —

On faceted surfaces, the attachment kinetics are often limited by the nucleation of new molecular terraces, as described by classical nucleation theory. For a simple monomolecular solid surface, crystal growth is driven by a chemical potential jump at the interface, such as a nonzero supercooling or supersaturation. A general polynuclear growth model gives the normal growth velocity $v_n \approx A\mu \exp{-S\beta a /\mu k T_{surf}}$, where $S \approx$ 1 is a geometrical factor, β is the step free energy at the crystal interface, $a = c^{-1/3} \approx$ 0.32 nm is the molecular size, μ is ice the chemical potential jump at the interface, k is the Boltzmann factor and T_{surf} is the interface temperature. We prefer this particular functional form so that $v_n \approx A\mu$ (the Wilson-Frenkel law) when $\beta \to 0$ and the exponential term goes to unity, although we expect that A may depend weakly on μ.

For growth from liquid water, $\mu \approx a^3 \rho_{ice} L_{sl} T_{surf}/T_m$, giving $v_n \approx K_T T_{surf} \approx$ $A_{sl} T_{surf} e^{-T_0/T_{surf}}$ with $kT_0 = S\beta^2/\rho_{ice} a L_{sl}$ (12, 48). The prefactor A_{sl} is determined by how sl fast liquid molecules diffuse into position to join the solid lattice, which can be roughly estimated by the Einstein-Stokes relation $A_{sl} \approx L_{sl} \rho_{ice} a/6\pi \eta_{eff} T_m$, where η_{eff} is the dynamic viscosity for liquid near the surface. The kinetics of liquid water near an ice surface is nontrivial, and it is possible that η_{eff} may differ from the normal bulk viscosity.

For ice growth from water vapor ,we have $\mu \approx \sigma_{surf} kT$ for $\sigma_{surf} \ll 1$, giving $v_n \approx \alpha v_{kin}$ $\sigma_{surf} \approx A_v \sigma e^{-\sigma_0/\sigma_{surf}}$ with $\sigma = S\beta^2 a^2/k^2 T^2$. For fast kinetics on rough surfaces, we expect $A_{sv} \approx 1$, although this may not be the case if nucleation is not the only factor that limits growth.

☐ ICE / WATER KINETICS —

In many observations of ice growth from liquid water reported in the literature, the growth rates are strongly limited by the diffusion of latent heat generated at the growing crystal surface, and in this case the bath supercooling T_{bath} can be markedly different from T_{surf} at the interface. Because the interfacial temperature T_{surf} is the essential variable governing growth kinetics, for now we restrict our attention to experiments where heat-diffusion effects have been carefully considered and modeled to determine growth velocities $v_n(T_{surf}) = K_T T_{surf}$.

Figure 4 shows an overview of K_T measurements for pure liquid water as a function of supercooling. Growth of the basal surface is generally well described by a 2D nucleation process at small super-coolings. By aligning the ice c axis parallel to the axis of a thin capillary tube, the effects of heat diffusion are relatively small and calculable, allowing the basal attachment kinetics to be extracted directly from growth data . The basal kinetics line in **Figure 4**, given by $K_T = 7.3 \times 10^{-4} \exp(-0.23/T_{surf})$, approximately describes experimental measurements after correcting for heat-diffusion effects.For $T_{surf} > 0.1°C$ the corrections becomes so large that extracting the basal attachment kinetics becomes unreliable for the measurements cited. Using nucleation theory, the basal kinetics curve yields a step free energy of $\beta_{basal,sl} \approx (5.6 \pm 0.7) \times 10^{-13}$ J/m for the basal ice/water interface. This appears to be a robust result, as two independent experiments measured nearly identical T_0 values.
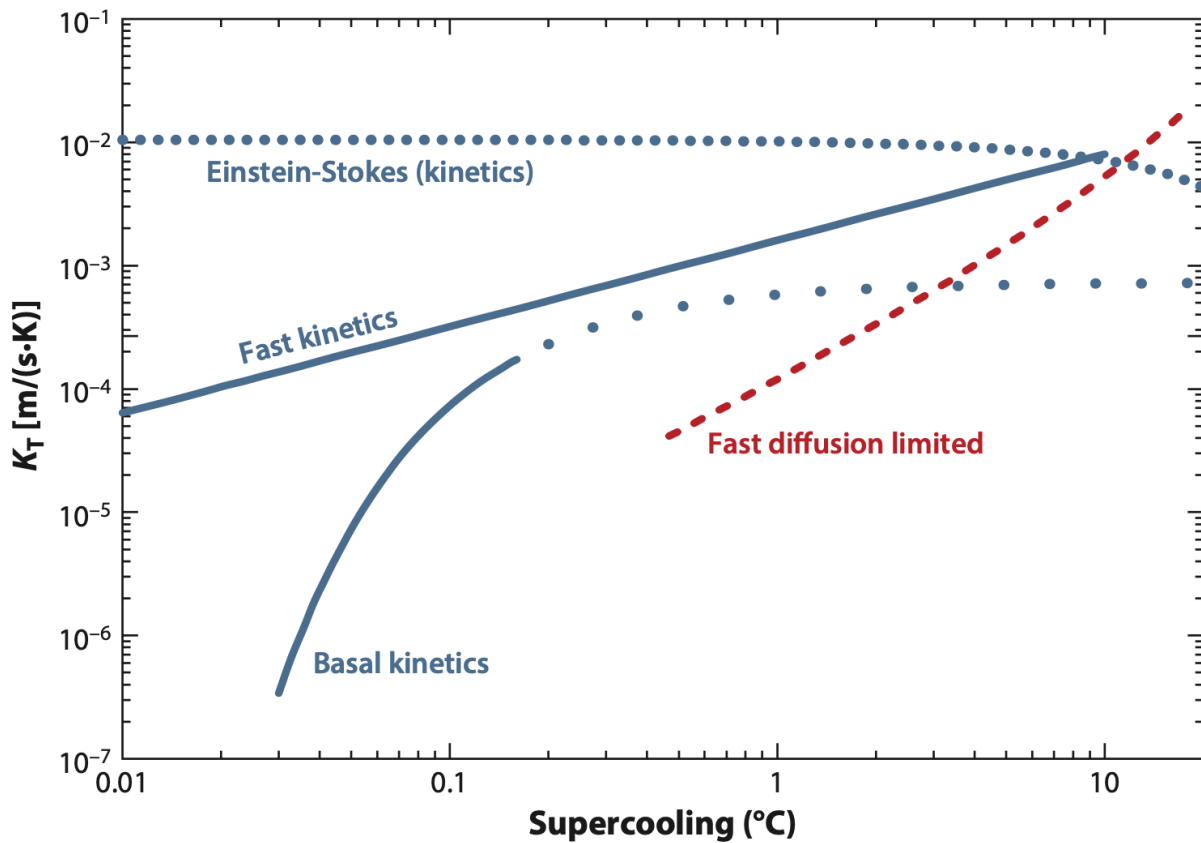
The blue curves in this plot show the kinetic attachment coefficient $K_T (T_{surf})$ for ice growth from liquid water, for the basal facet [labeled basal kinetics] and for fast-growing unfaceted surfaces [fast kinetics]. The thick portion of the basal line derives from ice growth data, and the dotted portion is extrapolated to larger T_{surf}. The other blue dotted

curve shows K_T from Einstein- Stokes theory . For comparison, the red dashed curve shows K_T (T_{bath}) for the tips of growing dendrites . Note that the abscissa in this graph does double duty, equal to either T_{surf} or T_{bath}

The curve labeled fast kinetics refers to measurements of fast-growing ice surfaces in capillary tubes. These data are of substantially poorer quality than the basal growth data, so this curve is only a rough approximation. The growth morphologies were likely needle-like crystals growing along the inner walls of (relatively large) capillary tubes, thus reducing heat-diffusion corrections to some degree. Modeling this process is problematic, however, so we cannot estimate systematic errors arising from residual diffusion effects, although they may be substantial.

Also plotted is $K_T(T_{surf})$ from the Einstein-Stokes model described above, assuming η_{eff} is equal to the measured bulk viscosity of water as a function of temperature. Ther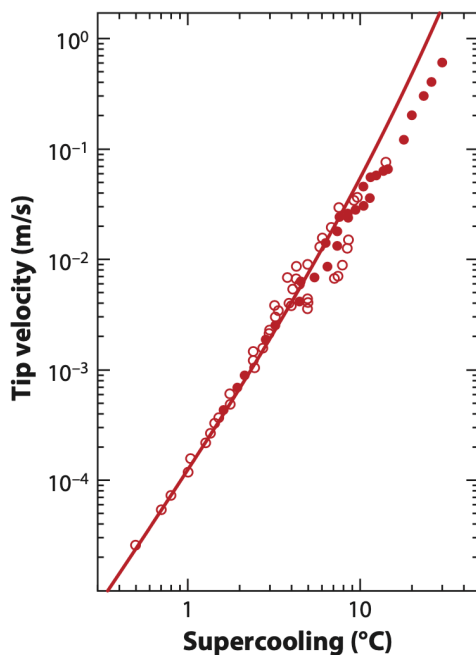e are surprisingly few modern experiments on ice growth rates from which kinetic attachment coefficients can be extracted, leaving much opportunity for improvement in this area. At high growth rates, however, great care must be exercised to separate kinetics-limited growth from diffusion-limited growth, perhaps requiring novel experimental techniques to obtain reliable measurements

Fig. 4

VII APPLICATIONS — AVALANCHE PREDICTION

Avalanches are masses of snow and ice that slide down mountains. In the past, predicting such movements was mostly a matter of guesswork. To make better predictions, scientists are now analyzing ice crystals, studying snow-stability patterns, and looking for connections between weather and avalanche danger. Protecting people from the dangers of avalanches is a major goal of this research.

☐ SNOW CRUNCH —

Snowflakes appear soft and delicate as they fall. But once the icy flakes get packed together on the ground, they can turn into a destructive force of nature.

An ice block that measures 3 feet on each side can weigh 1,000 pounds. The snow in an avalanche measuring 3 feet deep, 100 yards long, and 50 yards across could weigh a crushing 5 million pounds.

As an avalanche moves, it picks up nearly anything in its path, including rocks and other debris that can smack a skier senseless. Once a person is buried, it can be nearly impossible to get out of the snow without help. Suffocation often follows.

Snowflakes are crystals of ice.

Depending on the temperature of the air and ground, snow crystals change size and shape. Some stick. Some don't. Snow can be soft, slippery, wet, or icy, and it can change quickly as weather conditions change.

Experienced skiers know how tricky snow can be. They have various words to describe the types of snow they may run into: sugar, corn, powder, grapple, champagne, foof, mashed potatoes, Sierra cement, and more.

☐ LOOSE SNOW —

Avalanches often occur after storms, when loose snow abounds. Bigger storms present bigger hazards.

The slopes where avalanches are most likely to occur are between 35 and 45 degrees in steepness. Snow slides down steeper slopes before it can settle into place. On flatter slopes, avalanches have a smaller chance of gaining momentum.

Some places are especially likely to have avalanches. Near such areas, ski patrollers dig pits and look for signs that the layers of snow might be unstable. If it is, the patrollers set off explosives to create avalanches before people can do it by mistake.

The avalanches that are hardest to predict occur when there's a wide temperature range within the snowpack.

It might, for example, be –20 degrees Celsius near the snow surface but around 0 degrees C nearer the ground. Heat and water vapor then flow from warmer to colder areas. This destabilizes lower layers of snow.

☐ REAL CAUSE BEHIND AVALANCHES —

First, the snowpack structure needs to be suitable. That means having some sort of weak layer or weak interface in the snowpack. When we have these fairly warm, sunny days in the mountains—below freezing, but the kind of days that everybody likes skiing around—the solar short-wave energy that the sun is putting out can actually penetrate and warm the snowpack, a couple of centimeters deep. Simultaneously, there is a cooling effect that acts right on the surface of the snow, which is next to cold air. Imagine the surface of the snowpack trying to get colder while a couple of centimeters down, the snow is trying to get warmer. So we get this difference in temperature. Nature is constantly trying to even out differences in temperature. Since the original, strong configuration of snow grains is so insulating, heat movement from the warm spot to the cold surface is slow but continuous. This movement of heat within the snowpack also takes water vapor from the crystals in the warm spot. The water vapor flows until it hits the cold spot, and then it condenses out, but as ice, into a new grain of snow.

This rearranges the snow crystals within a 2-centimeter zone. We're not adding any snow; we're not taking away any snow; but we're taking the structure that's there and using vapor movement resulting from that difference in temperatures to rearrange the microstructure of the snow into a more efficient pathway for heat to move. Unfortunately, this also produces a mechanically weaker structure. Then that weak layer is eventually buried by some stronger layer, which we call a slab. The slab is a hard, cohesive layer of snow that can be very large, spanning an entire face of a mountain. And now it's resting on top of a weak foundation. It's like building a house on a foundation of potato chips.

Finally some trigger, like a skier or a snowboarder or a snowmobiler, puts enough force or stress onto the snowpack to break the weak layer, and causes that slab to start riding over the top as it goes downhill. Sometimes a slab avalanche can break several hundred meters wide.

☐ **BONDS BETWEEN SNOW-CRYSTALS —**

Bonds are what join two or more individual snowflakes or snow grains together. If we have two grains of snow that are half a millimeter in diameter each, I might call the area that joins those two grains together, maybe a quarter millimeter in diameter, a kind of bridge or bond. It's all one continuous piece of ice, but there is this restriction or narrowing that is a bonded area. Rounded grains tend to form a cohesive snowpack, and are usually associated with slabs. The orientation of bonds are random in that case, without a preferred orientation. When we get into faceted layers of snow, the bonds can start to arrange themselves in a particular pattern. We think that when the bonds between snow grains are oriented randomly, it is difficult to propagate a fracture through the snow—there is no clear, obvious path within the microstructure. However, in a weak, faceted layer of snow, the patterned alignment of bonds can provide a quick,

straight, and easy path for fractures to propagate, thus leading to total failure of the layer with far less energy. When the snow is easier to fracture, avalanches are much more likely to occur with lighter and more sensitive triggers.

VIII SIMULATION RESULTS

☐ **Abstract :** By using the afore mentioned theoretical models, a simulation model was developed by means of Cellular Automata Models on Wolfram.

☐ **Code Used :**

```
Clear[vertexFunc]
vertexFunc = Compile[{{para, _Real, 1}},
   Module[{center, ratio},
    center = para[[1 ;; 2]];
    ratio = para[[3]];
    {Re[#], Im[#]} + {{1, -(1/2)},{0, Sqrt[3]/2}}.Reverse[{-1,
1} center + {3, 0}] & /@
                (ratio 1/Sqrt[3] E^(I \[Pi]/6) E^(I Range[6] \
[Pi]/3))
    ],
   RuntimeAttributes -> {Listable}, Parallelization -> True,
   RuntimeOptions -> "Speed"
   (*, CompilationTarget->"C"*)];
   Clear[displayfunc]
displayfunc[array_, ratio_] := Graphics[{
   FaceForm[{ColorData["DeepSeaColors"][3]}],
   EdgeForm[{ColorData["DeepSeaColors"][4]}],
   Polygon[vertexFunc[Append[#, ratio]] & /@ Position[array,
1]]
   }, Background -> ColorData["DeepSeaColors"][0]]

stateSet = Tuples[{0, 1}, 6] // Rest;
gatherTestFunc = Function[lst, Sort[RotateLeft[lst, # - 1] & /@
Flatten[Position[lst, 1]]]];
stateClsSet = Sort /@ Gather[stateSet, gatherTestFunc[#1] ==
gatherTestFunc[#2] &];
stateClsSetHomogeneous = ArrayPad[#, {{0, 6 - Length@#}, {0,
0}}] & /@ stateClsSet;
Clear[ruleFunc2Comp]
ruleFunc2Comp = With[{
                    stateClsSetHomogeneous =
stateClsSetHomogeneous,
                    seedSet = RandomInteger[{0, 1000}, 1000],
                    pFreeze = {1, 0.2, 0.1, 0, 0.2, 0.1, 0.1,
0, 0.1, 0.1, 1, 1, 0},
                    pMelt = {0, 0.7, 0.5, 0.5, 0, 0, 0, 0.3,
0.5, 0, 0.2, 0.1, 0}
```
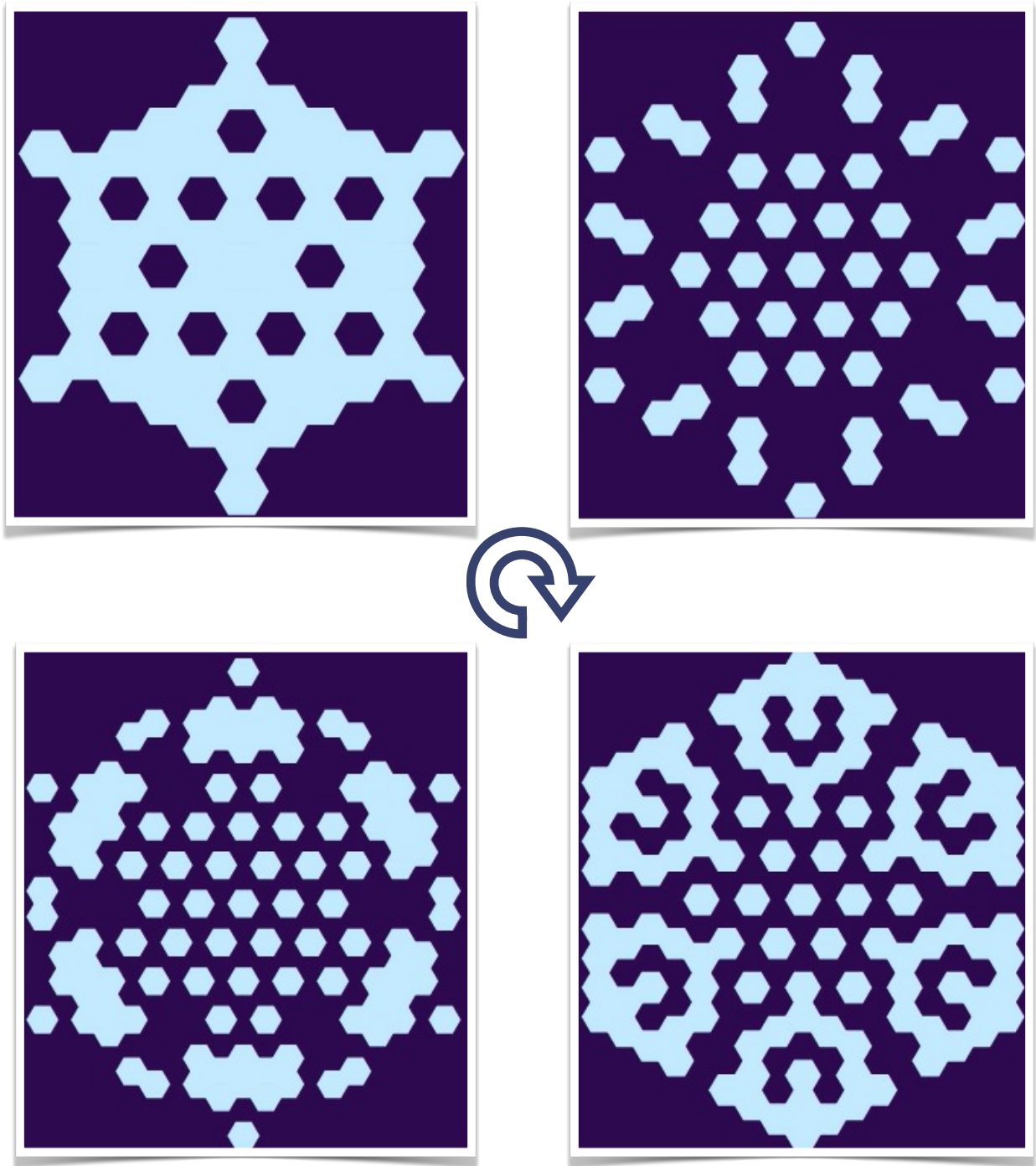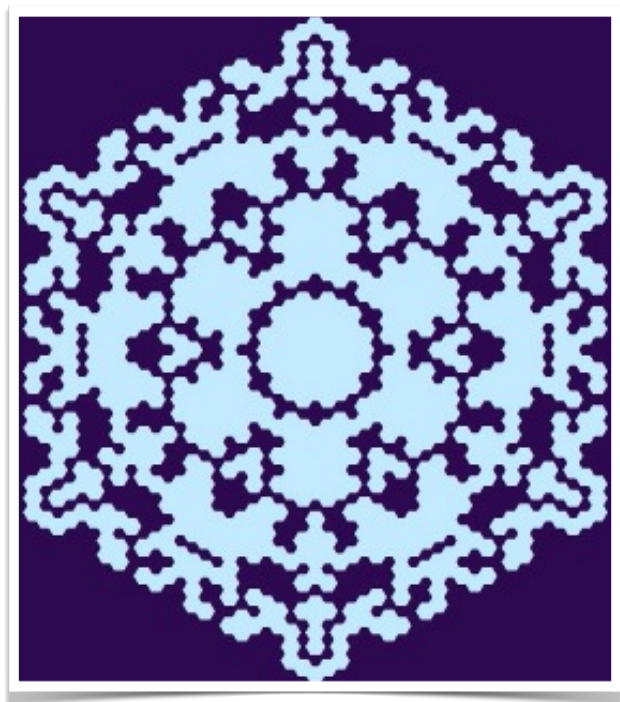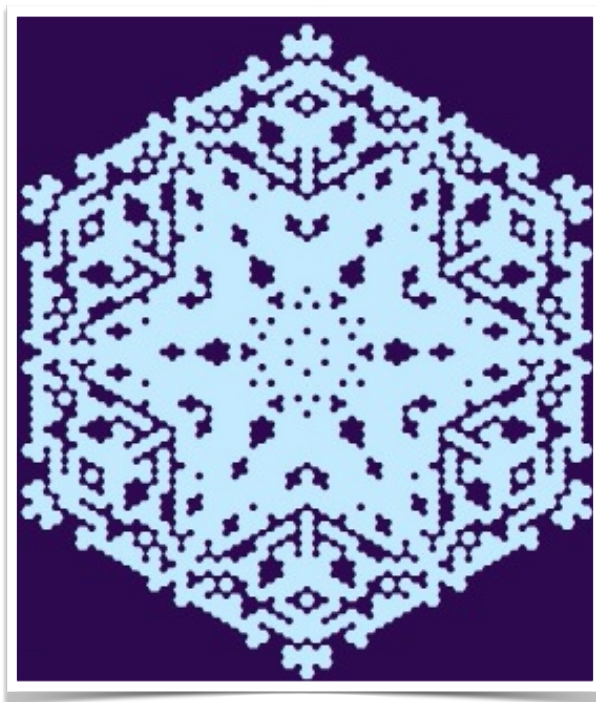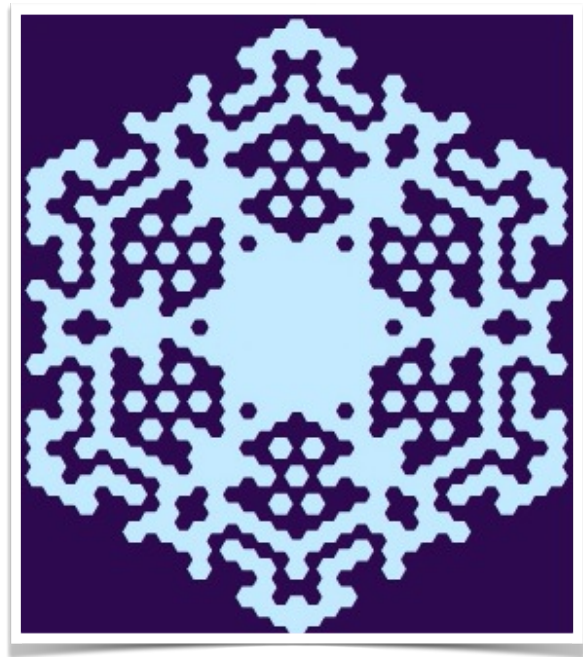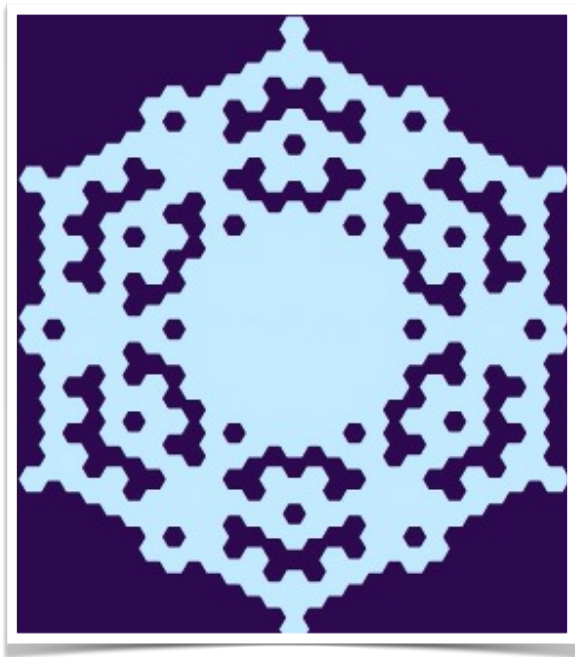
```
                          },
    Compile[{{neighborarry, _Integer, 2}, {step, _Integer}},
              Module[{cv, neighborlst, cls, rand},
                    cv = neighborarry[[2, 2]];
                    neighborlst = {#[[1, 2]], #[[1, 3]], #[[2,
3]], #[[3, 2]], #[[3, 1]], #[[2, 1]]}
&[neighborarry];
                    If[Total[neighborlst] == 0, cv,
                        cls = Position[stateClsSetHomogeneous,
neighborlst][[1, 1]];
                        SeedRandom[seedSet[[step +
1]]];
                        rand = RandomReal[];
                        Boole@If[cv== 0, rand < pFreeze[[cls]],
rand > pMelt[[cls]]]
                      ]
                    ],
          RuntimeAttributes -> {Listable}, Parallelization ->
True, RuntimeOptions -> "Speed"(*,CompilationTarget ->
"C"*)
            ]
    ];
dataSet = Module[{rule,
                  initM = {{{0, 0, 0}, {0, 1, 0}, {0, 0, 0}},
0},
                   rspec = {1, 1},
                   tmin = 0, tmax = 50, dt = 1
                  },
                  rule = {ruleFunc2Comp, {}, rspec};
                  CellularAutomaton[rule, initM, {{tmin, tmax,
dt}}]
                  ];
Manipulate[
    Rotate[displayfunc[dataSet[[k]], .99], 90\[Degree]],
    {k, 1, Length[dataSet], 1}]
```
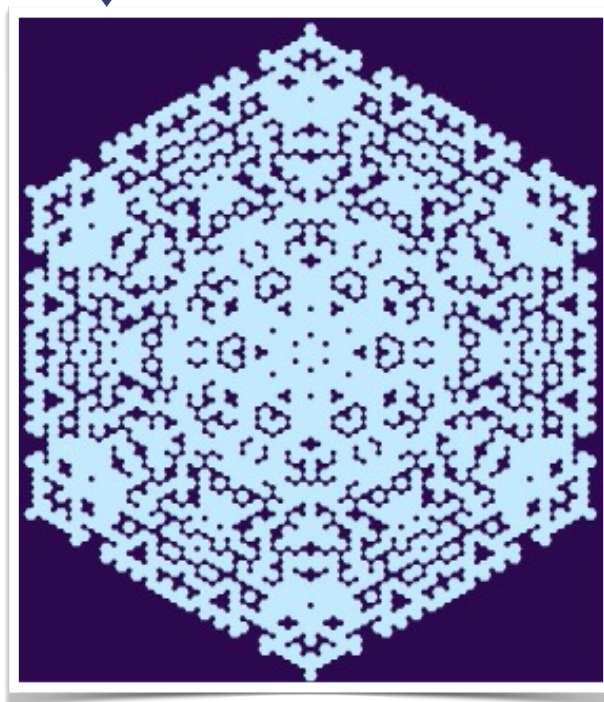
☐ **SIMULATION RESULTS :**

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Fig. 13.

the oth

ese pha

slow time scales.

jump-up and jump-down pieces correspond to solutions o

ns. These are obtained by simply setting $\epsilon = 0$ in (24). The

ns are:

$$V' = Y$$
$$Y' = cY - f(V) + W$$
$$W' = 0$$

at W must be constant along this solution. For the jump-up (

$V \equiv 0$ and look for a solution of the first two equations of

A traveling wave solution of (7) is a solution of the form $(v(x,t), w(x,t)) = (V(z), W(z))$, $z = x + ct$; that is, a traveling wave solution corresponds to a solution that propagates with constant shape and velocity. The velocity is c as in not known. We also assume that the traveling wave solution satisfies the boundary conditions $\lim_{z \to \pm\infty}(V(z), W(z)) = (0, 0)$.

We may note that a traveling wave solution corresponds to a solution of the first system of ordinary differential equations

$$V' = Y$$
$$Y' = cY - f(V) + W$$
$$W' = \frac{\epsilon}{c}(V - \gamma W)$$

together with the boundary conditions

$$\lim_{z \to -\pm\infty}(V(z), Y(z), W(z)) = (0, 0, 0)$$

Hence, a traveling wave solution corresponds to a homoclinic orbit of a first o system. This homoclinic orbit will exist only for special values of the velocity c.

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