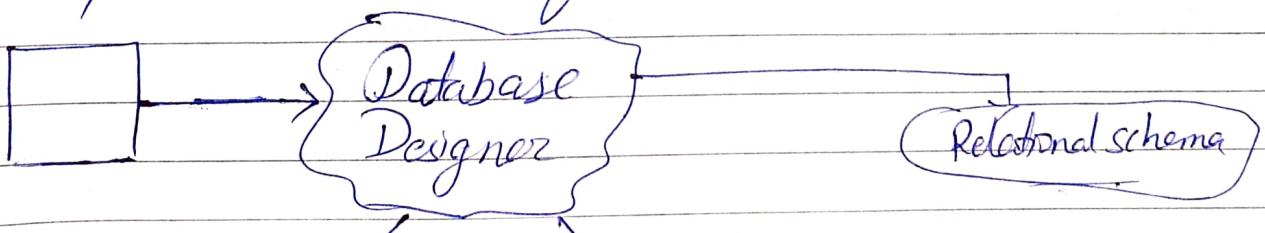


Entity-Relationship Model.

- Design Process
- Data Modelling
- Modelling of the constraint
- Entity Relationship diagram.
- Design issues
- Weak Entity.
- Extended Entity Relationship features.
- Design of a bank database.
- Reduction to Relational Schema.
- Database Design.

* Design Process *

- First principle - To understand the actual need of the end user.
- Second principle - Data model needs to represent the enterprise into a set of relational schema.
- Third Principle - Understand the functional requirement of the enterprise.



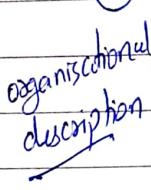
Two diff. kinds of decision → Business Decision. (what attributes one needs to store)

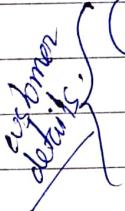
Business Decision → Computer Science Decision (How we can distribute the attribute across the relational schema.)

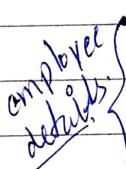
Physical design (tells us about the physical layout of the database).

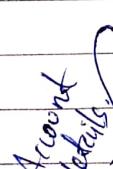
- Data structure to be used for storage
- Block size for data transfer and so on...

* Enterprise Description (Bank).

- ①  Bank is organised into Branch.
Each branch is identified by a unique name.
Bank monitors its assets.

- ②  Customers are identified by the customer-id value.
Customer may take loans.
Customer is associated with a banker.

- ③  Employees are identified by Emp-id value.
Other information regarding the employees.

- ④  Two types of account
(a) Saving Account
(b) Current Account.
Account operation details, a customer can have more than one account.
Each account is maintained by unique account no.

All account details.

- Balance & most recent date of use
- Saving account has an interest rate.

loan
details

- (5) → A loan can be held by one or more customers
- Identified by a unique loan No.
 - Bank keeps track of loan amount & loan payment details.
 - Loan payment No does not uniquely identify, what is called as a particular payment.
 - date & payment.

The final goal is to come up with a set of relational schema.

We have to adopt a key design approach.
To use a data model for modelling the data.

* Entity Relationship Model

- Models an enterprise as a collection of entities and relationships.

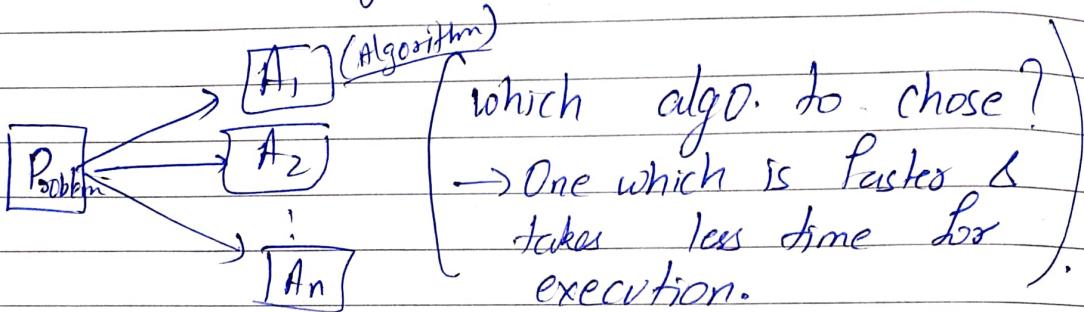
(a) Entities → 'thing' or an 'object' in the enterprise that is distinguishable from the others.

Eg. - Employee, loan, account, customer, branch etc.

→ Each entity is described by a set of attributes.

(b) Relationship → It is an association among several entities.

This model represents an enterprise at a logical level by a diagram. This is called as Entity-Relationship diagram (E-R diagram).



- ① Entity Set
- ② Relationship Set.
- ③ Attribute.

* Entity Set → Set of entities of similar kind.

Eg. Set of Person, Set of company.

Instructor (Ins_id, name, dept_name, salary).
Course (course_id, title, credit)

Fig-1

Instructors		Students	
Ins_id	Name	Roll-No	Name.
11235	John	151CS2019	Soham

advised by
↓
relationship.

→ "Advised-by" is a relationship associated with two Instructor entities Instructors & Students.

A relationship is a mathematical relation with ~~two~~ $n \geq 2$ many entities.

$$\{ (e_1, e_2, \dots, e_n) : e_1 \in E_1, e_2 \in E_2, \dots, e_n \in E_n \}$$

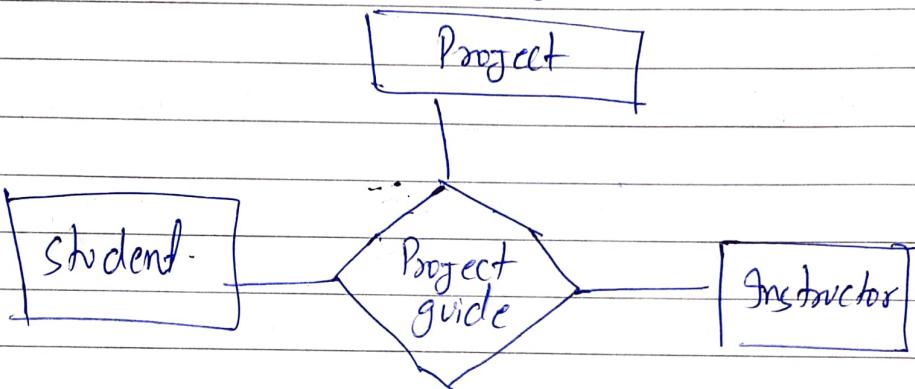
In Fig-1 $(11235, 151CS2019) \in \text{Advised by}$

Instructors		Student		
Ins_id	Name	advised by	Roll-No	Name
11	x	14 th Sep	111	abc
12	y		112	abcd
13	z		113	abcde
14	q		114	abcdef
15	b			

An additional attributes may be associated with relationship.

- Degree of Relationship → defined as no. of different kinds of involved in the relationship.

- 1) Binary Relationship → Involves two diff. kinds of entities.
- 2) Ternary Relationship → It is an association among three entities.



Hence, (student, project & instructor) are entity and project-guide is relationship.

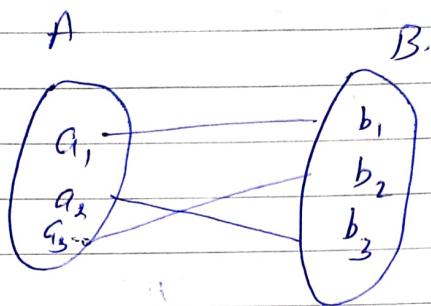
* Mapping the cardinality constraints

⇒ Express the no. of const entities to which another entity can be related / associated via a relationship set. It is useful for describing binary relation.

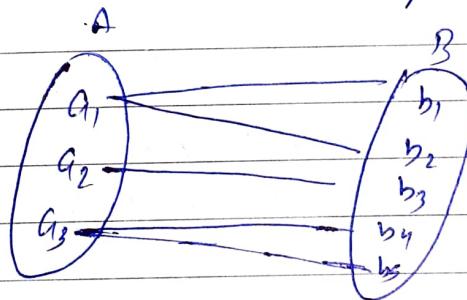
→ Cardinality Constraints.

- (1) One to One
- (2) One to Many
- (3) Many to One
- (4) Many to many.

* One to One

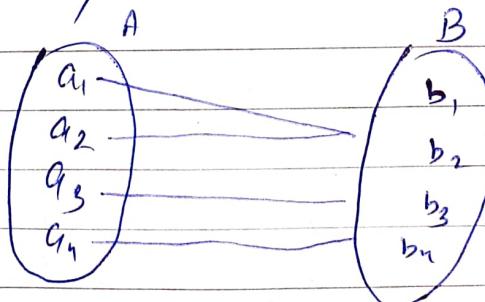


* One to Many.

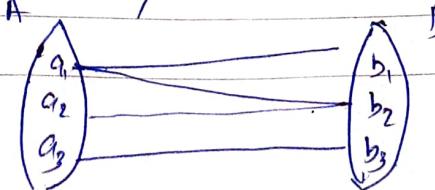


In this relationship, one occurrence of an entity relates to many occurrences in another entity.

* Many to One



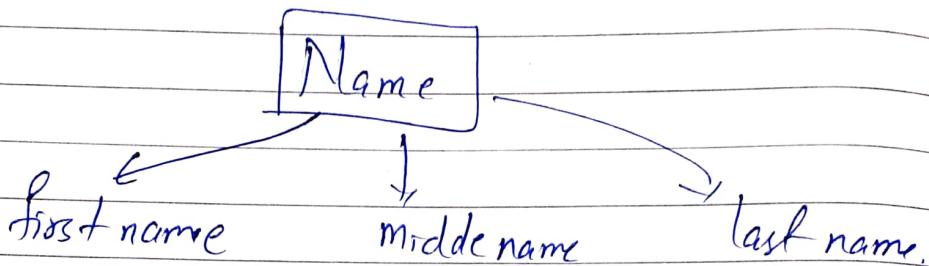
* Many to Many.



* Attributes :-

- 1) Simple v/s Composite Attribute.
- 2) Single valued v/s Multivalued Attribute.
- 3) Derived Attributes.

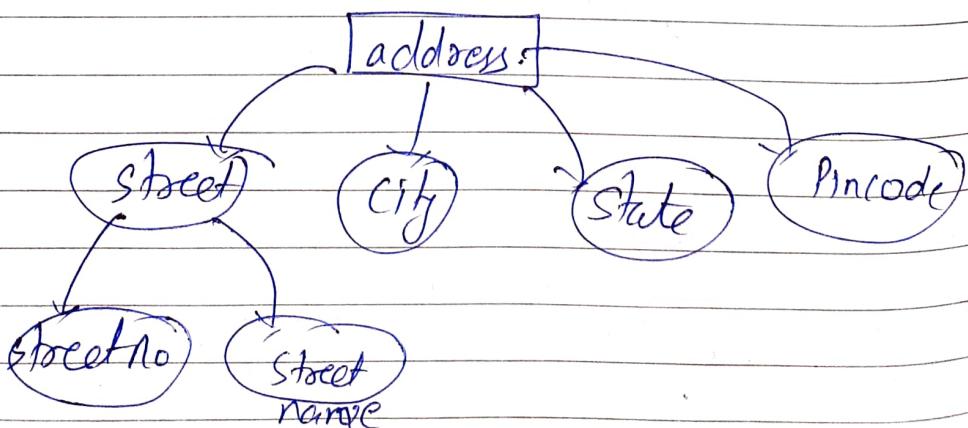
- Composite Attributes can be splitted into simple attributes.



→ It can be splitted into components.

- Simple Attribute → An attribute that can not be subdivided into components.

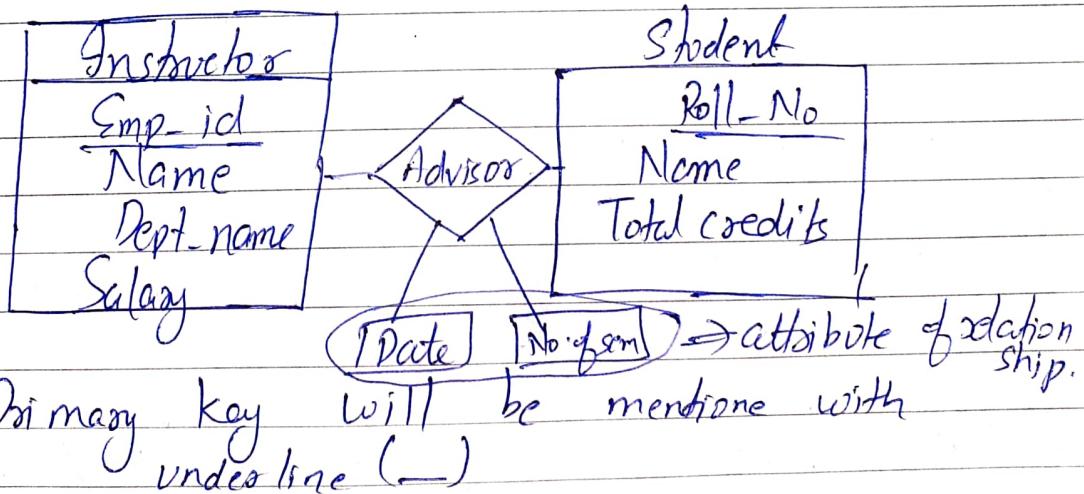
* Hierarchical Attributes



- * Derived Attributes → That can be derived from other attributes.
eg. total & average marks of student.
- * Single Valued → Attributes which take up only a single value for each entity instance. Eg. age of student
- * Multi-valued attribute → Attributes which take up more than a single value for each entity instance.
Eg. Phone Number of a student.

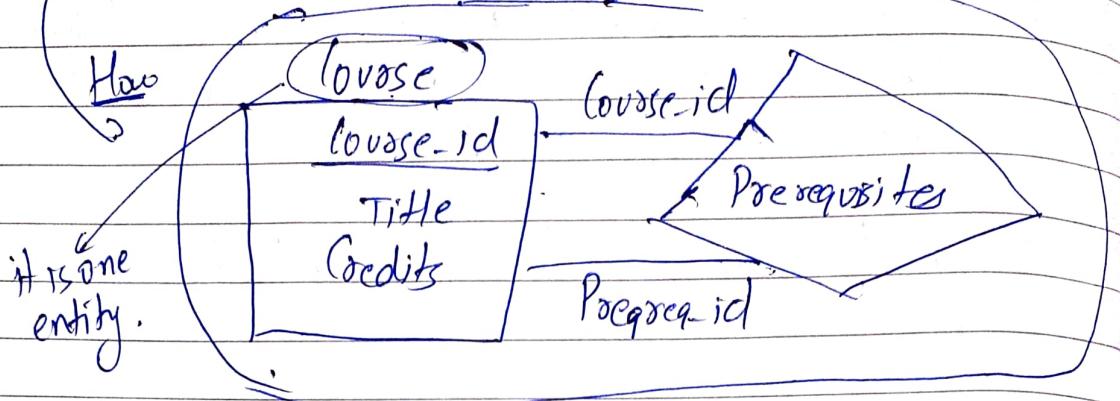
* Entity Relationship Diagram

- * Entity sets → Entities are represented as by rectangles. Their attributes are listed on in each line.



Relationship : \rightarrow It is represented by diamond.

Entities involved in a relationship
need not be distinct.



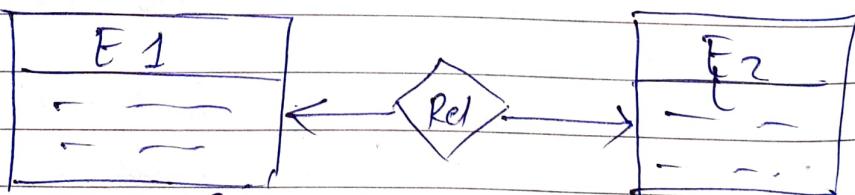
This type of entities relationship are called binary relationship.

* Cardinality. Constraint (Sign).

$\circ \rightarrow$ (One).

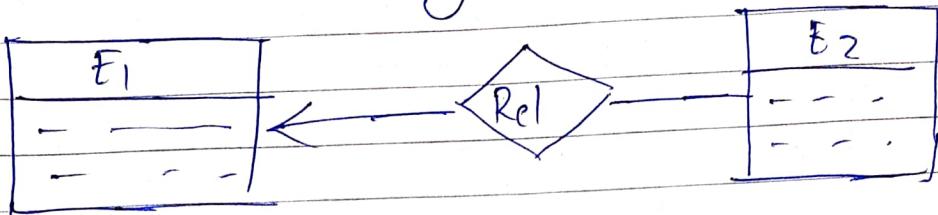
$\circ -$ (many).

* $\circ \triangleright$ One to One.



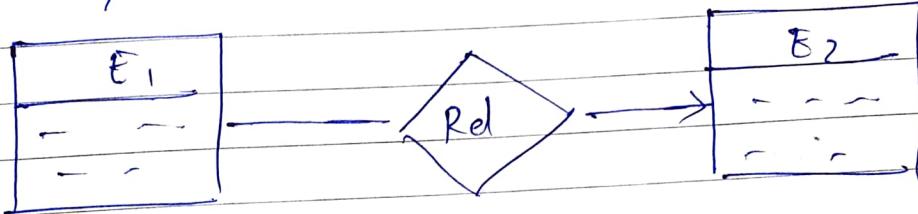
One entity from E₁ is involved with one entity from E₂ and vice versa.

2) One to Many.

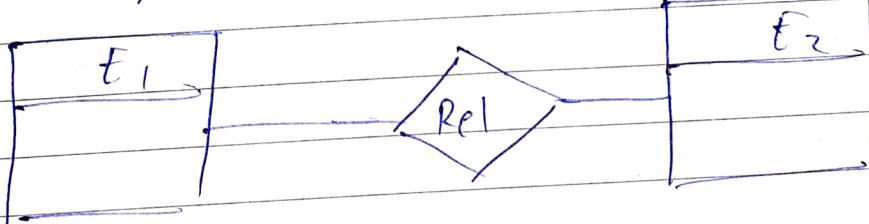


One Entity from E_1 is involved with many entity from E_2 .

3) Many to One



4) Many to Many.



* How to represent participation of diff. entities in an entity set in ER diagram.

→ Two types of participation



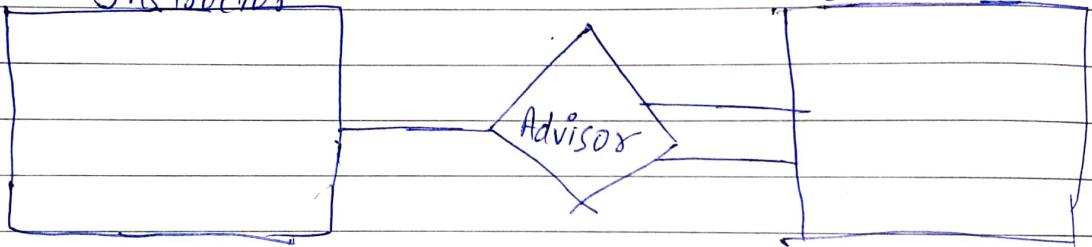
Total Participation

Every entity in entity set participates in at least one relationship

Partial participation

Instructor

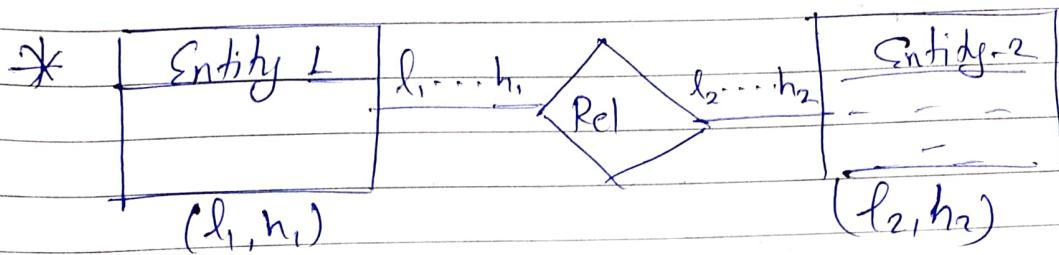
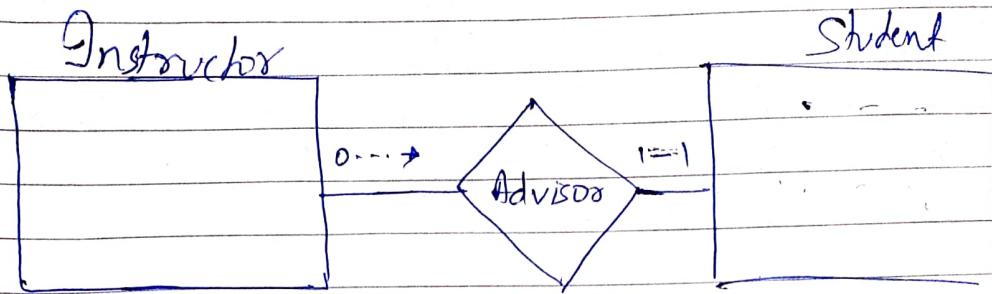
Student



Every student must have an advisor however an instructor may not advise any student

* More complex constraints.

→ Instructors can advise zero(0) or more students. A student must have one(1) advisor and cannot have multiple advisor.

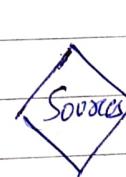


if the upper limit is not known, then it is represented by * (*)

<u>Instructor</u>	
Name	Emp-id
first_name	
middle_name	
last_name	
Address	
Street	
Street_no	
Street_name	
City.	
State	
Phone_No	
date of birth	

- Weak Entity Sets.
- ① Course ↗ (course_id, year)
 - ② Sections ↗ (section_id, sem)

<u>Course</u>	
course_id	
course_name	
Emp-id	



<u>Section</u>	
Sec_id	
Semester	
Year	

Weak Entity → If the attributes associated with an entity in an entity set is not sufficient to uniquely identify an entity, then this is called as weak entity.



Section

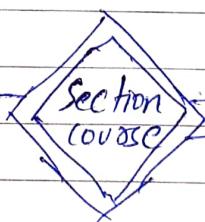
sec_id
semester
year

* Representation of a weak Entity in E-R diagram.

→ A weak entity is represented via a double rectangle.

→ The relationship of weak entity with identifying strong entity is depicted by diagonal.

Course
Course_id
title
credits



Section
sec_id
semester
year

ER diagram of a Balancing System.

- ① Bank has got customers
- ② Bank has got branches
- ③ Branches are identified by name, branch-no, address
- ④ Customers are identified by customer-id, phone-no, address
- ⑤ One customers may have more than one account
- ⑥ Customers can avail loans
- ⑦ Loans are identified by loan-id, loan-type & amount.
- ⑧ Accounts & loans are related to a particular branch.

Ans- Step-1: Identify the entities.

Branch, Bank, Customer, loan, Amount.

These are entities.

Step 2: Identify the relationship among the entities

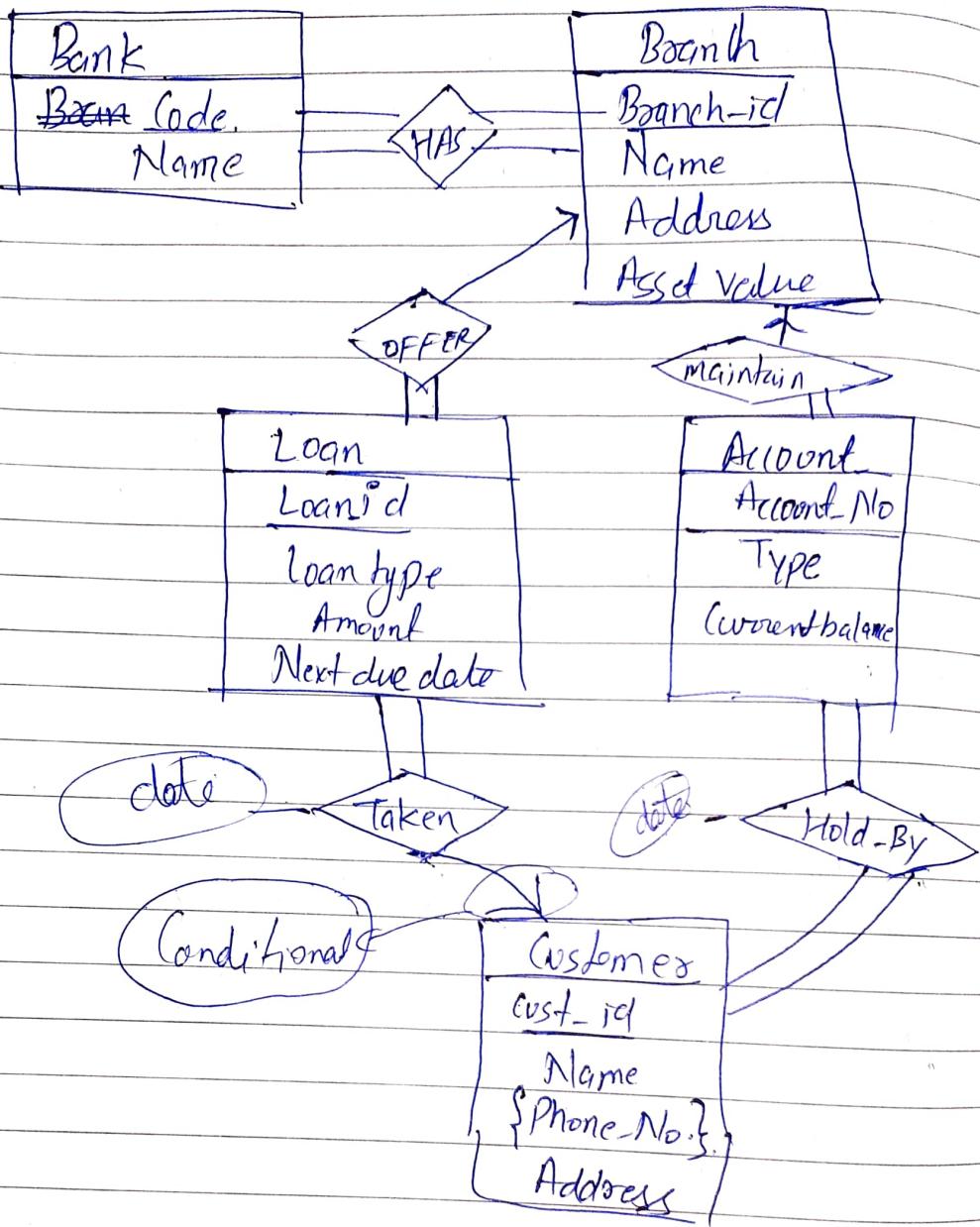
Entity 1	Entity 2	Relationship
Bank	Branch	HAS
Branch	loan	MAY OFFER
Customer	loan	Taken
Customer	Account	Hold by
Account	Branch	Maintain

* Add one more column to represent cardinality & participation constraints.

Date _____

Page _____

Step-3: Apply the rules of ER diagram to connect entities with the relationship.



* ER diagram of a university.

- ① Divided into different departments.
- ② Department offers courses.
- ③ Instructors and students belong to different departments.

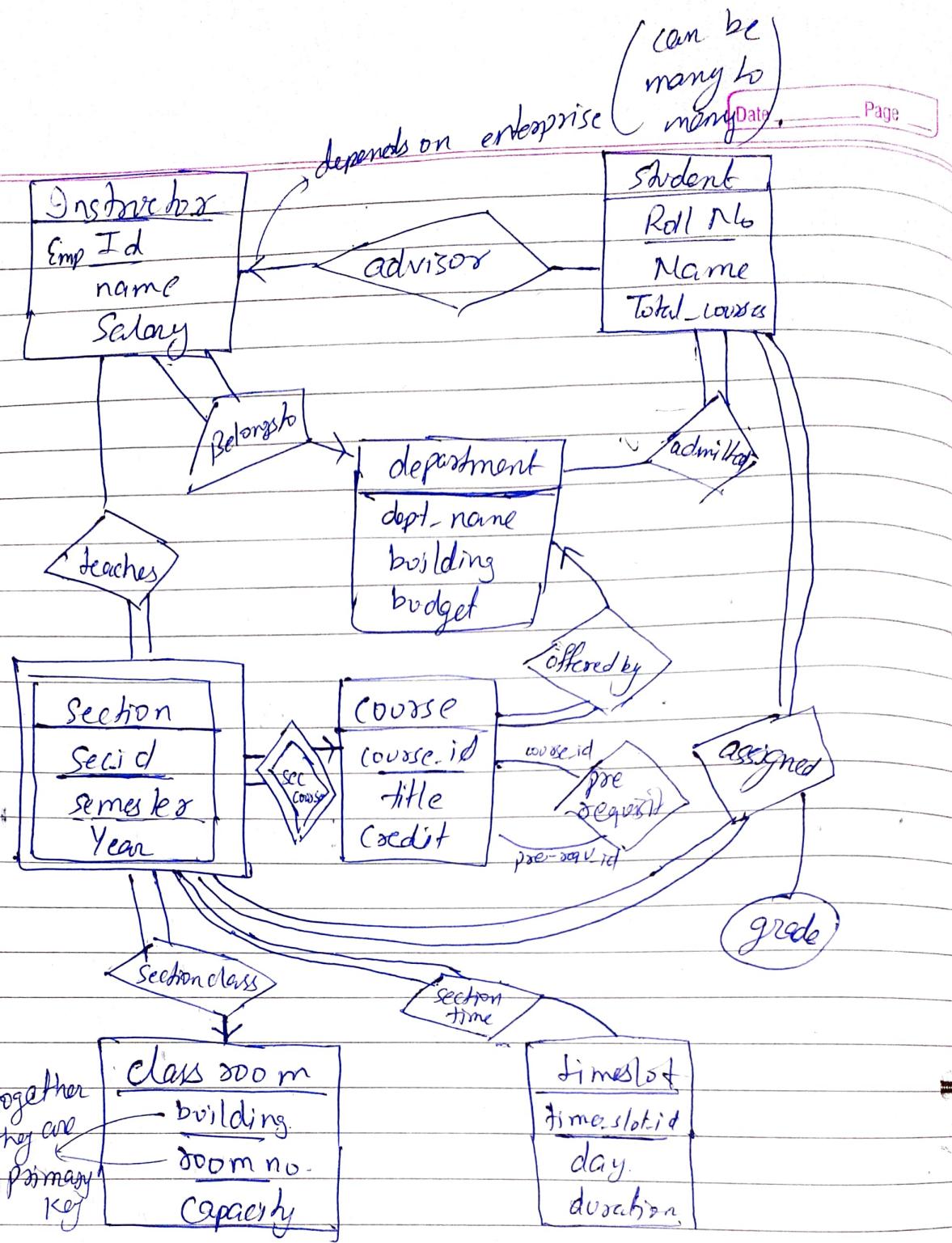
- (4) classes are divided into different sections
- (5) different sections are into different time slots & class-room.
- (6) different courses have diff. pre-requist requirements.

Ans:- Step 1: Identify the id entities.

Department, Instructors, students, sections, time-slots, class-room, courses.

Step -2: Identify the relationship among the entities.

Entity -1	Entity -2	Relationship.
Department	Instructors	Belongs To
Department	Students	Adm P Hld
Instructor	Students	Advised by
Instructor	Section	teaches.
Section	Course	Sec. course
Department	Course	Dep offered by.
Sections	class-room	section-class.
section	time-slot	section-time.
Student	Section	Assigned.
COURSE		pre-requist.

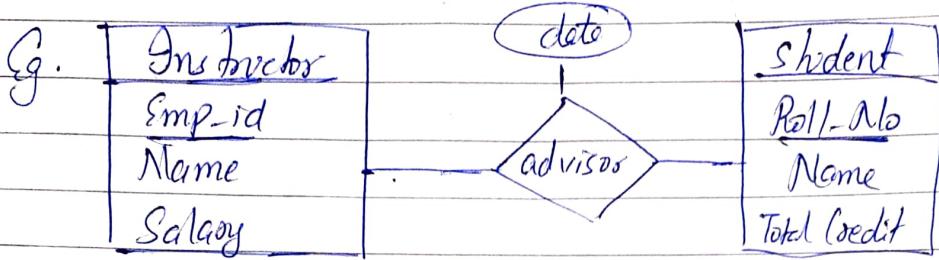


* Reduction to Relational Schemas

→ A database which conforms to an E-R diagram can be represented by a collection of relational schemas.

Representing Relationship

- As many to many relationship is represented as a schema with the following attributes:
 - Primary key for both entities.
 - Attribute associated with the relationship itself



Advisor (Emp-id, Roll-No, date)

⇒ How to deal with multi-valued attributes? → like Phone No?

Ans → A multi-valued attribute for an entity E is represented as a separate schema. It will be corresponding to primary key of E and an attribute corresponding to multivalue attribute M.

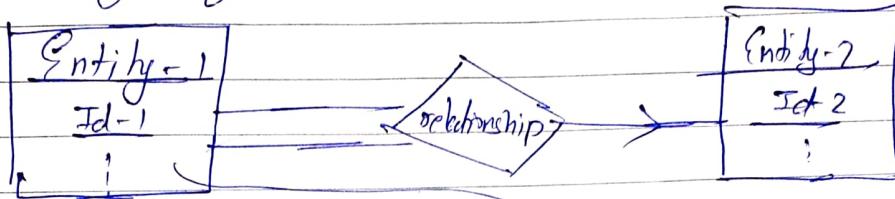
inst-phone = (Emp-id, Phone Number)

Eg: An instructor with Emp-id 122 has phone No abc & xyz

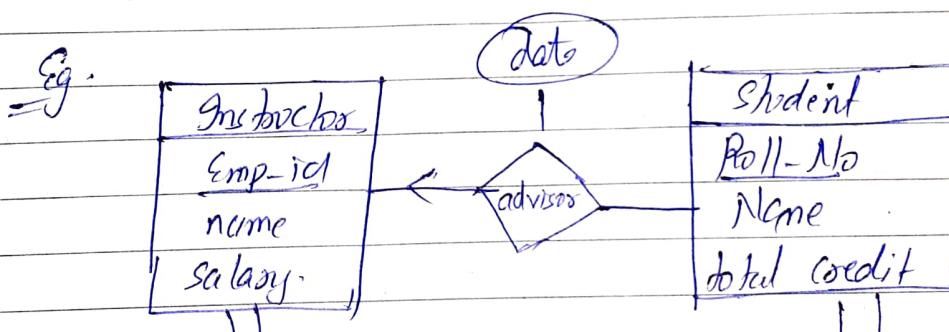
(122, abc), (122, xyz) ∈ inst-phone

- Many to One / One to Many.

⇒ Many to One or (One to many) relationship that are total on many side can be represented by adding an entire attribute to many side containing the primary key of "one" side.



relationship { Id-1, Id-2, ... }



Department
Dept_Name
building
budget

These dept. name
will act as a
foreign key

Rule-1

Instructor (Emp_id, Name, Salary)
Student (Roll_No, Name, total_credit, dept_name)

Department (Dept_Name, building, budget)

Advisor (Emp-id, Roll-No, date)

* One to One

→ for one to one relationship either side can be chosen as many and act accordingly.

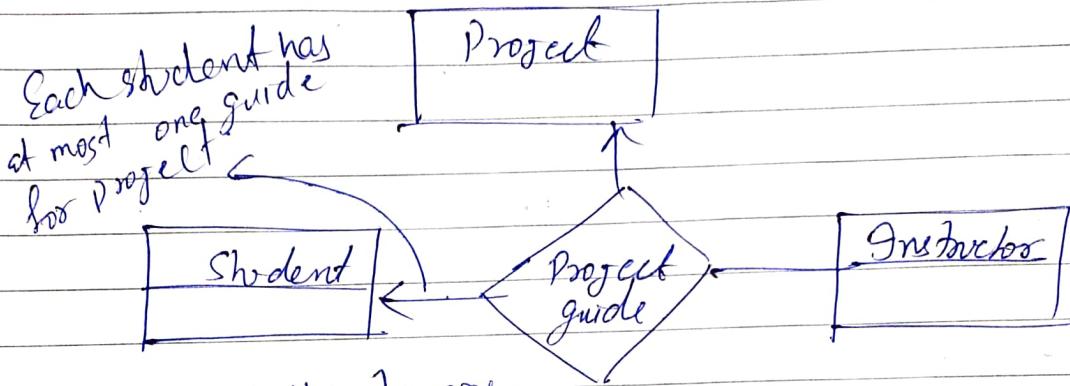
→ An extra attribute can be added to many side entity.

* Draw the relational schemas of university data base.

- ① Student (Roll-No, name, total credit, dept-name)
- ② Instructor (Emp-id, Name, Salary, dept-name)
- ③ Department (Dept.Name, building, budget)
- ④ Advisor (Roll-No, Emp-id, Date)
- ⑤ Section (Louse-id, sec-id, semester, year)
- ⑥ Course (course-id, title, credit, dept-name)
- ⑦ Teaches (Emp-id, course-id, Sec.id, Semester, Year)
- ⑧ Assigned (Roll-No, course-id, sec-id, semester, year, grade)
- ⑨ Time slot (timeslot-id, day, duration)
- ⑩ Prequisite (course-id, Preerequisite-id)
- ⑪ Class room (building, Room-No, capacity)

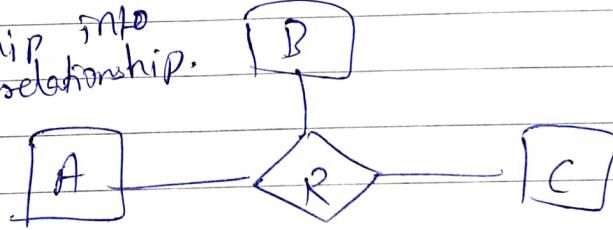
* Ternary Relationship

Though most of the relationship that we encounter in ER diagram are binary, still in many situations it is more convenient to represent relationship as non-binary (in particular ternary).



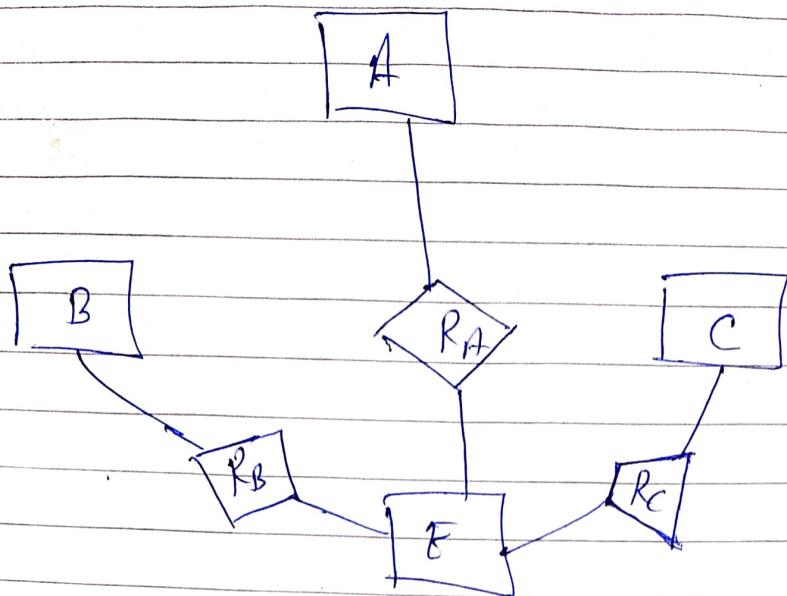
→ Break this ternary

relationship into
binary relationship.



→ Replace **R** by an entity set **E** with 3 relationship sets.

- R_A relating E and A
- R_B relating E & B
- R_C relating E & C .

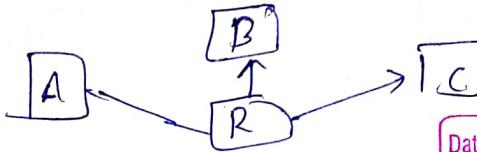


Ternary Relationship is broken into three binary relationships.

→ Steps for conversion.

- ① Create an identifying attribute for entity E and all the attributes of the relationship R are added to E.
- ② For each relationship $(a_i, b_i, c_i) \in R$,
 - a new entity (e_i) in the entity set E is created
 - add $(e_i, a_i) \in R_A$
 - add $(e_i, b_i) \in R_B$
 - add $(e_i, c_i) \in R_C$

We allow at most one arrow in a ternary relationship to indicate a cardinality constraint.

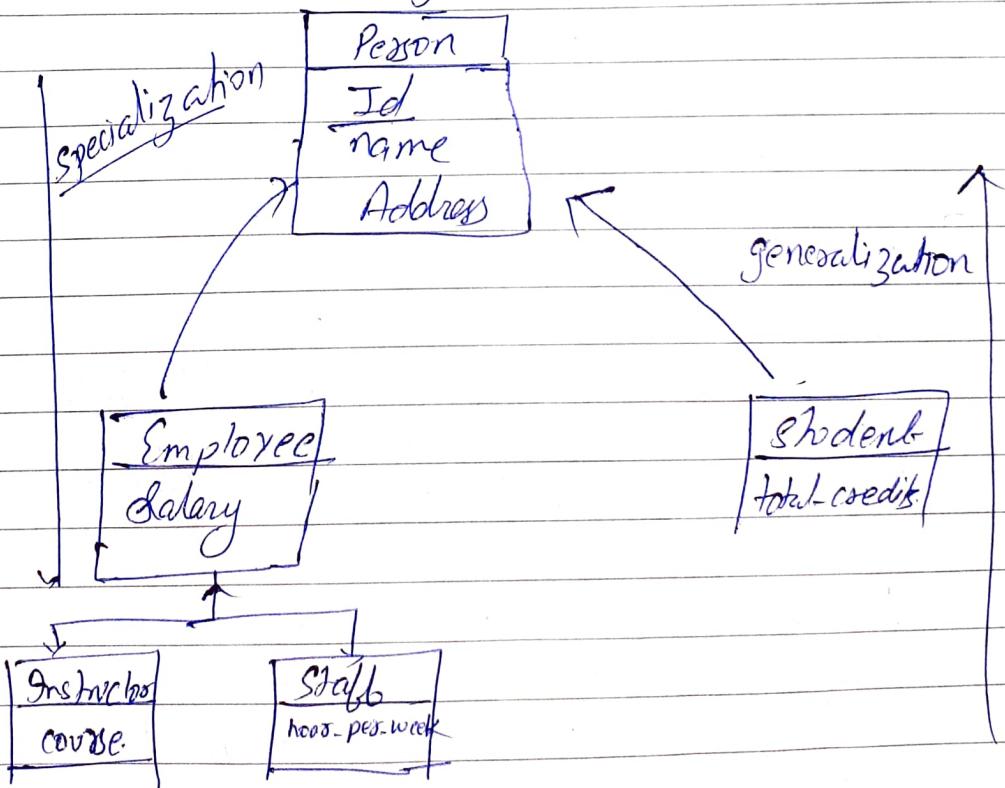


Interpretation -1: Each entity is associated with a unique entity from B or C.

Interpretation -2: Each pair of entity from (A,B) is associated with a unique entity from C. Similarly (A,C) is associated with an entity from B.

* Specialization & Generalization *

In the top down design process, we often may allow subgrouping within an entity set itself.



Attribute Inheritance → A lower level entity set inherits all the attributes of the higher level entity set to which it is linked.

* Overlapping & Disjoint

⇒ Employee & Student are overlapping as student who is doing PHD is an employee as well as student	Instructor & staff are disjoint
----------------------------------------------------------------------------------------------------	---------------------------------

Q How to convert generalization & specialization to relationship schema?

- Ans → (i) Form a schema for the higher level entity
 (ii) Form a schema for each lower level entity set and "include primary key of higher level entity set and the local attribute."

Person (ID, name, address)

Employee (ID, salary)

Student (ID, bal credit)

Instructor (ID, course)

Staff (ID, hours per week)

Advantages & Disadvantages (Easy to handle)

* Draw relationship schema for banking database.

Bank (code, name, address)

Branch (Branch_id, name, address, asset_value)

Loan (loan_id, loan_type, amount, ~~branch_id~~)

Account (Acc_No, type, balance, Branch_id)

Customer (cust_id, name, address)

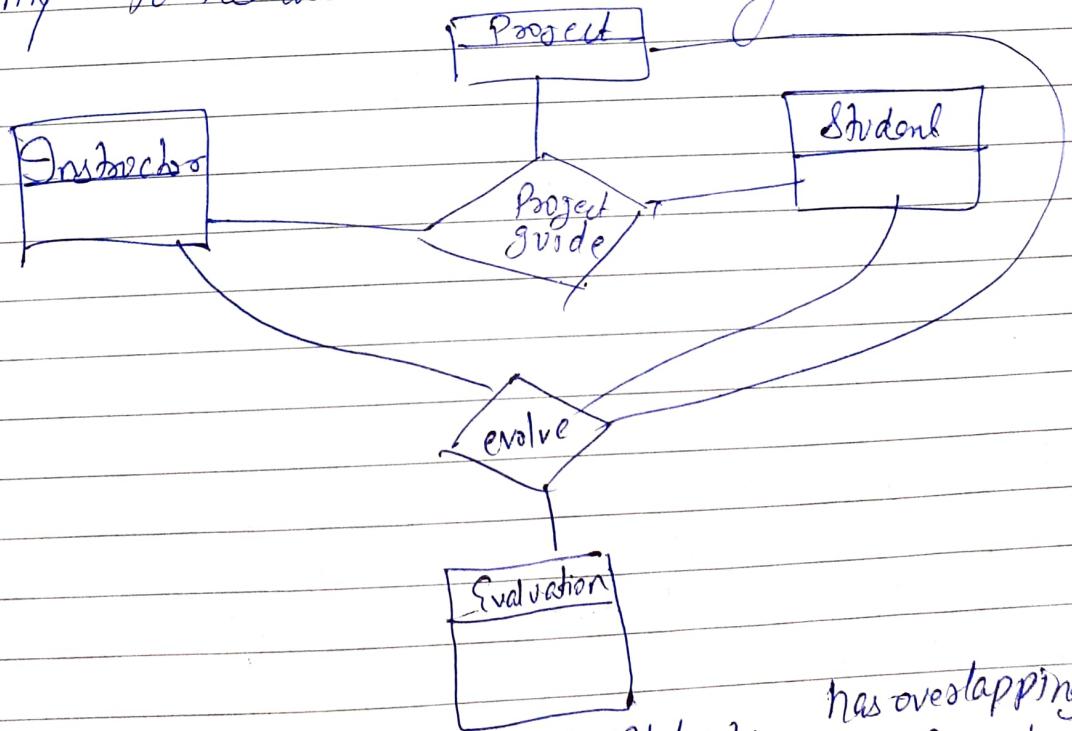
Taken (cust_id, loan_id, date)

Held By (cust_id, account_No, date)

Has (code, branch_id)

* Aggregation \Rightarrow

Only to reduce data redundancy.

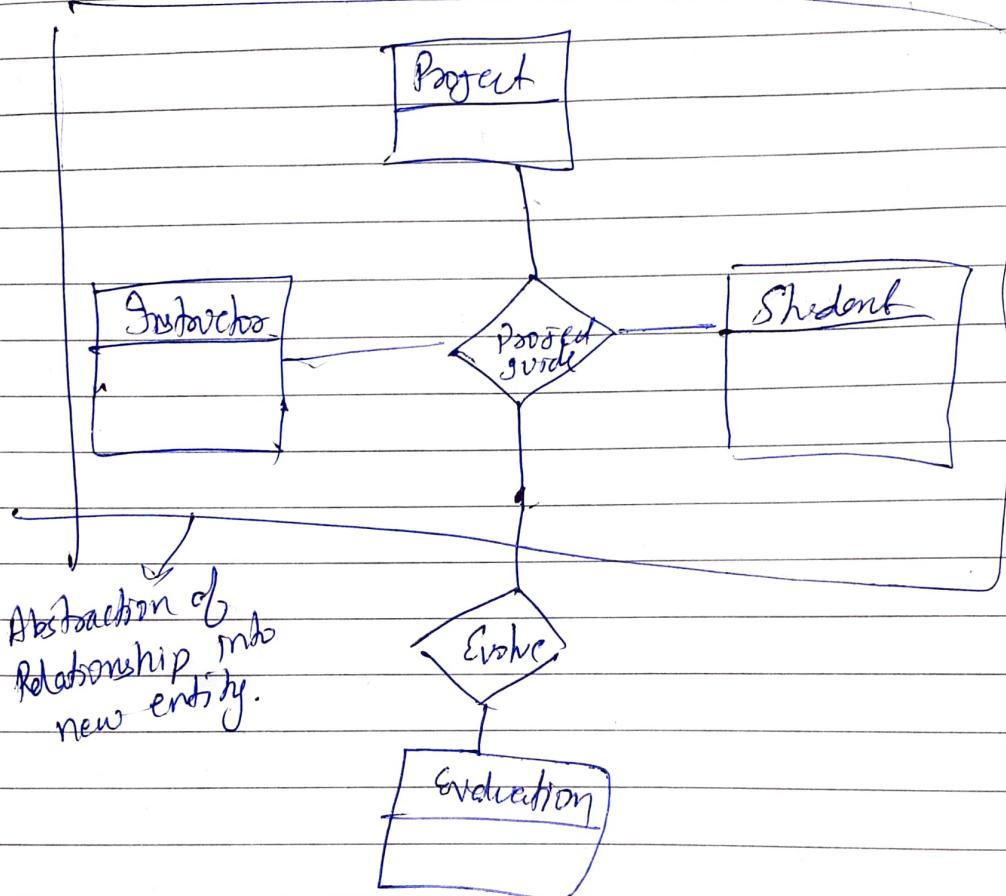


evolve & Project guide Relationship has overlapping information.
no nols.

Eliminate this redundancy via aggregation

Choose one from following steps \Rightarrow have to be followed to reduce data redundancy \Rightarrow

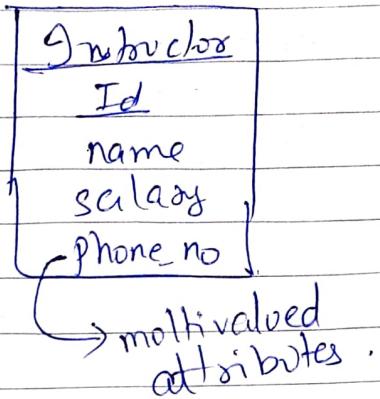
- 1) Treat relationship as an abstract entity
- 2) Allow relationship b/w relationships.
- 3) Abstraction of Relationship into new entity



`Evolve (Student_id, Inst_id, Proj_id, Ev_id, grade)`

Here Project guide data is redundant.
So we will not draw any schema for project-guide.

* Entities v/s Attributes



↓
Here we will create one entity of (phone_no & id) to reduce redundancy.

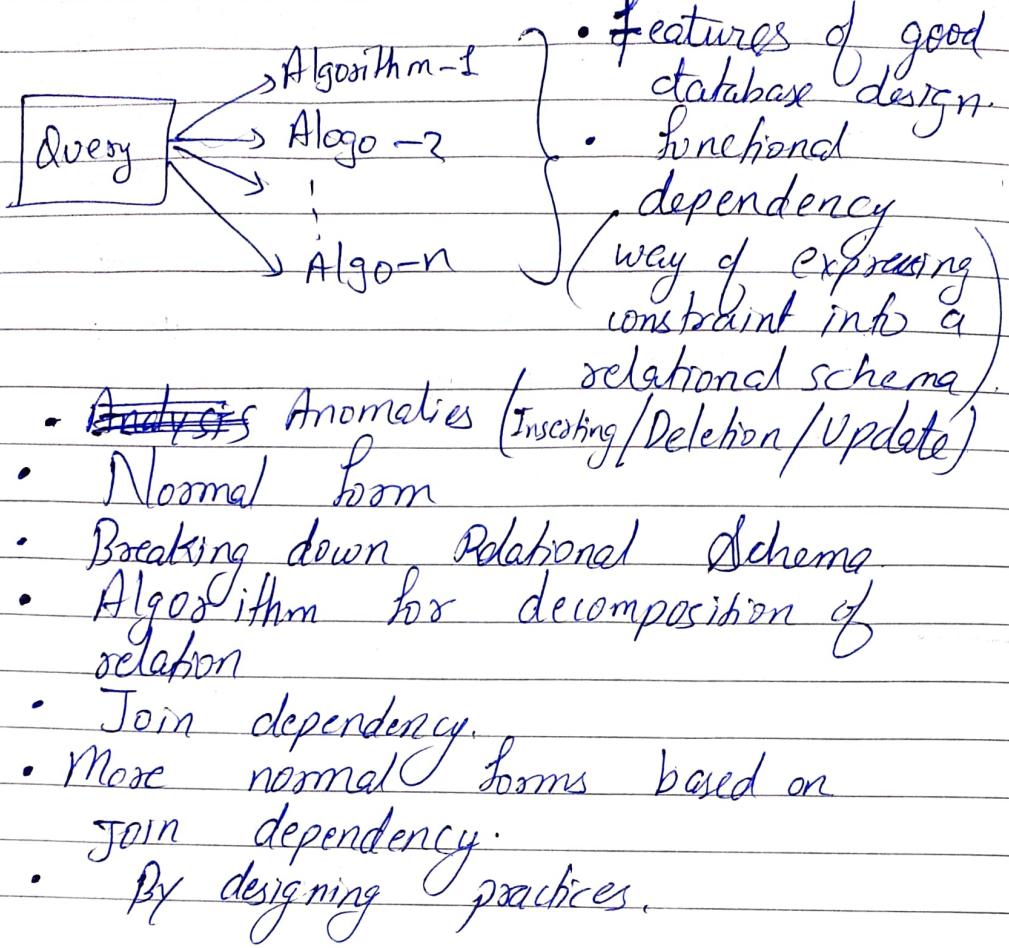
↑
Phone_No (Id, Phone_no).

Module -3

Functional Dependency & Database Normalization

Date _____

Page _____



* Employee (E-name, Aadhar_id, address, D.no)
 * Department (D-name, D.no, Manager_Aadhar_id)

Emp-Depar† (E-name, Aadhar_id, address, D.no,
 D-name, Manager_Aadhar_id)

→ What are advantages & disadvantages
 → Advantages

↳ in operation where we have to
 merge two tables (Employee & Department)
 we will be saved from natural join.

→ Disadvantages.

(i) Replication of same information
 ↳ data redundancy will increase.

(ii) Interpretation of NULL is different
 & difficult. it can be data not provided, data not available, etc. NULL values also occupy space.

Emp - Depart						
E-name	Aadhar-id	address	D-No	D-name	man-Aadhar-id	
John	xxx 3456	Bihar	D1065	Bio-Eng.	xxx abcd	
NULL	NULL	NULL	D1066	Arts	xxx 1304	

Query. → Suppose a new department has been created. with following details of (D-No, D-name & Man-Aadhar-id).

→ since Aadhar-id is a primary key, it can be null so it will be a ("Insertion Anomaly".)

Query : if John gets deleted, then the information of Department will be lost as he is the only guy with that department. So this is called deletion anomaly.)

{ Query: suppose we have to update manager's Aadhar-id, we have to update everywhere in the table.

→ (updation anomaly)

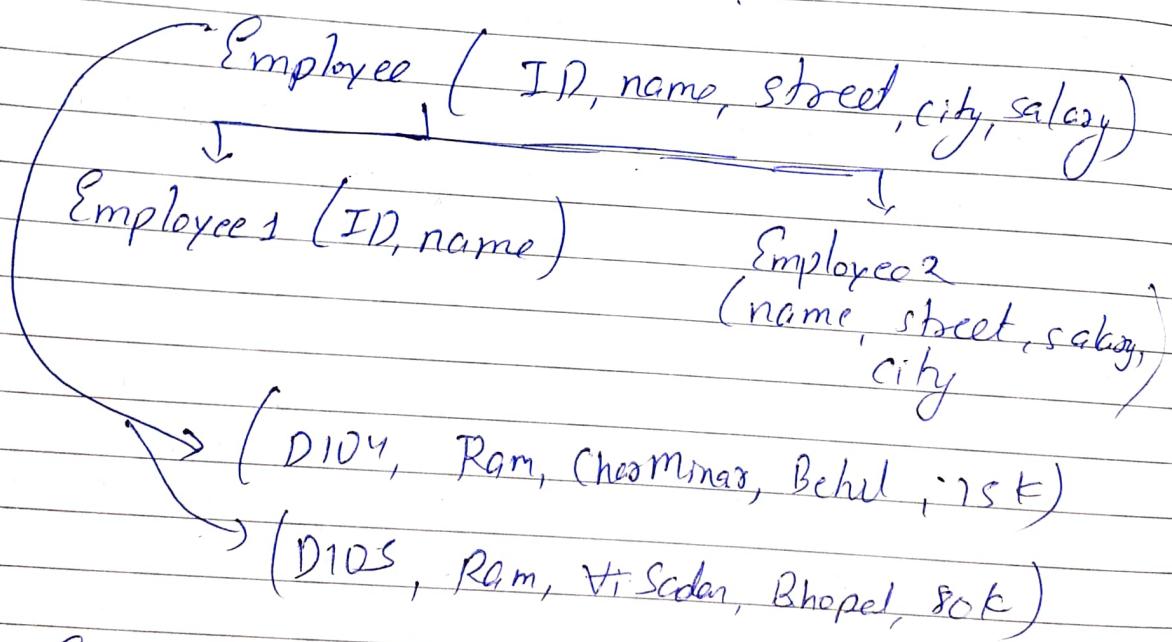
→ It is highly probable that the database will be in the inconsistent state after update operation.

→ Solution is we can split the table, such that data redundancy will not happen.

* The only way to avoid data redundancy is to decompose a relational schema into smaller schemas.

The no. of attributes are smaller.

How it can be done?



Employee-1	
ID	Name
D104	Ram
D105	Ram

Employee-2			
Name	Street	City	Salary
Ram	Chorminar	Behul	75K
Ram	Scoder	Bhopal	80K

Employee 1 \bowtie Employee 2.

We will get 4 tuples.

Id	Name	Street	City	Salary
D104	Ram	Charminar	Bhopal	75k ✓
D104	Ram	Sadar	Bhopal	80k } xx
D105	Ram	Charminar	Bhopal	75k
D105	Ram	Sadar	Bhopal	80k ✓

Two additional tuples are appearing when we have done natural join which is incorrect.

* Let R be a relational schema and R_1, R_2 form a decomposition of R .

$$\textcircled{1} \quad R = R_1 \cup R_2.$$

This even holds when we divide into more than two schemas:

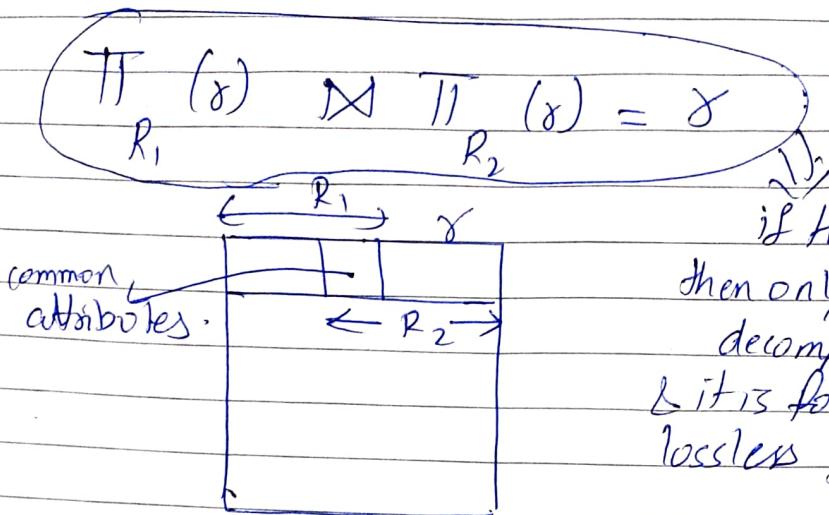
$$\Rightarrow R = R_1 \cup R_2 \cup R_3 \dots R_n$$

if we have decomposed R into R_1, R_2, \dots, R_n

(2) Lossless join property.
 We say that the decomposition is a lossless decomposition if there is no loss of information

by replacing R with $R_1 \times R_2$.

$\delta(R)$ \rightarrow set of attributes
name of schema



if this is valid
then only we can
decompose it.
& it is following
lossless join property.

A decomposition is lossy if natural
join of projection, we get a
proper superset of original
relation.

Formally $\rightarrow \delta \subset \Pi_{R_1}(\delta) \bowtie \Pi_{R_2}(\delta)$

Eg. $\delta(A, B, C) \rightarrow \delta_1(A, B)$
 $\delta_2(B, C)$

δ		
A	B	C
α	1	α
β	2	β

δ_1	
A	B
α	1
β	2

δ_2	
B	C
1	A
α	β

So here we will do natural join

$$\Pi_{A,B}(\gamma) \bowtie \Pi_{B,C}(\gamma) = \gamma'$$

A	B	C
α	1	12
β	2	13

This is an example of lossless join property.

* Normalization theory.

- To decide whether a particular relation (γ) is in good form or not. (less amount of dependency).
- If this is not then how it can be decomposed into set of relational schema such that
 - ① Each decomposed relation is in good form.
 - ② The decomposition is a lossless decomposition.
- Normalization theory depends on
 - ① functional dependency.
 - ② multivalued dependency.

① functional Dependency →

- There are variety of constraints in real world.
- functional dependency gives a mechanism to define a new set of constraints which are not possible using 'domain, not-null, primary key, foreign key etc' ~~constraints~~.
- An instance of relation that satisfies all such real world constraints is called legal instance.

* Primary key : A subset of attributes that can uniquely identify a tuple in a relation.

Let δ' be a relational schema with ' R ' as set of attributes and $\alpha \subseteq R$ & $\beta \subseteq R$. The functional dependency $\alpha \rightarrow \beta$ holds on R if and only if for any legal relation $\delta'(R)$, whenever any tuples t_1, t_2 of R & agree on the attributes α , they also agree on attributes β .

	α	β		
$t_i \rightarrow$				
$t_j \rightarrow$				

where $t_i \neq t_j$.

if $\alpha \rightarrow \beta$ & $t_1[\alpha] = t_2[\alpha]$

then $t_1[\beta] = t_2[\beta]$.

Eg.

δ	
A	B
1	4
1	5
3	2

Q. does $A \rightarrow B$ holds?
Ans - No.

Q. does $B \rightarrow A$ hold?
Ans - Yes, because

we didn't find any tuples
in which A attribute has diff. value.

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* functional dependency :-

$\delta(R)$, $\alpha \subseteq R$, $\beta \subseteq R$

$\alpha \rightarrow \beta$ holds if for any pair of
tuples t_1 & t_2 if $t_1[\alpha] = t_2[\alpha]$
then $t_1[\beta] = t_2[\beta]$.

* $f \rightarrow$ set of given functional dependency

$$f = \{ A \rightarrow B, B \rightarrow C \}$$

if there A values
are same then B value
also same.

if their B values are same
then C value will
also be same.

Superkey \rightarrow Uniquely identify a tuple.

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Given a set of functional dependencies, another set of functional dependencies are basically implied.

	A	B	C
t ₁	1	4	6
t ₂	1	4	6
t ₃	2	4	8

\Rightarrow from here we can say that $A \rightarrow C$ also holds true.

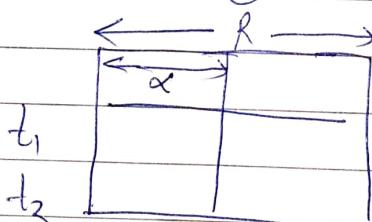
Implied set of functional dependencies = F'
Given set of functional dependencies = F

$$F^+ / F^C = F \cup F'$$

The set of functional dependencies that can be derived from given set of Functional dependencies is called the closure of F' .

* Keys in terms of Functional Dependency:-

- $\gamma(R)$, $\alpha \subseteq R$, we call α is the superkey of γ if and only if $\alpha \rightarrow R$



* Candidate key : α is a candidate key for R if and only if

$$\textcircled{1} \quad \alpha \rightarrow R$$

$$\textcircled{2} \quad \text{no } \beta \subset \alpha, \beta \rightarrow R$$

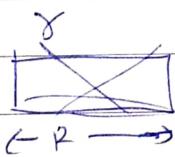
* Functional dependency is a generalised way to impose constraints on a relational schema.

Inst - dept (ID, name, salary, dept_name, building, budget).

$$\begin{array}{l} \text{dept_name} \rightarrow \text{building} \quad \checkmark \\ \text{ID} \rightarrow \text{building} \quad \checkmark \\ \text{dept_name} \rightarrow \text{salary} \quad \times \times \end{array}$$

* Use of functional dependency :-

- if a relation r is legal under any given set of functional dependency then we can say that r satisfies F .



Eg.

$$F = \{A \rightarrow B\}$$

A	B	C
1	2	6
1	2	6
2	3	7

if we want to add (2, 10, 11) then it can't be inserted due to functional dependency constraints.

* FD tests if they are legal under a given set of functional dependency (FD).

A	B	C
1	u	3
3	1	4
2	u	3

$$F = \{ A \rightarrow B \}$$

↳ we can add (5, 4, 6).

By looking at table ($B \rightarrow C$ holds).

A specific name instance of a relational schema may hold/satisfy a FD even if it is not defined.

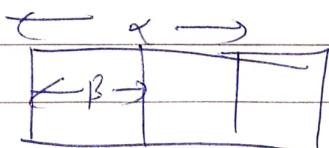
* Trivial Functional Dependency

→ A FD is trivial if it is satisfied by all instances of a relation. (if both values are same)

- ID, name \rightarrow name.
- name \rightarrow name.

Id	name	street	Phone No
D101	Ram		
D101	Ram		

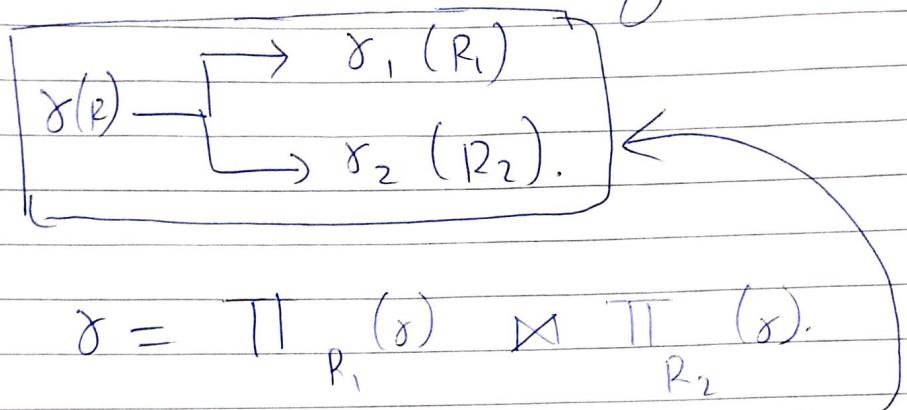
In general for any FD $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$



Eg. $FD \Rightarrow \left\{ \underbrace{A, B}_{\prod} \rightarrow C \right\}$

for any tuples if (both value, A&B are same, then their C values will be same).

* Lossless Decomposition in terms of FD.



A decomposition of R into $\boxed{(R_1, R_2)}$ is lossless if atleast one of the following dependencies is D in F^+ .

F^+ : Closure of F

$$\textcircled{1} \quad R_1 \cap R_2 \rightarrow R_1$$

$$\textcircled{2} \quad R_1 \cap R_2 \rightarrow R_2$$

Eg. $\gamma(A, B, C) \rightarrow \begin{cases} \gamma_1(A, B) \\ \gamma_2(B, C) \end{cases}$

$$F = \left\{ A \rightarrow B, B \rightarrow C \right\} \xrightarrow{\text{then}} C \xrightarrow{\text{also}} A \rightarrow C \text{ holds.}$$

$$F^C = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}.$$

$$R_1 \cap R_2 = B.$$

$B \rightarrow C$ holds
as it is given.

$$B \rightarrow A B$$

\downarrow \downarrow

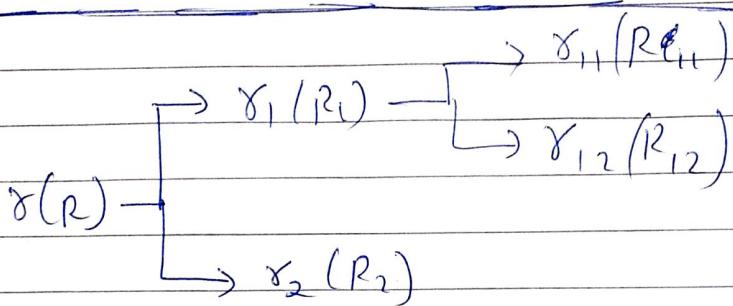
$$(R_1 \cap R_2) \rightarrow R_1$$

$$B \rightarrow BC$$

\downarrow

$$(R_1, R_2) \quad R_2$$

This is a lossless decomposition



* functional dependency theory Read Mgt

How to compute F^+ for some given F ?

Aomstong's Axioms

- Reflexive rule: If $\beta \subseteq \alpha$ then $\alpha \rightarrow \beta$ holds
(Trivial functional dependency)

- Augmentation Rule: If $d \rightarrow \beta$ hold & γ is another set of attribute then $\gamma d \rightarrow \gamma \beta$ also holds.

- Transitivity Rule: If $\alpha \rightarrow \beta$ holds & $\beta \rightarrow \gamma$ holds,
then $\alpha \rightarrow \gamma$ also holds.

These rules are sound. Means they generate only functional dependencies that actually hold.

These rules are complete. Means they generate all functional dependencies.

* Let $\delta(F)$:

$$F \longrightarrow F^+$$

Let F^A Actual functional dependencies which are holding on δ for given F .

Sound does not mean this.

$$F^+ \subset F^*$$

(complete also does not mean this) F

Eg. $\delta(A, B, C, G, H, I)$

$$F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$$

$$\begin{aligned} \text{Sol: } & A \rightarrow B & \dots & ① \\ & A \rightarrow C & \dots & ② \\ & CG \rightarrow H & \dots & ③ \\ & CG \rightarrow I & \dots & ④ \\ & B \rightarrow H & \dots & ⑤ \end{aligned}$$

Applying transitivity on ① & ⑤

$A \rightarrow H$ also holds

→ ⑥

* Argument G on both sides

$$AG \rightarrow CG \quad \text{--- } (7)$$

* Applying transitivity on (7) & (4)

$$AG \rightarrow I \quad \text{--- } (8)$$

* Applying transitivity on (7) & (3)

$$AG \rightarrow H \quad \text{--- } (9)$$

* Argumenting CG on both sides of (1)

$$CG \rightarrow CGI \quad \text{--- } (10)$$

* Argumenting I on both sides of (3)

$$CGI \rightarrow HI \quad \text{--- } (11)$$

* Applying transitivity on (10) & (11)

$$CG \rightarrow HI \quad \text{--- } (12)$$

\therefore Additional Rules :-

- Union Rule \rightarrow if $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds then $\alpha \rightarrow \beta\gamma$ holds.

- Decomposition Rule \rightarrow if $\alpha \rightarrow \beta\gamma$ holds, then $(\alpha \rightarrow \beta)$ and $(\alpha \rightarrow \gamma)$ holds.
- Pseudo transitivity Rule \rightarrow if $\alpha \rightarrow \beta$ holds and $\beta\gamma \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds.

* Proof of Union Rule.

$\star \quad \alpha \rightarrow \beta$ holds.

so augmenting α on both sides.

$$\alpha \rightarrow \alpha\beta \dots \textcircled{1\Delta}$$

$\star \quad \alpha \rightarrow \gamma$ holds.

so augmenting β on both sides.

$$\alpha\beta \rightarrow \beta\gamma \dots \textcircled{2\Delta}$$

Applying transitivity on $\textcircled{1\Delta}$ & $\textcircled{2\Delta}$

$$\boxed{\alpha \rightarrow \beta\gamma} \text{ also holds}$$

* Proof of decomposition Rule

$\textcircled{3\Delta} \dots \alpha \rightarrow \beta\gamma$ holds {given}.

$\textcircled{3\Delta} \dots \beta\gamma \rightarrow \beta$ also holds {by reflexivity}

$\textcircled{4\Delta} \dots \beta\gamma \rightarrow \gamma$ also holds {by reflexivity}.
from transitivity. from $\textcircled{3\Delta}$ & $\textcircled{4\Delta}$

$\alpha \rightarrow \beta$ holds & $\alpha \rightarrow \gamma$ also holds.

* Proof of Pseudo Transitivity

$\alpha \rightarrow \beta$ holds \rightarrow (1) {given}

$\beta \gamma \rightarrow \delta$ holds \cdots (2) {given}

To show $\alpha \gamma \rightarrow \delta$

Sol. \rightarrow Augmenting γ on both sides
of (1)

$\alpha \gamma \rightarrow \beta \gamma \cdots$ (3)

Transitivity $\stackrel{\text{on}}{\downarrow}$ (2) $\&$ (3)
 $\boxed{\alpha \gamma \rightarrow \delta}$

Input: $r(R)$ & $F \rightarrow$ given set of FD
Output: F^c

{ for each FD, $F \in F^+$, we will apply
reflexivity & augmentation rule on it. F
and update the innocent list with
new set of Functional dependencies

For each pair of FD, $F_1, F_2 \in F^c$

if F_1 & F_2 can be combined using transitivity
then add the resultant FD to F^+

Repeat the process until F^+ does not change.

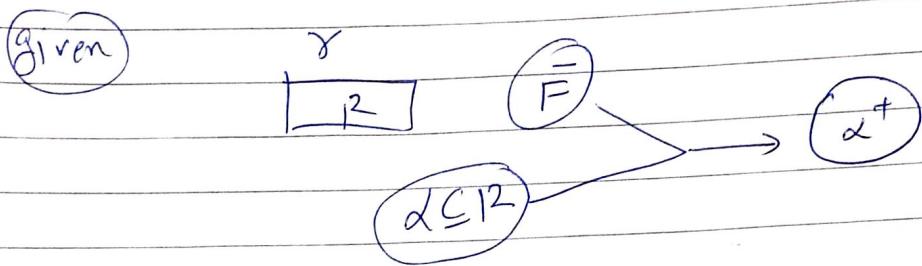
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* Closure of Attribute Set

Given a set of attributes α and a set of FDs, the closure of α under $F(\alpha^+)$ is the set of attributes that are functionally determined by α under F .



Given a set of attributes α , the closure of α with respect to functional dependency F is denoted as set of attributes that are functionally determined by α .

The closure of α is denoted by α^+

Eg. $\alpha \cup \underbrace{(A, B, C, G, H, I)}_R$

$$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, (G \rightarrow I), B \rightarrow H \}$$

$\alpha = \{ A, G \}$, Now we want to compute α^+ .

$$\underline{\text{Sol:}} \rightarrow \underline{\alpha} = \{D, G\} \rightarrow \alpha^+ = \{D, G\}$$

$$\downarrow A \rightarrow B, A \rightarrow C$$

$$\alpha^+ = \{A, B, C, G\}$$

$$\downarrow B \rightarrow H$$

$$\alpha^+ = \{A, B, C, G, H\}$$

$$\downarrow C \rightarrow I$$

$$\alpha^+ = \{A, B, C, G, H, I\}$$

$$\alpha^+ = R$$

The set of attributes in α^+ is equal to set of attributes of F.

Q Is AG a candidate key?

Sol:- Let us take closure of A^+

$$A^+ = \{A\}$$

$$\downarrow A \rightarrow B, A \rightarrow C$$

$$= \{A, B, C\}$$

$$\downarrow B \rightarrow H$$

$$\boxed{A^+ = \{A, B, C, H\}}$$

$$A^+ \neq R$$

$$\therefore A^+ = \{G\}$$

A_G is a candidate key ✓

Candidate key \rightarrow Minimal superkey is candidate key.

* Algorithm for computing Attribute closure.

Inputs : $\delta(R)$, $\alpha \subseteq R$ & F

Output : α^+

~~node~~ {

```

results ← {}
while δ changes to results do.
  for each β → δ ∈ F
    if β ⊆ result then
      result ← result ∪ β
end; end end
  
```

* Use of attribute closure

① Testing for superkey :-

To test α is a superkey we compute α^+ & checks whether it is equal or to R or not. If it is equal to R, then it is a superkey.

② Testing for functional dependency :-

$\alpha(p)$

F

$\alpha \rightarrow \beta$

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Check whether $\beta \subseteq \alpha^+ \rightarrow \text{compt } \alpha^+$
Check whether $\alpha \rightarrow \beta$ holds on & on no.

③ Computing closure of $F = (F^+)$

For each $\gamma \subseteq R$, we just find the
 γ^+ and to each $S \subseteq \gamma^+$, we get
FD, $\gamma \rightarrow S$.

* Canonical cover of given set of FD

Database ($\alpha^+ = 0$)

↓ tuples

after

update

Database ($\alpha^+ = 1$)

if F is very very large,
can we reduce F ??

$F' \subseteq F$

$F' \rightarrow$ (canonical) cover of F

$(F')^+ = (F^+)$

Here F' is of smaller size

* Extraneous Attribute (in F)

→ An attribute of FD is said to be extraneous if we can remove that attribute without changing F^+ .

$\gamma(P), F$

$\gamma(A, B, C, D)$.

Strong $F = D$.

$$F \cdot D = \{ AB \rightarrow C, A \rightarrow C \}$$

if $AB \rightarrow C$ holds, it does not imply $A \rightarrow C$ holds.

$$F = \left\{ \underbrace{AB \rightarrow C}_{①}, \underbrace{A \rightarrow D}_{②}, \underbrace{D \rightarrow C}_{③} \right\}$$

By applying transitivity on ② & ③.
 $A \rightarrow C$.

there is no need of AB in $AB \rightarrow C$.

So we can say that B is an extraneous attribute.

* Removing an attribute from left part of FD means that FD becomes stronger.