Name:- Pulkit Bansal
Section:- G
Branch:- C.S.E (core)
Roll No.:- 30
Univ. Roll No.:- 2014789

Tutorial Sheet-4

Ans ① $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2$

$k = \log_2 3 = 1.58$

$n^2 > n^{1.58}$

$\therefore \Theta(n^2)$

Ans ② $T(n) = 4T(n/2) + n^2$

$k = \log_2 4 = 2$

$n^2 = n^2$

$\therefore \Theta(n^2 \log n)$

Ans ③ $T(n) = T(n/2) + 2^n$

Here, $F(n)$ is not a polynomial.

$\therefore$ We ~~cannot~~ apply Master theorem.

Ans ④ $T(n) = 2^n T(n/2) + n^n$

Here, this recurrence iteration can't be solved using Master's Method.

Ans ⑤ $T(n) = 16T(n/4) + n$

$k = \log_4 16 = 2$

$n^2 > n$

$\therefore \underline{\Theta(n^2)}$

Ans ⑥ $T(n) = 2T(n/2) + n\log n$

$\therefore$ case $= 1$ (by normal Master theorem)

New by using extended Master's theorem.

$T(n) = aT(n/b) + \Theta(n^k \log^p n)$.

$a = b^k$, $2 = 2$.

$T(n) = \Theta(n \log_2 2 \log^{1+1} n)$

$\Theta(n \log^2 n)$

Ans ⑦ $T(n) = 2T(n/2) + n\log^{-1} n$

using extended Master's theorem.

$T(n) = aT(n/b) + \Theta(n^k \log^p n)$

$a = 2$, $b = 2$ $k = 1$, $b = -1$

$h = -1$

$\therefore T(n) = \Theta(n \log_2^2 \log\log n)$

$= \Theta(n \log\log n)$

Ans ⑧ $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \quad b = 4$

$\text{case} = \log_4 2 = 0.5$

$n^{0.5} < n^{0.51}$

$\therefore O(n^{0.51})$

Ans ⑨ $T(n) = 0.5T(n/2) + n^{-1}$

As $a < 1$, $\therefore$ Master's theorem cannot apply here.

Ans ⑩ $T(n) = 16T(n/4) + n!$

$k = \log_4 16 = 2$

$n^2 < n!$

$\therefore O(n!)$

Ans ⑪ $T(n) = 4T(n/2) + \log n$

here $a = 4$, $b = 2$ $k = 0$, $h = 1$

$\therefore$ using Extended Master's theorem.

$T(n) = aT(n/b) + O(n^k \log^n n)$

$a > b^k$

$4 > 2^0$

$\therefore \Theta(n^{\log_2 4})$

$\therefore O(n^2)$

Ans (12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

Here a is not constant. So, Master theorem can not apply here.

Ans (13) $T(n) = 3T(n/2) + n$

$k = \log_2 3 = 1.58$

$n^{1.58} > n$

$\therefore \Theta(n^{1.58})$

Ans (14) $T(n) = 3T(n/3) + \sqrt{n}$

$k = \log_3 3 = 1$

$n^1 > \sqrt{n}$

$\therefore O(n)$

Ans (15) $T(n) = 4T(n/2) + n$

$k = \log_2 4 = 2$

$n^2 > n$

$\therefore \Theta(n^2)$

Ans (16) $T(n) = 3T(n/4) + n \log n$

using extend Master's theorem;

$T(n) = a\,T(n/b) + \Theta(n^k \log^n n)$

Here $a = 3$; $b = 4$, $k = 1$, $n = 1$

$a < b^k$

$3 < 4^1$

$h >= 0$

$\therefore T(n) = \Theta(n^k \log^k n)$

$= \Theta(n \log n)$

Ans (17) $T(n) = 3T(n/3)$

$c = \log_3 3 = 1$

$n^1 = n$

$\therefore \Theta(n \log n)$

Ans (18) $T(n) = 6T(n/3) + n^2 \log n$
using extended Master's theorem

$T(n) = a(T(n/b) + a(n^k \log^k n)$

$a = 6, b = 3, k = 2, h = 1$

$a < b^k$

$6 < 3^2$

$h >= 0$

$\therefore T(n) = \Theta(n^k \log^k n)$

$= \Theta(n^2 \log n)$

Ans (19) $T(n) = 4T(n/2) + n\log^{-1} n$

using Master's theorem extended.

$$T(n) = aT(n/b) + \Theta(n^k \log^k n)$$

$a = 4,\ b = 2,\ k = 1,\ h = 1.$

$$a > b^k$$

$$4 > 2^1$$

$$\therefore T(n) = \Theta(n \log_2 4)$$

$$= \Theta(n^2)$$

Ans (20) $T(n) = 64T(n/8) - n^2 \log n$

As $f(n)$ is negative.

$\therefore$ We can now apply Master's Theorem.

Ans (21) $T(n) = 7T(n/3) + n^2$

using Master's theorem

$k = \log_3 7 = 1.97$

$$n^{1.97} < n^2$$

$$\therefore \Theta(n^2)$$

Ans (22) Here, we can apply Master's theorem.

* * * * *