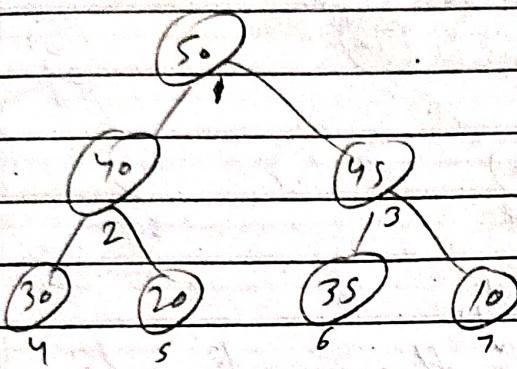
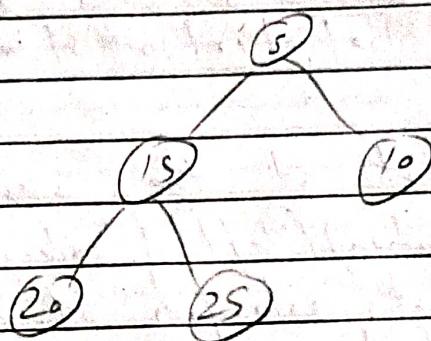


Heap

a) Max Heap



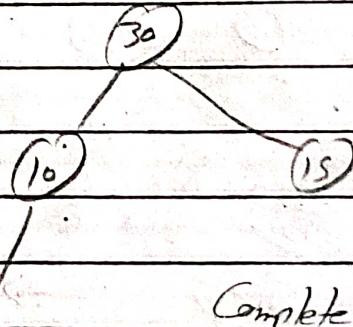
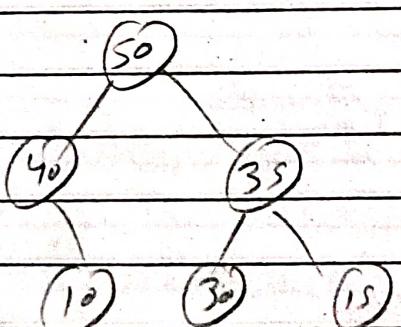
b) Min Heap



Heap is a complete Binary tree that satisfies a heap property.

↳ Don't have compulsion to the leaf node, that they must be have 2 children.

↳ Element positioning start from left.



Not Complete Binary tree

Complete Binary tree
Max heap

A =	0	1	2	3	4	5	6	7
	x	50	40	45	30	20	35	10

Node = ;

left child = $2 \times i$

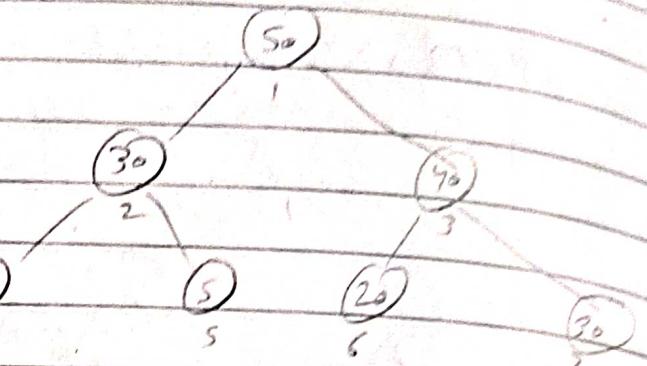
Parent(i) = $\lfloor \frac{i}{2} \rfloor$

Right child = $2 \times i + 1$

Insert (Max Heap)

Insert : 60

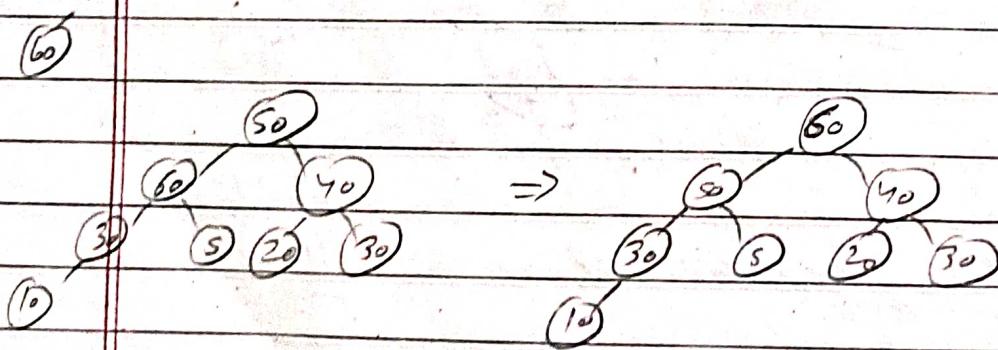
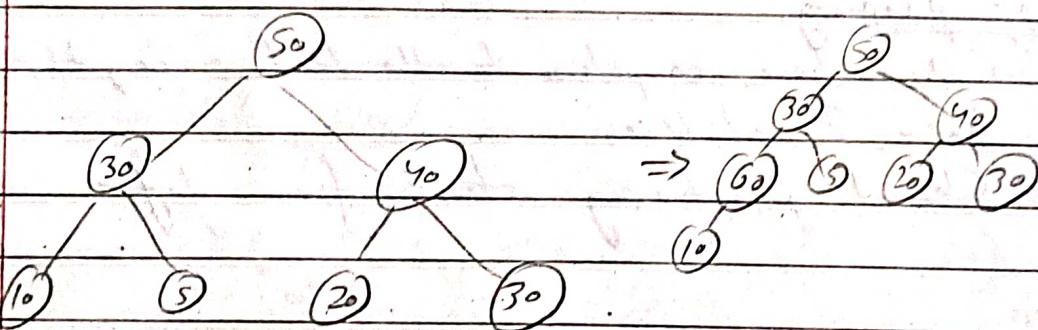
1	2	3	4	5	6	7
50	30	40	10	5	20	30



Step 1: Insert at the (insertion position) (left of node s.t.) that satisfies heap property.

Step 2: check if parent is smaller than insertion value ; swap

Step 3: Repeat until reaches at index 1.



1	2	3	4	5	6	7	8
60	50	40	30	5	20	30	10

Time Complexity = $O(\log(n))$

Traversal = Bottom to Top

void insert (A[], n, value) {

$n = n + 1$ // we are adding new element

$A[n] = \text{value}$;

int i = n;

while ($i > 1$) {

int parent = $i / 2$;

if ($a[\text{parent}] < a[i]$) {

swap (A, parent, i);

$i = \text{parent}$;

} else {

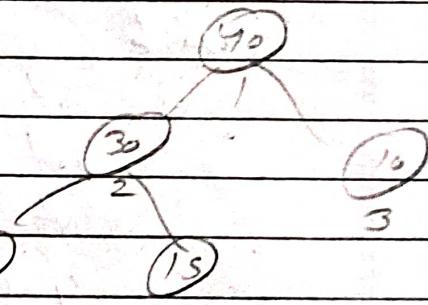
return;

}

3

Delete (Max Heap)

A = [40 | 30 | 10 | 20 | 15]



We care about top most ele.

Step 1: Remove Root

Step 2: Remove last node and insert it into root.

Step 3: Check for childs (Top Max) if parent smaller; swap.

Step 4: Repeat until n

Time Complexity = $\log(n)$

Traverser = Top to Bottom

$$A = \boxed{1 \quad 2 \quad 3 \quad 4 \quad 5} \\ A = \boxed{40 \quad 30 \quad 10 \quad 20 \quad 15}$$

$$A = \boxed{1 \quad 2 \quad 3 \quad 4 \quad 5} \\ A = \boxed{1 \quad 30 \quad 10 \quad 20 \quad 15}$$

$$A = \boxed{1 \quad 2 \quad 3 \quad 4 \quad 5} \\ A = \boxed{15 \quad 30 \quad 10 \quad 20 \quad \cancel{15}}$$

$$A = \boxed{1 \quad 2 \quad 3 \quad 4 \quad 5} \\ A = \boxed{30 \quad 15 \quad 10 \quad 20 \quad \cancel{15}}$$

$$A = \boxed{1 \quad 2 \quad 3 \quad 4 \quad 5} \\ A = \boxed{30 \quad 20 \quad 10 \quad 15 \quad \cancel{8}}$$

void delete(A[] , n) {

 A[1] = A[n];

 n = n - 1;

 i = 1;

 while (i < n) {

 int left = A[2 + i];

 int right = A[2 + i + 1];

 int larger = left > right ? 2 + i : 2 + i + 1;

 if (A[i] < A[larger]) {

 swap(A, i, larger);

 i = larger;

 }

 else {

 return;

 }

}

}

Time Complexity = $O(\log(n))$

Heapify

Way 1:

If we add by our method, it will take $O(n \log n)$

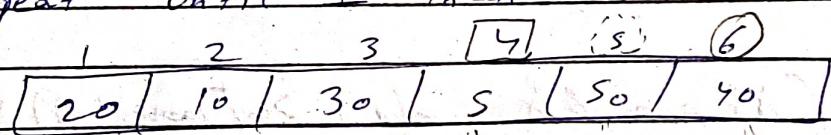
Way 2:

Heapify $O(n)$

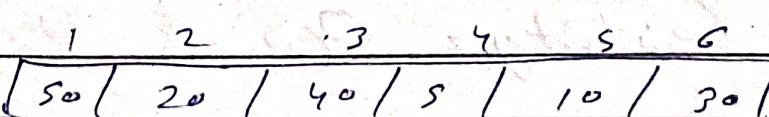
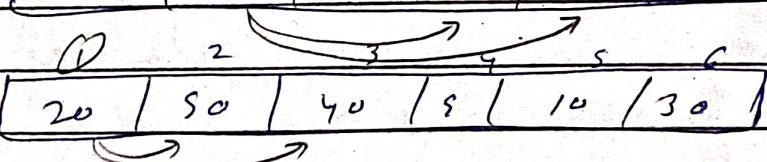
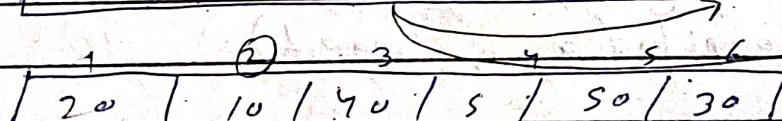
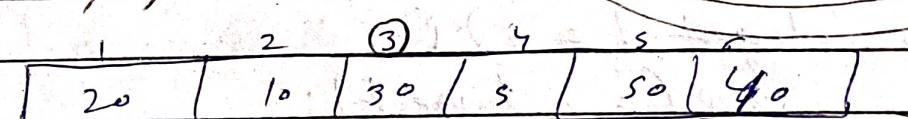
Theat There can be many configuration of a heap.

Step 1: Start from last element and check if its child is greater; swap

Step 2: Repeat until 1 index



Check = 6, 5, 4.



Max Heap

Improvement - We can start from $\frac{n}{2}$ instead of n .
Bcz for leaf node we are not swapping at all.

buildheap (int a[], int n) {

 for (int i = n / 2; i >= 0; i--) {

 heapify (a, n, i);

 }

void heapify (int a[], int n, int i) {

 int largest = i;

 int l = 2 * i;

 int r = 2 * i + 1;

 if (l < n && a[l] > a[largest]) {

 largest = l;

 }

 if (r < n && a[r] > a[largest]) {

 largest = r;

 }

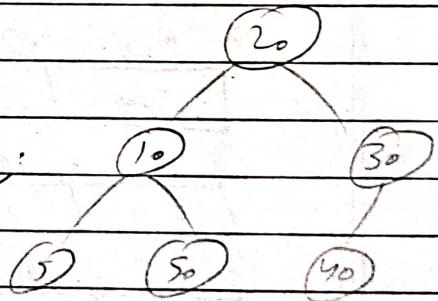
 if (largest != i) {

 swap (a, i, largest);

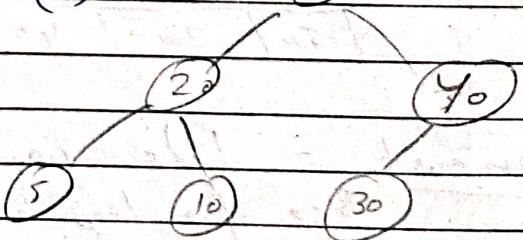
 heapify (a, n, largest);

 }

3



Time Complexity = $O(n)$



Heap Sort

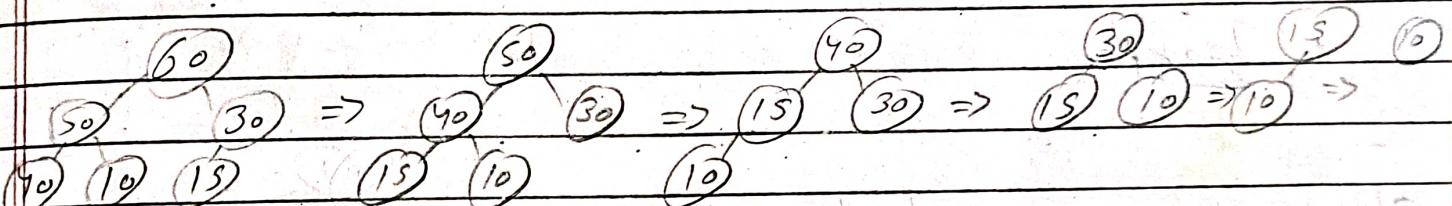
1	2	3	4	5	6
40	10	30	50	60	15

$O(n)$ ↓ Convert into a heap

1	2	3	4	5	6
60	50	30	40	10	15

$O(n \log n)$ ↓ Delete ele one by one

1	2	3	4	5	6
10	20	30	40	50	60



$\underbrace{n}_{\text{Deletion}}$ $\underbrace{(\log n)}_{\text{Deletion}}$

void heapSort (int a[], int n) {
 $O(n)$ build heap (a, n); // build heap
 $O(n \log n)$ for (int i = n; i > 1; i--) {
 print(a[0]); swap (a, 1, i); // swap and making ans
 heapify (a, i - 1, 1); // sort remaining
 3 } } // sort from

