The Potential Field Approach And Operational Space Formulation In Robot Control

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Abstract

The paper presents a radically new approach to real-time dynamic control and active force control of manipulators. In this approach the manipulator control problem is reformulated in terms of direct control of manipulator motion in operational space, the space in which the task is originally described, rather than controlling the task's corresponding joint space motion obtained after geometric and kinematic transformation. The control method is based on the construction of the manipulator end effector dynamic model in operational space. Also, the paper presents a unique real-time obstacle avoidance method for manipulators and mobile robots based on the "artificial potential field" concept. In this method, collision avoidance, traditionally considered a high level planning problem, can be effectively distributed between different levels of control, allowing real-time robot operations in a complex environment. Using a time-varying artificial potential field, this technique has been extended to moving obstacles. A two-level control architecture has been designed to increase the system real-time performance. These methods have been implemented in the COSMOS system for a PUMA 560 robot arm. We have demonstrated compliance, contact, sliding, and insertion operations using wrist and finger sensing, as well as real-time collision avoidance with moving obstacles using visual sensing.

1. Introduction

Conventional manipulator control, providing only linear feedback compensation to control joint positions independently, cannot meet the high accuracy and performance needed in precision manipulator tasks. Addressing this problem, much research has been directed at developing and modelling the dynamic equations of joint motion. Typical models relate joint variables to generalized torques and by necessity force the resulting control scheme to have two levels:

- Task description transformation level to express tasks in terms of joint coordinates;
- Joint space dynamic control level to calculate generalized force commands.

This first stage of transforming the task description is time consuming and prone to problems near kinematic singularities. Additionally, dealing with the dynamic compensation problem leads to high computational complexity in real-time control. Furthermore, the very approach of joint space control is ill-suited for active force control, an ability which is crucial in robot assembly tasks.

Robot collision avoidance, on the other hand, has typically been a component of higher levels of control in hierarchical robot control systems. It has been treated as a planning problem, and research in this area has focused on the development of collision-free path planning algorithms [1-4]. These algorithms aim at providing the low level control with a path that will enable the robot to accomplish its assigned task free from any risk of collision.

From this perspective, the function of low level control is limited to the execution of elementary operations for which the paths have been precisely specified. The robot's interaction with its environment is then paced by the time-cycle of high level control, which is generally several orders of magnitude slower than the response time of a typical robot. This places limits on the robot's real-time capabilities for precise, fast, and highly interactive operations in a cluttered and evolving environment. We will show, however, that it is possible to greatly extend the function of low level control and to carry out more complex operations by coupling environment-sensing feedback with the lowest level of control.

Increasing the capability of low level control has been the impetus for the work on real-time obstacle avoidance that we discuss here. Collision avoidance at the low level of control is not intended to replace high level functions or to solve planning problems. The purpose here is to make better use of low level control capabilities in performing real-time operations. At this low level of control, the degree or level of competence [5] will remain less than that of higher level control.

The operational space formulation is the basis for the application of the potential field approach to robot manipulators. This formulation has its roots in the work on end-effector motion control and obstacle avoidance [6] that has been implemented for an MA23 manipulator at the Laboratoire d'Automatique de Montpellier in 1978. The operational space approach has been formalized by constructing its basic tool, the equations of motion in the operational space of the manipulator end-effector. Details of this work have been published elsewhere elsewhere [7,8,9]. In this paper, we review the fundamentals of the operational space formulation and the artificial potential field concept.

2. Operational Space Formulation

An operational coordinate system is a set x of m_0 independent parameters describing the manipulator end-effector position and orientation in a frame of reference \mathcal{R}_0 . For a non-redundant manipulator, these parameters form a set of configuration parameters in a domain of the operational space and constitute, therefore, a system of generalized coordinates. The kinetic energy of the holonomic articulated mechanism is a quadratic form of the generalized velocities

$$T(\mathbf{x}, \dot{\mathbf{x}}) = \frac{1}{2} \dot{\mathbf{x}}^T \Lambda(\mathbf{x}) \dot{\mathbf{x}}; \tag{1}$$

where $\Lambda(\mathbf{x})$ designates the symmetric matrix of the quadratic form, *i.e.* the kinetic energy matrix. Using the Lagrangian formalism, the end-effector equations of motion are given by

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\mathbf{x}}}) - \frac{\partial L}{\partial \mathbf{x}} = \mathbf{F};\tag{2}$$

where the Lagrangian $L(\mathbf{x}, \dot{\mathbf{x}})$ is

$$L(\mathbf{x}, \dot{\mathbf{x}}) = T(\mathbf{x}, \dot{\mathbf{x}}) - U(\mathbf{x}); \tag{3}$$

and $U(\mathbf{x})$ represents the potential energy of the gravity. \mathbf{F} is the operational force vector. These equations can be developed [7,8] and written in the form

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{4}$$

where $\mu(\mathbf{x}, \dot{\mathbf{x}})$ represents the centrifugal and Coriolis forces, $p(\mathbf{x})$ the gravity forces.

The control of manipulators in operational space is based on the selection of F as a command vector. In order to produce this command, specific forces Γ must be applied with joint-based actuators. The relationship between F and the joint forces Γ is given by

$$\mathbf{\Gamma} = J^T(\mathbf{q}) \; \mathbf{F}; \tag{5}$$

where q is the vector of the n joint coordinates, and J(q) the Jacobian matrix.

The extension of the operational space approach to redundant manipulators is presented in [7,8,9].

3. End-Effector Dynamic Decoupling

While in motion, a manipulator is subject to highly nonlinear inertial, centrifugal, Coriolis, and gravity forces. Dynamic decoupling, which is achieved by compensating for these forces, requires their evaluation using the manipulator dynamic model. Considering the complexity of these models, real-time dynamic control of manipulators has been viewed as a computationally expensive approach. This problem is yet more acute in operational space, since the corresponding equations of motion are relatively more complex than those of joint motion.

Addressing this problem, we designed a two-level control system architecture based on isolating the configuration dependent coefficents in the dynamic model. The load of real-time computation of these coefficents can then be paced by the rate of configuration changes, which is much lower than that of the mechanism dynamics.

Using the dynamic model (4), the decoupling of the end-effector motion in operational space is achieved by

$$\mathbf{F} = \Lambda(\mathbf{x})\mathbf{F}^* + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}); \tag{6}$$

where \mathbf{F}^* represents the command vector of the decoupled end-effector, which becomes equivalent to a *single unit mass*.

Let $[\dot{q}\dot{q}]$ and $[\dot{q}^2]$ be

The joint torque vector corresponding to the operational space command vector (6) can be developed [7,8] following the structure

$$\Gamma = J^{T}(\mathbf{q})\Lambda(\mathbf{q})\mathbf{F}^{*} + \tilde{B}(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}(\mathbf{q})[\dot{\mathbf{q}}^{2}] + \mathbf{g}(\mathbf{q}); \tag{8}$$

 $\tilde{B}(\mathbf{q})$, $\tilde{C}(\mathbf{q})$, and $\mathbf{g}(\mathbf{q})$ are the $n \times n(n-1)/2$, $n \times n$, and $n \times 1$ matrices of the joint forces under the mapping into joint space of the end-effector Coriolis, centrifugal, and gravity forces, respectively.

The dynamic decoupling of the end-effector can thus be obtained using the configuration dependent dynamic coefficients $\Lambda(\mathbf{q})$, $\tilde{B}(\mathbf{q})$, $\tilde{C}(\mathbf{q})$ and $g(\mathbf{q})$.

Furthermore, the rate of computation of the end-effector position, a costly step since it involves evaluations of the manipulator geometric model, can be reduced by integrating an operational position estimator into the control system. Finally, the control system (see Figure 1) has the following architecture:

- A low rate parameter evaluation level: updating the end-effector dynamic coefficients, the Jacobian matrix, and the geometric model.
- A high rate servo control level: computing the command vector using the estimator and the updated dynamic coefficents.

4. Active Force Control

Precise control of applied end-effector forces is crucial to accomplish advanced robot assembly tasks. An extensive research effort has been devoted to manipulator force control problems. Accommodation [10], joint compliance [11], active compliance [12], passive compliance, and hybrid position/force control [13] are among the various methods proposed. Active force control has been generally based on kinematic considerations, and has been treated within the framework of joint space control systems. However, task specification for motion and applied forces, wrist and finger force sensing feedback, and dynamics, are closely linked to the end-effector. Active force control can be naturally addressed in the framework of operational space control systems.

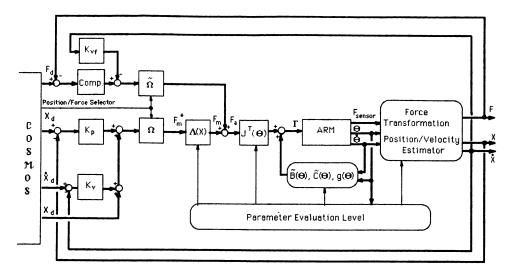


Figure 1: Operational Space Control System Architecture

A unified operational command vector can be used for end-effector dynamic decoupling, motion, and active force control [9]. The corresponding joint force vector is

$$\Gamma = J^{T}(\mathbf{q})[\Lambda(\mathbf{q})(\Omega \mathbf{F}_{m}^{*} + \widetilde{\Omega} \mathbf{F}_{s}^{*}) + \widetilde{\Omega} \mathbf{F}_{a}^{*}] + \widetilde{\mathbf{b}}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}). \tag{9}$$

where \mathbf{F}_m , \mathbf{F}_a are the operational command vectors of motion and active force control. \mathbf{F}_s^* represents the vector of end-effector velocity damping that acts in the direction of desired forces. Ω is the generalized position and force specification matrix. The control system architecture is shown in Figure 1, where k_{vf} is a velocity gain in \mathbf{F}_s^* .

5. The Artificial Potential Field Approach

We present this method in the context of manipulator collision avoidance. Its application to mobile robots is straightforward. The philosophy of the artificial potential field approach can be schematically described as follows:

The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end-effector, and obstacles are repulsive surfaces for the manipulator parts.

Let us first consider the collision avoidance problem of a manipulator end-effector with a single obstacle \mathcal{O} . If \mathbf{x}_d designates the goal position, the control of the manipulator end-effector with respect to the obstacle \mathcal{O} can be achieved by subjecting it to the artificial potential field

$$U_{art}(\mathbf{x}) = U_{\mathbf{x}_d}(\mathbf{x}) + U_{\mathcal{O}}(\mathbf{x}). \tag{10}$$

This leads to the following expression of the potential energy in the Lagrangian (3)

$$U(\mathbf{x}) = U_{art}(\mathbf{x}) + U_g(\mathbf{x}); \tag{11}$$

where $U_g(\mathbf{x})$ represents the gravity potential energy. Using Lagrange's equations (2), and taking into account the end-effector dynamic decoupling (6), the command vector \mathbf{F}_{art}^* of the decoupled end-effector that corresponds to applying the artificial potential field U_{art} (10) can be written as

$$\mathbf{F}_{art}^* = \mathbf{F}_{\mathbf{x}_d}^* + \mathbf{F}_{\mathcal{O}}^*; \tag{12}$$

with

$$\mathbf{F}_{\mathbf{x}_{d}}^{*} = -\mathbf{grad}[U_{\mathbf{x}_{d}}(\mathbf{x})]
\mathbf{F}_{O}^{*} = -\mathbf{grad}[U_{O}(\mathbf{x})]$$
(13)

 $\mathbf{F}_{\mathbf{x}_d}^*$ is an attractive force allowing the point \mathbf{x} of the end-effector to reach the goal position \mathbf{x}_d , and \mathbf{F}_O^* represents a Force Inducing an Artificial Repulsion from the Surface of the obstacle (FIRAS, from the French), created by the potential field $U_O(\mathbf{x})$. $\mathbf{F}_{\mathbf{x}_d}^*$ corresponds to the proportional term, i.e. $-k_p(\mathbf{x}-\mathbf{x}_d)$, in a conventional PD servo, where k is the position gain. The attractive potential field $U_{\mathbf{x}_d}(\mathbf{x})$ is simply

$$U_{\mathbf{x}_d}(\mathbf{x}) = \frac{1}{2} k_p (\mathbf{x} - \mathbf{x}_d)^2. \tag{14}$$

 $U_O(\mathbf{x})$ is selected such that the artificial potential field $U_{art}(\mathbf{x})$ is a positive continuous and differentiable function which attains its zero minimum when $\mathbf{x} = \mathbf{x}_d$. The articulated mechanical system subjected to $U_{art}(\mathbf{x})$ is stable. Asymptotic stabilization of the system is achieved by adding dissipative forces proportional to $\dot{\mathbf{x}}$. Let k_v be the velocity gain; the forces contributing to the end-effector motion and stabilization are of the form

$$\mathbf{F}_{m}^{*} = -k_{p}(\mathbf{x} - \mathbf{x}_{d}) - k_{v}\dot{\mathbf{x}}. \tag{15}$$

This command vector is inadequate to control the manipulator for tasks that involve large end-effector motion toward a goal position without path specification. For such a task, it is better for the end-effector to move in a straight line, with an upper speed limit.

Rewriting equation (15) leads to the following expression, which can be interpreted as specifing a desired velocity vector in a pure velocity servo-control.

$$\dot{\mathbf{x}}_d = \frac{k_p}{k_v} (\mathbf{x}_d - \mathbf{x}). \tag{16}$$

Let V_{max} designate the assigned speed limit. The limitation of the end-effector velocity magnitude can then be obtained [14] by

$$\mathbf{F}_{m}^{*} = -k_{v}(\dot{\mathbf{x}} - \nu \dot{\mathbf{x}}_{d}); \tag{17}$$

where

$$\nu = min(1, \frac{V_{max}}{\sqrt{\dot{\mathbf{x}}_d^T \dot{\mathbf{x}}_d}}). \tag{18}$$

With this scheme the velocity vector $\dot{\mathbf{x}}$ is controlled to be pointed toward the goal position while its magnitude is limited to V_{max} . The end-effector will then travel at that speed, in a straight line, except during the acceleration and deceleration segments or when it is inside the repulsive potential field regions of influence.

6. FIRAS Function

The artificial potential field $U_O(\mathbf{x})$ should be designed to meet the manipulator stability condition and to create at each point on the obstacle's surface a potential barrier which becomes negligible beyond that surface. Specifically, $U_O(\mathbf{x})$ should be a non-negative continuous and differentiable function whose value tends to infinity as the end-effector approaches the obstacle's surface. In order to avoid undesirable perturbing forces beyond the obstacle's vicinity, the influence of this potential field must be limited to a given region surrounding the obstacle.

Using analytic equations $f(\mathbf{x}) = 0$ for obstacle description, the first artificial potential field function we used [6] was based on the values of the function $f(\mathbf{x})$

$$U_{\mathcal{O}}(\mathbf{x}) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{f(\mathbf{x})} - \frac{1}{f(\mathbf{x}_0)} \right)^2 & \text{if } f(\mathbf{x}) \le f(\mathbf{x}_0); \\ 0 & \text{if } f(\mathbf{x}) > f(\mathbf{x}_0). \end{cases}$$
(19)

The region of influence of this potential field is bounded by the surfaces $f(\mathbf{x}) = 0$ and $f(\mathbf{x}) = f(\mathbf{x}_0)$, where \mathbf{x}_0 is a given point in the vicinity of the obstacle and η a constant gain. This potential function can be obtained very simply in real-time since it does not

require any distance calculations. However, this potential is difficult to use for asymmetric obstacles, where the separation between an obstacle's surface and equipotential surfaces can vary widely.

Using the shortest distance to an obstacle \mathcal{O} , we have proposed [7] the following artificial potential field

$$U_{\mathcal{O}}(\mathbf{x}) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)^2 & \text{if } \rho \leq \rho_0 ;\\ 0 & \text{if } \rho > \rho_0. \end{cases}$$
 (20)

where ρ_0 represents the limit distance of the potential field influence and ρ , the shortest distance to the obstacle \mathcal{O} . The selection of the distance ρ_0 will depend on the end-effector operating speed V_{max} and on its deceleration ability. End-effector acceleration characteristics are discussed in [15].

Any point of the robot can be subjected to the artificial potential field. A *Point Subjected* to the *Potential* is called a PSP. The control of a PSP with respect to an obstacle O is achieved using the FIRAS function

$$\mathbf{F}_{(\mathcal{O}, psp)}^* = \begin{cases} \eta(\frac{1}{\rho} - \frac{1}{\rho_0}) \frac{1}{\rho^2} \frac{\partial \rho}{\partial \mathbf{x}} & \text{if } \rho \leq \rho_0 ; \\ 0 & \text{if } \rho > \rho_0 ; \end{cases}$$
 (21)

where $\frac{\partial \rho}{\partial \mathbf{x}}$ denotes the partial derivative vector of the distance from the PSP to the obstacle

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y} \frac{\partial \rho}{\partial z} \right]^{T}.$$
 (22)

The joint forces corresponding to $\mathbf{F}_{(0,psp)}^*$ are obtained using the Jacobian matrix associated with this PSP. Observing (6) and (12), these forces are given by

$$\Gamma_{(\mathcal{O},psp)} = J_{psp}^{T}(\mathbf{q})\Lambda(\mathbf{x})\mathbf{F}_{(\mathcal{O},psp)}^{*}.$$
(23)

7. Obstacle Geometric Modeling

Obstacles are described by the composition of *primitives*. A typical geometric model base includes primitives such as a point, line, plane, ellipsoid, parallelepiped, cone, and cylinder. The first artificial potential field (19) requires analytic equations for the description of obstacles. For primitives such as a *parallelepiped*, *finite cylinder*, and *cone*, we have developed analytic equations representing envelopes which best approximate the primitives' shapes.

The surface, termed an n-ellipsoid, is represented by the equation

$$\left(\frac{x}{a}\right)^{2n} + \left(\frac{y}{b}\right)^{2n} + \left(\frac{z}{c}\right)^{2n} = 1; \tag{24}$$

and tends to a parallelepiped of dimensions (2a,2b,2c) as n tends to infinity. A good approximation is obtained with n=4, as shown in Figure 2.

A cylinder of elliptical cross section (2a, 2b) and of length 2c can be approximated by the so-called n-cylinder equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^{2n} = 1. \tag{25}$$

The analytic description of primitives is not necessary for the artificial potential field (20), since the continuity and differentiablity requirement is on the shortest distance to the obstacle. The primitives above, and more generally all convex primitives, comply with this requirement.

Determining the orthogonal distance to an n-ellipsoid or to an n-cylinder requires the solution of a complicated system of equations. To avoid this costly computation, a variational procedure for the distance evaluation has been developed. The distance expressions for other primitives are presented in [16].

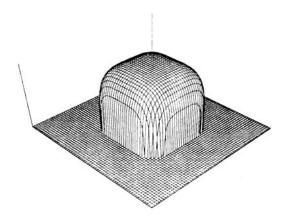


Figure 2: An n-ellipsoid with n = 4

8. Robot Obstacle Avoidance

An obstacle \mathcal{O}_i is described by a set of primitives $\{\mathcal{P}_p\}$. The superposition property (additivity) of potential fields enables the control of a given point of the manipulator with respect to this obstacle by using the sum of the relevant gradients

$$\mathbf{F}_{\mathcal{O}_{i},psp}^{*} = \sum_{p} \mathbf{F}_{(\mathcal{P}_{p},psp)}^{*}. \tag{26}$$

Control of this point for several obstacles is obtained using

$$\mathbf{F}_{psp}^* = \sum_{i} \mathbf{F}_{(O_i, psp)}^*. \tag{27}$$

It is also feasible to have different points on the manipulator controlled with respect to different obstacles. The resulting joint force vector is given by

$$\Gamma_{obstacles} = \sum_{i} J_{psp_{j}}^{T}(\mathbf{q}) \Lambda(\mathbf{x}) \mathbf{F}_{psp_{j}}^{*}.$$
 (28)

Specifying an adequate number of PSPs enables the protection of all of the manipulator's parts. An example of a dynamic simulation for a redundant 4 dof manipulator operating in the plane [Khatib and Le Maitre 1978] is shown in the display of Figure 3. The artificial potential field approach can be extended to moving obstacles, since stability of the mechanism persists with a continuously time-varying potential field.

The manipulator obstacle avoidance problem has been formulated in terms of collision avoidance of links, rather than points. Link collision avoidance is achieved by continuously controlling the link's closest point to the obstacle. At most, n PSPs then have to be considered. Additional links can be artificially intoduced or the length of the last link can be extended to account for the manipulator tool or load. In an articulated chain, a link can be represented as the line segment defined by the Cartesian positions of its two neighboring joints. In a frame of reference \mathcal{R} , a point m(x, y, z) of the link bounded by $m_1(x_1, y_1, z_1)$ and $m_2(x_2, y_2, z_2)$ is described by the parametric equations

$$x = x_1 + \lambda(x_2 - x_1) y = y_1 + \lambda(y_2 - y_1) z = z_1 + \lambda(z_2 - z_1)$$
(29)

The problem of obtaining the link's shortest distance to a parallelepiped can be reduced to that of finding the link's closest point to a vertex, edge, or face. The analytic expressions of

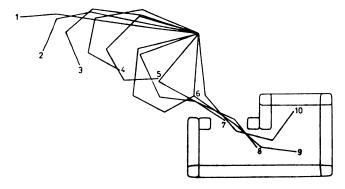


Figure 3: Displacement of a 4 dof manipulator inside an enclosure.

the link's closest point, the distance, and its partial derivatives for a parallelepiped, cylinder and cone are given in [16].

9. Joint Limit Avoidance

The potential field approach can be used to satisfy the manipulator internal joint constraints. Let \underline{q}_i and \overline{q}_i be respectively the minimal and maximal bounds of the i^{th} joint coordinate q_i . q_i can be kept within these boundaries by creating barriers of potential at each of the hyperplanes $(q_i = \underline{q}_i)$ and $(q_i = \overline{q}_i)$. The corresponding joint forces are

$$\gamma_{\underline{q}_{i}} = \begin{cases}
\eta(\frac{1}{\underline{\ell}_{i}} - \frac{1}{\underline{\ell}_{i(0)}})\frac{1}{\underline{\ell}_{i}^{2}} & \text{if } \underline{\ell}_{i} \leq \underline{\ell}_{i(0)}; \\
0 & \text{if } \underline{\ell}_{i} > \underline{\ell}_{i(0)};
\end{cases}$$
(30)

and

$$\gamma_{\overline{q}_{i}} = \begin{cases} -\eta(\frac{1}{\overline{\rho}_{i}} - \frac{1}{\overline{\rho}_{i(0)}})\frac{1}{\overline{\rho}_{i}^{2}} & \text{if } \overline{\rho}_{i} \leq \overline{\rho}_{i(0)}; \\ 0 & \text{if } \overline{\rho}_{i} > \overline{\rho}_{i(0)}; \end{cases}$$
(31)

where $\underline{\rho}_{i(0)}$ and $\overline{\rho}_{i(0)}$ represent the distance limit of the potential field influence. The distances $\underline{\rho}_i$ and $\overline{\rho}_i$ are defined by

$$\frac{\rho_i}{\overline{\rho}_i} = q_i - q_i \\
\overline{\rho}_i = \overline{q}_i - q_i$$
(32)

10. Level of Competence

The potential field concept is indeed an attractive approach to the collision avoidance problem, and much research has recently been focused on its applications to robot control [17-19]. However, the complexity of tasks that can be achieved with this approach is limited. In a cluttered environment, local minima can occur in the resultant potential field. This can lead to a stable positioning of the robot before reaching its goal. While local procedures can be designed to exit from such configurations, limitations for complex tasks will remain. This is because the approach has a *local* perspective of the robot environment.

Nevertheless, the resulting potential field does provide the global information necessary, and a collision-free path, if attainable, can be found by linking the absolute minima of the potential. Linking these minima requires, however, a computationally expensive exploration of the potential field. This goes beyond the real-time control we are concerned with here, but can be considered as an integrated part of higher level control. Work on high level collision-free path planning based on the potential field concept has been investigated in [20].

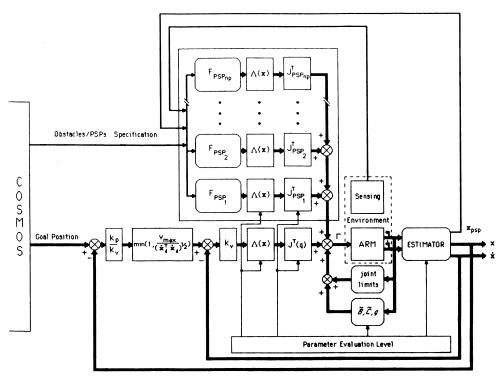


Figure 4: Control System Architecture.

11. Real-Time Implementation

Finally, the global control system integrating the potential field concept with the operational space approach has the following structure

$$\Gamma = \Gamma_{motion} + \Gamma_{obstacles} + \Gamma_{joint-limit}; \tag{33}$$

where

$$\mathbf{\Gamma}_{motion} = J^{T}(\mathbf{q})\Lambda(\mathbf{q})\mathbf{F}_{m}^{*} + \tilde{B}(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}(\mathbf{q})[\dot{\mathbf{q}}^{2}] + \mathbf{g}(\mathbf{q}); \tag{34}$$

The control system architecture is shown in Figure 4 where np represents the number of PSPs. The Jacobian matrices $J_{psp_j}^T(\mathbf{q})$ have common factors with the end-effector Jacobian matrix $J^T(\mathbf{q})$. Thus, their evaluation does not require significant additional computation.

12. Applications

This approach has been implemented in an experimental manipulator programming system COSMOS (Control in Operational Space of a Manipulator-with-Obstacles System). Using a PUMA 560 and a Machine Intelligence Corporation vision module, demonstration of real-time collision avoidance with links and moving obstacles [21] have been performed.

We have also demonstrated real-time end-effector motion and active force control operations with the COSMOS system, using wrist and finger sensing. These include contact, slide, insertion, and compliance operations [22].

In the current multiprocessor implementation (PDP 11/45 and PDP 11/60), the rate of the servo control level is 225 Hz while the coefficient evaluation level runs at 100 Hz.

13. Summary and Discussion

We have presented our operational space formulation of manipulator control which provides the basis for this obstacle avoidance approach, and have described the two-level architecture designed to increase the real-time performance of the control system. The operational space formulation has been shown to be an effective means of achieving high dynamic performance in real-time motion control and active force control of robot manipulators for complex assembly tasks. In addition, the complex transformation of the task into joint coordinates, required in conventional joint space control approaches, is eliminated.

Further, we have described the formulation and the implementation of a real-time obstacle avoidance approach based on the artificial potential field concept. In this approach, collision avoidance, generally treated as high level planning, has been demonstrated to be an effective component of low level real-time control.

The integration of this low level control approach with a high level planning system seems to be one of the more promising solutions to the obstacle avoidance problem in robot control. With this approach, the problem may be treated in two stages:

- at high level control, generating a global strategy for the manipulator's path in terms of intermediate goals (rather than finding an accurate collision-free path);
- at the low level, producing the appropriate commands to attain each of these goals, taking into account the detailed geometry and motion of manipulator and obstacle, and making use of real-time obstacle sensing (low level vision and proximity sensors).

By extending low level control capabilities and reducing the high level path planning burden, the integration of this collision avoidance approach into a multi-level robot control structure will improve the real-time performance of the overall robot control system. Potential applications of this control approach include moving obstacle avoidance, collision avoidance in grasping operations, and obstacle avoidance problems involving multi-manipulators or multi-fingered hands.

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