**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Given that

mean =45

standard deviation =8

the work will started after 10 min

hence the mean would be 45+10=55

the car will be ready after one hour (x)=60

hence z=(60-55)/8=0.625

From z table, z value will be 0.73237

The probability that the service manager cannot meet his commitment=1-0.73237

0.2676

So the option B is correct.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: The given statement is false.

Explanation:

The probability of employees older than 44:

X=44

Mean=38

Std=6

1-Stats.norm.cdf(44,38,6)=1-0.8413

= 0.1587

Now, find the probability of employees between 38 and 44:

Stats.norm.cdf (44,38,6) -stats.norm.cdf (38,38,6)

=0.8413-0.5

=0.3413

The probability of employees age between 38 to 44 is more than employees age more than 44.

So the statement is **false**.

1. A training program for employees under the age of 30 at the center would be

Ans: the statement is **TRUE**

Explanation :

Calculate the probability of employees age under 30 :

X=30

Mean=38

Std=6

Stats.norm.cdf(30,38,6)=0.0912

The total number of employees age under 30 is 0.0912\*400=36.48

Hence the statement is true

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: According to the central limit theorem, any large sum of the identically and independent distributed random variable is going to be approximately normal

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Option D is correct.

Explanation:

Given that

Mean=100

Std=20

The probability of random variable taking the value between them is 0.99

Hence,

Z value at 99%= stats.norm.ppf(0.995)=2.5758

2.2758\*20+100=151.51

(-2.5758)\*20+100=48.484

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Ans: The above problem having 2 profits

Total number of profits=2

Mean= [profit 1+ profit 2]

=5+7 =12

Std=(9+16)

=sqrt(25)=5

Mean in rs = 12\*45 =540

Std in rs = 5\*45 = 225

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range for 95% :-

Stats.norm.interval(0.95,540,225)

Range is rs (99.088,908.991)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

the 5th percentile:

Z score we need to find the value of 0.5000-0.050=0.4500

We are getting the value of -1.645

The 5th percentile of profit= mean+(-1.645)\*std

=540-(1.645\*225)

=540-370.125

=169.87= 170 in million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: probability of 1st division making loss = stats.norm.cdf(0,5,3)

=0.0479

Probability of 2nd division making loss= stats.norm.cdf(0,7,4)

= 0.04005