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High-Frequency Trading Strategy Based on Deep Neural Networks

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For my beloved mother, father and sisters.
Thanks for always being there for me.

Abstract

Recent conceptual and engineering breakthroughs in Machine Learning (ML), particularly in Deep Neural Networks (DNN), have revolutionized the Computer Science field and have been responsible for astonishing breakthroughs in computer vision, speech recognition, facial recognition, transaction fraud detection, automatic translation, video object tracking, natural language processing, and robotics, virtually disrupting every aspect of our lives. The financial industry has not been oblivious to this revolution; since the introduction of the first ML techniques, there have been efforts to use them as financial modeling and decision tools rendering in some cases limited and other in cases useful results, but overall, not astonishing results as in other areas. Some of the most challenging problems for ML come form finance, for instance, price prediction whose solution will require not only the most advanced ML techniques but also other non-standard and uncommon methods and techniques, giving the origin of a new field called Financial ML, whose name has been coined by Lopez de Prado last year.

Today, many hedge funds and investment banks have ML divisions, using all kinds of data sources and techniques, to develop financial modeling and decision tools. Consequently, ML is a part of the present and probably will be the future of the financial industry. In this thesis, we use the Deep Neural Networks (DNN) and Recurrent Neural Networks (RNN), two of the most advanced ML techniques, whose learning capabilities are enhanced using the representational power of the Discrete Wavelet Transform (DWT), to model and predict short-term stock prices showing that these techniques allow us to develop exploitable high-frequency trading strategies.

Since high-frequency financial (HF) data are expensive, difficult to access, and immense (Big Data), there is no standard dataset in Finance or Computational Finance. Therefore, the chosen testing dataset consists of the tick-by-tick data of 18 well-known companies from the Dow Jones Industrial Average Index (DJIA). This dataset has 348.98 millions of transactions (17 GB) from January 2015 to July 2017.

After a long iterative process of data exploration and feature engineering, several features were tested and combined. The tick-by-tick data are preprocessed and transformed using the DWT with a Haar Filter. The final features consist of the sliding windows of two variables: one-minute pseudo-log-returns (the logarithmic difference between one-minute average prices) and the features generated by the DWT.

These transformations, which are non-standard data transformations in finance, will better represent the high-frequency behavior of Financial Time Series (FTS). Moreover, the DNN predicts the next one-minute pseudo-log-return that can be transformed into the next predicted one-minute average price. These prices will be used to build a high-frequency trading strategy that buys (sells) when the next one-minute average price prediction is above (below) the last one-minute closing price.

Results show that (i) the proposed DNN achieves a highly competitive prediction performance in the price prediction domain given by a Directional Accuracy (DA) ranging from 64% to 72%. (ii) The proposed strategy yields positive profits, a max draw-down less or equal to 3%, and an annualized volatility ranging from 3% to 9% for all stocks.

The main contribution is the innovative approach for predicting FTS. It includes the combination of the advanced learning capabilities of the Deep Recurrent Neural Networks (DRNNs), the representational power in frequency and time domains of the DWT, and the idea of modeling time series through average prices.

Keywords: Short-term price Forecasting, High-frequency financial data, High-frequency Trading, Algorithmic Trading, Deep Neural Networks, Discrete Wavelet Transform, Computational Finance, Algorithmic trading.

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- Arévalo A., Hernandez G. (2017). Forecasting of One-minute Average Prices using Deep Architectures. Quinto Coloquio Doctoral de la Facultad de Ingeniería. Universidad Nacional de Colombia. Bogotá, Colombia.

- Arévalo A., León D., Hernandez G. (2019). Hierarchical Portfolio Construction using Inverse-Variance Weighting. Septimo Coloquio Doctoral de la Facultad de Ingeniería. Universidad Nacional de Colombia. Bogotá, Colombia.

Chapter 1

Introduction

Financial Markets are essential for the economy and society. They offer several advantages that are crucial and central for economic development, which will eventually result in an improvement in life quality [84, 45, 32, 98, 100]: Financial Markets can transform saving into investment, improve the allocation of capital, affect the saving rate, provide liquidity for critical actors to grow their businesses, increase efficiency for price discovery and reduce costs by enabling a competitive environment among market participants, generate jobs and give back more benefits to society.

However, our understanding of the markets is still very weak. Although we have a large number of theory books with very elegant formulas supported by rigorous demonstrations, which are quite difficult to follow by non-mathematicians, these usually have unrealistic suppositions and restrictions that do not model the actual behavior of the markets. When hedge funds and investors try to create or improve new trading strategies, the expected behavior of those strategies is to fail, because markets do not behave as theoretically expected. And that is the same reason why only a few traders become successful and make big fortunes, while most fail. The few who achieve success, have something in common: They are not systematic and are governed by subjective and empirical criteria [28].

1.1 Algorithmic Trading

With the evolution of computers, financial markets also evolved. Since the last century, human participants began to be replaced by machines. Now the traditional hedge funds are moving to the quantitative world, a world that unlike the traditional one, is completely systematic and is supported entirely by algorithmic trading strategies.

Algorithmic trading, defined as the use of automated programs that make trading orders, has been widely used in the middle of the previous century [14]. Another more technical definition of algorithmic trading is an online algorithm that opens and closes automatically trading positions, seeking to make the best decision without knowing the future. This type of

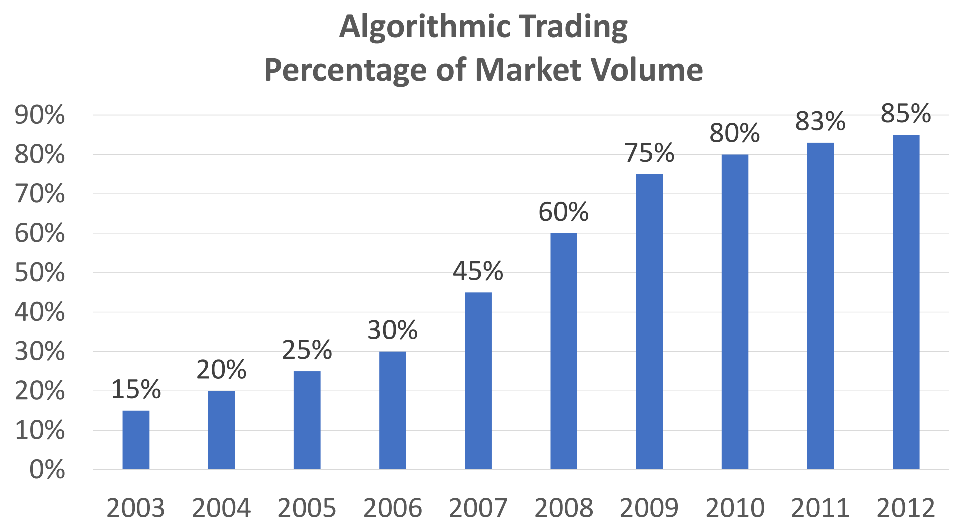


Figure **1-1**: Algorithmic Trading: Percentage of Market Volume. Source: [43].

problems is studied in Online Algorithms, Stochastic Dynamic Programming, and Stochastic Optimal Control. Moreover, these algorithms can work at several time scales:

- Low-Frequency (LF): Days, Weeks, Months or more.
- Medium-Frequency (MF): Hours or Minutes.
- High-Frequency (HF): Seconds, Milliseconds or less. At this time-scale, humans have a disadvantage given the fact that machines are faster and can process much information within a short time.

Figure **1-1** shows the percentage of American equities market volume originated by machines. The percentage increased considerably from 15% in 2003 to 85% in 2012 [43]. According to the 2015 Algorithmic Trading Survey [105], algorithmic trading offers several advantages (Figure **1-2**):

- The main reason for using algorithms is the ease of use (16.3%): Given the technological advances, there are now specialized algorithms in electronic trading platforms as Quantopian and JForex by Dukascopy Bank, which allow non-experts in programming, to build their algorithms for buying or selling stocks and currencies. Moreover, these platforms allow performing back-testing over available historical data, to simulate and evaluate algorithm performance in previous market conditions. The back-testing can give an idea of how the strategy might perform in the future.
- Reduced market impact (11.3%): Due to the possibility of sending large orders in small batches, the market impacts are reduced. This advantage is closely related to anonymity (13.3%). The small batches are invisible, facing the huge volume available.

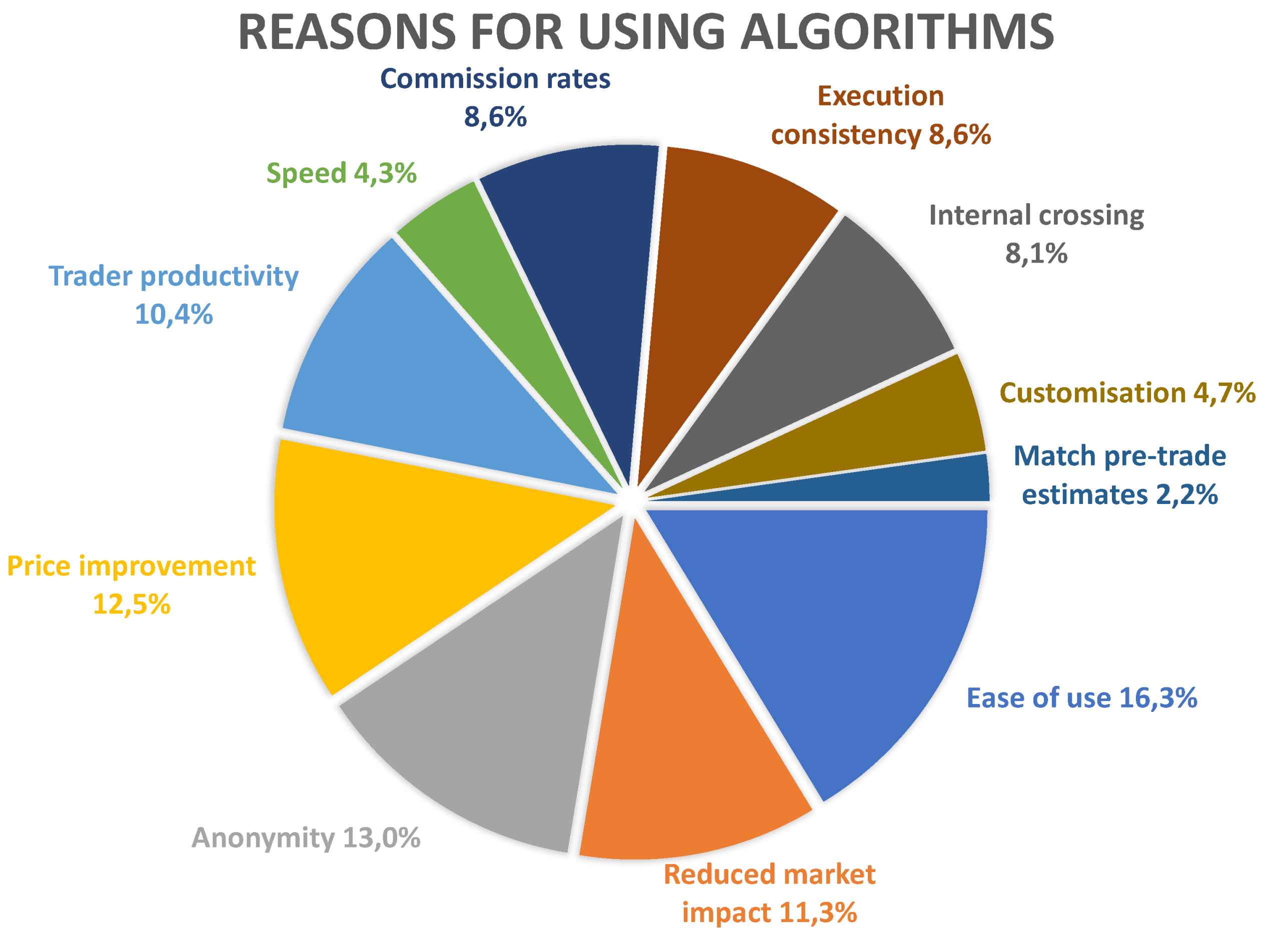


Figure **1-2**: Reasons for Using Algorithmic Trading. Source: Modified from [105].

- Price improvement (12.5%): The price discovery, described as the market process whereby new information is impounded into asset prices, can be performed efficiently. Furthermore, trades are executed at the best current price, because algorithms can track the market conditions. So, they can decide at any time and immediately execute it within milliseconds.

- Trader Productivity (10.4%): the risk of manual errors in placing the trades is reduced. Besides, humans cannot make decisions on short timescales and consider all the available information, but machines can. This advantage is closely related to speed (4.3%): With algorithmic trading, multiple market conditions can be automatically checked on small-scale times.

- Commission rates (8.6%): Cost transactions can be reduced.

- Execution consistency (8.6%): Algorithms receive feedback from real-time data and can continuously track the trading orders and market status.

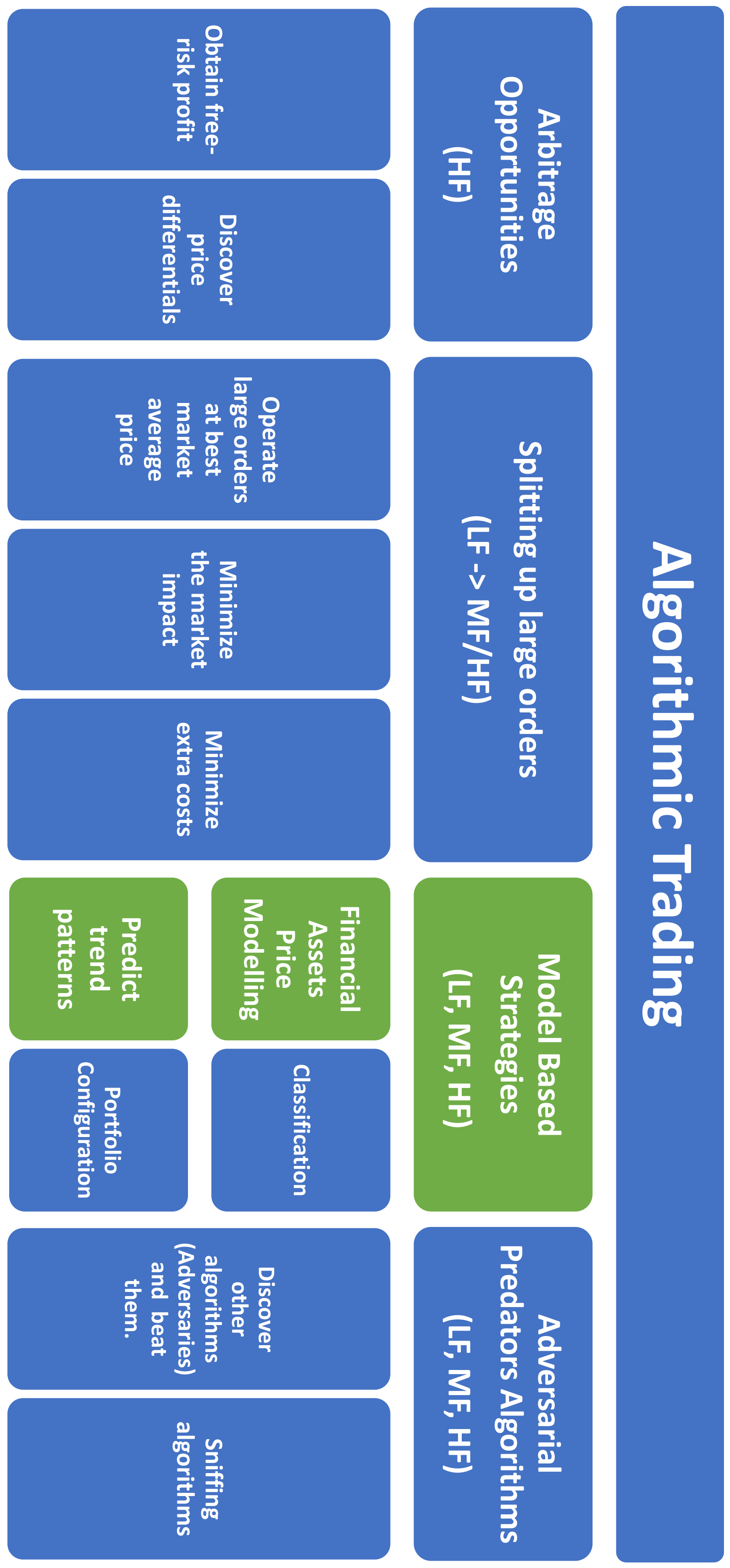

Algorithmic Trading
Arbitrage Opportunities (HF)
Obtain free-risk profit
Discover price differentials
Splitting up large orders (LF -> MF/HF)
Operate large orders at best market average price
Minimize the market impact
Minimize extra costs
Model Based Strategies (LF, MF, HF)
Financial Assets Price Modelling
Classification
Predict trend patterns
Portfolio Configuration
Adversarial Predators Algorithms (LF, MF, HF)
Discover other algorithms (Adversaries) and beat them.
Sniffing algorithms

Figure **1-3**: Algorithmic Trading. Based on [5, 18, 23, 97].

Financial markets have caught a lot of attention and interest from a large number of investors around the world. Figure **1-3** presents several types of algorithmic trading that practitioners have widely used. There are four important families: Arbitrage opportunities, algorithms for splitting up large orders, model-based strategies, and adversarial predators algorithms.

Arbitrage opportunities, which operate at an HF scale, consist of discovering price differentials between markets or exchanges rates. These strategies are risk-free profit and require speed. For instance, if an asset is cheaper in the Market A than in the Market B, an investor buys the asset in the Market A, and after the investor sells the asset rapidly in the Market B, the investor will earn the price differential minus transaction costs. Given the fact that the investor can calculate the final profit before making the transaction, this strategy is risk-free. Given the conditions of the perfect market and a large number of market participants, the arbitrage opportunities are increasingly challenging to find [3].

Additionally, not all strategies seek to gain profits. Other strategies also seek to avoid impacts on the market. When someone needs to operate large orders, this agent must be careful, if the agent sends the entire order, this action may cause price changes sharply, given the laws of supply and demand. It may produce overruns and losses.

Therefore, there is another kind of algorithms that break-up large orders, so the large order is hidden in small batches, and every batch can be operated at best market average price, minimizing the market impact and extra costs. Those strategies are composed of two components: the signal maker that operates at an LF scale, and the executor of the order that performs the corresponding buy/sell orders at an MD or HF scale [48].

Also, there are algorithms specialized in detecting other algorithms and sniffing their *modus operandi*, to take advantage and beat them. Those algorithms are called Beyond Trading Algorithms or adversarial predators algorithms, which can operate at any time scale (LF, MD, HF).

The model-based strategies, which operate at any time scale (LF, MD, HF), seek to model some financial markets dynamics to identify behavior patterns and take advantage of them, or even, to classify financial assets into different categories to build a robust portfolio and hedge the risk. The modeling and predicting prices of financial assets have proved to be arduous tasks that remain unsolved [25, 74]. This is mainly due to two reasons: First, financial markets exhibit a nonlinear, complex, stochastic, non-stationary and heteroskedastic behavior [65, 69, 87]. And second, markets also are a source of big data, then the task of handling financial data becomes a tricky challenge that requires high technical knowledge in computer science [28].

The most important approaches for price prediction are those based on simulation, analytical models, and computational intelligence techniques. Since the 1800s, analytical approaches (classical statistical techniques and stochastic processes modeling) have dominated the prediction arena [25, 65, 77]. Even though those classical models have been successful for handling time series in many disciplines, they have very limited precision and accuracy and low out-sample performance in the financial time series prediction [25].

Research during the last 20 years shows that computational intelligence approaches (more specifically the techniques based on Machine Learning) are more effective in financial time series tasks than analytical approaches [111, 59, 104, 85, 93, 16, 67, 116, 106, 34, 63, 21, 9, 64, 114, 81, 61, 19, 6, 55, 60, 109]. Computational Intelligence includes a wide variety of techniques: Decision trees, Dynamic Bayesian Networks, Hidden Markov Models, Support Vector Machines (SVMs), Kernel methods, Artificial Neural Networks (ANNs), and recently, Deep Learning (DL), which has emerged as a useful technique for asset price modeling. DL has proven to be able to learn complex representations of high-dimensional data and has demonstrated significant results in [27, 36, 118, 33, 22, 31, 7, 99, 12, 39].

For the construction of portfolios, investment portfolio management theory (Markowitz's Portfolio Optimization, Stochastic portfolio theory, Risk parity, among others) and clustering algorithms (Hierarchical clustering, k-means clustering, among others) are usually used.

Recent conceptual and engineering breakthroughs in Machine Learning (ML), particularly in Deep Learning (DL) whose lead developers Bengio, Hinton and LeCun were awarded last year with the most prestigious award in Computer Science, the ACM A.M. Turing Award [2], have revolutionized the Computer Science field and have been responsible for astonishing breakthroughs in computer vision, speech recognition, facial recognition, transaction fraud detection, automatic translation, video object tracking, natural language processing, and robotics, virtually disrupting every aspect of our lives. The financial industry has not been oblivious to this revolution. However, there have not been astonishing results in the financial industry as in other areas.

Now it is recognized that some of the most challenging problems for ML come from Finance, for instance, price prediction, whose solution will require not only the most advanced ML techniques but also require to develop their methods and techniques, giving the origin of a new field called Financial ML, whose name has been coined by Lopez de Prado last year [28]. Consequently, ML is a part of the present and probably will be the future of the financial industry. This transformation is also propelling developments in financial theory by looking for answers of why and how the ML models can capture more effectively the complex behavior of financial markets.

1.2 Problem Statement and Motivation

Financial markets play a vital role in our society since they affect economic growth, and in turn, can improve the people's quality life [84, 45, 32, 98, 100]. Consequently, it is critical to research and to improve our understanding of Financial Market Behaviour. Nevertheless, the main problems of research in financial markets are: First, there is too much secrecy in the industry. Second, there is no standard public data, then experiments and works are difficult to replicate or compare. And finally, the worst of all problems: the industry and academia are on different paths.

Furthermore, the funds are moving to the world of algorithmic trading, motivated by the machine learning revolution that has taken place in each of the aspects of our daily life. While it is true that it is a revolution which still does not stop. On many occasions machine learning is overrated and expectations are quite inflated. As a result, many quantitative hedge funds fail, not because the techniques do not work, but they are misused [28].

The main motivation of this thesis is to help build a bridge between the financial industry and academic research, providing experience and new approaches to improve understanding of the markets, and showing results that ML techniques allow us to develop exploitable HF trading strategies. Moreover, this thesis seeks to be a public base work for academic and industrial research in financial machine learning. This field is a new multidisciplinary world that is being born and will revolutionize the financial industry [28].

1.3 Hypothesis

The task of FTS prediction can be very challenging and quite frustrating. Like many data science projects, this work started because there was some data available. Our case was Apple's tick-by-tick price series during the financial crisis of 2008. This dataset was more difficult because it was of the real market in conditions of very high volatility (to see more information about this dataset, see [7, 8]).

Naively, we tried to solve the problem of predicting short-term time series with an end-to-end approach, that is, placing data as input to a machine learning model and letting the model do everything. Although we used all the available models available in Caret package (R) and scikit-learn package (Python) from Generalized Linear Models, Kernels, Support Vector Machines, Decision Trees to Neural Networks (Multilayer perceptron, Elman network and Jordan network), the out-sample performance always ranged from 50% to 54% of DA.

Frustrated by the results, we tried to create very deep neural networks with *H2O* (an open-source software for big data analysis) that received huge time windows of prices, and although the training times took days, the out-sample results did not improve significantly.

After reviewing the academic literature, we found many works adding unrealistic procedures like smoothing all the time series, removing data outliers, over-fitting, among others. But in a few papers, we saw something interesting: first, to include preprocessed information like Rolling averages, technical indicators. And second, to add dimension reduction procedures like PCA, auto-encoders, among others. We tried this new approach and the out-sample performance increased to %58.

The feature engineering approach significantly improved the results but it was not enough. After a long iterative process of data exploration and feature engineering, several features were tested and combined. The found features were $3n+2$ values: the current time (hour and minute), the last n one-minute pseudo-log-returns, the last n one-minute standard deviations of prices and the last n one-minute trend indicators, where n is the window size.

The trend indicator is calculated as the slope a of a fitted linear model on tick-by-tick prices within the same minute. Let P be the price, t be the time in milliseconds within the particular minute, and a and b be the linear model parameters. This statistical measurement is defined as follows:

$$P = at + b \tag{1-1}$$

A small slope, close to 0, means that during the next minute, the price is going to remain stable. A positive value means that the price is going to rise. A negative value means that the price is going to fall. Change is proportional to distance value compared to 0; if the distance is too far from zero, the price will rise or fall sharply. Figure **1-4** shows an example of the trend indicator.

We proposed a model based on a deep multi-layer network that includes those features. And this model achieved a %65 of DA [7, 8]. These non-conventional inputs proved to be definitive for improving model performance.

The inclusion of the time as an additional input, suggested that there are specific patterns with dynamic temporal dependencies in financial time series. However, a multi-layer network usually learns static patterns over time [49]; therefore, a more powerful model that supports temporal dynamics in its architecture (like Recurrent Neural Networks) could perform better. Furthermore, those works proved feature engineering is a critical tool for enhancing the model learning capabilities.

The major advantage of the trend indicator is to allow modeling the stock behavior at an HF scale. Since the trend indicator was a key feature, we tried to improve it. The first attempt was to make the linear model out of a polynomial of greater degree. Figure **1-5** shows an example.

However, this improvement did not work because the parameters of greater degrees of the trend indicator were very close to 0 in all cases. We had to add a pre-processing step of orthogonalize the polynomial and then to apply the linear model fitting. This orthogonalization was key for increasing the model performance.

Hence, the hypotheses of this work were to use Deep Recurrent Neural Networks (DRNN), such as Long Short-Term Models (LSTM) [52] and Gated Recurrent Units (GRU) [24], and to enhance their learning capabilities with the Discrete Wavelet Transform (DWT) as an input feature generator. Wavelets are useful for feature discovery [90, 80]. Unlike Fourier transforms, they provide a decomposition of a signal, or time series, with a resolution in the frequency and time domains. Wavelets are a useful way to extract key features from signals to reproduce them without having to save or keep the complete signal [108]. Moreover, the Haar Wavelet, which is the most simple orthogonal one, will be used for this work. Besides, they have been used successfully in handling FTS [90, 80].

This work is an exploratory study that will focus on model-based strategies for price predicting, specifically on those based on Recurrent Neural Networks (RNNs), which are a subtype of ANNs that includes Deep Learning (DL) paradigms and techniques. The proposed

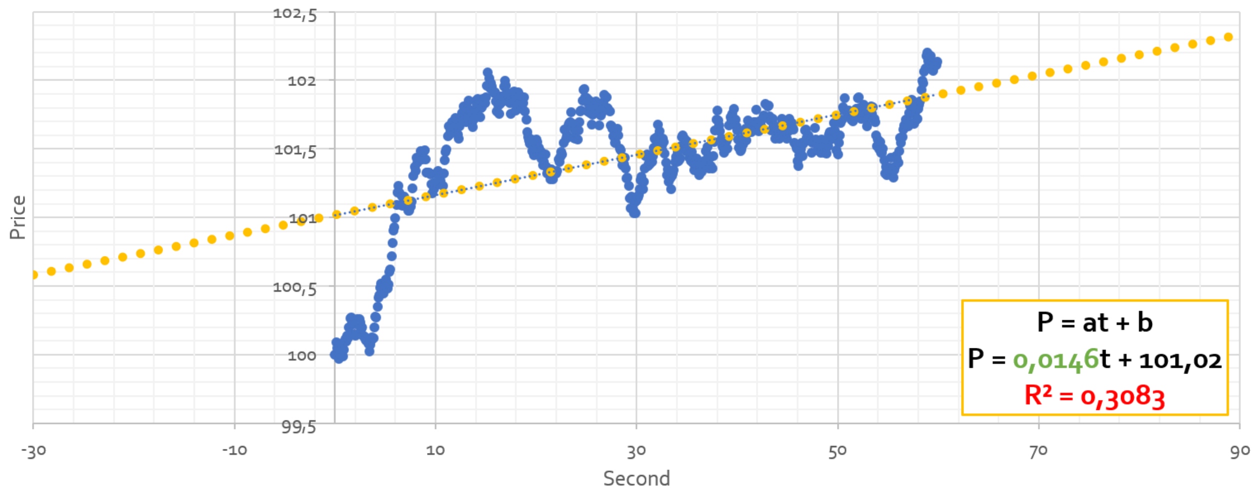


Figure **1-4**: Trend Indicator (Version I): The slope of a linear model.

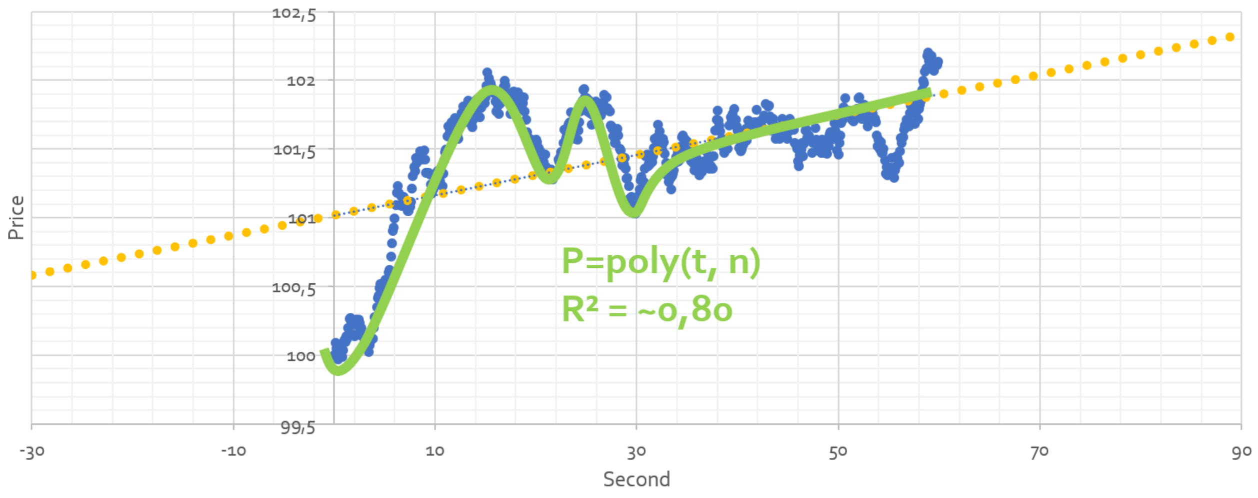


Figure **1-5**: Trend Indicator (Version II): The parameters of linear model.

strategy will use the tick-by-tick time series (HF data) of 18 companies from the Dow Jones Industrial Average Index (DJIA). We expect that wavelets in conjunction with DL allow improving the stock prices predictions on high-frequency data concerning previous results achieved in [8, 7].

1.4 Thesis Structure

This document continues as follows:

- Chapter 2 (Financial Time Series) presents background theory on Financial Time Series (FTS). This chapter collects a characterization of FTS and shows a brief overview of financial time series predictors.

- Chapter 3 (Wavelets) presents a background in wavelets analysis for FTS, emphasizing in the Haar wavelet and the Discrete Wavelet Transform (DWT). This chapter also shows the advantages of using this kind of transformation in FTS.
- Chapter 4 (Deep Neural Networks) presents background theory on DNNs and describes some common ANNs architectures, emphasizing Recurrent Neural Networks (RNNs). It also shows a brief timeline of ANNs and some remarkable applications in Finance.
- Chapter 5 (DNN Short-Term Price Predictor) proposes an innovative predictor based on DNN, DWT, and pseudo-log-returns. This chapter formalizes the concept of pseudo-log-returns and exposes the reason for using this non-standard data transformation.
- Chapter 6 (High-Frequency Strategy) presents the proposed trading strategy, which uses high-frequency data and the predictor described in Chapter 5.
- Chapter 7 (Experiments and Results) presents the experiments and obtained results for assessing the proposed predictor and the high-frequency strategy. The predictor is compared against three different alternatives of DNN architectures, ARIMA model, a Random Walk (RW) simulation, and other seven techniques of similar complexity: Linear regression, Ridge, Lasso, Bayesian ridge, Stochastic Gradient Descent (SGD) Regressor, Decision trees and Support Vector Regression (SVR). Besides, this chapter presents a comparison of the proposed strategy with three other types: Buy & Hold Strategy, Relative Strength Index (RSI) strategy with six different time windows, and one-minute random trading strategy.
- Chapter 8 (Conclusions) shows conclusions that range from the use of the DWT, the inclusion of DL, advantages of the feature engineering, the power of the pseudo-log-returns and the suitability of liquid stocks for high-frequency trading strategies, to the discussion of the success of the proposed strategy.
- Finally, Chapter 9 (Possible Opportunities for Future Research) proposes recommendations for future research and possible extensions. It is a good starting point to extend and improve this work.

Chapter 2

Financial Time Series

The study of time series is a universal topic and interesting for all practitioners and researchers who perform quantitative analyses on data indexed by time. Time series are used in any domain of knowledge that involves temporal measurements of a phenomenon. There are many properties and types of time series, but the most valuable ones are the stochastic series which are the realization of a stochastic process [107]. This chapter presents background theory on Financial Time Series (FTS) and shows a brief overview of FTS predictors.

2.1 Time Series

A time series X is an ordered collection of temporal observations X_t indexed by an index set T [40]. A time series is formally defined as follows:

$$X = \{X_1, X_2, \dots\} = \{X_t : \quad t \in T\} \tag{2-1}$$

The study of time series focuses on two areas [79]:

- **Time series analysis:** This area focuses on the study of methods and techniques for analyzing and characterizing series behavior.
- **Time series prediction:** This area focuses on the study of models for predicting aptly future observations of a time series X based on previously observed values. These models produce an approximate time series Y which is expected to be as close as possible to the actual time series X.

Given that this work focuses on time series prediction and proposes a prediction model, it is required to define how to measure the quality of an estimator or model. According to the reviewed literature, the most common measures in Finance are [57]:

- **Mean Squared Error (MSE):** It measures the average of the squares of the errors (the difference between all predicted time series values Y_t and expected time series values X_i). Although it is widely used in Finance and many other disciplines, it has a serious disadvantage for handling outliers and variables scales. Hence, it is a scale-dependent measure.

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(X_t - Y_t)^2 \tag{2-2}$$

- **Directional Accuracy (DA):** It measures the percentage of the predicted directions (Up, Down, None) that match with the ones of the time series X. Since DA is unaffected by outliers and variables scales, it is a scale-independent measure. As a result, this measure is widely used in Finance [96].

$$DA = \frac{100\%}{n-1}\sum_{t=2}^{n} 1_{sign(X_t - X_{t-1}) = sign(Y_t - T_{t-1})} \tag{2-3}$$

where 1_A is an indicator function, defined as follows:

$$1_A = f(x) = \begin{cases} 1, & A = \text{True} \\ 0, & A = \text{False} \end{cases} \tag{2-4}$$

and *sign* is a sign function, defined as follows:

$$sign(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0) \end{cases} \tag{2-5}$$

2.2 Financial Time Series

A Financial Time Series (FTS) is a collection of financial data points such as asset price, traded volume, earnings reports, business health data, or any transformation of the previous ones.

In recent years, great interest has been generated in the FTS prediction field, since a precise and accurate prediction of asset prices or stock market indexes is an indispensable tool for investment decision-making. However, FTS are characteristically stochastic and non-stationary [47, 1, 103]. Stochastic refers to the property of being randomly determined, whereas, non-stationary refers to the characteristic of having means and variances that change over time.

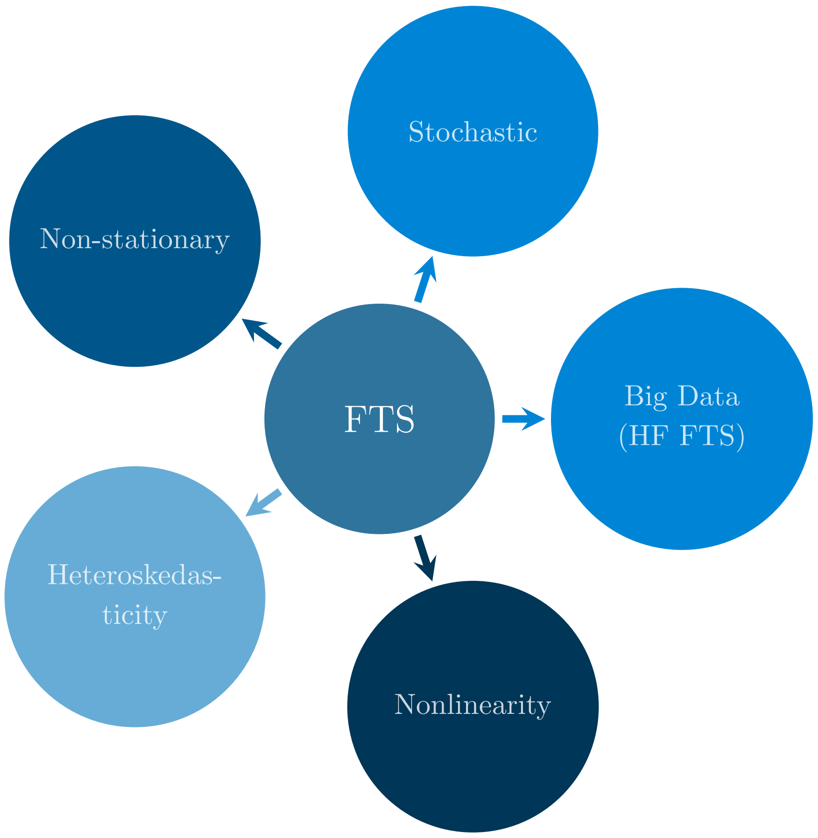


Figure **2-1**: Financial Time Series Characteristics.

As a result of the stochastic and non-stationary properties, an FTS has a probability density function that changes over time. Moreover, an FTS is conditional heteroscedastic, namely the variability of an FTS across time is dependent on other variables. Hence, FTS prediction is considered one of the most challenging tasks of the time series prediction arena. In many cases, the study of FTS has inspired the evolution of the forecasting methods, which apply to time series in general [107, 47, 26].

Furthermore, FTS collect data of price changes over time due to the interaction of market participants and their expectations, therefore, it is a hard challenge, because financial markets exhibit the seven properties inherent of complex systems [83]: "Historicity, Irreversible, Thoroughgoing change, Propagation across multiple levels, Multiple modes of projection, Emergence and Sensibility to externalities". Moreover, financial markets are a source of big data, then FTS becomes a tricky challenge to manipulate all this massive amount of information. Figure **2-1** shows the summary of FTS characteristics.

These characteristics are shaped by the action of many market participants acting actively in financial markets at different times, with a variety of investments size, making them difficult to analyze. It is common to transform them into a difference series, such as log-returns, in order to obtain a stationary time series and thus reduce somewhat its complexity and difficulty.

Log-return is a continuously compounded return on an investment. Let p_t and p_{t-1} be the current and previous closing price, respectively.

$$\begin{aligned} R_t &= \ln \frac{p_t}{p_{t-1}} \cdot 100\% = (\ln p_t - \ln p_{t-1}) \cdot 100\% \\ p_t &= p_{t-1} \cdot e^{\frac{R_t}{100\%}} \end{aligned} \tag{2-6}$$

Figure **2-2** shows a summary of quantitative models for price prediction. There are three important families of quantitative models for price prediction: Simulation approaches, Analytical approaches, and Computational intelligence. Simulation approaches are Game theory simulations and Monte Carlo simulations.

2.3 Price Prediction Arena

Analytical approaches consist mainly of statistical models and stochastic processes modeling. Statistical models were popular in the forecasting arena since the 1800s [77]. They include many classic models such as Linear Regression (LR), Exponential Smoothing (ES), Autoregressive (AR), Moving Averages (MA), Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Conditional Heteroskedasticity (GARCH).

Econometric models have dominated the time series prediction arena, specifically, linear statistical methods such as Auto-Regressive Integrated Moving Average (ARIMA). This model consists of the combination of the models AR and MA, and the statistical technique of differentiation [15].

The AR model represents a random process in which the output variable depends linearly on its previous values plus a white noise[78]. The white noise ε_t is a random signal which has equal intensity at any frequencies [17]. The $AR(p)$ model is defined as follows:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t \tag{2-7}$$

where φ_i is the i-th parameter of the model, c is a constant and p the order of the autoregressive model (number of time lags).

The MA model represents a random process in which the output variable depends linearly on the current and previous values of a white noise signal [78]. The $MA(q)$ model is defined as follows:

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2-8}$$

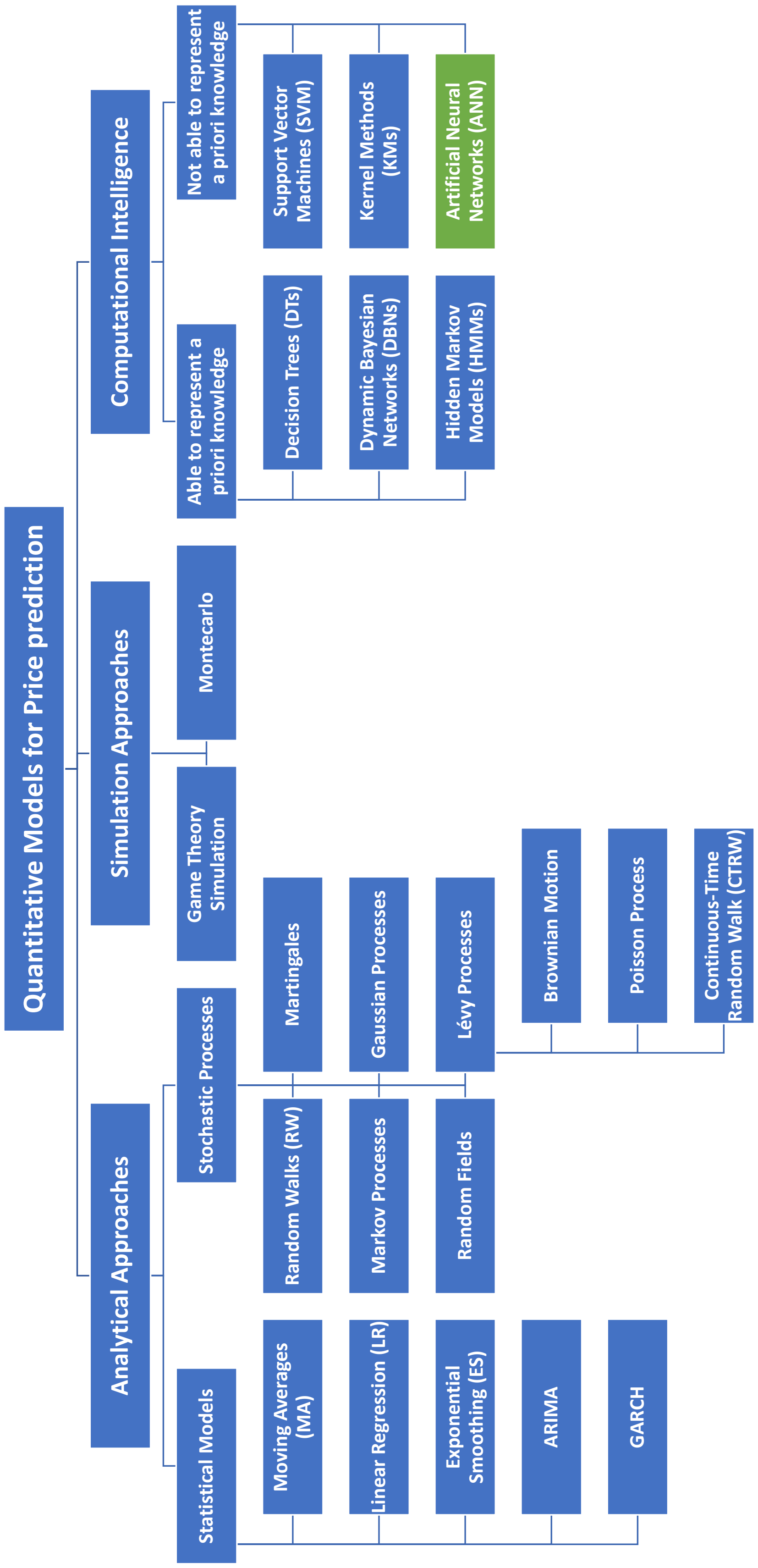


Figure **2-2**: Quantitative Models for Price Prediction. Based on [25, 65, 77].

where θ_i is the i-th parameter of the model, μ is the expectation of X_t and q the order of moving average model (number of lagged errors in the prediction equation).

The ARMA model consists on the combination of the AR and MA models [112]. The $ARMA(p,q)$ model is defined as follows:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2-9}$$

The differentiation technique is a time series transformation that makes it stationary by eliminating trend and seasonality [15]. The first-order differentiation is defined as follows:

$$X'_t = X_t - X_{t-1} \tag{2-10}$$

whereas, the second-order differentiation is defined as follows:

$$X''_t = X'_t - X'_{t-1} = (X_t - X_{t-1}) - (X_{t-1} - X_{t-2}) = X_t - 2X_{t-1} + X_{t-2} \tag{2-11}$$

An ARIMA model is a set of discrete time linear equations with noise, defined as follows:

$$ARIMA(p,d,q) = \left(1 - \sum_{k=1}^{p} \alpha_k L^k\right)(1-L)^d X_t = \left(1 - \sum_{k=1}^{q} \beta_k L^k\right)\epsilon_t. \tag{2-12}$$

where L is the lag operator and d the degree of the differencing model (number of non-seasonal differences needed to satisfy the stationary condition).

ARIMA models have been successful in the prediction of general time series. However, they are not able to capture all financial markets complexities and non-linear relationships [25]. Although ARIMA models usually perform well in in-sample datasets, they have a poor performance in out-sample datasets.

Besides, stochastic processes consist of discrete-time and continuous-time models like Random walks (RW), Martingales, Markov processes, Gaussian processes, Random fields, and Lévy processes, which are a generalization of Brownian motion, Poisson processes and Continuous-Time Random Walk (CTRW).

Furthermore, the Brownian motion process, also known as the Wiener process in honor of Norbert Wiener, is one of the best known Lévy processes, has been widely used for modeling price dynamics. Thus, it plays a vital role in quantitative finance [62].

In 1965, Eugene Fama published a prominent work about the consistency of the random walk model and Efficient Market Hypothesis (EMH) [37]. EMH was considered a paradigm theory in Finance by the middle of the 1970s arguing that asset prices capture and reflect all the available information.

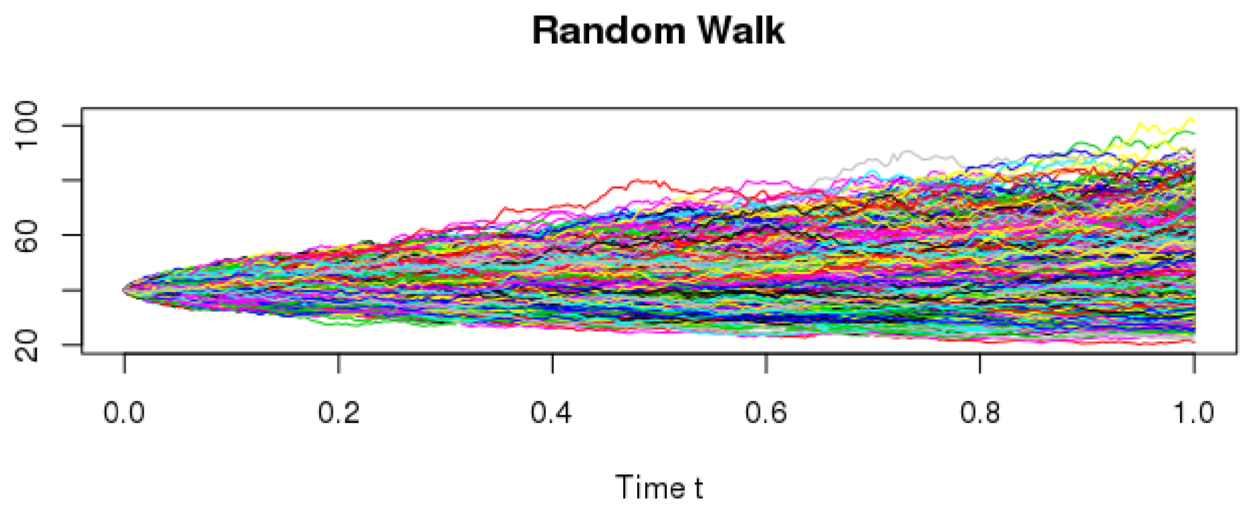


Figure **2-3**: Simulation of 100 Random Walks with Parameters $\mu = 0.16$, $\sigma = 0.50$ and $P_0 = 40$.

Several studies appeared in the 1930s [72, 38] and noticed that changes in stock prices seem to follow a fair-game pattern, the second underpinning of the EMH. This hypothesis has led to the random walk hypothesis, first exposed by Bachelier in 1900 [11], which states that stock prices are random, hence, are unpredictable. A consequence is clear, is not possible to "beat the market" and the prices follow a fair value.

Figure **2-3** shows several simulations of a random walk based on a stochastic process with drift and diffusion components, i.e. Brownian motion proposed as a benchmark model to compare the proposed approach for FTS prediction.

The premise is that investors react instantaneously to any additional information revealed, thus eliminating opportunities to obtain benefits persistently. Therefore, prices always fully reveal the information available and no profit can be obtained from information based trading [70]. It leads to a random walk where the market is more efficient. The EMH is the foundation of the theory that stock prices could follow a random walk. However, there is no consensus on whether the market follows this paradigm or not. Our intuition is that the market does not follow the EMH and is predictable to a certain degree that can be exploited by a trading strategy.

As a result of the poor performance in out-sample datasets of classical techniques, other techniques like machine learning methods have risen as an important alternative to handle FTS, since those techniques can learn and predict the complex market patterns from data. Research during the last 20 years shows that Computational intelligence approaches are more effective in predicting FTS than Analytical approaches [59, 19, 109, 61].

Besides, Computational intelligence has two subfamilies: the models that can represent a priori knowledge and those that cannot. For instance, Decision trees, Dynamic Bayesian Networks (DBNs), and Hidden Markov Models can represent a priori expert knowledge, whereas

Support Vector Machines (SVMs), Kernel methods, Artificial Neural Networks (ANNs) are like black boxes which can learn complex representations, but it is not known precisely what it does inside [26, 25, 65, 77]. ANNs were a favorite theme in data analysis during the 1980s. In 1988, the first known application of ANN for price prediction was presented by [111]. They used ANNs for predicting IBM daily stock returns.

Recently, Deep Learning (DL) models have demonstrated greater effectiveness in several tasks (classification and prediction), in different domains such as video analysis, audio recognition, text analysis, and image processing. After the DL boom in 2009 [56], the number of works related to finance has increased considerably.

Hence, DL has emerged as a useful technique for asset price modeling because it has proven to be able to learn complex representations of high-dimensional data. This work will focus on Deep Neural Networks (DNNs) like deep MLPs and RNNs, which are ones of the most widely used techniques in DL.

Chapter 3

Wavelets

Given the fact that financial markets exhibit non-linear and cyclical behaviors [94], wavelets are useful for feature discovery [90, 80]. Unlike Fourier transforms, they provide a decomposition of a signal, or time series, with a resolution in the frequency and time domains. Wavelets are a useful way to extract key features from signals in order to reproduce them without having to save or keep the complete signal [108]. They have been used successfully for handling FTS [90, 80].

A wavelet is a wave function with an average value of zero, which begins at zero, increases, and then decreases back to zero. Therefore, wavelet has a finite duration [42]. Moreover, wavelets are described by the mother wavelet function $\psi(t)$ and the scaling function $\varphi(t)$. The function $\psi(t)$ is a band-pass filter, whereas the function $\varphi(t)$ is a filter that ensures that all the spectrum is covered.

3.1 Haar Wavelet

On 1909, Alfréd Haar proposed the first and the simplest possible wavelet: the Haar wavelet [46], as shown in Figure **3-1**. Its mother wavelet function $\psi(t)$ is defined as follows:

$$\psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3-1}$$

whereas its scaling function $\varphi(t)$ is defined as follows:

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3-2}$$

The Haar wavelet has a technical disadvantage: It has discontinuities and hence it is not a differentiable function. However, its discontinuities are an advantage for FTS analysis

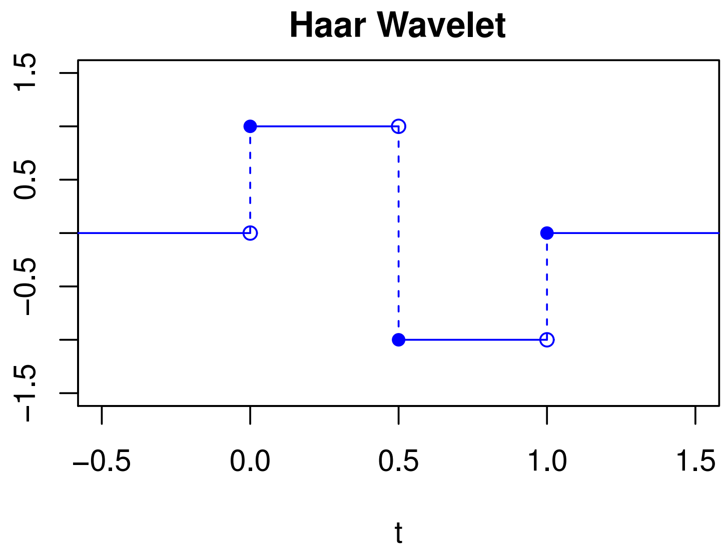


Figure **3-1**: Haar Wavelet.

because the discontinuities perfectly fit to handle signals with sudden transitions and jumps. Moreover, the Haar wavelet is able to approximate uniformly any continuous real function by linear combinations of $\varphi(2^k t + l)$, where $k \in \mathbb{N}$ and $l \in \mathbb{Z}$ [108].

3.2 Discrete Wavelet Transform

The wavelet transform could be either continuous or discrete. Since financial time series are discrete, the Discrete Wavelet Transform (DWT) is more suitable to handle FTS than the Continuous Wavelet Transform (CWT) [108].

The DWT consist of the following steps [108, 86]: First, a signal x is passed simultaneously through a low-pass filter with impulse response g and a high-pass filter h. The filters g and h are the dilated (by powers of two), reflected, and normalized versions of the mother wavelet and scaling function, respectively. The filters are defined as follows:

$$h[n] = \frac{1}{\sqrt{2^j}} \psi\left(\frac{-t}{2^j}\right) \tag{3-3}$$

$$g[n] = \frac{1}{\sqrt{2^j}} \varphi\left(\frac{-t}{2^j}\right) \tag{3-4}$$

Second, given the fact that the filters g and h remove the half of the signal frequencies, the half of the resulting samples can be discarded according to Nyquist's rule, that is, the resulting signal can be sub-sampled by 2. Finally, the high-pass filter produces the detail

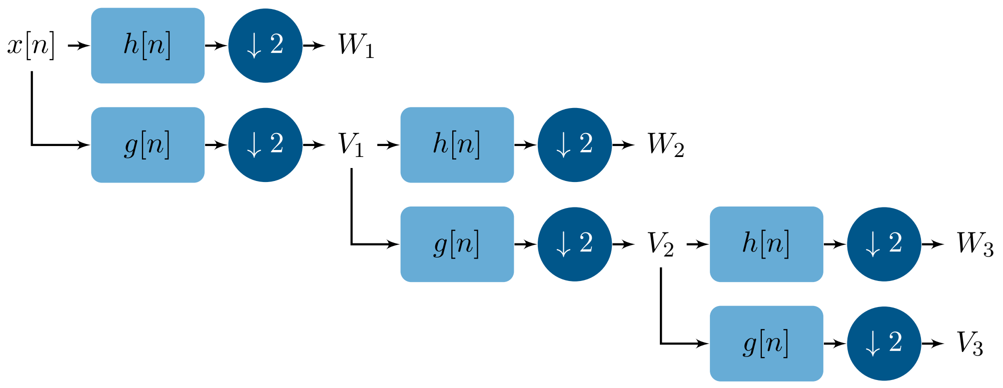


Figure **3-2**: Cascading and Filter Banks.

or wavelet coefficients W, whereas the low-pass filter produces the approximation or scaling coefficients V. The resulting signals, which are half the length of the original signal x, are defined as follows:

$$W = y_{\text{high}} = (x * h) \downarrow 2 = \sum_{k=-\infty}^{\infty} x[k]h[2n-k] \tag{3-5}$$

$$V = y_{\text{low}} = (x * g) \downarrow 2 = \sum_{k=-\infty}^{\infty} x[k]g[2n-k] \tag{3-6}$$

The previous transformation can be repeated multiple times in order to further increase the frequency resolution. The first filter outputs will be named Level 1 coefficients, then the transformation is applied again to the Level 1 scaling coefficients producing the Level 2 coefficients, and so on. The recursive transformation, which is presented in Figure **3-2**, is described as:

$$V_n = \begin{cases} (x * g) \downarrow 2 & n = 1 \\ (V_{n-1} * g) \downarrow 2 & \text{otherwise} \end{cases} \tag{3-7}$$

$$W_n = \begin{cases} (x * h) \downarrow 2 & n = 1 \\ (V_{n-1} * h) \downarrow 2 & \text{otherwise} \end{cases} \tag{3-8}$$

However, it has some disadvantages [50]. First, it requires that the length of the data be 2^j. Second, it is not shift-invariant. Finally, it may shift data peaks, causing wrong approximations when compared to the original data.

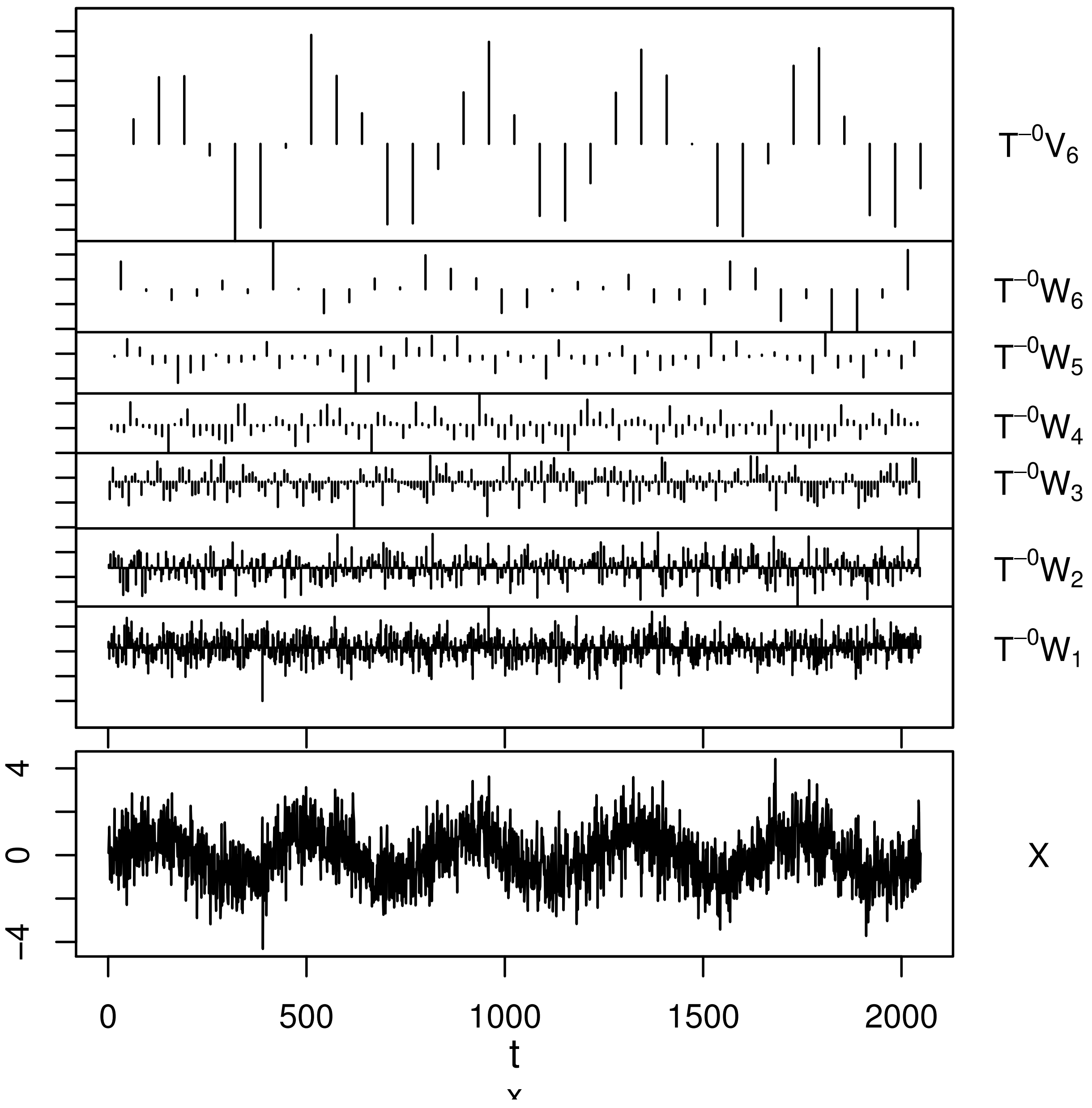


Figure **3-3**: DWT Decomposition.

Figure **3-3** shows an example of the DWT decomposition over a sinusoidal signal with noise. The signal is decomposed for six levels, and the figure shows the original signal, the six levels wavelets coefficients W and the last level scaling coefficients V.

Moreover, wavelets possess additional advantages that help to overcome the FTS properties [50]: First, it does not require a strong assumption about the data generation process. Second, it provides information in both time and frequency domains. Finally, it can locate discontinuities in the data.

Chapter 4

Deep Neural Networks

Artificial Neural Networks (ANNs) are a hot topic in intelligent systems, machine learning, data science, and other disciplines. This chapter presents the background theory on DNNs and describes some common ANNs architectures, emphasizing Recurrent Neural Networks (RNNs). It also shows a brief timeline of ANNs and some important applications in finance.

4.1 Artificial Neurons

ANNs are inspired by the biological neural networks on animals' brains. Like their biological analogous, they are a collection of connected units called neurons, where each neuron shares information with other ones.

An artificial neuron performs a weighted aggregation of all its input signals (each input is multiplied by a weight W). Then, it shifts the resulting signal adding a bias b, and applies an activation function, which is generally nonlinear (like Binary step, Sigmoid, Hyperbolic tangent, Arctangent, Softsign, Rectified Linear Unit (ReLU), Gaussian, among others). Finally, it propagates a new output signal to other neurons [49]. Figure **4-1** shows the structure of an artificial neuron.

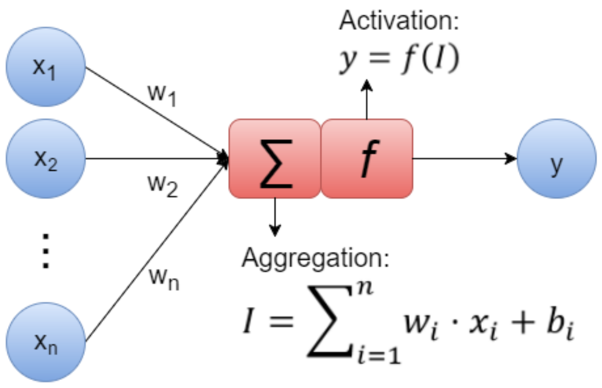
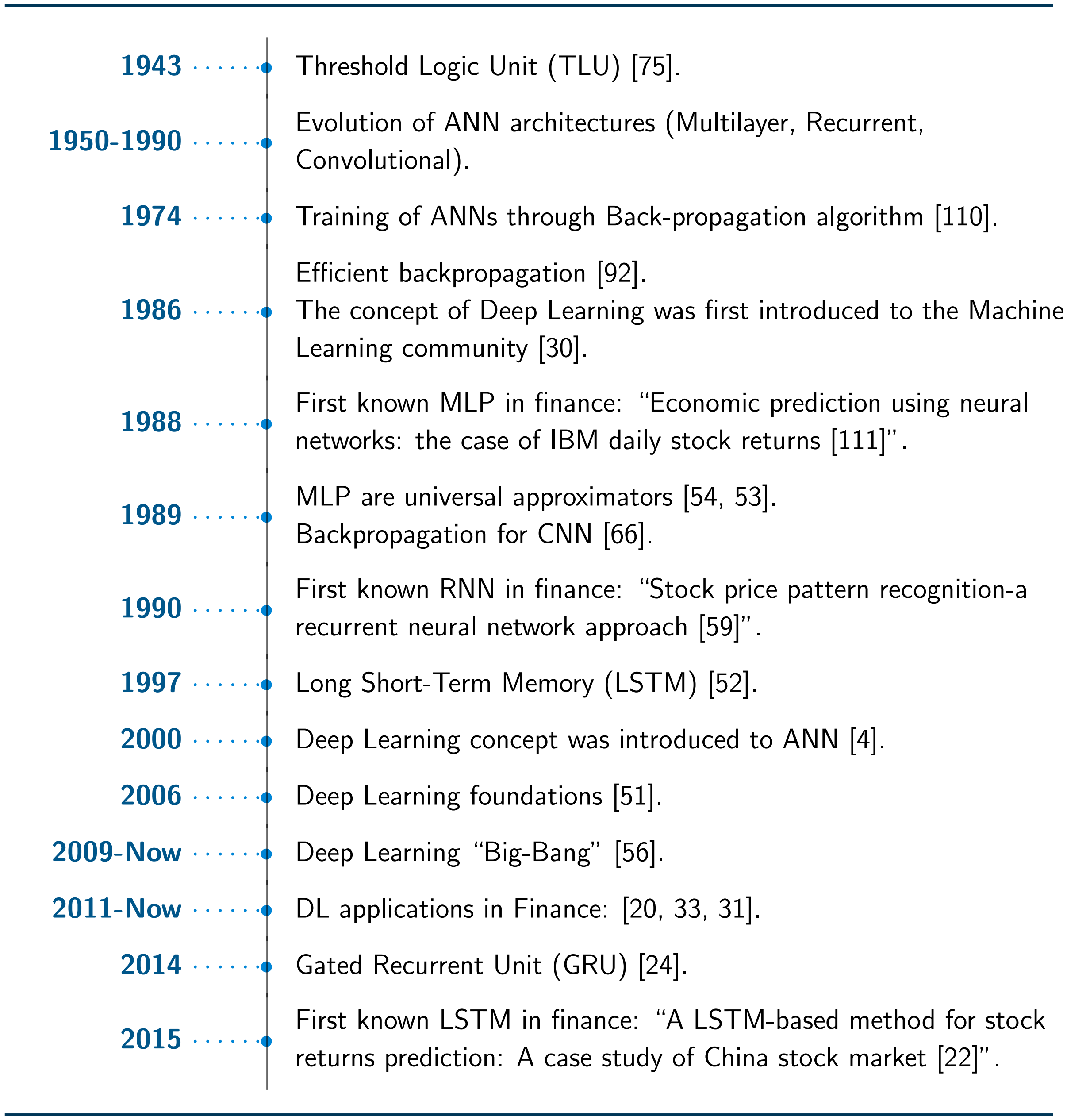


Figure **4-1**: Artificial Neuron.

ANNs have been widely used to predict FTS since the 1980s, due to its ability to identify and learn intricate patterns from data in high dimensional spaces [41, 65, 87, 101].

4.2 Artificial Neural Networks Evolution

Figure **4-2** shows a brief timeline about neural networks and some of its applications in Finance.



1943	Threshold Logic Unit (TLU) [75].
1950-1990	Evolution of ANN architectures (Multilayer, Recurrent, Convolutional).
1974	Training of ANNs through Back-propagation algorithm [110].
1986	Efficient backpropagation [92]. The concept of Deep Learning was first introduced to the Machine Learning community [30].
1988	First known MLP in finance: "Economic prediction using neural networks: the case of IBM daily stock returns [111]".
1989	MLP are universal approximators [54, 53]. Backpropagation for CNN [66].
1990	First known RNN in finance: "Stock price pattern recognition-a recurrent neural network approach [59]".
1997	Long Short-Term Memory (LSTM) [52].
2000	Deep Learning concept was introduced to ANN [4].
2006	Deep Learning foundations [51].
2009-Now	Deep Learning "Big-Bang" [56].
2011-Now	DL applications in Finance: [20, 33, 31].
2014	Gated Recurrent Unit (GRU) [24].
2015	First known LSTM in finance: "A LSTM-based method for stock returns prediction: A case study of China stock market [22]".

Figure **4-2**: Artificial Neural Networks in Finance.

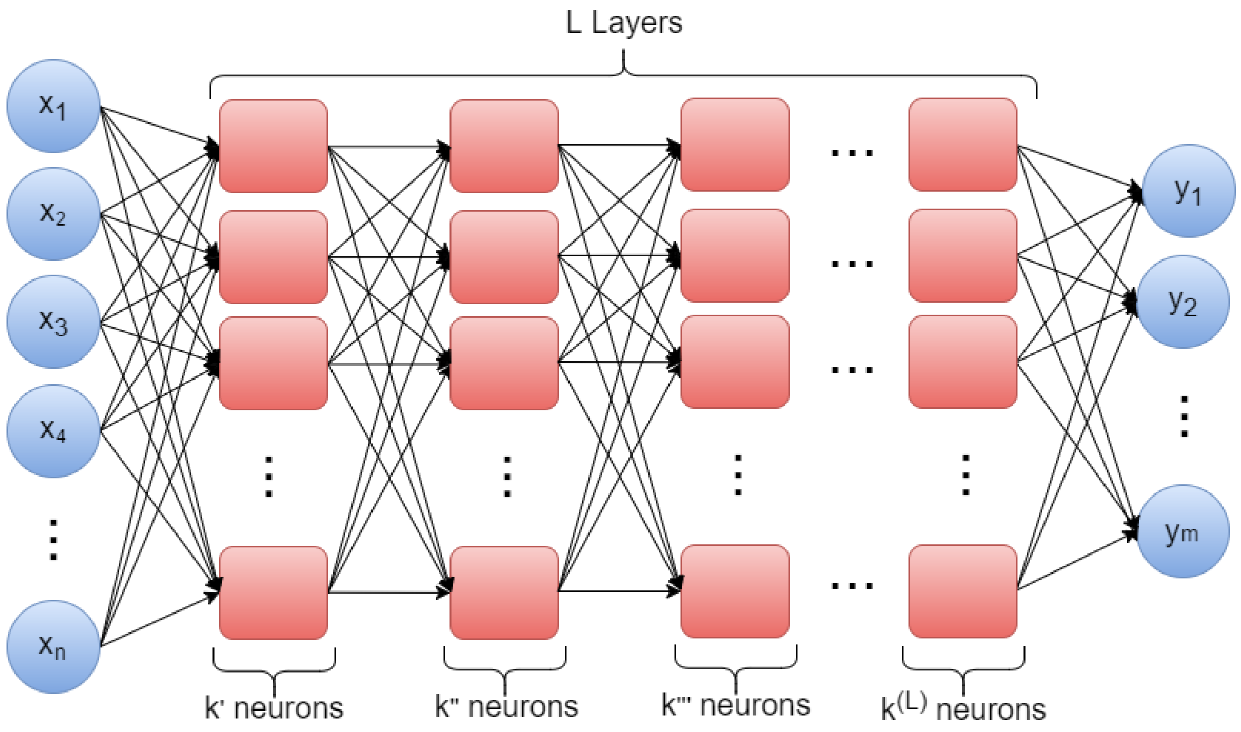


Figure **4-3**: Multilayer Perceptron.

In 1943, the Threshold Logic Unit (TLU) or Linear Threshold Unit was proposed in [75]. A TLU is the first neuron unit model. It performs a weighted sum of the boolean inputs and returns a boolean value if the sum exceeds a predefined linear threshold.

From the 1950s to 1990s, multiple networks architectures were proposed, but all lacked efficiency for learning algorithms. One of the first emerging ANN architectures was the Feed-forward Neural Network (FNN). This kind of ANN consists of multiple connected artificial neurons in such a way that there are no loops. In this way, the input signals always go forward from input to output neurons.

There are many FNN sub-types, but the most common and famous architecture is the Multilayer Perceptron (MLP). An MLP organizes its neurons in stacked layers of neurons. Each layer is composed of neurons fully-connected to all the neurons in the next layer. There are three kinds of layers: The input layer that contains all input neurons, the output layer that contains all outputs neurons, and the hidden layers that contain all intermediate neurons [49]. Figure **4-3** shows an MLP.

Convolutional Neural Networks (CNNs) are a variation of MLPs, whose layers are not fully connected, but are locally connected [66]. As CNNs have fewer connections and parameters; it requires minimal processing to be fitted.

In 1974, Werbos was the first author who described the training of ANNs through the back-propagation algorithm, which consists of the following steps [49, 91, 110]:

1. To initialize the weights matrices and biases vectors with random values.
2. To compute the error between the network's outputs and the desired outputs at the output neurons.
3. To calculate the descent gradient based on the weights and biases that allows minimizing the error for all layers from output to input.
4. To update weights and biases according to gradients.
5. Repeat steps 2-4 until max iterations are reached, or another stopping criterion is satisfied.

The idea behind this algorithm is that the error is propagated backward from the output layer to all neurons in the hidden layers that contributed to the error. However, on deep networks, namely with many layers, the error is diluted exponentially, and the gradient only modifies the last layers, whereas the first layers remain unmodified [49].

ANNs were a favorite theme in data analysis during the 1980s. During this decade, many promising works began to appear in all areas of knowledge, such as "Backpropagation Applied to Handwritten Zip Code Recognition" [66]. However only since 1988, the first known application of ANN in finance "Economic prediction using neural networks: the case of IBM daily stock returns" was published in [111].

In 1989, the universal approximation theorem was published. It demonstrated that a multilayer feed-forward network is a universal approximator that can approximate any linear or non-linear function [29, 54]. In the same year, LeCun also proposed an efficient back-propagation algorithm for CNN [66].

One of the first successful applications of ANNs was to predict sunspots. This problem had been studied since the last century and ANNs showed superiority over the available models in 1990 [68]. However, ANNs possess limitations related to finding good weights and biases that enable them to approximate the target data correctly. This happens because the training process is actually an optimization process. As a result, it gets stuck on local minimums, and the ANN does not rapidly converge to an optimal solution. Additionally, over-training causes over-fitting, which means the ANN memorizes the data and then it fails to generalize it [95, 115].

4.3 Deep Learning

In 2000, the concept of Deep Learning (DL), which was adopted from neuroscience, was first introduced to ANNs in [4]. However, this concept was first introduced to the Machine Learning community in 1986 [30]. It emerged as a novel way of making sparse and layered representations of data. These ideas were inspired by the way the visual system works:

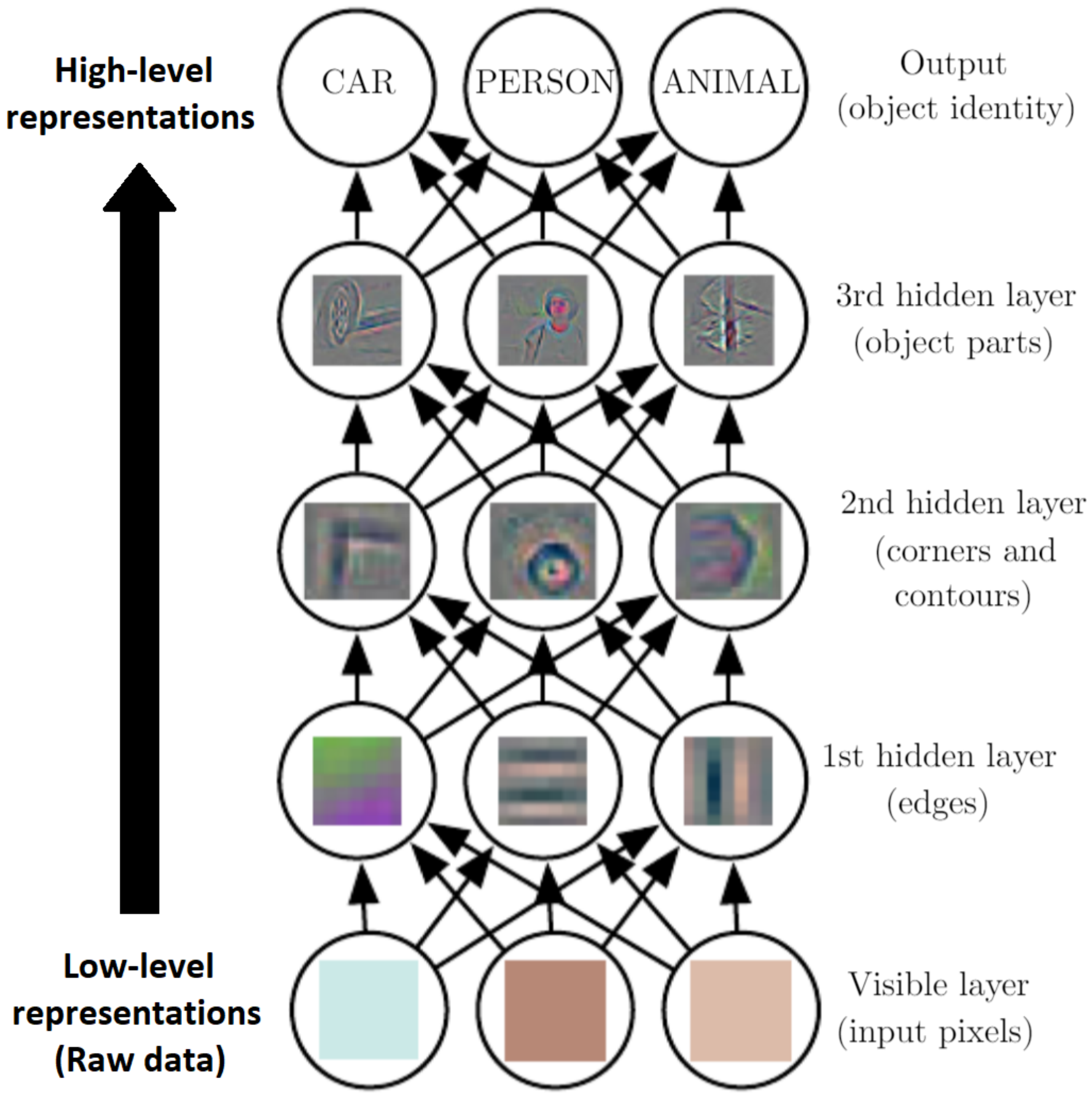

High-level representations
CAR
PERSON
ANIMAL
Output (object identity)
3rd hidden layer (object parts)
2nd hidden layer (corners and contours)
1st hidden layer (edges)
Low-level representations (Raw data)
Visible layer (input pixels)

Figure **4-4**: Deep Learning Representation. Modified from [44].

the light (input signal) enters through the retina, and the signal is processed layer by layer by the visual cortex. In each layer, the signal is transformed from low-level to high-level representations [82]. Figure **4-4** shows an example of the DL representation.

Despite the high interest in ANNs during the '80s, it decreased considerably due to severe problems of training algorithms, until 2006 when DL concepts were successfully implemented in [51]. They showed how to effectively train a deep MLP combining supervised and unsupervised methods. First, each layer at a time is pre-trained using unsupervised Restricted Boltzmann Machines (RBM). Then, the whole network is fine-tuned using the backpropaga-

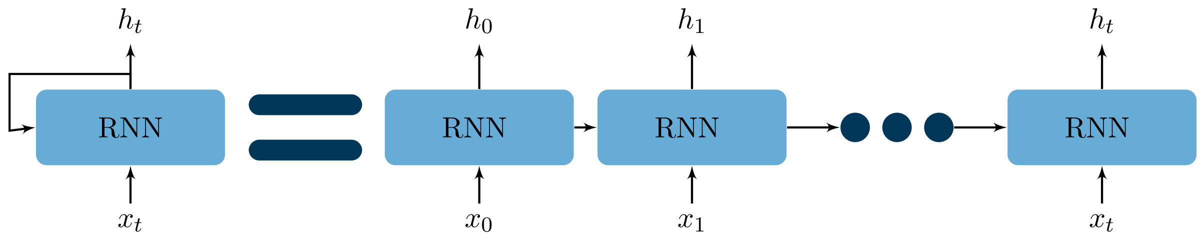


Figure **4-5**: An Unrolled Recurrent Neural Network.

tion algorithm. In this way, the training algorithm converges numerically, and a deep MLP can learn complex patterns by generalizing hierarchical and sparse representations of data.

The DL "Big-Bang" started in 2009, thanks to the advances in hardware, specifically in Graphics Processing Units (GPUs) [56]. GPUs are suitable for linear algebra operations involved in training algorithms. Hence, the use of GPUs increased the speed of training deep-learning model by more than 100 times [89, 13]. Even this difference has been increased more by the recent advances in GPUs.

DL models have demonstrated greater effectiveness in several tasks (classification and prediction) in different domains such as video analysis, audio recognition, text analysis, and image processing. Currently, DL is a hot topic of public interest because it has faced and solved many problems like text translation, speech recognition, motion tracking, machine failure prediction, among others. DL has revolutionized every aspect of our daily life. Despite its advantages, DL applications in computational finance are still limited [10, 20, 33, 102, 113, 107].

4.4 Recurrent Neural Networks

Recurrent Neural Networks (RNNs) eliminate the FNN restriction of its lack of loops; as each neuron has feedback connections from its output to input. Given that this kind of network receives feedback by the output data, this connection allows the network to store a state in an internal memory, which also allows it to model a dynamic temporal behavior [95, 52, 76]. Figure **4-5** shows the RRN architecture. If an RNN is unrolled, it is equivalent to an infinite MLP; therefore, it is a deep natural architecture.

In general, RNNs are specialized in the identification of patterns through time, because they can store previous states in its internal memory. However, RNNs are more difficult to train than simple MLPs, because they are more complex than MLPs. The most simplest RNN is the Jordan network [58] and the Elman network [35]. Both are three-layer networks with only a hidden network. Elman network is defined as [35]:

$$\begin{aligned} h_t &= \sigma_h(W_h x_t + U_h h_{t-1} + b_h) \\ y_t &= \sigma_y(W_y h_t + b_y) \end{aligned} \tag{4-1}$$

whereas, Jordan network is defined as [58]:

$$\begin{aligned} h_t &= \sigma_h(W_h x_t + U_h y_{t-1} + b_h) \\ y_t &= \sigma_y(W_y h_t + b_y) \end{aligned} \tag{4-2}$$

where x_t, h_t and y_t are the input, hidden layer and output vectors, respectively. W and U are the network weights matrices; b is the biases vector; and finally, σ_h and σ_y are the activation functions.

Unfortunately, these networks have some issues related to learning too long-term time dependencies and vanishing and exploding gradient problems. As a result, training algorithms were not able to converge numerically. In 1997, the Long Short-Term Memory (LSTM), a kind of RNN, was proposed in [52] and solved some issues of traditional RNN. LSTMs are capable of learning in a balanced way both long and short-term dependencies [44]. A common LSTM unit is composed of a cell and three gates (an input, output and forget gate) [95].

$$\begin{aligned} f_t &= \sigma_g(W_f x_t + U_f h_{t-1} + b_f) \\ i_t &= \sigma_g(W_i x_t + U_i h_{t-1} + b_i) \\ \hat{c}_t &= \sigma_h(W_c x_t + U_c h_{t-1} + b_c) \\ o_t &= \sigma_g(W_o x_t + U_o h_{t-1} + b_o) \\ c_t &= \begin{cases} 0 & t = 0 \\ f_t \circ c_{t-1} + i_t \circ \hat{c}_t & \text{otherwise} \end{cases} \\ h_t &= \begin{cases} 0 & t = 0 \\ o_t \circ \sigma_h(c_t) & \text{otherwise} \end{cases} \end{aligned} \tag{4-3}$$

where x_t and h_t are the input and output vectors, respectively. f_t, i_t and o_t are the forget, input and output gate activation vectors, respectively. c_t is the cell state vector. σ_g and σ_h are a sigmoid function and a hyperbolic tangent, respectively. The operator $\circ$ denotes the Hadamard product and finally, W, U and b are parameter matrices and vectors. Figure **4-6** presents the LSTM architecture.

Additionally, models that stack multiple LSTM are more effective and successful for specific tasks such as automatic language translation, because the process of stacking involves applying the concept of DL. The multiple LSTM can make a better representation of the input data, resulting in the learning of high-level representations. The first known application of an LSTM in finance, "A LSTM-based method for stock returns prediction: A Case Study of China Stock Market", was published in [22].

In 2014, an LSTM variation, called Gated Recurrent Unit (GRU), was proposed in [24]. A GRU, which is a gating mechanism in RNNs, is simpler and easier to train than LSTM because it has fewer parameters to be trained than an LSTM. A GRU combines and unifies

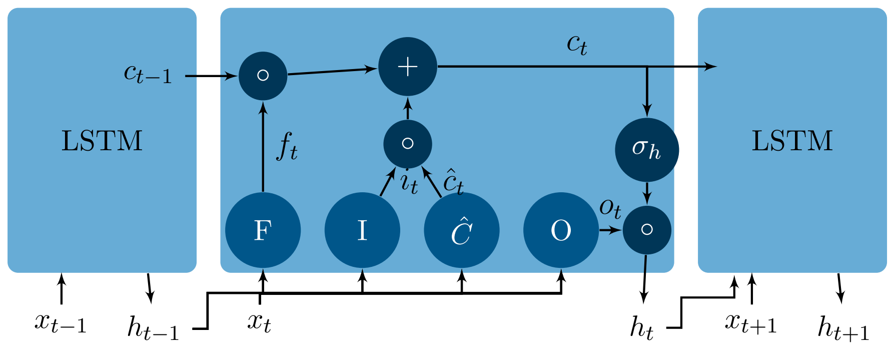


Figure **4-6**: LSTM Architecture

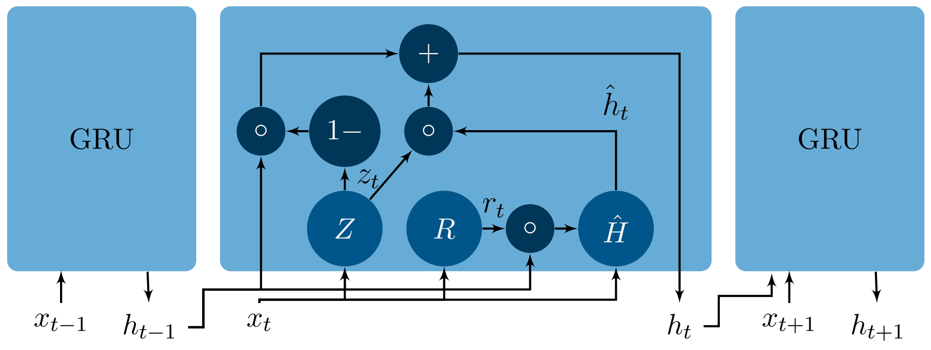


Figure **4-7**: GRU Architecture

some LSTM gates and is defined as:

$$
\begin{aligned}
z_t &= \sigma_g(W_z x_t + U_z h_{t-1} + b_z) \\
r_t &= \sigma_g(W_r x_t + U_r h_{t-1} + b_r) \\
\hat{h}_t &= \sigma_h(W_h x_t + U_h(r_t \circ h_{t-1}) + b_h) \\
h_t &= \begin{cases} 0 & t = 0 \\ (1 - z_t) \circ h_{t-1} + z_t \circ \hat{h}_t & \text{otherwise} \end{cases}
\end{aligned}
\tag{4-4}
$$

where x_t and h_t are the input and output vectors, respectively. z_t and r_t are the update and reset gate activation vectors, respectively. σ_g and σ_h are the sigmoid function and the hyperbolic tangent function, respectively. The operator $\circ$ denotes the Hadamard product and, finally, W, U and b are parameter matrices and vectors. Figure **4-7** presents the GRU architecture.

4.5 Artificial Neural Networks in Finance

Given these characteristics, DL has emerged as a useful technique for asset price modeling because it has proven to be able to learn complex representations of high-dimensional data.

Table **4-1**: Artificial Neural Networks Applications in Finance

Year	**Applications**	**Techniques**
1988	[111]	Feedforward Neural Network
1990	[59]	Recurrent Neural Network
1996	[104]	Recurrent Neural Network
1998	[85]	Feedforward Neural Network + Fuzzy Inference System
	[93]	Recurrent and Probabilistic Neural Networks
2000	[16, 67]	Multilayered Feedforward Neural Network
2001	[116]	Feedforward Neural Network + Wavelet Transforms
	[106]	Recurrent Neural Networks + GARCH
	[34]	Feedforward Neural Network
2002	[63]	Neuro-Fuzzy Network
2004	[21]	Feedforward Artificial Neural Network
2005	[9, 64]	Feedforward Artificial Neural Network
	[114]	Recurrent Neural Network + EGARCH
2006	[81]	Feedforward Neural Network
2007	[61]	Recurrent Neural Network
2009	[19, 6]	Feedforward Artificial Neural Network + Genetic Algorithm
2011	[55, 60]	Recurrent Neural Network
2012	[109]	Recurrent Neural Network
2013	[27]	Deep Neural Network
	[36]	Artificial Neural Networks + Genetic Algorithms
2014 2015	[118]	Deep Belief Networks
	[33]	Convolutional Neural Network
	[22]	Long Short-Term Memory
2016	[31]	Convolutional Neural Networks + Long Short-Term Memory
	[7]	Deep Neural Network
	[99]	Recurrent Neural Network + PCA
2017	[12]	Long Short-Term Memory + Autoenconders
2018	[39]	Long Short-Term Memory

On DL arena, the most simple architecture is MLP, but it has demonstrated significant results in [7, 8]. Table **4-1** presents some successful works using ANN for FTS classification and prediction. However, the number of related works is limited. As a result, the industry of algorithmic trading needs better models to predict price trends with greater accuracy and precision, since market dynamics exhibit complex and stochastic behavior.

During the last years, the number of works has increased thanks to the advances in data processing and the accessibility to financial data. It is important to remember that financial data are costly and difficult to obtain.

Furthermore, the tick-by-tick is largely composed of many transactions at the same price and few ones with small changes (price jumps) under normal market conditions. Those changes, which occur without high variance, have a step-style; prices change intermittently between two prices (the best bid and best ask quotes). Wavelets like the Haar filter (the simplest orthogonal wavelet), can exploit this property because they are ideal for modeling this kind of behavior. This work focuses on Deep Neural Networks (DNNs) like deep MLPs and RNNs, which are the most widely used techniques in DL. Wavelets, in conjunction with DNNs, allow improving the stock prices predictions on HF data concerning previous results achieved in our previous work: [8, 7]. In the next chapter, we will present the proposed model that uses wavelets as an input feature generator.

Chapter 5

DNN Short-Term Price Predictor

This chapter presents the proposed predictor based on a DNN whose inputs are generated by the DWT. It also presents the used dataset, shows the data exploration and data preprocessing, and finally, describes the proposed predictor.

5.1 Dataset Description

The dataset consists of tick-by-tick data, from January 1st, 2015 to July 31th, 2017 of 18 companies from the Dow Jones Industrial Average (DJIA). The DJIA, which is the second-oldest U.S. market index, consists of 30 major companies based in the United States. For this work, only normal trading hours (between 9:30:00 am and 3:59:59 pm EST) were considered. Figure **5-1** shows the candle chart of one stock, and Table **5-1** shows a complete dataset description.

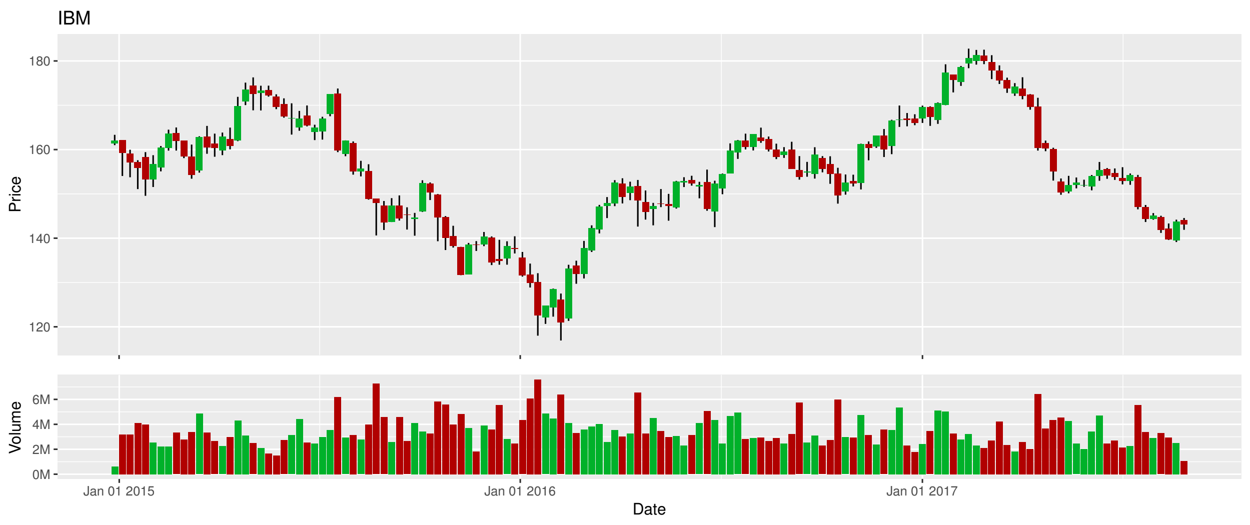


Figure **5-1**: Stock: International Business Machines Corporation (IBM).

Table **5-1**: Dataset Description.

Company	**Industry**	**Exchange**	**Symbol**	**# Ticks (in Millions)**	**File size**
Apple	Consumer electronics	NASDAQ	AAPL	146.98 M	7.1 GB
American Express	Consumer finance	NYSE	AXP	4.15 M	197 MB
Boeing	Aerospace and defense	NYSE	BA	3.55 M	168 MB
Caterpillar	Construction and mining equipment	NYSE	CAT	4.59 M	218 MB
Cisco Systems	Computer networking	NASDAQ	CSCO	58.11 M	2.8 GB
Chevron	Oil & gas	NYSE	CVX	7.1 M	340 MB
DuPont Corporation	Chemical industry	NYSE	DD	3.66 M	170 MB
Walt Disney	Broadcasting and entertainment	NYSE	DIS	6.34 M	304 MB
General Electric	Conglomerate	NYSE	GE	4.76 M	224 MB
Goldman Sachs	Banking, Financial services	NYSE	GS	3.2 M	151 MB
The Home Depot	Home improvement retailer	NYSE	HD	4.35 M	206 MB
IBM	Computers and technology	NYSE	IBM	3.59 M	173 MB
Intel	Semiconductors	NASDAQ	INTC	70.47 M	3.4 GB
Johnson & Johnson	Pharmaceuticals	NYSE	JNJ	5.77 M	278 MB
JPMorgan Chase	Banking	NYSE	JPM	7.9 M	375 MB
Coca-Cola	Beverages	NYSE	KO	3.72 M	174 MB
3M	Conglomerate	NYSE	MMM	2.63 M	127 MB
ExxonMobil	Oil & gas	NYSE	XOM	8.13 M	386 MB
	Total			**348.98 M**	**17 GB**

5.2 Dataset Exploration

Tick-by-tick series of liquid stocks under normal market conditions compose the dataset. As prices in financial markets can vary dramatically in a long-term, but it would be expected that this kind of time series has very few sudden jumps in an HF scale. However, the selected stocks exhibit the following characteristics in a typical one-minute period:

- Depending on the stock, many transactions occur with a few milliseconds of difference within the same minute. For instance, there are 555 AAPL transactions per minute and 10 MMM transactions per minute on average.
- The tick-by-tick prices do not usually change concerning the previous one, that is, prices have very low volatility at high-frequency. On average, half of the transactions are traded at the same previous price.
- There are not many different prices inside a minute. For instance, there are 222 CSCO Transactions which are traded at 12 possible prices on average.
- Price jumps are not very wide. Prices usually change a few cents depending on the stock liquidity.

Table **5-2**: Minute Statistics of High-Frequency Data.

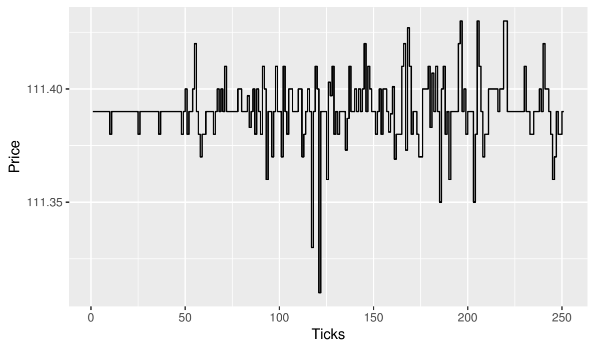
Ticker	**Avg. ticks per minute**	**Median # of different prices per minute**	**Percentile 75% different prices per minute**	**Avg. streaks (same price) per minute**	**% of ticks with the same price per minute**	**Avg. # of one-cent price jump per minute**
AAPL	555	46	64	33	67.27	30
AXP	16	4	5	2	58.91	2
BA	14	5	7	3	43.67	2
CAT	18	5	7	3	51.68	3
CSCO	222	12	16	3	77.51	3
CVX	27	6	9	5	51.72	5
DD	14	4	5	2	55.58	2
DIS	24	5	7	4	55.53	4
GE	19	3	4	1	72.49	1
GS	13	5	8	3	38.23	1
HD	17	5	7	3	45.91	3
IBM	14	5	7	3	43.18	2
INTC	269	13	17	3	78.73	3
JNJ	22	5	7	4	56.08	4
JPM	30	5	7	3	64.81	3
KO	15	3	4	1	69.20	0
MMM	10	4	6	2	41.64	1
XOM	31	5	8	5	60.72	5

Table **5-2** illustrates different measures for all the gathered data. In general, the behavior is very similar among these stocks. Stocks with lower liquidity exhibit bigger jumps between ticks, that is, more volatility. In contrast, stocks with higher liquidity exhibit smaller jumps sizes, usually one cent jump when prices change between ticks. For most of the stocks, the greater percent of ticks in a one-minute interval occurs at one single price, corresponding to the normal market conditions mentioned above.

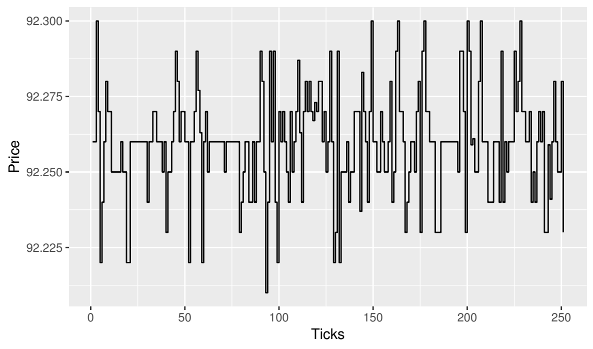
In summary, as the average number of ticks per minute increases, the percentage of ticks at the same price per minute also increases. Figures **5-2**a and **5-2**b show the behavior of high liquid and low liquid stocks. Both kinds of stocks have a very similar shape: a step-style (prices that change intermittently between the best bid and best ask quotes). As a result, the wavelet decomposition is suitable for representing the tick-by-tick data.

The Haar wavelet discontinuities perfectly fit to handle those sudden transitions since the Haar mother wavelet also has that step-style. Therefore, the resulting data representation will contain a lot of useful compressed information about HF price behavior. The wavelets coefficients will be used as DNN inputs. It is important to note that applying this transformation enriches the data since the resulting time series is represented regarding the frequency domain with temporal resolution; hence, the signal has properties from both time and frequency domains.

Furthermore, some financial stocks usually have a linear correlation with other stocks at



(a) High Liquid Stock Steps.



(b) Low Liquid Stock Steps.

Figure **5-2**: Stock Steps: High Frequency Behavior.

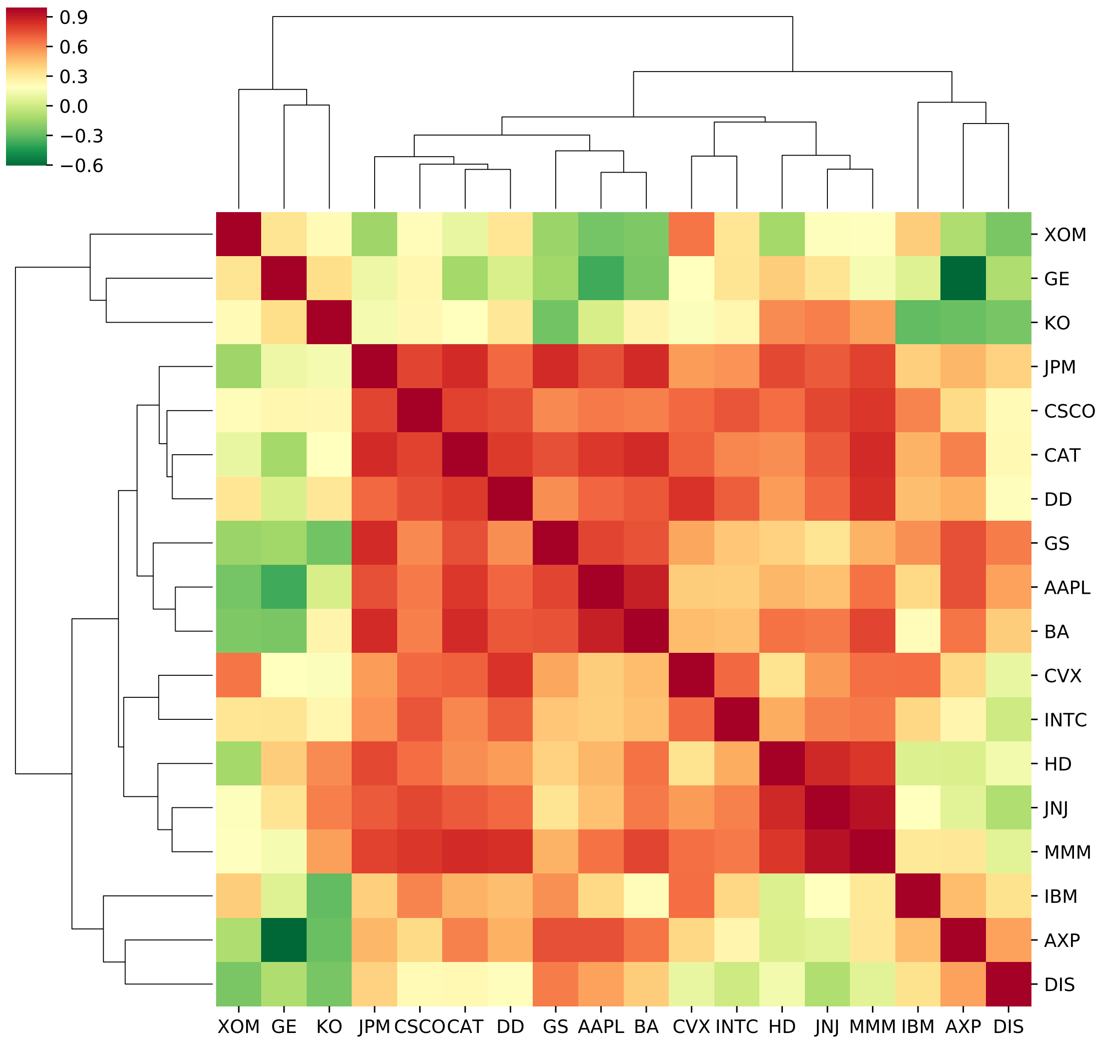


Figure **5-3**: Correlations of the One-minute Closing Price Series

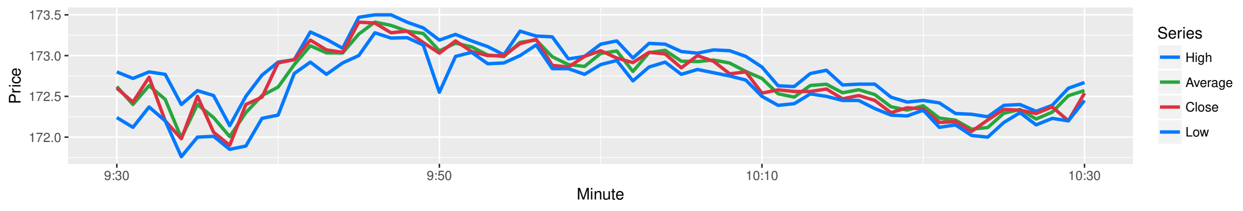


Figure **5-4**: Example of 60 One-minute prices.

lower frequency scale. Figure **5-3** shows the dataset correlation map of one-minute closing price series. It is interesting that JPM, CSCO, CAT, DD, GS, AAPL, BA, CVX, INTC, HD, JNJ and MMM make up a cluster. It implies they are highly correlated between them. Meanwhile, GE and XOM are highly anti-correlated. As a result, all the stocks within a cluster have similar dynamics that are transversal to all in the same cluster, and a DNN can learn these regularities and patterns.

5.3 Pseudo-log Returns

Figure **5-4** shows an example of 60 one-minute highest, average, closing and lowest prices. The average price is the best descriptor of the current price status because the highest or lowest prices usually are within a confidence range of the average price. Therefore the next highest and lowest prices can be estimated using an average price, whereas, the closing price can be any value close to the average price, which sometimes coincides with the highest or lowest price. The highest, lowest and closing prices are highly exposed to noise and external market dynamics, unlike the average price which has a more stable behavior. Depending on the number of ticks per minute, the average price is more stable, that is, one-minute average prices of a highly traded stock are more stable than those of a lowly traded stock.

As the objective of this work is to develop the best possible predictor, the average prices are more adequate than closing prices [7, 8]. In such a way, the traded prices will match at some time with the expected average during the next minute. Consequently, this work introduces an innovative approach: to use average prices as inputs and outputs of the predictor.

For this reason, this work introduces the concept of pseudo-log-return, which is a homologous transformation in log-returns. It is computed as the logarithmic difference (log of quotient) between average prices of consecutive periods from the same timescale (minutes, seconds, among others). Let $\widehat{p_t}$ and $\widehat{p_{t-1}}$ be the current and previous one-minute average price, respectively.

$$\begin{aligned}\widehat{R_t} &= \ln \frac{\widehat{p_t}}{\widehat{p_{t-1}}} \cdot 100\% = (\ln \widehat{p_t} - \ln \widehat{p_{t-1}}) \cdot 100\% \\ \widehat{p_t} &= \widehat{p_{t-1}} \cdot e^{\frac{\widehat{R_t}}{100\%}}\end{aligned} \tag{5-1}$$

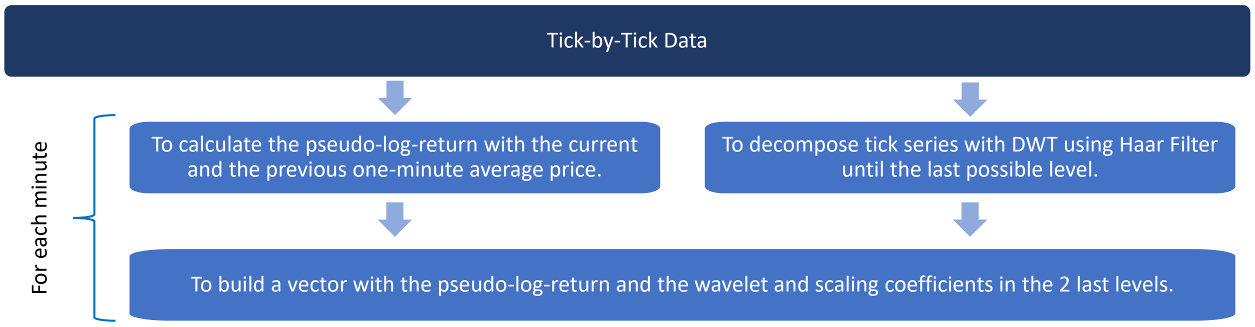

Tick-by-Tick Data
For each minute
To calculate the pseudo-log-return with the current and the previous one-minute average price.
To decompose tick series with DWT using Haar Filter until the last possible level.
To build a vector with the pseudo-log-return and the wavelet and scaling coefficients in the 2 last levels.

Figure **5-5**: Data Preprocessing

Pseudo-log-returns, which are a non-standard data transformation, offer more quality information on the status of an asset price. Given the fact that averages are more stables than noisy tick-by-tick prices, they are less affected by outliers and noise than log-returns.

5.4 Dataset Preprocessing

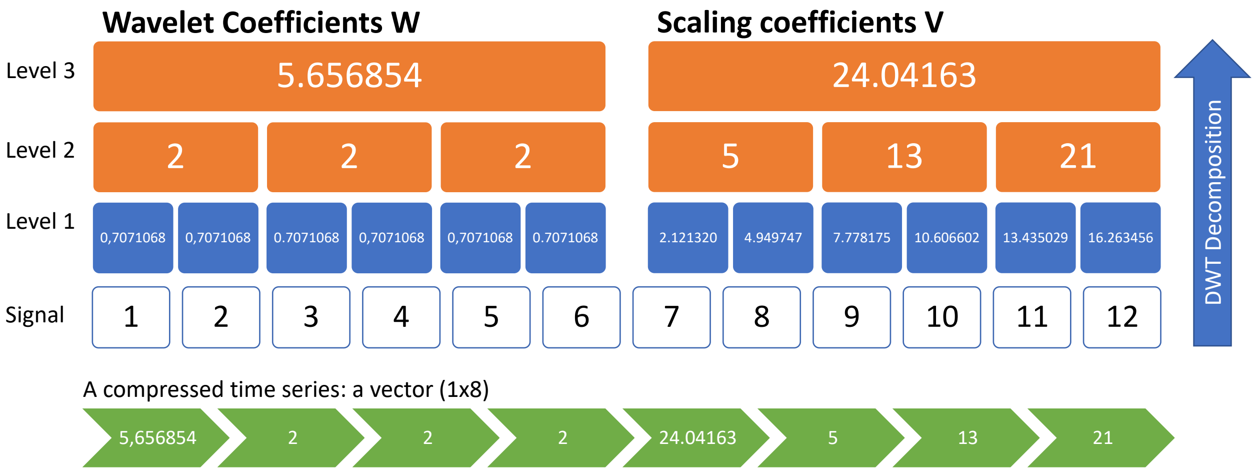
Figure **5-5** shows the preprocessing summary. All data are summarized at a one-minute level. First, the tick-by-tick time series is grouped by the minute. Second, the one-minute average prices are computed. Third, the pseudo-log-returns are computed using the current and previous one-minute average prices. Fourth, all transactions within the minute are compressed in a vector of 8 values using the DWT with a Haar filter.

The compression process is described as follows. First, the log-returns of the tick-by-tick series are computed. Second, the resulting time series is decomposed by the DWT with a Haar filter until the possible levels. Finally, the wavelet and scaling coefficients in the last two levels are selected. The final result is a vector of eight values: four wavelet coefficients and four scaling coefficients. Figures **5-6**a and **5-6**b show examples of the decomposition of the numbers ranging from 1 to 12 and from 1 to 8, respectively.

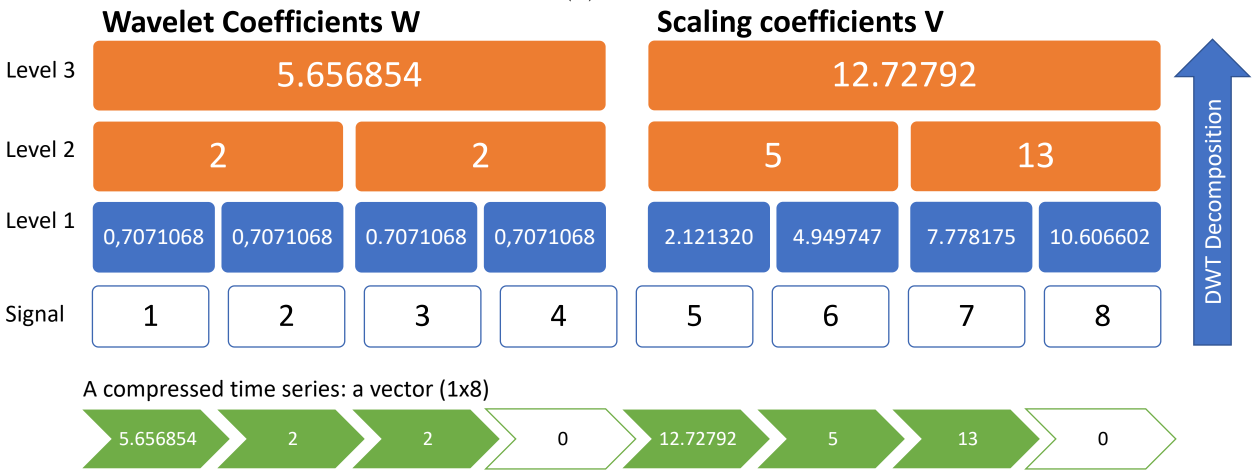
Since the signal is sub-sampled by 2 in each decomposition, the last level will be composed of only two coefficients: a wavelet coefficient and a scaling coefficient. However, the penultimate level could be composed of four (Figure **5-6**b) or six (Figure **5-6**a) coefficients depending on the number of samples of the previous level, if it was or nor a power of two. In this case, there are only four coefficients and two zeros are appended to the vector in order to obtain eight values. Finally, a vector that is composed of the pseudo-log-return and the wavelet and scaling coefficients in the two last levels is built for each minute.

5.5 Modeling

After a long iterative process of data exploration and feature engineering, several features were tested and combined. The final features consist of the sliding windows of two variables:



(a) Case I.



(b) Case II.

Figure **5-6**: Compressed Tick-by-Tick Series using DWT.

one-minute pseudo-log-returns and one-minute compressed tick-by-tick vectors with length 8. Each vector is the result of transforming tick-by-tick transactions recorded in a particular minute using a DWT and a Haar filter. Several DNN architectures will be tested in Chapter 7. The used degrees of freedom were the number of previous observations (sliding window size from 2 to 60 minutes), number of layers (from 1 to 10), number of neurons per layer and type of activation function (Tanh, Maxout, Rectifier).

The final architecture was chosen taking account the balance between efficiency and accuracy. Figure **5-7** shows the network architecture sketch with five hidden layers (27, 22, 17, 11 and 6 neurons using *Tanh* activation function, respectively) and a window size of 3.

Chapter 7 will show a comparison between the proposed DNN, the Auto-regressive Integrated Moving Average (ARIMA) model, three different alternatives of DNN architectures, a Random Walk (RW) simulation, and other seven classic machine learning techniques of similar complexity.

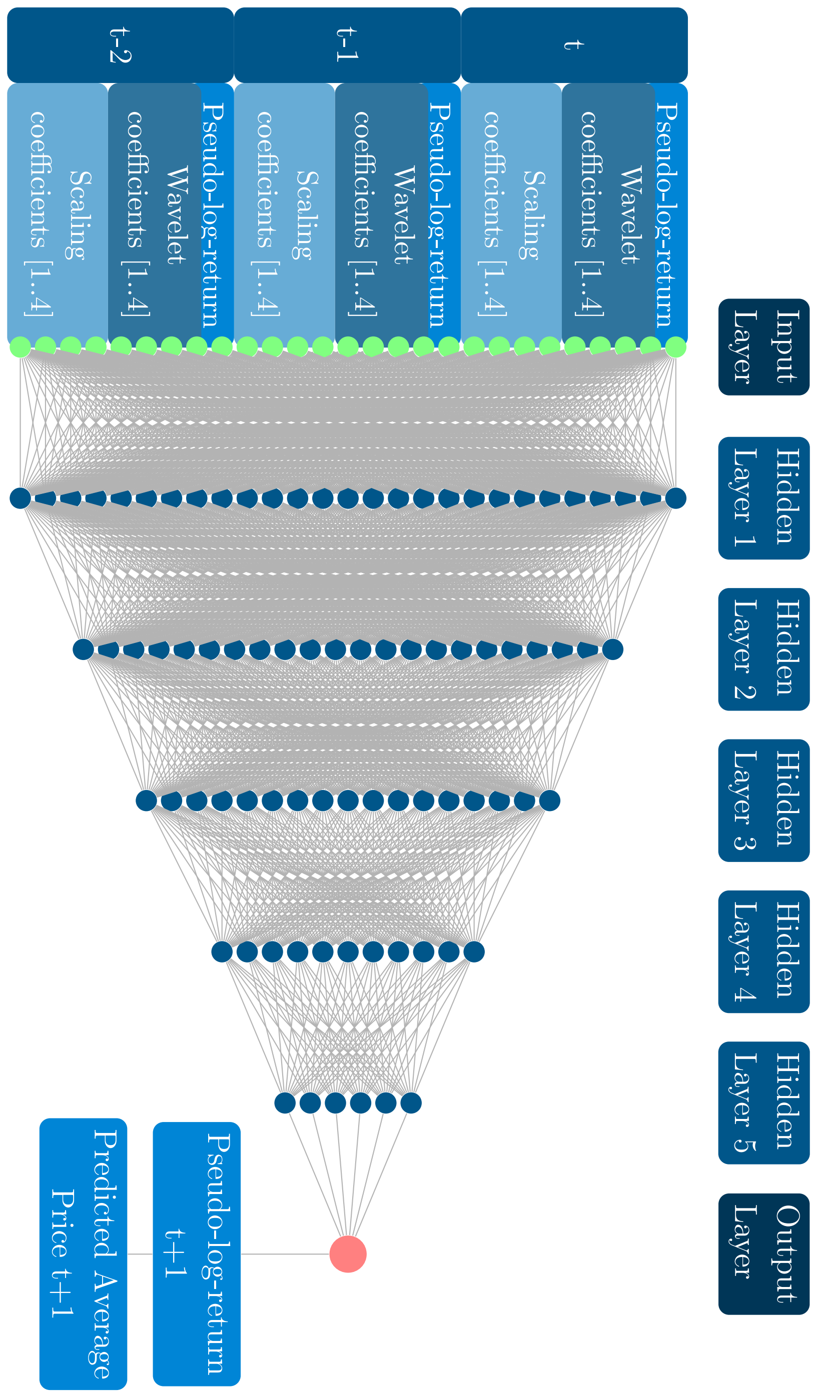

t-2
t-1
t
Scaling coefficients [1..4]
Wavelet coefficients [1..4]
Pseudo-log-return
Scaling coefficients [1..4]
Wavelet coefficients [1..4]
Pseudo-log-return
Scaling coefficients [1..4]
Wavelet coefficients [1..4]
Pseudo-log-return
Input Layer
Hidden Layer 1
Hidden Layer 2
Hidden Layer 3
Hidden Layer 4
Hidden Layer 5
Output Layer
Pseudo-log-return t+1
Predicted Average Price t+1

Figure **5-7**: Deep Architecture.

Chapter 6

High-Frequency Strategy

This chapter presents the proposed high-frequency trading strategy that the described short-term price predictor will use. This strategy consists of a variation of one presented in our previous work [7, 8].

6.1 Baseline Strategy

A high-frequency trading strategy based on a DNN, which predicts the next one-minute average price, was proposed in [7, 8]. It was tested over the AAPL stock during the financial crisis of 2008 and yielded extraordinary profits. Figure **6-1** shows the strategy flowchart.

This strategy operates at high-frequency time scale. It opens a position at the beginning of each minute according to the price target (the predicted one-minute average price) and closes the position when the price reaches the target at any moment or when the minute ends (in case of the price never yields the target price). It opens the position in the following way: If the next predicted average price is above/below the last closing price, it always buys/sells a stock. This strategy is simple but has several issues in a real environment. For instance, it does not consider:

- **Non-liquid periods:** the strategy could buy at high prices or sell at low prices during non-liquid periods because there is not enough supply or demand to trade and get profits.

- **Sharply price variations:** The strategy waits until the end of the minute to close its position in order to stop the looses. However, if the price changes sharply and goes to the opposite side during that minute, this strategy is vulnerable to volatility and potential losses. Besides, the strategy has an asymmetric behavior, when the price reaches the target price, the strategy cuts the earnings. Meanwhile, when the price never reaches the target price and goes to the wrong side, there are no stop-loss mechanisms.

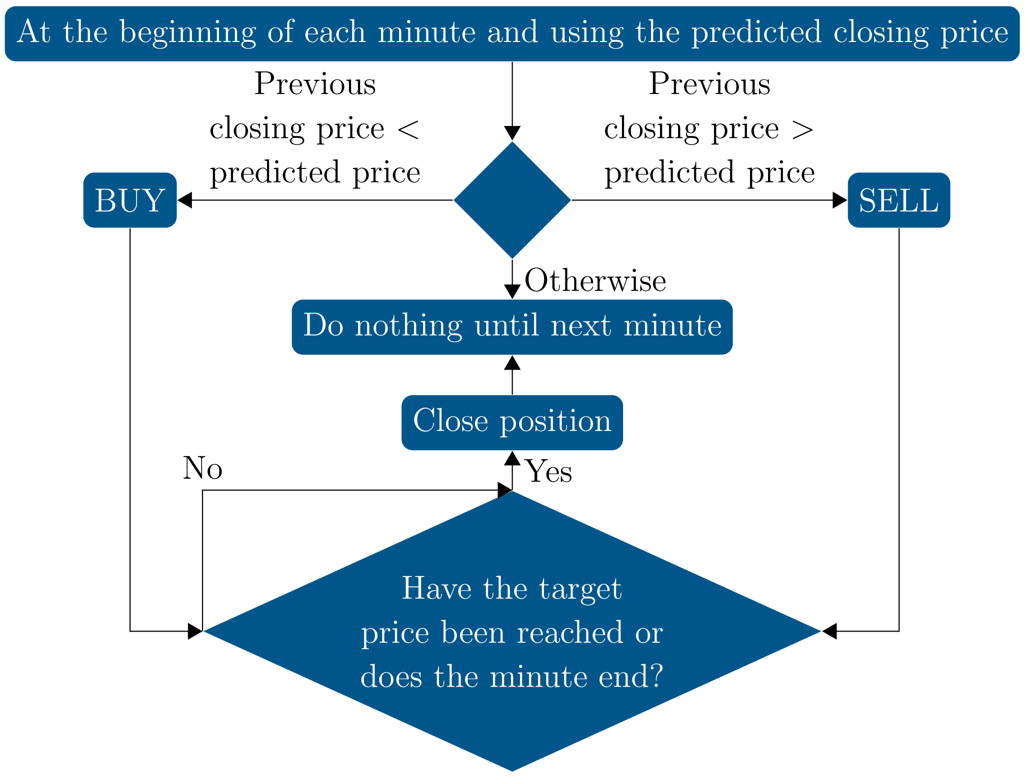

At the beginning of each minute and using the predicted closing price
Previous closing price < predicted price
Previous closing price > predicted price
BUY
SELL
Otherwise
Do nothing until next minute
Close position
Yes
No
Have the target price been reached or does the minute end?

Figure **6-1**: Strategy Flowchart.

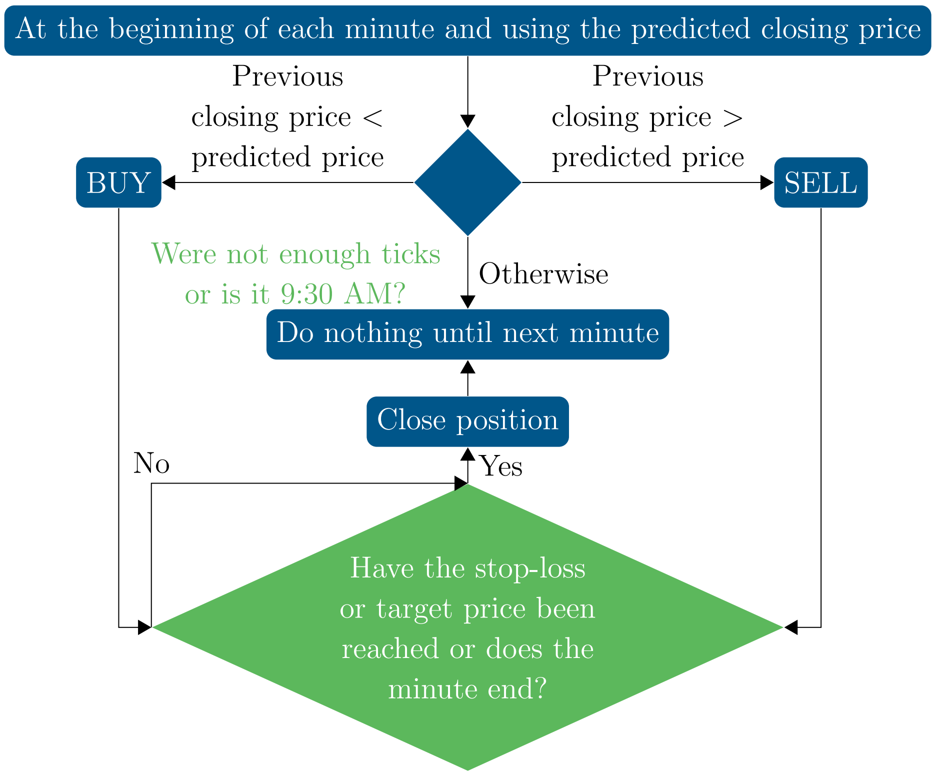

At the beginning of each minute and using the predicted closing price
Previous closing price < predicted price
Previous closing price > predicted price
BUY
SELL
Were not enough ticks or is it 9:30 AM?
Otherwise
Do nothing until next minute
Close position
Yes
No
Have the stop-loss or target price been reached or does the minute end?

Figure **6-2**: Modified Strategy Flowchart.

- **Bearish or bullish market openings:** News or earnings reports, which usually are published before market openings, have a strong impact on the markets and can make them have a bullish or bearish expectation. Then, the strategy will operate based on old data from the previous day and its behavior may be unexpected and risky since data will not be reliable about the actual market status.

6.2 Proposed Strategy

This work presents a modified version of the previous strategy. Figure **6-2** shows the new flowchart, which includes three main changes:

First, it only trades if the last minute had at least n transactions. There is no guarantee of knowing if the next minute will be enough liquid to trade. However, under the assumption that the last minute was liquid, then the probability that the next minute will also be enough liquid is high. For this work, the selected criterion to determine if a minute is liquid or not is that the amount of transactions during that minute is greater than the 25th percentile. This threshold can be easily calculated from historical data.

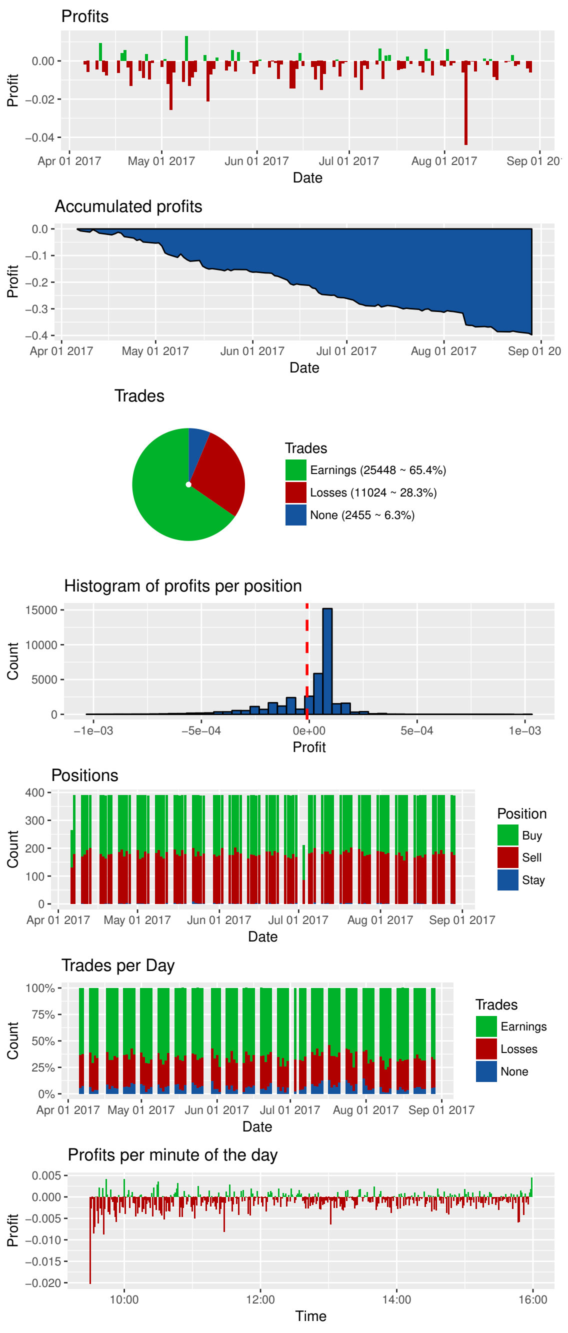
Second, if the strategy opens a position and the price goes in the opposite direction and exceeds a threshold, the strategy will close its position. For this work, the threshold was defined as the current price minus the distance from the target price to the current price. For example, if the current price is \$1000 and the target price is \$1001, the stop-loss price will be \$999. Meanwhile, if the current price is \$1000 and the target price is \$999, the stop-loss price will be \$1001. In consequence, the strategy becomes symmetric.

And finally, it does not trade on the first minute of the day. Thereby, the strategy avoids volatility and risks associated with market openings.

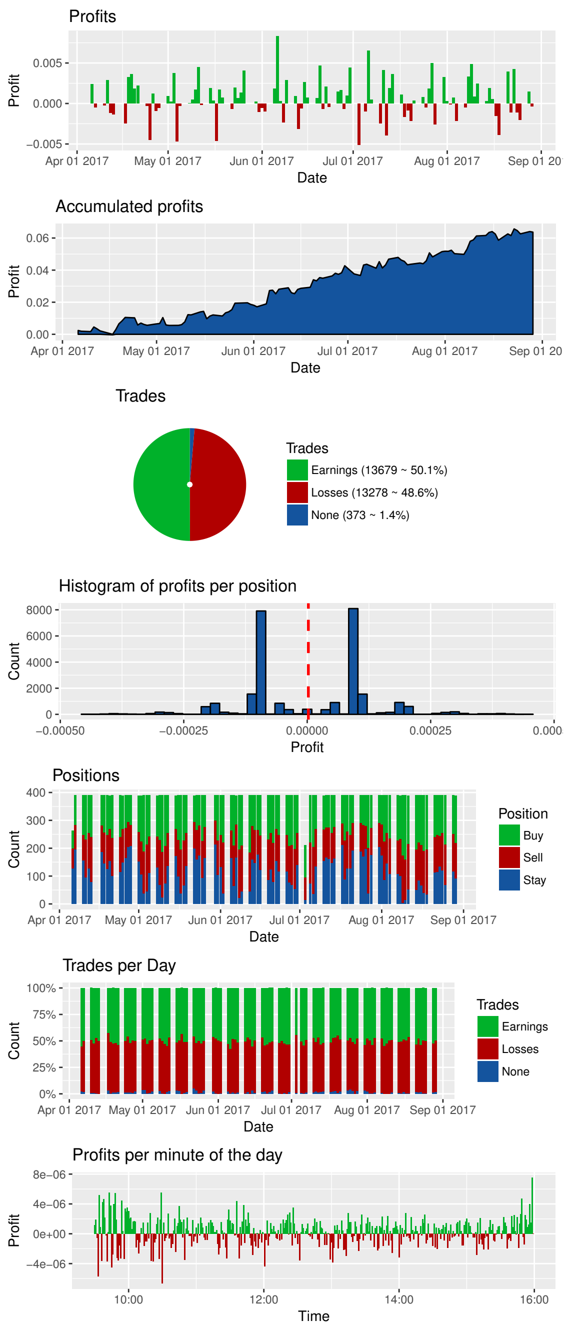
6.3 Simulation of the Proposed Strategy

Those changes have a high impact on the performance of the strategies. Figure **6-3** shows the old strategy performance compared to the new strategy performance during a trading simulation using the same testing period from the previous chapter (approximately five months). This simulation did not take into account transaction and spread costs. For each stock, it starts with 1 dollar and reinvests all the available money in the following operations.

The accumulated profits varied a lot: the new strategy had a positive profit, contrary to the old strategy. On average, the old strategy made a lot of good trades (%65), but it lost too much money (until -2% per operation) on a few others. The left histogram shows an asymmetric behavior and the right one shows a symmetric-like behavior. It is evident that the stop-loss mechanism implemented by the new strategy had a positive effect on its performance. Furthermore, the charts on the last row show that the old strategy lost too much money (until -2% on average) during the first minute of the day. As a result, it proves



(a) Old Strategy



(b) New Strategy

Figure **6-3**: Performance Comparison of the Strategies: The Walt Disney Company (DIS).

that refraining from trading during the market opening can avoid potential losses and also reduces exposure to risk. Moreover, Chapter 7 will show a comparison between the proposed HF trading strategy and three types of trading strategies: A Buy & Hold Strategy, an RSI Strategy with six different time windows, and a one-minute Random Trading Strategy.

Chapter 7

Experiments and Results

This chapter presents the experimental setting and the corresponding results of the proposed DNN short-term price predictor and the proposed high-frequency trading strategy.

7.1 Experimental Setting: Short-Term Price Predictor

Table **7-1** shows the experimental setting overview for testing the proposed short-term predictor and for comparing it against different DNN architectures, classical econometric and statistical models (ARIMA and RW) and classical machine learning techniques (Linear regression, Ridge, Lasso, Bayesian ridge, SGD regressor, Decision trees, and SVR). The following subsections present the proposed experiments in more detail.

Multi-Layer Perceptron

In [7, 8], a MLP was proposed. It also predicts the next one-minute pseudo-log-return. Its input features are $3n + 2$ values: the current time (hour and minute), the last n one-minute pseudo-log-returns, the last n one-minute standard deviations of prices and the last n one-minute trend indicators, where n is the window size. The trend indicator is calculated as the slope a of a fitted linear model on tick-by-tick prices within the same minute.

The DNN has one input layer, L hidden layers, and one output layer. The number of neurons in each layer depends on the input number I as well as on hidden layers number L. In each hidden layer, the number of neurons decreases with a constant factor of $\frac{1}{L}$. For example, with five hidden layers: Each layer will have I, $\lceil\frac{4}{5}I\rceil$, $\lceil\frac{3}{5}I\rceil$, $\lceil\frac{2}{5}I\rceil$, $\lceil\frac{1}{5}I\rceil$ neurons respectively. For example, with three hidden layers: Each layer will have I, $\lceil\frac{2}{3}I\rceil$, $\lceil\frac{1}{3}I\rceil$ respectively. The output layer always has one neuron. All neurons in hidden layers use a *Tanh* activation function, whereas the output neuron uses a *Linear* activation function.

Several MLPs architectures were tested. The used degrees of freedom were the number of previous observations (sliding window size from 2 to 60 minutes), number of layers (from 2 to 10), number of neurons per layer, type of activation function (Tanh, Maxout, Rectifier). The

Table **7-1**: Experimental Setting: Short-Term Price Predictor

Technique	**Inputs**	**Explored Freedom Degrees**	
		Window Size	**Hidden Layers**
Multi-Layer Perceptron	1. Time (Hour and Minute) 2. One-minute pseudo return. 3. One-minute standard deviation. 4. Trend indicator.	2 - 60	2 - 10
ARIMA	1. One-minute pseudo returns	1 - 5	*NA*
RW		*NA*	*NA*
Multi-Layer Perceptron + DWT	1. One-minute pseudo return. 2. Compressed tick-by-tick time series by DWT.	2 - 60	2 - 10
LSTM + DWT			
GRU + DWT			
Linear Regression			*NA*
Ridge			
Lasso			
Bayesian Ridge			
SGD Regressor			
Decision Trees			
SVR			

final architecture was chosen taking account the balance between efficiency and accuracy. Let n be 3 and the architecture be composed of 5 hidden layers, and each layer with 11, 8, 7, 5, 3 neurons, respectively. In each hidden layer, the number of neurons decreases with a constant factor of $\frac{1}{5}$.

H_2O, which is open-source software for big-data analysis, was used to train the MLPs. The ADADELTA training method was selected because it offers several benefits. Given that ADADELTA is a per-dimension adaptive learning rate method for gradient descent, it is not necessary to manually search parameters for gradient descent and is also robust to large gradients and noise. The chosen ADADELTA parameters were $\rho = 0.9999$ and $\epsilon = 10^{-9}$. The MLPs were trained during 50 epochs.

ARIMA

ARIMA is the most used analytic technique for forecasting financial time series. Even though it has good performance over in-sample datasets, it has bad performance over out-sample

Table **7-2**: ARIMA Models.

Symbol	**ARIMA**			**AIC**	**Symbol**	**ARIMA**			**AIC**
	p	**d**	**q**			**p**	**d**	**q**	
AAPL	3	0	2	-512971.11	GS	4	0	0	-545029.58
AXP	5	0	0	-591849.07	HD	4	0	1	-616573.05
BA	5	0	0	-576961.03	IBM	5	0	0	-637538.48
CAT	4	0	0	-508757.06	INTC	4	0	0	-557499.88
CSCO	5	0	0	-565012.80	JNJ	2	0	3	-716969.38
CVX	5	0	0	-537075.58	JPM	2	0	3	-531586.55
DD	5	0	0	-539193.59	KO	5	0	0	-756890.50
DIS	1	0	2	-588161.11	MMM	4	0	0	-708079.02
GE	3	0	2	-621440.67	XOM	5	0	0	-613883.51

datasets. The performed process was as follows:

- First, time series were rescaled to a logarithmic scale to improve stabilization of strong growth trends.
- Second, a large number of ARIMA models were fitted with the Augmented Dickey-Fuller Test, the Auto and Cross-Covariance and Correlation Function Estimation (ACF) and the Partial Auto and Cross-Covariance and Correlation Function Estimation (PACF).
- Finally, for each dataset, the ARIMA model with the lowest AIC is chosen. Table **7-2** shows the ARIMA models fitted with the R package *forecast*.

Random Walk Simulation

For each dataset, 10 random walks were simulated. The parameters μ and σ were calculated using a sliding window that starts with the first price P_0 to the last known price P_t in order to avoid look-ahead bias. Moreover, the method performance was only measured over the out-of-sample dataset.

Multi-Layer Perceptron with Discrete Wavelets Transform

Several DNN architectures were tested. The used degrees of freedom were the number of previous observations (sliding window size from 2 to 60 minutes), number of layers (from 1 to 10), number of neurons per layer, type of activation function (Tanh, Maxout, Rectifier). The final architecture was chosen taking account the balance between efficiency and accuracy.

In summary, the selected features are the one-minute pseudo-log-returns and the compressed tick-by-tick time series (8 values per minute). The selected sliding window size is 3. Therefore, the network input is a vector of 27 values, whereas, the network output is the next one-minute pseudo-log-return. Moreover, the final architecture has one input layer, five hidden layers (27, 22, 17, 11 and 6 neurons using $Tanh$ activation function, respectively) and one output layer (1 neuron using $Linear$ activation function). The number of neurons in each layer decreases with a constant factor of $\frac{1}{5}$.

TensorFlow, an open-source machine learning framework, was used to train the MLPs. Adam optimization was selected. The mean squared error was chosen as the loss function. The default parameters were used for the training process. The MLPs were trained during 200 epochs.

Recurrent Neural Networks with Discrete Wavelets Transform

Furthermore, a GRU and an LSTM were trained using the same proposed method inputs (the last three one-minute pseudo-log-returns and the last three compressed tick-by-tick time series (8×3)) and the same proposed method's output (the next one-minute pseudo-log-return). All neurons use a *tanh* activation function, except the output neuron which uses a *linear* function. Each architecture only has five hidden layers in order to reduce the model's complexity and decrease training times. Therefore, each hidden layer is composed of 27, 22, 17, 11 and 6 units. The number of neurons decreases with a constant factor of $\frac{1}{5}$.

TensorFlow was also used to train the RNNs. Adam optimization was selected. The mean squared error was chosen as the loss function. The default parameters were used for the training process. The RNNs were trained during 200 epochs.

Classic Statistical and Machine Learning Techniques

In order to make a more robust benchmark that compares the results with another approach of similar complexity. The following seven statistical and machine learning techniques for regression were trained:

Linear Models

They model the output like a linear combination of the inputs variables.

$$\hat{y}(w, x) = w_0 + w_1 x_1 + \cdots + w_p x_p \tag{7-1}$$

- **Linear Regression:** It fits by minimizing the residual sum of squares between the observed outputs and the linear approximation predictions.

$$\min_{w} ||X_w - y||_2^2 \tag{7-2}$$

- **Ridge:** It imposes a penalty on the size of coefficients.

$$\min_{w} ||X_w - y||_2^2 + \alpha ||w||_2^2 \tag{7-3}$$

- **Lasso:** It imposes a penalty on the size of coefficients.

$$\min_{w} \frac{1}{2n_{samples}} ||X_w - y||_2^2 + \alpha ||w||_1^2 \tag{7-4}$$

- **Bayesian Ridge:** It estimates a probabilistic model within the context of Bayesian inference.

- **SGD Regressor:** It fits by minimizing a regularized empirical loss with Stochastic Gradient Descent (SGD).

Decision Trees

They are one of the favorite supervised machine learning techniques. The Decision Trees, which are commonly used for classification and regression problems, infers a set of simple decisions rules from the variable inputs.

SVR

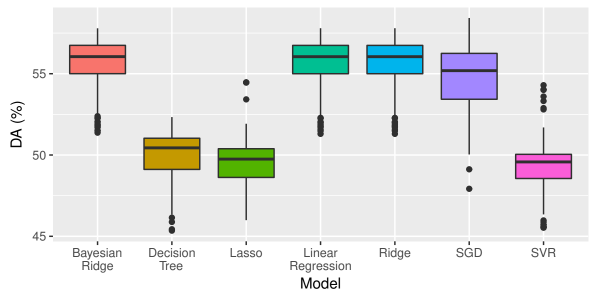
The Support Vector Regression (SVR) is a supervised machine learning technique based on Support Vector Machines (SVM). They construct a set of hyperplanes in a high-dimensional space that segments the space. Although they are a powerful technique, they do not scale well with large datasets.

The chosen input variables were the last n observations (one-minute pseudo-log-returns), and the output variable was the next one-minute pseudo-log-return. The explored window sizes n were 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 30 and 60. These models were trained with the Python library *scikit-learn* using the default parameters.

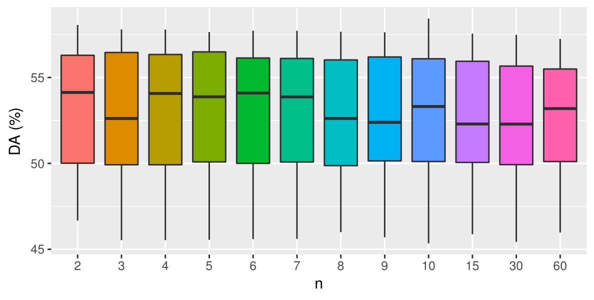
7.2 Experiment and Results: Price Predictor

Each dataset described in the previous section was preprocessed and divided into two parts: first 85% is an in-sample dataset, and the remaining 15% is an out-sample dataset. For each symbol and ANN model, ten ANNs were trained, and then the average error was computed and reported. Overall, all the model results were homogeneous and stable.

Figure **7-2** shows the Performance of DNN vs. ARIMA and RW. After seeing the performance of the models over the out-sample datasets, machine learning techniques had a much better performance compared to ARIMA and Random Walks. The superiority of DNNs is clear over traditional and theoretical methods such as ARIMA and RW. Figures **7-1**a and



(a) DA per Model.



(b) DA per Window Size (n).

Figure **7-1**: Performance of the Classic Statistical and Machine Learning Techniques.

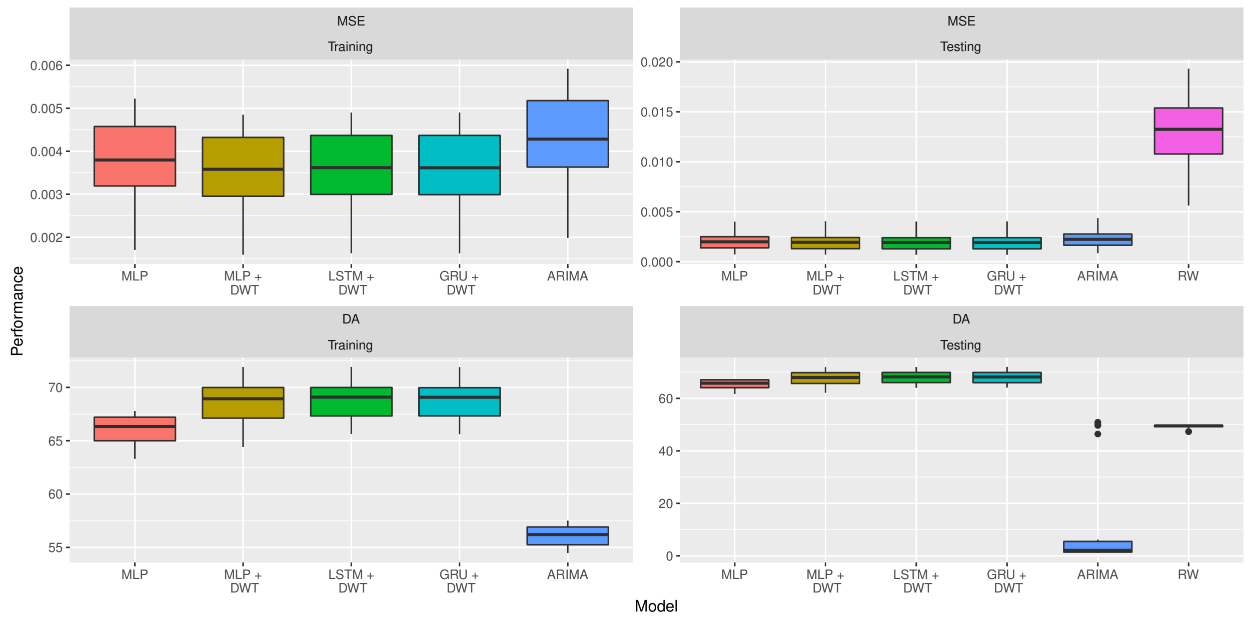


Figure **7-2**: Deep Neural Networks vs. ARIMA and Random Walk.

7-1b show the seven techniques performance. The DA achieved during the testing phase by these techniques ranges from 45.34% to 58.42%. Although these techniques are approaches with similar complexity like DNNs, they are not able to compete with the proposed model that uses the power of the wavelets decomposition and the DL paradigm.

Figure **7-3** shows the performance of the DNNs. Meanwhile, networks using DWT are slightly better than the one without DWT; On average, MLP without DWT, MLP with DWT, GRU with DWT and LSTM with DWT achieved an MSE of 0.002026, 0.001963, 0.001941, 0.001939 and a DA of 65.27%, 67.38%, 67.74%, 67.72%, respectively. In contrast, ARIMA achieved an MSE of 0.002287 and a DA of 12.48%, RW achieved an MSE of 0.01271 and a DA of 49.26%. The best model was GRU, though its performance is almost equal to LSTM. Figure **7-4** shows the performance of the models per symbol during the training and testing phases. During the testing phase, the best symbol was APPL (DA=71.98% -

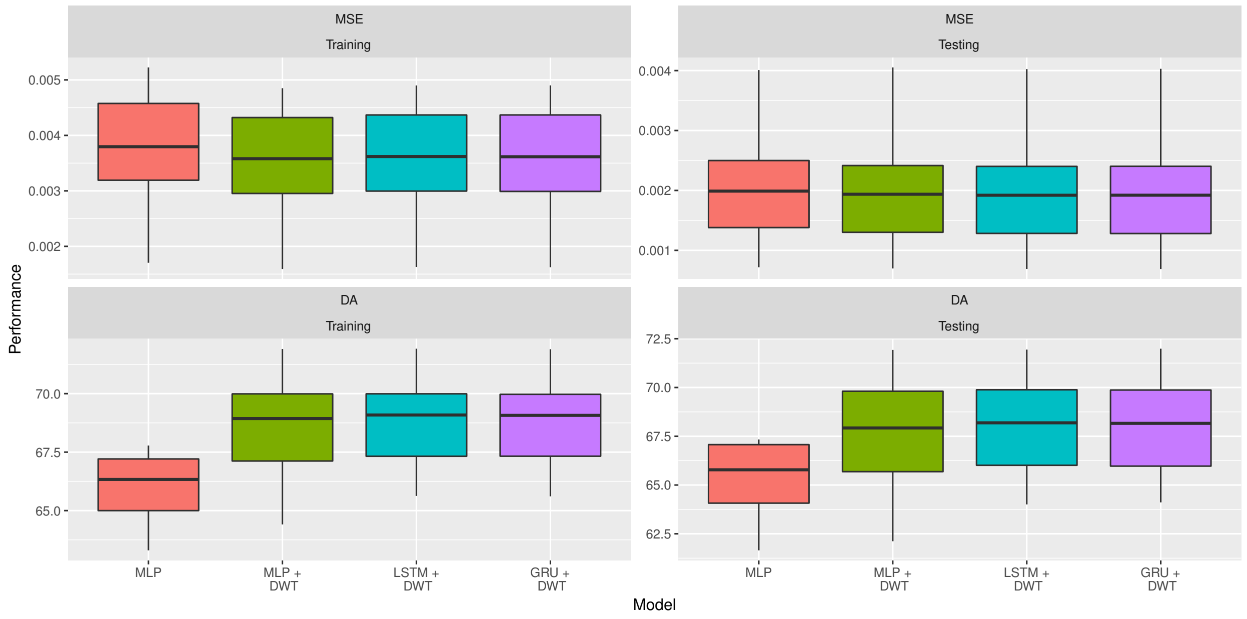


Figure **7-3**: Performance of the Deep Neural Networks.

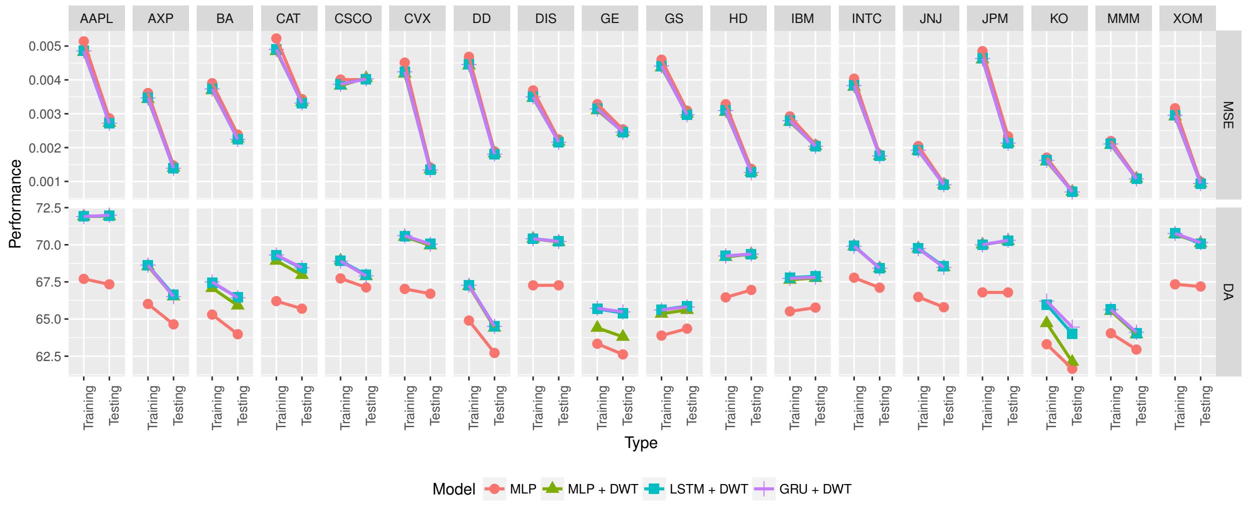


Figure **7-4**: Model Performance per Symbol during the Training and Testing Phases

MSE=0.0027), and the worst was MMM (DA=64.11% - MSE=0.0010), with the model GRU + DWT.

In order to ensure the quality of the results, 12 Paired Student's t-tests were carried out with a significance level α at 0.01. Table **7-3** shows the results. It can be concluded that MLP is better than RW and ARIMA, MLP with DWT is better than MLP, LSTM with DWT and GRU with DWT are better than MLP with DWT, but there is not enough evidence that GRU with DWT is better than LSTM with DWT. Otherwise, GRU is significantly better than LSTM, though it has the same performance as LSTM, GRU has less complexity and

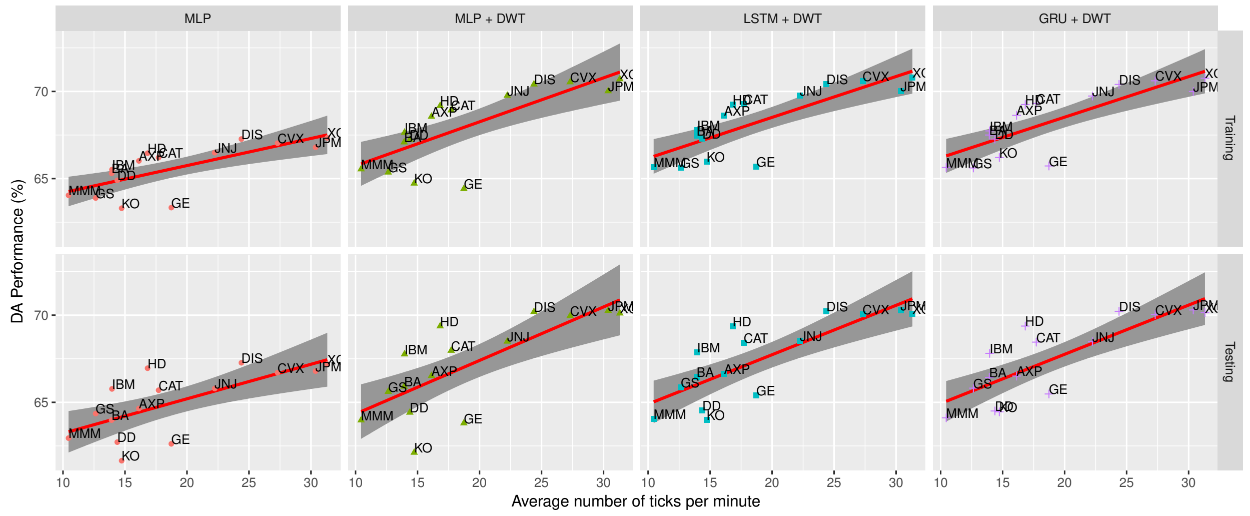


Figure **7-5**: Average Number of Ticks per Minute vs. DA Performance

Table **7-3**: Paired Student's t-tests: Significance Level α at 0.01.

Error	**Group 1**	**Group 2**	H_α	**p-value**	**Result**
DA	MLP	ARIMA	greater	3.5e-10	Accept H_α
	MLP	RW	greater	2.2e-16	Accept H_α
	MLP + DWT	MLP	greater	2.6e-08	Accept H_α
	GRU + DWT	MLP + DWT	greater	0.013	Accept H_α
	LSTM + DWT	MLP + DWT	greater	0.008	Accept H_α
	LSTM + DWT	GRU + DWT	greater	0.238	Reject H_α
MSE	MLP	ARIMA	less	6.1e-11	Accept H_α
	MLP	RW	less	8.3e-12	Accept H_α
	MLP + DWT	MLP	less	2.5e-05	Accept H_α
	GRU + DWT	MLP + DWT	less	0.002	Accept H_α
	LSTM + DWT	MLP + DWT	less	0.002	Accept H_α
	LSTM + DWT	GRU + DWT	less	0.954	Reject H_α

fewer parameters than LSTM, therefore, the training time is reduced. Table **7-4** shows the differences in time between the LSTM and GRU model on a laptop. As a result, GRU is more appropriate for use in a real environment.

DNNs can learn the market dynamics with good precision and accuracy over out-sample datasets. However, DA performance depends on the symbol. For instance, more liquid instruments like AAPL, CSCO, and INTC got much better performance with a DA close to 72%, 68%, and 68%, respectively. Otherwise, a less liquid instrument like MMM (10 ticks per minute) had a lower DA performance, close to 64%.

Table **7-4**: Training time.

	LSTM	**GRU**
Number of inputs	27	27
Number of neurons in the layer 1	27	27
Number of neurons in the layer 2	22	22
Number of neurons in the layer 3	17	17
Number of neurons in the layer 4	11	11
Number of neurons in the layer 5	6	6
Number of neurons in the output layer	1	1
Total trainable parameters	14,800	11,100
Total training time	119.79 s	100.55 s
Total time per prediction	60.04 μs	57.75 μs

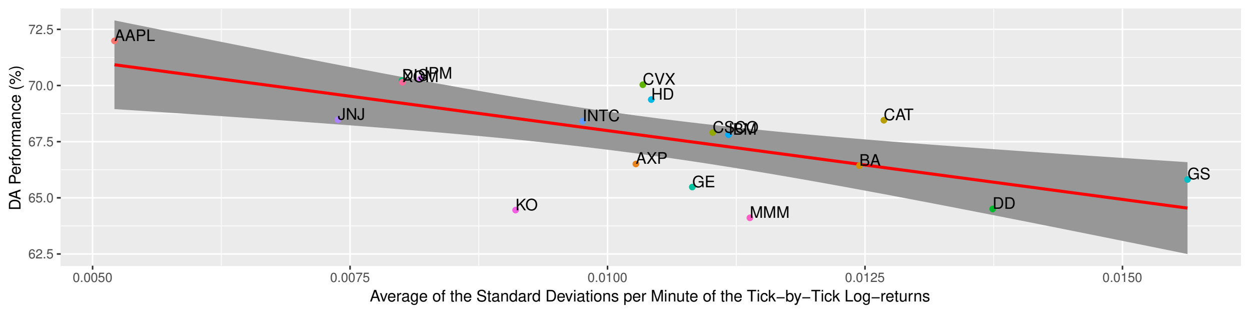


Figure **7-6**: Average of the Standard Deviations per Minute of the Tick-by-Tick Log-returns vs. DA Performance

Pearson and Spearman correlation tests were performed in order to verify the dependency between the average number of ticks per minute and the DA performance achieved by each machine learning model on all out-sample datasets. The Pearson correlation archived by MLP without DWT, MLP with DWT, GRU with DWT and LSTM with DWT on all symbols were 0.4407459, 0.4524886, 0.4695877 and 0.4648353, respectively. Meanwhile, the Spearman correlation achieved by MLP without DWT, MLP with DWT, GRU with DWT and LSTM with DWT over all symbols were 0.7684211, 0.7368421, 0.7649123 and 0.7701754, respectively.

The Spearman correlation suggests that there is a non-linear correlation between liquidity and DA Performance. Therefore, as liquidity is higher in certain periods, the proposed model has greater effectiveness. It is worth mentioning that the model effectiveness will be stuck at some unknown point; it does not matter how high the liquidity is, the proposed models will never reach 100% precision.

Figure **7-5** shows the relationship between the average number of ticks per minute and

DA performance for each machine learning model. Stocks with higher liquidity (AAPL, CSCO, and INTC) were excluded for better visualization. As shown in Figure **7-5**, the more traded the stock is, the model predicts it better. Figure **7-5** also draws a fitted Generalized Additive Model (GAM).

Besides, there is an inverse correlation between the average of the standard deviations per minute of the tick-by-tick log-returns, and the DA performance achieved by the best model (GRU with DWT) on all out-sample datasets. Pearson and Spearman correlation tests were -0.650861 and -0.6573787, respectively. As shown in Figure **7-6**, the proposed GRU model has greater effectiveness as lower as volatility inside a minute is.

7.3 High-Frequency Trading Strategy Simulation

Table **7-5** shows an overview of the experimental setting for testing the proposed high-frequency trading strategy. Table **7-6** shows the strategy performance during a trading simulation using the same testing period from the previous chapter (approximately five months) and the same parameters described in the previous chapter, i.e. it did not take into account transaction and spread costs. Investment in each stock starts with 1 dollar and reinvests all the available money in the following operations. Furthermore, Figures **7-8**, **7-9** and **7-10** show the three best-performing stocks, whereas Figures **7-11**, **7-12** and **7-13** show the three-worst performing stocks.

Moreover, there are three groups of stocks: the stocks with annualized returns greater than 100%, the ones with annualized returns between 5% and 100%, and those with less than 5%. Although DNNs had similar behaviors and remarkable forecasting accuracy, there are behavior dynamics at the high-frequency scale that are not captured by the aggregated one-minute series. Additionally, those dynamics are different for each stock.

The best group is composed of CSCO and INTC. Figures **7-8** and **7-9** show both performances. The achieved profits had exponential growth behavior. The strategy made 63.5% and 61.7% good trades, but 36.3% and 38.2% bad trades, respectively. Both average profits per position are positive. The strategy had a stable and homogeneous behavior for all testing days concerning the number of positions and the amount of buy (long) and sell (short) positions. Moreover, those two stocks yielded amazing profits.

The worst group is composed of CVX and DD. Figures **7-12** and **7-13** show both performances. Unlike the best group, these stocks had noisy behavior and larger draw-downs. The strategy made 50.22%, 49.91% and 48.40% good trades, but 49.59% and 48.04% bad trades, respectively. These stocks were also the only ones that had accumulated losses for a short time, though they were able to recover and yield earnings.

The third group is composed of the remaining 14 stocks. This group had a similar behavior compared to the best group. The main differences are that those stocks presented some draw-downs and yielded high earnings, but not as superior as those of the best group.

All stocks had an annualized volatility raging from 3.54% to 9.01%, with a mean of 5.53%

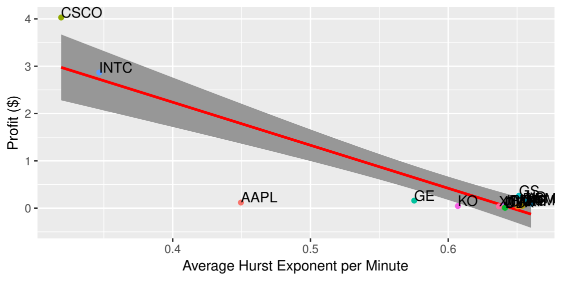
Table **7-5**: Experiment Setting: High-Frequency Trading Strategy

Parameter	**Value**
Number of stocks	18
Time period	From April 2017 to September 2017
Duration	Approximately 5 months
Transaction costs	No
Spread costs	No
Initial capital	1 USD
DNN retraining	No
Reinvest profits	Yes

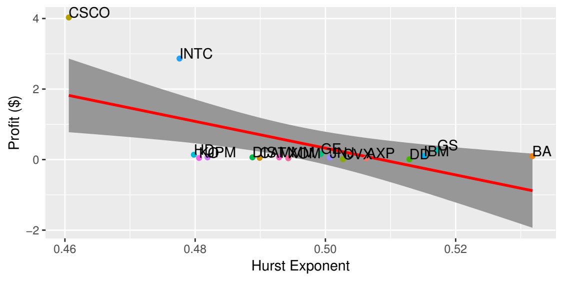
Table **7-6**: Simulation Results.

Stock	**Profit ($)**	**Annualized Returns (%)***	**Annualized Volatility (%)***	**Max Draw-down (%)**	**Earnings (%)**	**None (%)**	**Losses (%)**
CSCO	4.0297	5528.32	9.01	0.33	63.54	0.16	36.30
INTC	2.8653	2817.78	8.30	0.00	61.69	0.15	38.16
GS	0.2639	79.38	8.86	3.06	50.39	0.70	48.91
GE	0.1601	44.84	5.13	1.08	46.96	8.52	44.52
HD	0.1385	38.21	4.42	1.13	50.78	0.80	48.42
AAPL	0.1189	32.35	8.34	1.91	51.01	0.00	48.99
IBM	0.0985	26.42	4.34	1.20	50.01	1.41	48.58
BA	0.0976	26.16	5.54	1.54	50.10	1.25	48.65
MMM	0.0637	16.65	4.12	1.25	49.27	2.04	48.69
DIS	0.0637	16.65	3.76	0.58	50.05	1.36	48.58
JPM	0.0630	16.48	4.56	1.86	50.26	0.60	49.14
JNJ	0.0555	14.43	3.54	0.89	49.92	1.54	48.54
CAT	0.0490	12.66	5.53	1.75	49.90	1.05	49.04
KO	0.0452	11.66	4.01	2.15	45.88	9.76	44.36
XOM	0.0386	9.91	3.78	1.36	50.22	1.17	48.61
AXP	0.0362	9.27	4.16	2.13	48.65	3.89	47.46
CVX	0.0133	3.36	4.57	2.21	49.91	0.50	49.59
DD	0.0079	1.98	6.53	3.26	48.40	3.57	48.04

*Annualized rates assuming 252 trading days.



(a) Average Hurst Exponents per Minute



(b) Hurst Exponents

Figure **7-7**: Hurst Exponents vs. Strategy Profits.

and max draw-downs ranging from 0% to 3.26%, with a mean of 1.5%. Besides, there were more positive trades than negative ones for all stocks, because the proposed strategy has symmetric behavior.

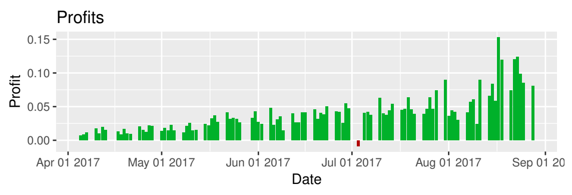
The profits are not correlated to the model DA, because the performance depends on the microstructure of the stocks at a High-Frequency scale. For this reason, the analysis expands to include the Hurst exponent, which is commonly used in fractal geometry and is a measure of long-term memory of time series [73]. This exponent ranges from 0 to 1. If it is lesser than 0.5, the time series is an anti-persistent time series (or mean-reverting series). If it is greater than 0.5, the time series is a persistent time series. Otherwise, if it is close or equal to 0.5, the time series is a Brownian time series (or random time series) [88].

Figure **7-7**a shows the relationship of the average of the computed Hurst exponents on the tick-by-tick log-returns per stock and minute and the strategy profits. According to the Pearson correlation coefficient (-0.8847), there is an inverse linear relationship between these variables. Coincidentally, the two stocks CSCO and INTC, which had amazing profits, have the two lowest average Hurst exponents per minute: 0.3189 and 0.3465, respectively. Meanwhile, the other stocks had an average Hurst exponent per minute that ranges from 0.44 to 0.66. It means that CSCO and INTC have a mean-reverting behavior within a normal minute, meanwhile, the other stocks are close to being Brownian time series within a minute.

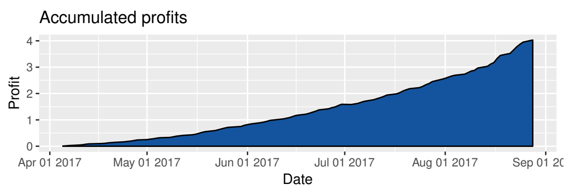
Moreover, Figure **7-7**b shows the relationship of the computed Hurst exponent on all tick-by-tick log-returns per stock and the strategy profits. According to the Pearson correlation coefficient (-0.5862), there is not a strong linear relationship between these variables. Since the behavior of the time series varies through time, the coefficients differ according to the computed averages.

7.4 Trading Strategy Benchmark

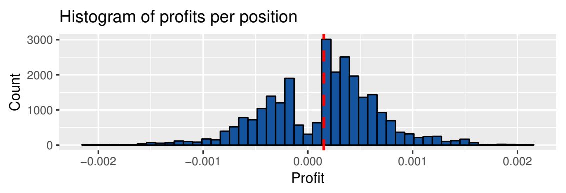
There is no standard benchmark on Finance; therefore, three types of strategies were simulated over the same testing datasets. Table **7-7** shows the experimental setting overview for



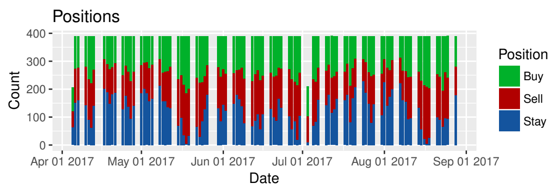
(a) Profits.



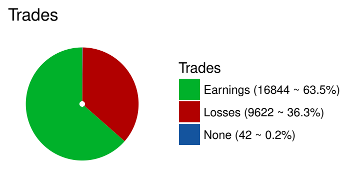
(b) Accumulated Profits.



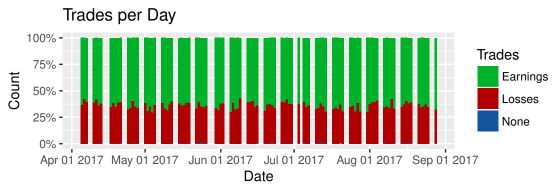
(c) Histogram of Profits per Position.



(d) Positions.

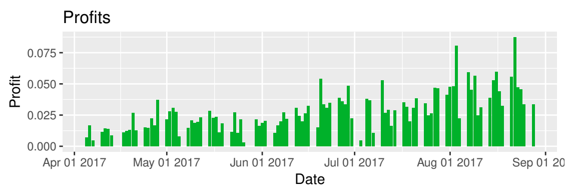


(e) Trades.

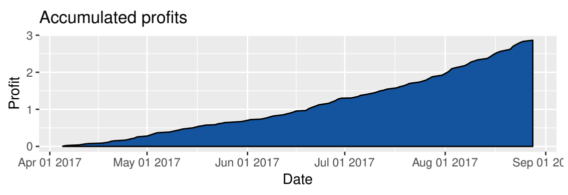


(f) Trades per Day.

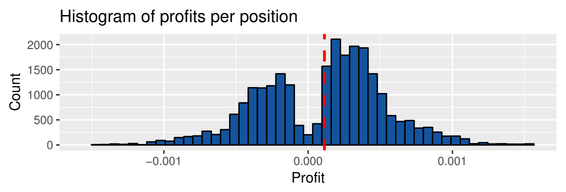
Figure **7-8**: The Best-Performing Stock: Cisco Systems, Inc. (CSCO).



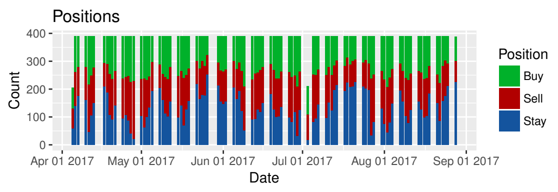
(a) Profits.



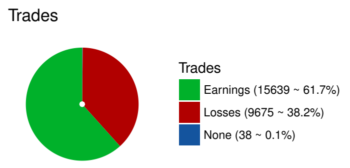
(b) Accumulated Profits.



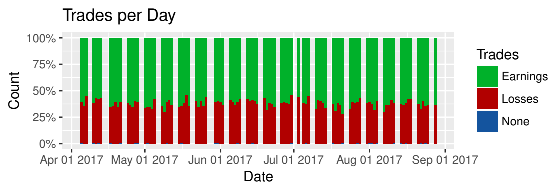
(c) Histogram of Profits per Position.



(d) Positions.

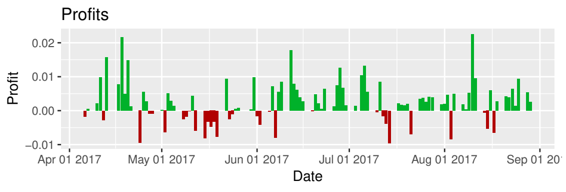


(e) Trades.

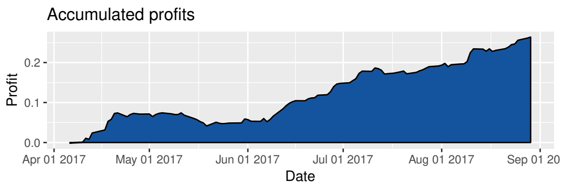


(f) Trades per Day.

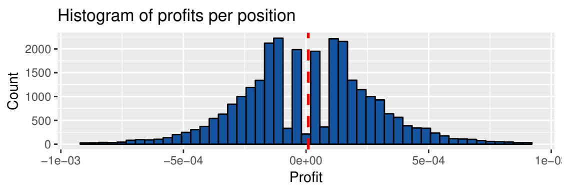
Figure **7-9**: The Second Best-Performing Stock: Intel Corporation (INTC).



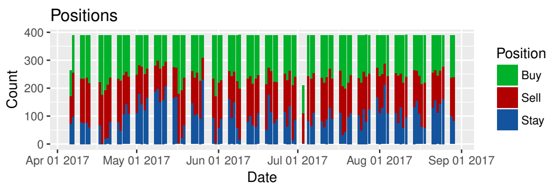
(a) Profits.



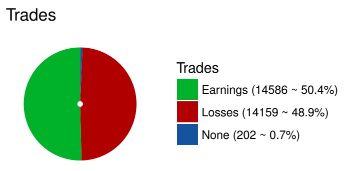
(b) Accumulated Profits.



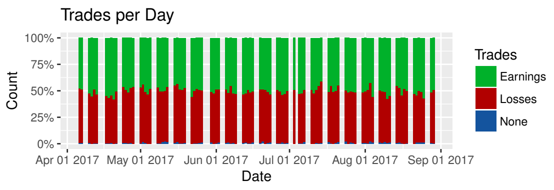
(c) Histogram of Profits per Position.



(d) Positions.

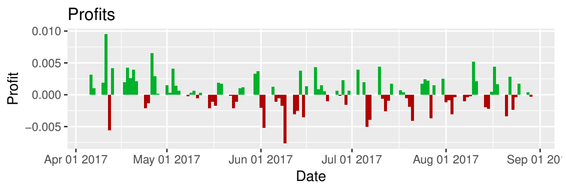


(e) Trades.

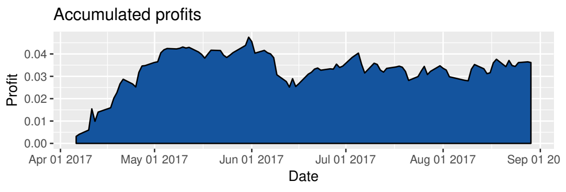


(f) Trades per Day.

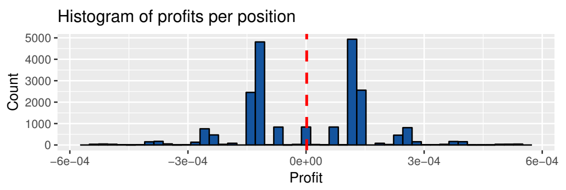
Figure **7-10**: The Third Best-Performing Stock: Goldman Sachs Group, Inc. (GS).



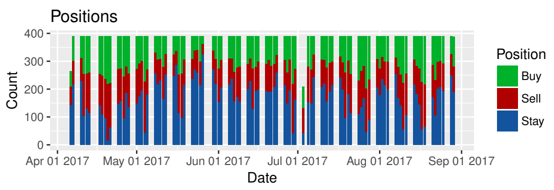
(a) Profits.



(b) Accumulated Profits.



(c) Histogram of Profits per Position.



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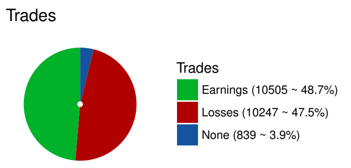
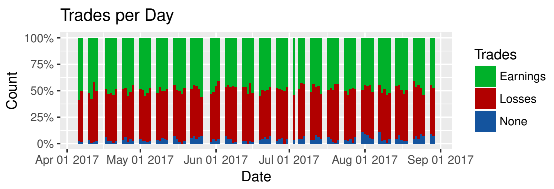
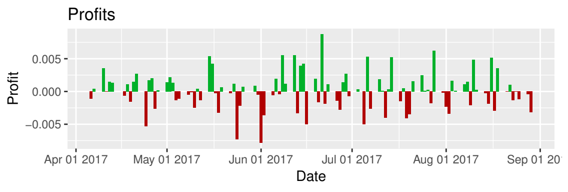


(e) Trades.

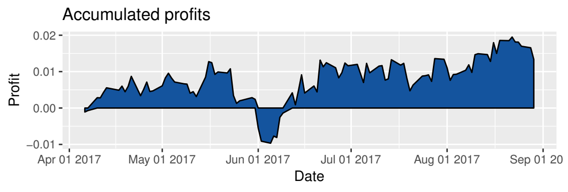


(f) Trades per Day.

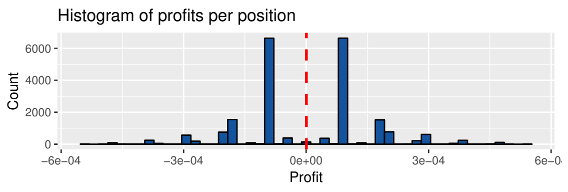
Figure **7-11**: The Third Worst-Performing Stock: American Express Company (AXP).



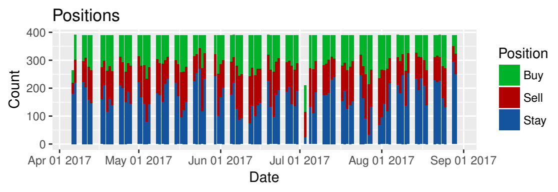
(a) Profits.



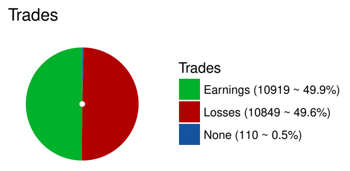
(b) Accumulated Profits.



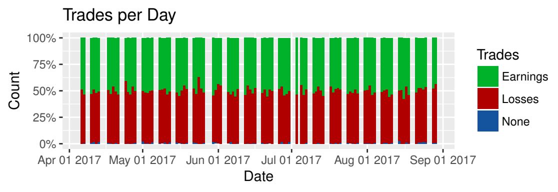
(c) Histogram of Profits per Position.



(d) Positions.

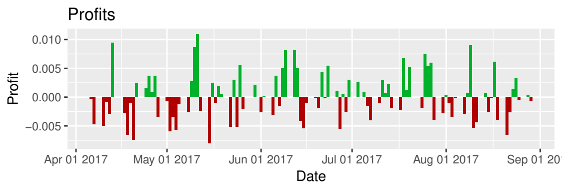


(e) Trades.

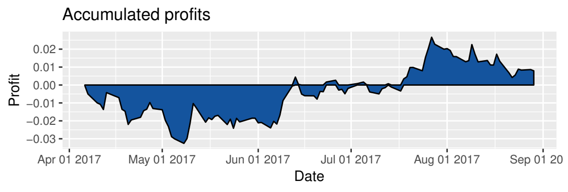


(f) Trades per Day.

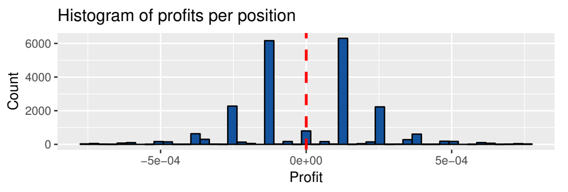
Figure **7-12**: The Second Worst-Performing Stock: Chevron Corporation (CVX).



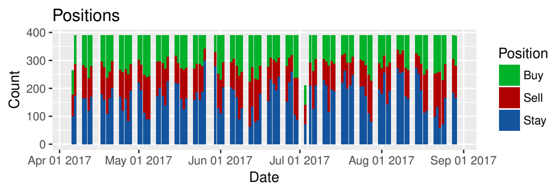
(a) Profits.



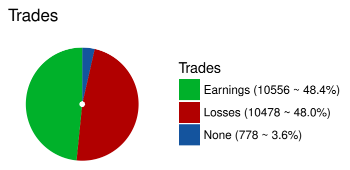
(b) Accumulated Profits.



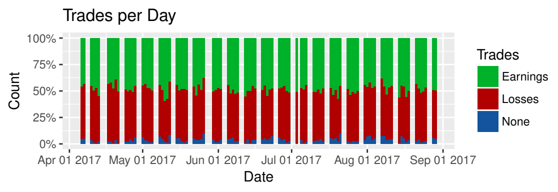
(c) Histogram of Profits per Position.



(d) Positions.



(e) Trades.



(f) Trades per Day.

Figure **7-13**: The Worst-Performing Stock: DuPont Corporation (DD).

Table **7-7**: Experiment Setting: Trading Strategy Benchmark

Strategy	**Number of stocks**	**Time period**	**Transaction costs**	**Spread costs**	**Initial capital**
Proposed Strategy	18	From April 2017 to September 2017 (Approximately 5 months)	No	No	1 USD
Buy & Hold					
RSI Strategy (4 mins.)					
RSI Strategy (9 mins.)					
RSI Strategy (14 mins.)					
RSI Strategy (19 mins.)					
RSI Strategy (24 mins.)					
RSI Strategy (29 mins.)					
Random Trading					

comparing the proposed high-frequency trading strategy against the three types of strategies. Table **7-8** shows the simulation results. Moreover, Figure **7-14** shows the accumulated profit per each strategy. The chosen strategies were:

Buy & Hold Strategy (BHS)

A BHS is a passive investment strategy in which a trader buys a stock (opens a long position), and holds it in his portfolio for a long time, while, he expects the stock value increases overtime. The BHS lost money with six stocks: INTC, GE, IBM, XOM, AXP, CVX. In contrast, the proposed strategy earned money with all stocks. If a portfolio had been created with 18 stocks and \$1 for each stock, the BHS would have to earn \$2.9897, and the proposed strategy would have to earn \$8.2353 (approximately 2.7 times more).

RSI Trading Strategy

For each minute, the Relative Strength Index (RSI) was computed using n one-minute periods. If the asset is overbought, the strategy sells the asset and then buys it when the minute ends. If the asset is oversold, the strategy buys the asset and then sells it when the minute ends. Otherwise, the strategy does not do anything until the next minute. The chosen n were 4, 9, 14, 19, 24 and 29, and the chosen oversold and overbought thresholds were 20 and 80, respectively.

All the RSI strategies yielded positive returns. The profit was lesser to the yielded by the proposed strategy and the Buy & Hold Strategy. The best performing configuration was $n = 4$, and the worst was $n = 29$.

Table **7-8**: Strategy Benchmark.

Stock	**Proposed Strategy**	**Buy & Hold**	**RSI Strategy**						**Random Trading**
			4 mins.	**9 mins.**	**14 mins.**	**19 mins.**	**24 mins.**	**29 mins.**	
CSCO	4.0297	0.1317	0.2930	-0.0427	0.0003	0.0387	0.0288	0.0278	-0.0152
INTC	2.8653	-0.0564	0.4479	0.1298	0.0231	0.0242	0.0231	0.0122	0.0123
GS	0.2639	0.1262	0.2435	0.0320	-0.0032	-0.0007	-0.0100	-0.0225	0.0031
GE	0.1601	-0.0372	0.2714	0.0866	0.0447	0.0259	0.0185	0.0211	0.0270
HD	0.1385	0.4229	0.0456	0.0262	-0.0007	-0.0091	-0.0180	-0.0172	-0.0034
AAPL	0.1189	0.4496	0.0875	0.0383	0.0402	0.0207	0.0142	0.0039	-0.0052
IBM	0.0985	-0.1132	0.0640	0.0077	-0.0032	0.0001	0.0028	0.0058	0.0059
BA	0.0976	0.8369	-0.0136	-0.0362	-0.0369	-0.0429	-0.0285	-0.0263	0.0148
MMM	0.0637	0.2366	0.0607	-0.0138	-0.0272	-0.0255	-0.0260	-0.0239	-0.0002
DIS	0.0637	0.0805	0.0372	0.0255	0.0148	0.0247	0.0193	0.0236	-0.0021
JPM	0.0630	0.4542	0.2267	0.0759	0.0315	0.0123	0.0105	0.0168	-0.0167
JNJ	0.0555	0.2560	0.0125	0.0566	0.0186	0.0121	0.0119	0.0044	0.0071
CAT	0.0490	0.2638	0.0704	0.0135	0.0146	0.0088	0.0009	-0.0055	0.0226
KO	0.0452	0.0750	0.2190	0.0549	0.0167	0.0145	0.0112	-0.0016	0.0071
XOM	0.0386	-0.1716	0.1510	0.0065	-0.0057	-0.0083	-0.0053	-0.0022	-0.0098
AXP	0.0362	-0.0860	0.0933	0.0321	0.0277	0.0226	0.0196	0.0120	-0.0033
CVX	0.0133	-0.0349	0.0335	-0.0004	0.0024	-0.0047	-0.0101	-0.0056	0.0022
DD	0.0079	0.1140	0.1432	0.0726	0.0414	0.0275	0.0323	0.0242	0.0143
Total	**8.2086**	**2.9481**	**2.4868**	**0.5651**	**0.1991**	**0.1409**	**0.0952**	**0.0470**	**0.0605**

One-minute Random Trading Strategy

For each minute, a long or short operation is opened and then closed when the minute ends. The decision is taken randomly. Depending on the run, the strategy can obtain earnings or losses. Therefore, 100 simulations were performed and the average profit was reported. This average profit is very close to 0.

Besides, the proposed strategy adds some advantages: First, the predictor of the strategy can be retrained in less than two minutes in standard hardware at any time (If it is required to reduce training times, hardware with better capabilities can be used). Second, the predictions can be performed in real-time. Third, the strategy is simple and can be implemented in any trading platform. And finally, the strategy has a symmetric behavior at opening and closings signals.

The proposed strategy has stable performance, though it is necessary to extend the analyzes to consider transaction costs, spreads, and liquidity in high-frequency. At the time of the development of this work, we did not have access to the best bid-ask series or limit order books. For this reason, we can not perform a perfect simulation that includes all those hidden costs such as the spread. Hence, we will estimate the average spread, and then, will make a small analysis of the strategy's performance when a constant spread cost is introduced. It is important to note that the spread is variable over time, and in order to

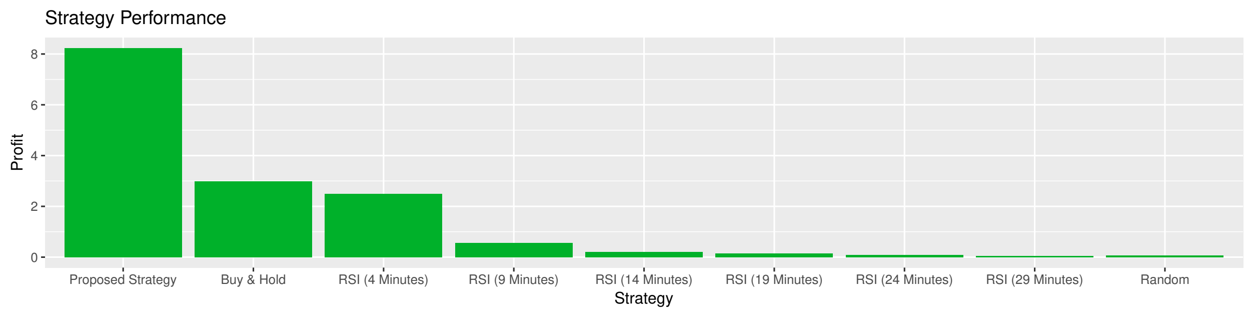


Figure **7-14**: Strategy Performance

make a more realistic simulation, the best bid/ask time series is required.

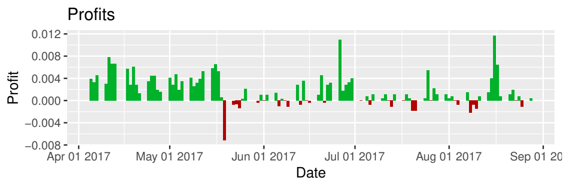
We estimated the average spread using the tick-by-tick series and the following technique: First, we calculated the price difference between ticks in absolute value. Second, we eliminated the zeros. And finally, we calculated the average over the remaining values. It is not a rigorous technique but it will allow us to analyze the behavior of the strategy with a constant estimated spread.

The estimated spreads range from USD 0.01 to USD 0.03 depending on the stock. Figure **7-17**a shows the relation between the estimated spread and the achieved DA by the GRU model. And figure **7-17**b shows the relation between the estimated spread and the achieved profit by the proposed strategy. On both charts, the variables do not have a correlation between them. As a result, it can be concluded that the predictor is really learning something, and the excellent results are not a product of the spread.

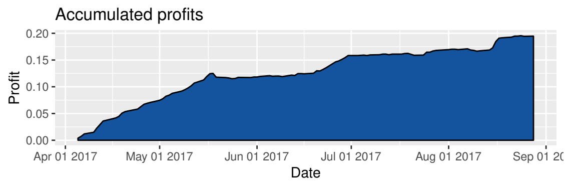
Figure **7-17**c shows the estimated spread vs. the average difference between closing prices and the predicted prices. The larger the spread, the larger the difference between the closing price and the estimated average price. This means that the model can predict values that are actionable by a trading strategy because the prediction is greater than the spread.

One of the conclusions of the thesis is that HF trading is suitable and advisable only for high liquidity shares. For this reason, we simulated the strategy with a constant spread of 0.01 USD using the tick-by-tick data from AAPL and CSCO. Figures **7-16** and **7-15** shows the obtained results. As expected, the profits go down, but the performance of the strategy does not obey the spread but by the behavior and micro-economic structure that each stock has in an HF time scale. The analysis of the microeconomic structure of each stock was outside the scope of the thesis. However, it is something quite interesting to research.

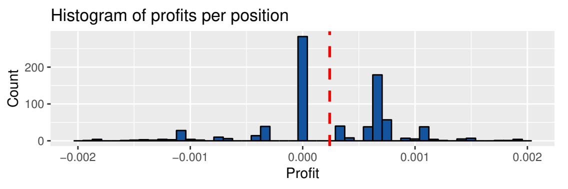
For a more realistic implementation and to avoid the spread cost, two prediction models must be created: one that predicts the average bid price and the other that predicts the average ask price. In this way, the decisions are made based on the prices available in the limit order books, and in case of identifying a short/long opportunity, there is no discount for profit spread.



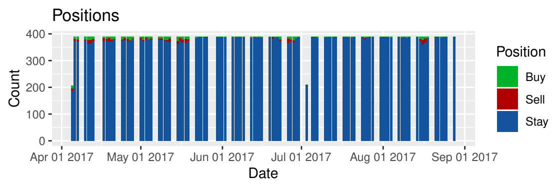
(a) Profits.



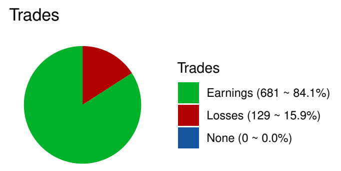
(b) Accumulated Profits.



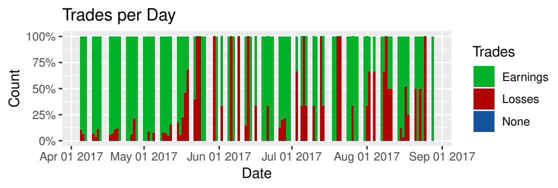
(c) Histogram of Profits per Position.



(d) Positions.

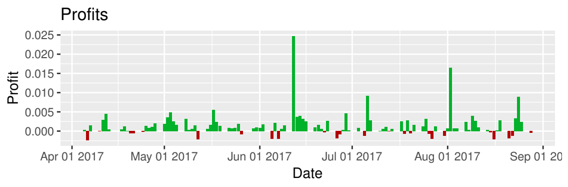


(e) Trades.

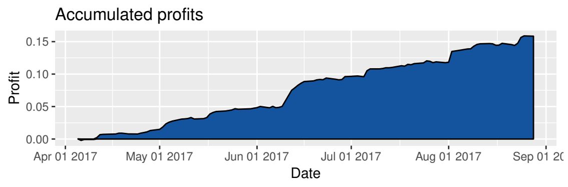


(f) Trades per Day.

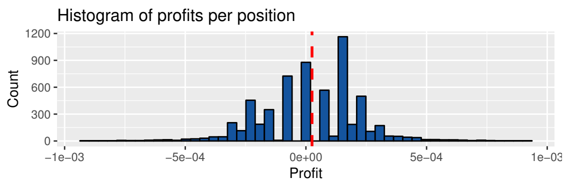
Figure **7-15**: Cisco Inc. (CSCO): Estimated spread USD 0.01



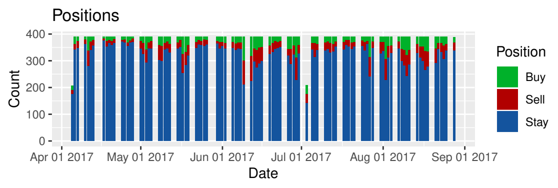
(a) Profits.



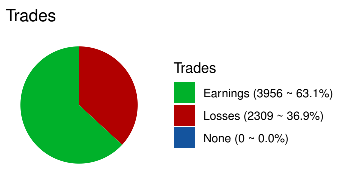
(b) Accumulated Profits.



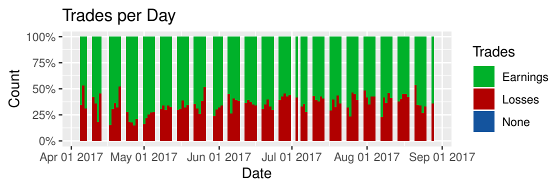
(c) Histogram of Profits per Position.



(d) Positions.

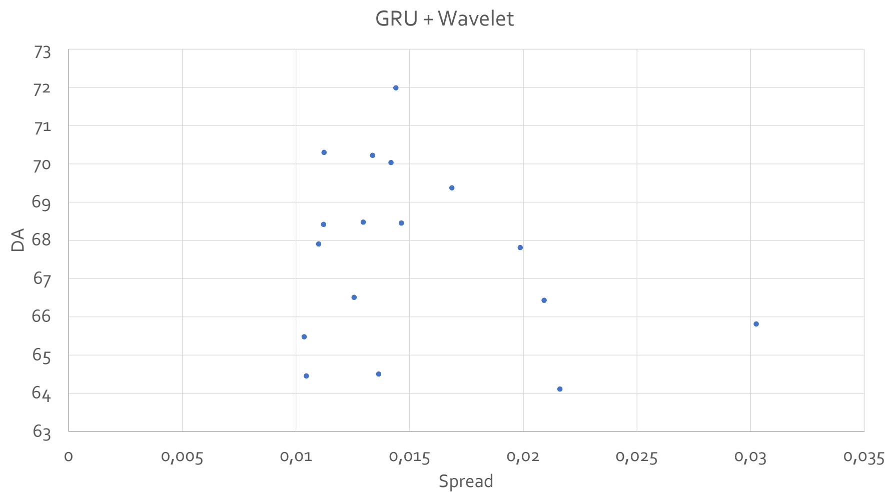


(e) Trades.

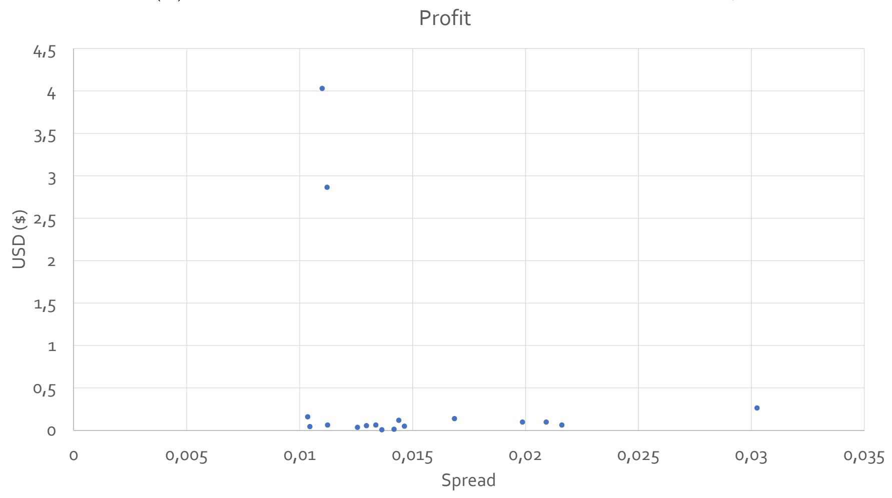


(f) Trades per Day.

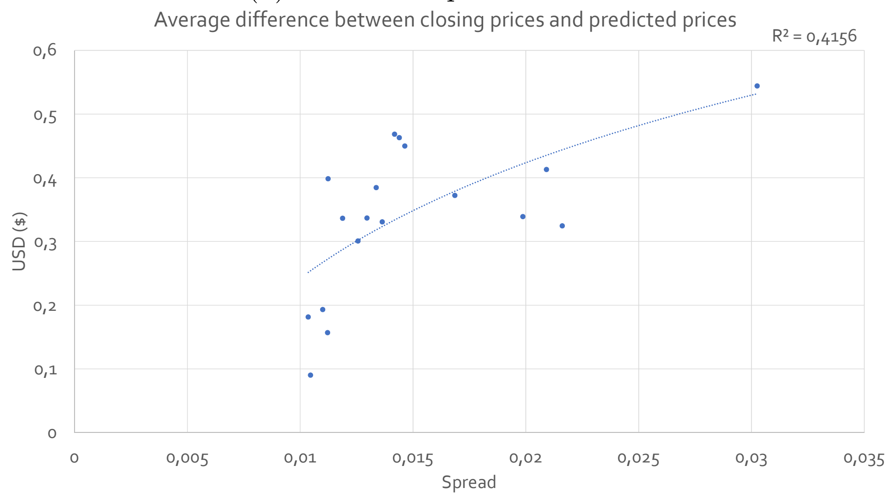
Figure **7-16**: Apple Inc. (AAPL): Estimated spread USD 0.01



(a) Estimated spread vs. Directional Accuracy



(b) Estimated spread vs. Profit



(c) Spread Analysis: Diff

Figure **7-17**: Spread Analysis

Chapter 8

Conclusions and Recommendations

The main motivation of this thesis was to help build a bridge between the financial industry and academic research, providing experience and new approaches to improve understanding of the markets. Moreover, this work seeks to be a public base work for academic and industrial research in financial machine learning, which is a new multidisciplinary world that is being born and will revolutionize the financial industry [28]. This new discipline needs non-standard procedures, techniques, and approaches (such as those presented in this document) that differs from the standard machine learning world. Therefore, works like this are needed to continue with this revolution.

The main contribution of this work is the proposed innovative approach for predicting FTS. It includes the combination of:

- The advanced learning capabilities of the Deep RNN (LSTM and GRU).
- The representational power in frequency and time domains of the DWT.
- And the idea of modeling time series through average prices.

Many works have studied the prediction of FTS and suggested to explore more learning techniques and complicated features [117, 71, 39, 12]. Being motivated by the previous reasons, this work explored DL models (more specifically, RNN like LSTM and GRU) and the combination of multiple features (the time, one-minute standard deviations, one-minute trend indicators, one-minute log-returns, Fourier coefficients, wavelet coefficients, among others).

Moreover, this work proposes a straightforward high-frequency trading strategy that uses the proposed DNN predictor. It supports the idea behind the pseudo-log-returns: Predicting an average price is easier than predicting a closing price, that is, the product of a random and noisy process. This strategy presented highly competitive results (see Chapter 7).

Besides, an average price can be better exploited in a trading strategy, than a closing price. The reason is that average prices predictions have less uncertainty than those of closing prices. Hence, making decisions with less uncertainty leads to trades with less risk and greater possibilities of carrying out successful trades. Below the main conclusions and recommendations will be presented:

Discrete Wavelet Transform

- The main advantage of using DWT is that the resulting time series is represented in the frequency domain with temporal resolution; hence, the signal has properties from both time and frequency domains.
- The Haar wavelet has a technical disadvantage: It has discontinuities and hence it is not a differentiable function. However, its discontinuities are an advantage for FTS analysis because they perfectly fit to handle signals with sudden transitions and jumps.
- Without the features of using the DWT, the DNN had not been able to learn the complex and intricate patterns of FTS. Therefore, DWT has proved to be an important tool for time series analysis and prediction.
- The proposed preprocessing method based on the DWT works with any signal of any length greater to 8. This property is useful for homogenizing data in fixed shape size because any machine learning technique can use this resulting data.
- Besides, this preprocessing method allows the proposed predictor to exploit and take advantage of the available information at an HF scale.
- The DWT is easy to program within any trading strategy since there are many robust packages and libraries in most popular programming languages.

Deep Learning

- The use of an RNN allowed achieving better results than a simple MLP because RNN has an internal memory that allows modeling temporal dynamics.
- The DL arena has a lot of robust techniques that can face the difficult task of predicting FTS and modeling market behavior because they can learn a deep hierarchical representation of data, in which the input is transformed from low-level to high-level representations.
- The proposed predictor can be trained in less than two minutes in standard hardware at any time (If it is required to reduce training times, hardware with better capabilities can be used). This property is useful for online retraining if required.

- GPUs increased the speed of training DL models because they are suitable for massive linear algebra operations involved in training algorithms.
- The use of adaptive optimizers guaranteed a fast convergence in the ANN parameters.

Feature Engineering

- It is important to note that feature engineering is an essential step on any machine learning project. Currently, the end-to-end approach, in which the models are based on raw data, and feature engineering is not required, is the most common DL approach in many disciplines. However, this approach is not recommended for finance applications (at least not with the current advances in hardware and machine learning techniques), due to the excessive complexity and properties of FTS.
- Due to restrictions when the strategy is operating at High-Frequency scale, the prediction model inputs must be able to be calculated fast.

Pseudo-log-returns

- Pseudo-log-returns, which are a non-standard data transformation, offer more quality information on the status of an asset price. Given the fact that averages are more stable than noisy tick-by-tick prices, they are less affected by outliers and noise than log-returns.
- Another approach would be to use the median that is more stable than the average. However, given the time and speed constraints at High-Frequency scale, it is more recommended to use the average prices, because the calculation of the median implies sorting the tick-by-tick prices and this process is performed in $O(n \log n)$. Meanwhile, the calculation of the average can be performed in $O(n)$.
- Besides, this work differs from others in the use of the concept of pseudo-log-returns. It allowed the proposed prediction model to achieve better performance since the average prices are less exposed to noise.
- Predicting average prices is an easier task than predicting closing prices. Depending on the financial asset, its liquidity and the amount of trades within a minute, the average becomes an estimator of greater confidence and supplies more valuable information about the market status. Therefore, it is easier for a DNN or any machine learning model to predict average prices than closing prices.
- Pseudo-log-returns are easy to implement in any programming language; therefore, they can be integrated into a trading strategy in any algorithmic trading platform.

Liquid stocks

- In this study, the proposed DNN achieved 72% of directional accuracy, predicting the Apple Inc. stock (AAPL), which has the highest average number of trades per minute within the dataset. Found evidence suggests that the performance of the prediction model has a non-linear correlation with the number of trades per minute. As the amount of ticks increases, the prediction also improves because the averages are more stable and reliable.

High-Frequency Trading Strategy

- An average price estimation can be better exploited in a trading strategy than a closing price estimation.
- There is evidence that not operating in all minutes can be crucial for the excellent performance of the strategy. The reason is that opening and closing positions in minutes of high volatility or low liquids can be a bad idea for HF strategies.
- Given the fact that there is no guarantee of knowing whether the next minute will be enough liquid to operate, in this work, a policy was suggested: do not trade if the traded volume of the previous minute does not exceed a threshold. This policy is straightforward but effective.
- Found evidence suggests that there is an inverse linear relationship of the average of the computed Hurst exponents on the tick-by-tick log-returns per minute and the strategy profits.

Chapter 9

Opportunities for Future Research

This work presented highly competitive results. There are many possible opportunities for future research that are derived from this work and seek to improve it:

Discrete Wavelet Transform

- The Haar wavelet discontinuities perfectly fit to handle signals with sudden transitions like FTS. However, a possible research opportunity would be to explore the performance of other wavelets (Daubechies, Coiflets, Symlets, Discrete Meyer, Biorthogonal, among others), as well as other discrete transforms like Discrete Chebyshev transform and Discrete cosine transform.

- It might also be interesting to include higher resolution levels as inputs to the DNN, in order to analyze if adding more wavelet and scaling coefficients, the prediction performance can improve.

Deep Learning

- Another possible opportunity for research is to explore other DL architectures like Deep Convolutional Neural Networks or Deep Belief Networks, or even other Machine Learning techniques that allow incorporating expert knowledge, such as Hidden Markov Model (HMM) and Dynamic Bayesian Networks.

Feature Engineering

- This research could also be extended to studying available data on Limit Order Books (LOB), volume time series, external news, earnings reports, technical indicators, among others. It could allow extracting more knowledge and identify more complex patterns from the market at an HF scale. With a better understanding of market status, the model will be able to make better decisions and yield higher and more stable profits.

Trading Strategy

- The presented strategy is simple and yielded good performance. However, it must be refined for implementing it in a real environment. It is necessary to extend the analysis including transaction costs and distribution, as can this can be crucial to the strategy viability.

- For deciding if a minute is suitable for trading, the proposed policy is straightforward, but effective, in order to handle the volatility or low liquids changing conditions. However, it can be refined by implementing reinforcement learning to determine if the next minute should be operated at the high-frequency scale. In summary, including reinforcement learning is an excellent possible research opportunity.

- It will be interesting to extend the analysis in order to consider what is the useful life of a machine learning model, and what would be the rate to retrain for a real implementation.

- Finally, a clustering algorithm or another technique can be used to choose a group of stocks and build a portfolio. So, the proposed method of this dissertation can be used to manage positions for each selected stock.

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