



pumas^{AI}

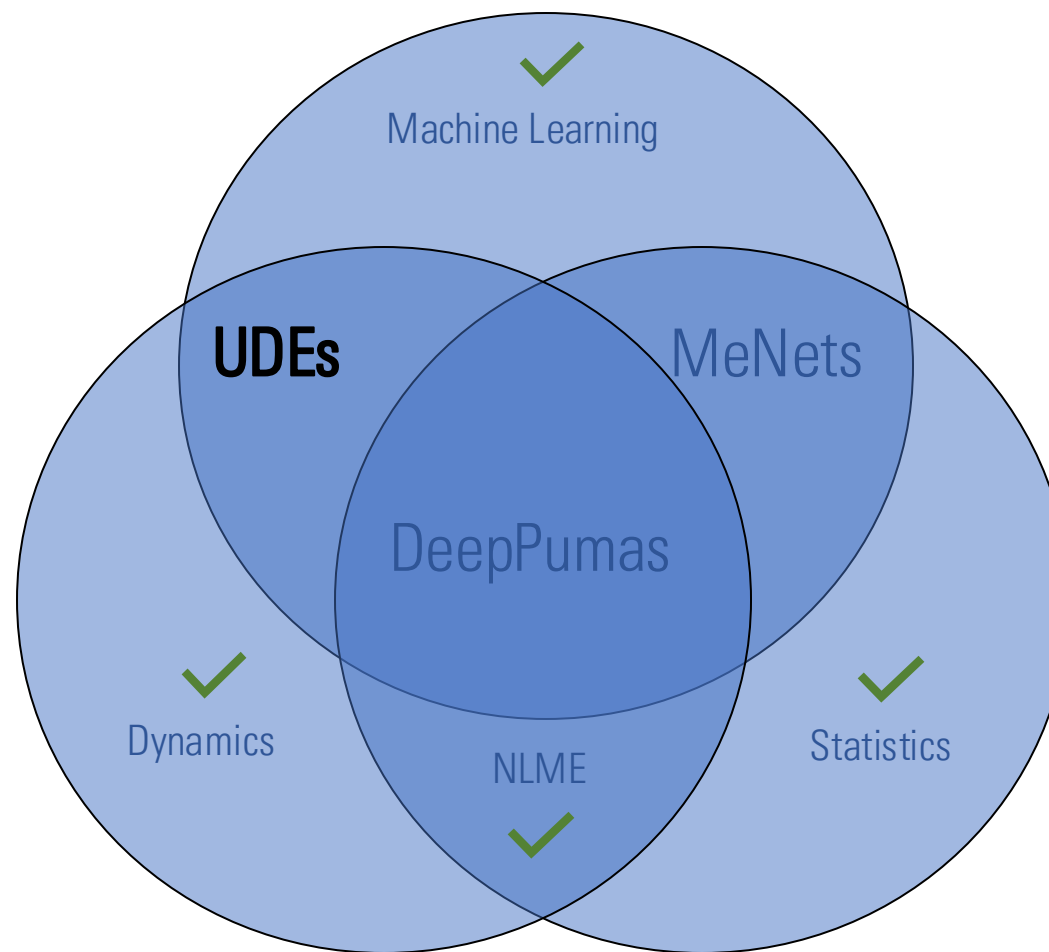


NeuralODEs and UDEs

Niklas Korsbo

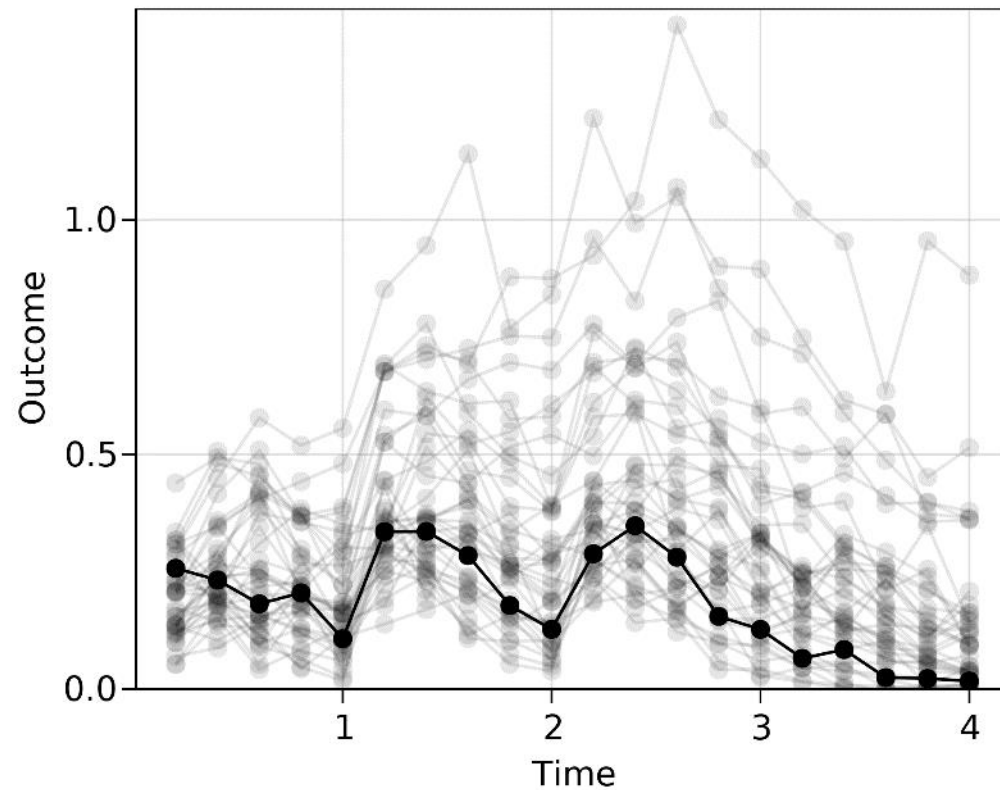


What to
make of
embedded
ML?





NLME WITH DEEPPUMAS



Typical values

$$\theta \in \mathbb{R}_+^3$$
$$\Omega \in \mathbb{R}_+^3$$

Patient data

Age
Weight



Random effects

$$\eta \sim \text{MvNormal}(\Omega)$$

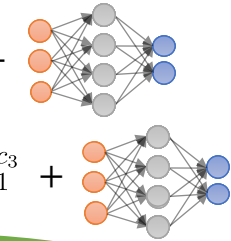
Universal
differential
equations
(UDEs)

Individual parameters

$$Ka_i = \theta_1 \cdot e^{\eta_{i,1}} + c_1 \cdot Age_i$$

$$CL_i = \theta_2 \cdot e^{\eta_{i,2}}$$

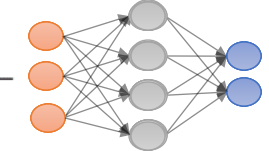
$$V_i = \theta_3 \cdot e^{\eta_{i,3}} + c_2 \cdot Weight_i^{c_3}$$



Dynamics

$$\frac{d[\text{Depot}]}{dt} = -Ka[\text{Depot}],$$

$$\frac{d[\text{Central}]}{dt} = Ka[\text{Depot}] -$$



Error model

$$Outcome \sim \text{Normal}(\text{Central}, \sqrt{\text{Central}} \cdot \sigma)$$

Universal Differential Equations and friends



2018 - "Neural Ordinary Differential Equations", Chen et al.

2020 - "Universal Differential Equations for Scientific Machine Learning", Rackauckas et al.

Neural ODE

$$\frac{d\mathbf{X}}{dt} = NN(\mathbf{X}(t), t)$$

Use a differential equation solvers as a scaffold for continuous time, continuous depth neural networks.

Similar to recurrent neural networks and ResNets

Universal differential equation (UDE)

$$\begin{aligned}\frac{dx}{dt} &= x \cdot y - NN(x) \\ \frac{dy}{dt} &= p - x \cdot y\end{aligned}$$

Insert universal approximators (like NNs) to capture terms in dynamical systems.

Scientific Machine Learning (SciML)

An abstract concept of mixing scientific modeling with machine learning.

Different encoded knowledge



- Number of states.

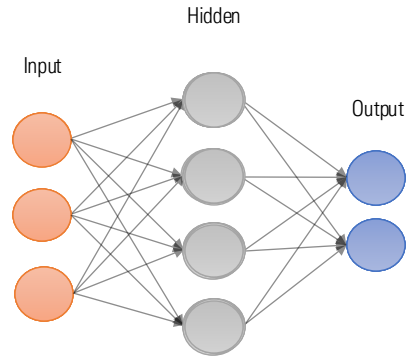
- Explicit knowledge of some terms
- Relationships (R independent of Depot!)

- Number of states.
- Relationships
- Dependence/independence of terms
- Conservation between Depot and Central

- Precise position of the unknown function.
- Precise input to the unknown function.
- Lots of knowledge!



UDEs – pretty simple, really



Mathematically: Just a function!

NNs are useable anywhere where you'd use a function!

- Decide where in the dynamics you have an unknown function.
- Decide what inputs this function may have.
- Fit everything in concert

The only hard part is building software for fitting but, with DeepPumas, that's not your problem!