Gyrosynchrotron code documentation, part II:
built-in analytical electron distributions

The built-in analytical electron distribution functions (with three exceptions, see below) are assumed to have the factorized form:

$$f(E,\mu)=u(E)g(\mu), \tag{1}$$

where E is the electron energy, $\mu=\cos\alpha$, and α is the electron pitch-angle. The functions $u(E)$ and $g(\mu)$ satisfy the normalization conditions

$$2\pi\int\limits_{E_{\min}}^{E_{\max}}u(E)\mathrm{d}E=n_{\mathrm{e/b}},\qquad \int\limits_{-1}^{1}g(\mu)\mathrm{d}\mu=1, \tag{2}$$

where $n_{\mathrm{e/b}}$ is the number density of either thermal or non-thermal electrons (depending on the distribution type). In a given volume element, the energy distribution $u(E)$ is specified by the parameter `ParmLocal[17]`, and the pitch-angle distribution $g(\mu)$ is specified by the parameter `ParmLocal[19]` (see the separate document `GScodeI.pdf`).

Currently, the following energy distributions are supported:

1. FFF (index = 0 or 1): free-free only *(always isotropic)*.
2. THM (index = 2): thermal.
3. PLW (index = 3): power-law over energy.
4. DPL (index = 4): double power-law over energy.
5. TNT (index = 5): continuous thermal-nonthermal (power-law) over energy.
6. KAP (index = 6): kappa-distribution.
7. PLP (index = 7): power-law over momentum.
8. PLG (index = 8): power-law over relativistic factor.
9. TNP (index = 9): continuous thermal-nonthermal (power-law) over momentum.
10. TNG (index = 10): continuous thermal-nonthermal (power-law) over relativistic factor.
11. TPL (index = 11): isotropic thermal + power-law over energy *(partially factorized)*.
12. TDP (index = 12): isotropic thermal + double power-law over energy *(partially factorized)*.

Currently, the following pitch-angle distributions are supported:

1. ISO (index = 0 or 1): isotropic.
2. ELC (index = 2): exponential (on the pitch-angle cosine) loss-cone.
3. GAU (index = 3): gaussian (on the pitch-angle cosine) loss-cone.
4. GAB (index = 4): directed gaussian (on the pitch-angle cosine) beam.
5. SGA (index = 5): directed super-gaussian (on the pitch-angle cosine) beam.

1 Energy distributions

Free-free only (FFF; index 0 or 1)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see below)*.
- `ParmLocal[17]` $= 0$ or 1.
- `ParmLocal[29]` $= n_\mathrm{H}$ is the neutral hydrogen number density, in cm^{-3} *(see below)*.
- `ParmLocal[30]` $= n_\mathrm{p}$ is the proton number density, in cm^{-3} *(see below)*.

Note: If either `ParmLocal[29]` or `ParmLocal[30]` are nonzero, they specify the number densities of neutral hydrogen n_H and protons n_p, respectively; the number density of thermal electrons is assumed to be $n_\mathrm{e} = n_\mathrm{p}$. Otherwise, if `ParmLocal[29]` = `ParmLocal[30]` = 0, the local plasma parameters are computed using the total plasma density n_0 = `ParmLocal[11]`, temperature T_0 = `ParmLocal[2]`, and Saha equation.

If this index is selected, only the free-free emission from thermal plasma with isotropic Maxwellian distribution is computed; the pitch-angle distribution index `ParmLocal[19]` has no effect. The free-free emission and absorption processes include contributions from electron-ion collisions and (at $T_0 < 160 \times 10^3$ K) electron-neutral-hydrogen collisions.

Note: If $T_0 > 0$, the free-free contribution of isotropic thermal plasma is always computed by default, in addition to the gyrosynchrotron emission from an analytical or/and numerically defined electron distribution.

Thermal distribution (THM; index 2)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[11]` $= n_0$ is the plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[17]` $= 2$.
- `ParmLocal[29]` $= n_\mathrm{H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_\mathrm{p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

Relativistic thermal distribution is given by the expression

$$u_\mathrm{THM}(\Gamma)\mathrm{d}\Gamma = \frac{n_e}{2\pi}\frac{\Gamma\sqrt{\Gamma^2-1}}{\theta K_2(1/\theta)}\exp\left(-\frac{\Gamma}{\theta}\right)\mathrm{d}\Gamma, \tag{3}$$

where n_e is the number density of the thermal electrons, Γ is the Lorentz-factor, $\theta = k_\mathrm{B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, k_B is the Boltzmann constant, and K_2 is the MacDonald function of the second order.

Note: If $T_0 < 0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Note: Although the background thermal plasma is present in most cases, its gyrosynchrotron emission is computed only if one explicitly selects the thermal (or a thermal/nonthermal, see below) electron distribution in the list of parameters.

Single power-law distribution over kinetic energy (PLW; index 3)

Used parameters:

- `ParmLocal[6]` $= E_{\rm min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[12]` $= n_{\rm b}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $= 3$.

Power-law distributions of the nonthermal electrons over kinetic energy $E = mc^2(\Gamma - 1)$ are widely used for interpretation of solar radio and hard X-ray emissions. These distributions are given by the expression

$$u_{\rm PLW}(E){\rm d}E = AE^{-\delta}{\rm d}E, \text{ for } E_{\rm min} < E < E_{\rm max}, \tag{4}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_{\rm b}}{2\pi}\frac{\delta - 1}{E_{\rm min}^{1-\delta} - E_{\rm max}^{1-\delta}}, \tag{5}$$

where $n_{\rm b}$ is the number density of the nonthermal electrons. The logarithmic normalization for $\delta = 1$ is not implemented; however, one can arbitrarily approach this case taking δ very close but slightly different from 1.

Double power-law distribution over energy (DPL; index 4)

Used parameters:

- `ParmLocal[6]` $= E_{\rm min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[8]` $= E_{\rm break}$ is the break energy, in MeV.
- `ParmLocal[9]` $= \delta_1$ is the low-energy power-law index.
- `ParmLocal[10]` $= \delta_2$ is the high-energy power-law index.
- `ParmLocal[12]` $= n_{\rm b}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $= 4$.

In this case, the electron spectrum consists of two parts (high-energy and low-energy), where both the high-energy and low-energy parts are described by power laws, but with different indices. This distribution (double power-law or broken ower-law) can be described by the following expression:

$$u_{\rm DPL}(E){\rm d}E = {\rm d}E \begin{cases} A_1 E^{-\delta_1}, & \text{for } E_{\rm min} < E \le E_{\rm break}, \\ A_2 E^{-\delta_2}, & \text{for } E_{\rm break} \le E < E_{\rm max}, \end{cases} \tag{6}$$

and 0 outside the range from $E_{\rm min}$ to $E_{\rm max}$. In the above expression, $A_1 E_{\rm break}^{-\delta_1} = A_2 E_{\rm break}^{-\delta_2}$ (to make the function continuous), $\delta_1 \neq 1$, and $\delta_2 \neq 1$. The normalization factor is given by

$$A_1^{-1} = \frac{2\pi}{n_{\rm b}}\left(\frac{E_{\rm min}^{1-\delta_1} - E_{\rm break}^{1-\delta_1}}{\delta_1 - 1} + E_{\rm break}^{\delta_2-\delta_1}\frac{E_{\rm break}^{1-\delta_2} - E_{\rm max}^{1-\delta_2}}{\delta_2 - 1}\right), \tag{7}$$

i.e., $n_{\rm b}$ is the total number density of nonthermal electrons between $E_{\rm min}$ and $E_{\rm max}$, and A_2 is found using the above continuity condition.

Thermal/nonthermal distribution over energy (TNT; index 5)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[3]` $= \varepsilon$ is the matching parameter ε.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[17]` $= 5$.
- `ParmLocal[29]` $= n_{\rm H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_{\rm p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

This distribution looks like a thermal distribution at low energies and a single power-law nonthermal distribution at high energies, with a smooth transition at some energy $E_{\rm cr}$, i.e.

$$u_{\rm TNT}(E){\rm d}E = {\rm d}E\left\{\begin{array}{ll} u_{\rm THM}(E), & \text{for } E \le E_{\rm cr}, \\ AE^{-\delta}, & \text{for } E_{\rm cr} \le E < E_{\rm max}, \end{array}\right. \tag{8}$$

and 0 for $E > E_{\rm max}$. In the above expression, $u_{\rm THM}(E)$ is the thermal distribution function (3), $A = u_{\rm THM}(E_{\rm cr})E_{\rm cr}^{-\delta}$ to make the function continuous, the matching point $E_{\rm cr}$ satisfies the condition $E_{\rm cr} < E_{\rm max}$, and $\delta > 1$. The matching point $E_{\rm cr}$ is defined as the energy corresponding to the momentum $p_{\rm cr}$

$$p_{\rm cr}^2 = \frac{p_{\rm THM}^2}{\varepsilon}, \tag{9}$$

where $p_{\rm THM}$ is the mean thermal momentum corresponding to the energy $k_{\rm B}T_0$, and the parameter ε specifies location of the turning point; the distribution becomes purely thermal when $\varepsilon < p_{\rm THM}^2/p^2(E_{\rm max})$.

The normalization condition is assumed to be the same as for the purely thermal distribution (3), which is valid for $\varepsilon \ll 1$; the thermal electron number density $n_{\rm e}$ is either equal $n_{\rm p}$ or computed using n_0, T_0 and Saha equation. The nonthermal electron number density $n_{\rm b}$ is computed using the above-mentioned continuity condition, and the total electron number density equals $n_{\rm e} + n_{\rm b}$.

Note: If $T_0 < 0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Kappa distribution (KAP; index 6)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[4]` $= \varkappa$ is the parameter $\varkappa$.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the electrons, in MeV.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[17]` $= 6$.

- `ParmLocal[29]` $= n_{\rm H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_{\rm p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

Another way of describing the smooth transition from the thermal distribution to a nonthermal tail is a so-called kappa distribution, which is widely used to quantify particle distributions in the interplanetary plasma. It is convenient to express the kappa distribution in terms of the Lorentz-factor Γ:

$$u_{\rm KAP}(\Gamma){\rm d}\Gamma = A\frac{\Gamma\sqrt{\Gamma^2-1}}{\theta^{3/2}\left[1+\dfrac{\Gamma-1}{(\varkappa-3/2)\theta}\right]^{\varkappa+1}}{\rm d}\Gamma \text{ for } E<E_{\max}, \tag{10}$$

and 0 otherwise. In the above expression, $\theta = k_{\rm B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, and $\varkappa$ is the distribution parameter. The normalization factor A is calculated numerically by using normalization condition (2) to provide the total electron number density equal to $n_{\rm e}$. Kappa distribution becomes purely thermal distribution when $\varkappa\to\infty$.

Note: If kappa distribution is selected, the free-free contribution is also computed using the formulae for isotropic kappa distribution.

Note: If $T_0<0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Power-law distribution over momentum (PLP; index 7)

Used parameters:

- `ParmLocal[6]` $= E_{\min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[7]` $= E_{\max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[12]` $= n_{\rm b}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $= 7$.

Power-law distribution of the nonthermal electrons over the absolute value of momentum is given by the expression

$$u_{\rm PLP}(p){\rm d}p = Ap^{-\delta}{\rm d}p \text{ for } p_{\min}<p<p_{\max}, \tag{11}$$

and 0 otherwise. The normalization constant A equals

$$A=\frac{n_{\rm b}}{2\pi}\frac{\delta-3}{p_{\min}^{3-\delta}-p_{\max}^{3-\delta}}, \tag{12}$$

where $n_{\rm b}$ is the number density of nonthermal electrons, $p_{\min}=p(E_{\min})$, and $p_{\max}=p(E_{\max})$; the case of $\delta=3$ is not implemented.

Power-law distribution over Lorentz factor (PLG; index 8)

Used parameters:

- `ParmLocal[6]` $= E_{\min}$ is the low-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[7]` $= E_{\max}$ is the high-energy cutoff of the accelerated electrons, in MeV.

- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[12]` $= n_\mathrm{b}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $= 8$.

Power-law distribution of the nonthermal electrons over Lorentz factor is given by the expression

$$u_\mathrm{PLG}(\Gamma)\mathrm{d}\Gamma = A\Gamma^{-\delta}\mathrm{d}\Gamma \text{ for } \Gamma_\mathrm{min} < \Gamma < \Gamma_\mathrm{max}, \tag{13}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_\mathrm{b}}{2\pi}\frac{\delta - 1}{\Gamma_\mathrm{min}^{1-\delta} - \Gamma_\mathrm{max}^{1-\delta}}, \tag{14}$$

where n_b is the number density of nonthermal electrons, $\Gamma_\mathrm{min} = \Gamma(E_\mathrm{min})$, and $\Gamma_\mathrm{max} = \Gamma(E_\mathrm{max})$; the case of $\delta = 1$ is not implemented.

Thermal/nonthermal distribution over momentum (TNP; index 9)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[3]` $= \varepsilon$ is the matching parameter ε.
- `ParmLocal[7]` $= E_\mathrm{max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[17]` $= 9$.
- `ParmLocal[29]` $= n_\mathrm{H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_\mathrm{p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

This distribution is similar to the thermal/nonthermal distribution over energy (TNT) with the only difference that the nonthermal part (at $E > E_\mathrm{cr}$) is described by the power-law distribution over the absolute value of momentum, that is

$$u_\mathrm{TNP}(p)\mathrm{d}p = \mathrm{d}p \begin{cases} u_\mathrm{THM}(p), & \text{for } p < p_\mathrm{cr}, \\ Ap^{-\delta}, & \text{for } p_\mathrm{cr} \le p < p_\mathrm{max}, \end{cases} \tag{15}$$

and 0 for $p > p_\mathrm{max}$. In the above expression, $u_\mathrm{THM}(p)$ is the thermal distribution function (3) expressed via momentum, p_cr is given by Eq. (9), $p_\mathrm{max} = p(E_\mathrm{max})$. Location of the matching point and the matching and normalization conditions are the same as for the TNT distribution.

Note: If $T_0 < 0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Thermal/nonthermal distribution over Lorentz factor (TNG; index 10)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the plasma temperature, in K.
- `ParmLocal[3]` $= \varepsilon$ is the matching parameter ε.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the accelerated electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[17]` $= 10$.
- `ParmLocal[29]` $= n_{\rm H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_{\rm p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

This distribution is similar to the thermal/nonthermal distribution over energy (TNT) with the only difference that the nonthermal part (at $E > E_{\rm cr}$) is described by the power-law distribution over the Lorentz factor, that is

$$u_{\rm TNG}(\Gamma){\rm d}\Gamma = {\rm d}\Gamma \begin{cases} u_{\rm THM}(\Gamma), & \text{for } \Gamma < \Gamma_{\rm cr}, \\ A\Gamma^{-\delta}, & \text{for } \Gamma_{\rm cr} \le \Gamma < \Gamma_{\rm max}, \end{cases} \tag{16}$$

and 0 for $\Gamma > \Gamma_{\rm max}$. In the above expression, $u_{\rm THM}(\Gamma)$ is the thermal distribution function (3) expressed via Lorentz factor, $\Gamma_{\rm cr} = \Gamma(p_{\rm cr})$, $p_{\rm cr}$ is given by Eq. (9), $\Gamma_{\rm max} = \Gamma(E_{\rm max})$. Location of the matching point and the matching and normalization conditions are the same as for the TNT distribution.

Note: If $T_0 < 0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Isotropic thermal + power-law over energy (TPL; index 11)

Used parameters:

- `ParmLocal[2]` $= T_0$ is the thermal plasma temperature, in K.
- `ParmLocal[6]` $= E_{\rm min}$ is the low-energy cutoff of the nonthermal electrons, in MeV.
- `ParmLocal[7]` $= E_{\rm max}$ is the high-energy cutoff of the nonthermal electrons, in MeV.
- `ParmLocal[9]` $= \delta$ is the power-law index.
- `ParmLocal[11]` $= n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[12]` $= n_{\rm b}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $= 11$.
- `ParmLocal[29]` $= n_{\rm H}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $= n_{\rm p}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

If this index is selected, the electron distribution function represents a sum of an isotropic thermal distribution and a (possibly anisotropic) single power-law distribution, i.e.

$$f_{\mathrm{TPL}}(E,\mu)=f_{\mathrm{THM}}(E)+u_{\mathrm{PLW}}(E)g(\mu), \tag{17}$$

where $f_{\mathrm{THM}}(E)$ is an isotropic distribution function with the energy dependence given by Eq. (3), $u_{\mathrm{PLW}}(E)$ is the single power-law energy distribution (4), and $g(\mu)$ is a pitch-angle distribution specified separately by the parameter `ParmLocal[19]` (i.e., the anisotropy factor is applied only to the nonthermal component). The thermal and nonthermal components are normalized independently, to provide the electron number densities of n_{e} and n_{b}, respectively; the total electron number density equals $n_{\mathrm{e}}+n_{\mathrm{b}}$.

Note: In contrast to the above mentioned thermal/nonthermal distributions, the TPL distribution is not made to be continuous; the thermal and nonthermal components can overlap in some range of energies. The gyrosynchrotron emissivities and absorption coefficients are computed separately for each component and then added together.

Note: If $T_0<0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

Isotropic thermal + double power-law over energy (TDP; index 12)

Used parameters:

- `ParmLocal[2]` $=T_0$ is the thermal plasma temperature, in K.
- `ParmLocal[6]` $=E_{\mathrm{min}}$ is the low-energy cutoff of the nonthermal electrons, in MeV.
- `ParmLocal[7]` $=E_{\mathrm{max}}$ is the high-energy cutoff of the nonthermal electrons, in MeV.
- `ParmLocal[8]` $=E_{\mathrm{break}}$ is the break energy, in MeV.
- `ParmLocal[9]` $=\delta_1$ is the low-energy power-law index.
- `ParmLocal[10]` $=\delta_2$ is the high-energy power-law index.
- `ParmLocal[11]` $=n_0$ is the thermal plasma density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[12]` $=n_{\mathrm{b}}$ is the number density of nonthermal electrons, in cm^{-3}.
- `ParmLocal[17]` $=12$.
- `ParmLocal[29]` $=n_{\mathrm{H}}$ is the neutral hydrogen number density, in cm^{-3} *(see note for FFF)*.
- `ParmLocal[30]` $=n_{\mathrm{p}}$ is the proton number density, in cm^{-3} *(see note for FFF)*.

This electron distribution is similar to the TPL distribution, but the (possibly anisotropic) nonthermal component has the double power-law energy dependence, i.e.

$$f_{\mathrm{TDP}}(E,\mu)=f_{\mathrm{THM}}(E)+u_{\mathrm{DPL}}(E)g(\mu), \tag{18}$$

where $f_{\mathrm{THM}}(E)$ is an isotropic distribution function with the energy dependence given by Eq. (3), $u_{\mathrm{DPL}}(E)$ is the double (or broken) power-law energy distribution (6), and $g(\mu)$ is a pitch-angle distribution specified separately by the parameter `ParmLocal[19]`.

Note: If $T_0<0$, the absolute value $|T_0|$ is used instead, but the free-free contribution is not computed.

If the energy distribution index differs from the above values ($0-12$) then the free-free only model (index 0) will be used by default.

2 Pitch-angle distributions

Isotropic distribution (ISO; index 0 or 1)

Used parameters:

- `ParmLocal[19]` $= 0$ or 1.

In this case, the electron distribution does not depend on pitch-angle, that is

$$g_{\mathrm{ISO}}(\mu) = \mathrm{const} = \frac{1}{2}. \tag{19}$$

Exponential loss-cone distribution (ELC; index 2)

Used parameters:

- `ParmLocal[19]` $= 2$.
- `ParmLocal[20]` $= \alpha_{\mathrm{c}}$ is the loss-cone boundary, in degrees.
- `ParmLocal[22]` $= \Delta\mu$ is the loss-cone boundary width.

Symmetric loss-cone distribution with exponential boundary is given by the expression

$$g_{\mathrm{ELC}}(\mu) = A \begin{cases} 1, & \text{for } |\mu| < \mu_{\mathrm{c}}, \\ \exp\left(-\dfrac{|\mu| - \mu_{\mathrm{c}}}{\Delta\mu}\right), & \text{for } |\mu| \geq \mu_{\mathrm{c}}, \end{cases} \tag{20}$$

where $\mu_{\mathrm{c}} = \cos\alpha_{\mathrm{c}} > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_{\mathrm{c}} + \Delta\mu - \Delta\mu \exp\left(\frac{\mu_{\mathrm{c}} - 1}{\Delta\mu}\right)\right]. \tag{21}$$

Gaussian loss-cone distribution (GLC; index 3)

Used parameters:

- `ParmLocal[19]` $= 3$.
- `ParmLocal[20]` $= \alpha_{\mathrm{c}}$ is the loss-cone boundary, in degrees.
- `ParmLocal[22]` $= \Delta\mu$ is the loss-cone boundary width.

Symmetric loss-cone distribution with gaussian boundary is given by the expression

$$g_{\mathrm{GLC}}(\mu) = A \begin{cases} 1, & \text{for } |\mu| < \mu_{\mathrm{c}}, \\ \exp\left[-\dfrac{(|\mu| - \mu_{\mathrm{c}})^2}{\Delta\mu^2}\right], & \text{for } |\mu| \geq \mu_{\mathrm{c}}, \end{cases} \tag{22}$$

where $\mu_{\mathrm{c}} = \cos\alpha_{\mathrm{c}} > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_{\mathrm{c}} + \frac{\sqrt{\pi}}{2}\Delta\mu \,\mathrm{erf}\left(\frac{1 - \mu_{\mathrm{c}}}{\Delta\mu}\right)\right], \tag{23}$$

where erf is the error function.

Gaussian beam distribution (GAU; index 4)

Used parameters:

- `ParmLocal[19]` $= 4$.
- `ParmLocal[21]` $= \alpha_0$ is the beam direction, in degrees.
- `ParmLocal[22]` $= \Delta\mu$ is the beam width.

Gaussian beam distribution is given by the expression

$$g_{\mathrm{GAU}}(\mu) = A \exp\left[-\frac{(\mu-\mu_0)^2}{\Delta\mu^2}\right], \tag{24}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and the parameter $\Delta\mu$ determines the beam angular width. The above expression represents the beam along the field line for $\alpha_0 = 0$ or $180°$, the transverse beam for $\alpha_0 = 90°$ (coincides with the GLC distribution with $\alpha_c = 90°$), and an oblique beam (or a hollow-beam) otherwise. The normalization factor A is given by

$$A^{-1} = \frac{\sqrt{\pi}}{2}\Delta\mu\left[\mathrm{erf}\left(\frac{1-\mu_0}{\Delta\mu}\right) + \mathrm{erf}\left(\frac{1+\mu_0}{\Delta\mu}\right)\right]. \tag{25}$$

Supergaussian beam distribution (SGA; index 5)

Used parameters:

- `ParmLocal[19]` $= 5$.
- `ParmLocal[21]` $= \alpha_0$ is the beam direction, in degrees.
- `ParmLocal[22]` $= \Delta\mu$ is the beam width.
- `ParmLocal[23]` $= a_4$ is the additional coefficient a_4.

This distribution is very similar to the GAU distribution near its maximum (μ_0) but decreases more rapidly at some angular distance from μ_0. Such a shape is achieved by adding a term with fourth degree of $(\mu - \mu_0)$ to the argument of exponent in (24), that is

$$g_{\mathrm{SGA}}(\mu) = A \exp\left[-\frac{(\mu-\mu_0)^2 + a_4(\mu-\mu_0)^4}{\Delta\mu^2}\right], \tag{26}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and the beam angular width and shape near the maximum are determined by the parameters $\Delta\mu$ and a_4. The normalization factor A is calculated numerically by using normalization condition (2).

If the angular distribution index differs from the above values ($0-5$) then the isotropic distribution (index 0) will be used.

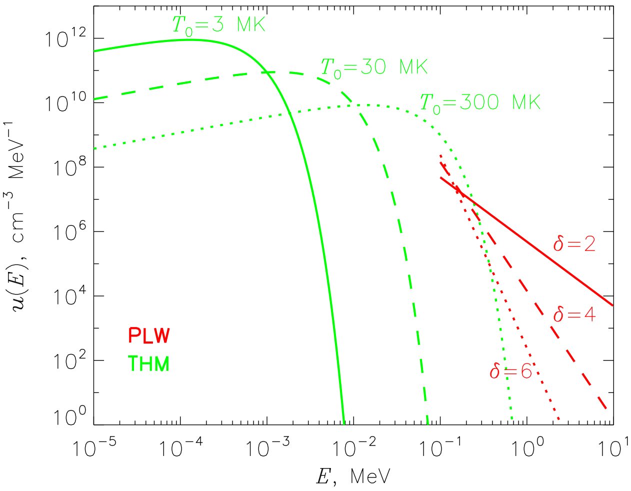

Figure 1: Thermal electron distribution (for $n_0 = 3 \times 10^9$ cm^{-3} and different electron temperatures) and single power-law electron distribution over kinetic energy (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and different power-law indices δ).

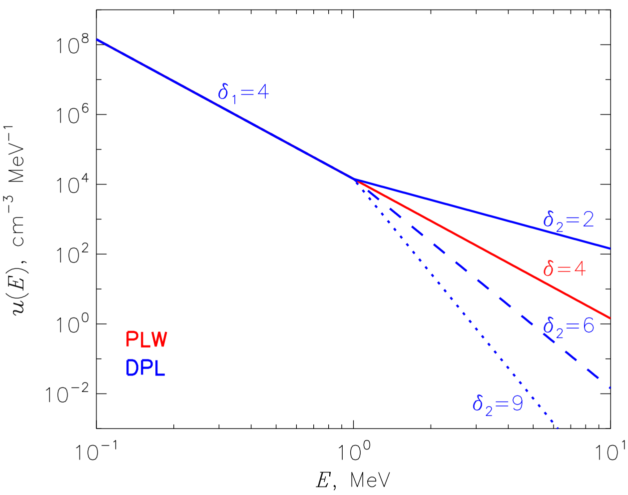

Figure 2: Double power-law electron distribution (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{break} = 1$ MeV, $E_\mathrm{max} = 10$ MeV, $\delta_1 = 4$, and different high-energy power-law indices δ_2). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

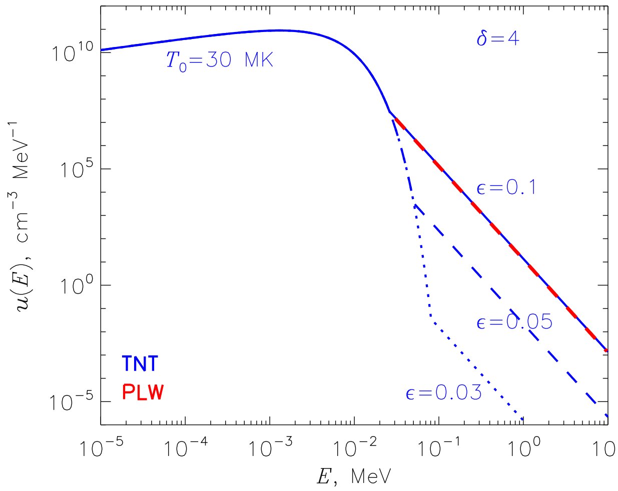

Figure 3: Thermal/nonthermal electron distribution over kinetic energy (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\delta = 4$, and different matching parameters ε). Red dashed line represents the nonthermal "tail" of the thermal/nonthermal distribution; for $\varepsilon = 0.1$, this "tail" behaves as the single power-law distribution with $n_\mathrm{b} = 10^6$ cm^{-3}, $E_\mathrm{min} = 0.03$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$.

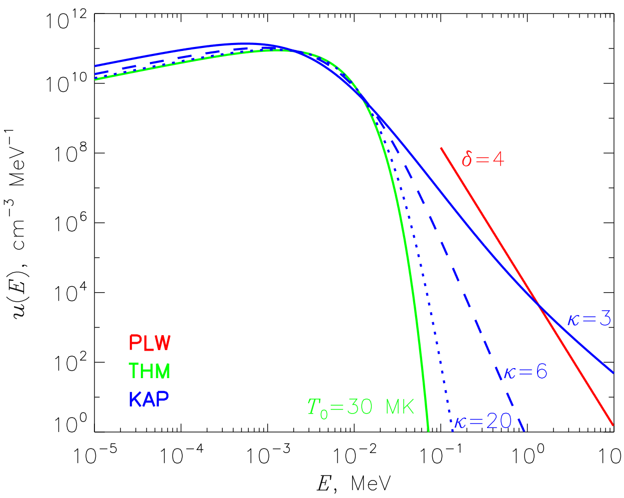

Figure 4: Kappa distribution (for $n_\mathrm{e} = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, and different values of the parameter $\varkappa$). Thermal distribution (for the same particle number density and temperature) and single power-law distribution (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$) are given for reference.

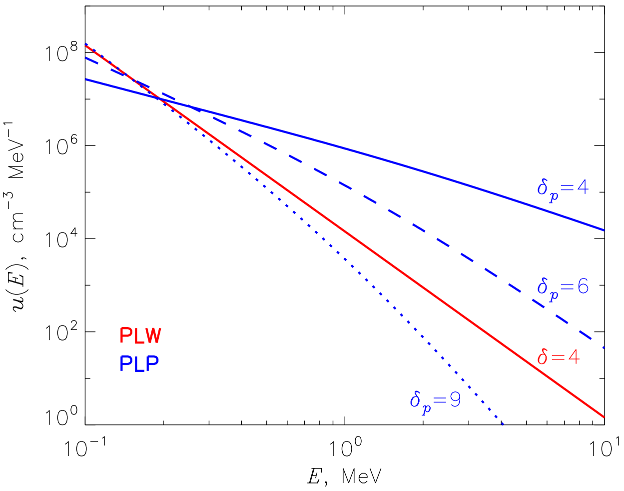

Figure 5: Power-law electron distribution over momentum (for $n_b = 3 \times 10^7$ cm^{-3}, $E_{min} = 0.1$ MeV, $E_{max} = 10$ MeV, and different power-law indices δ_p). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

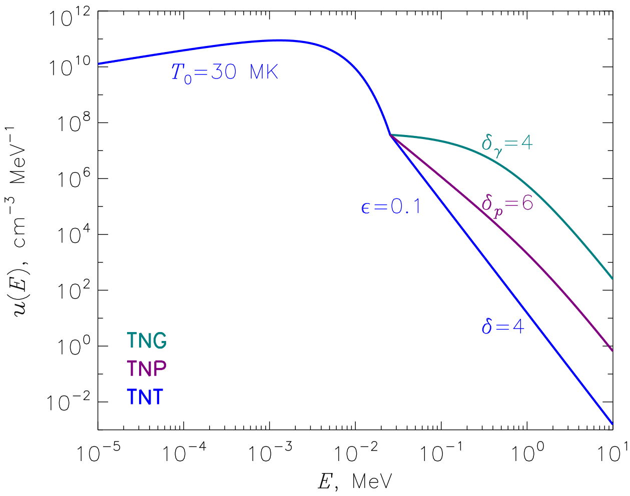

Figure 7: Different thermal/nonthermal electron distributions (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\varepsilon = 0.1$). All the distributions have different numbers of fast electrons above E_{cr}.

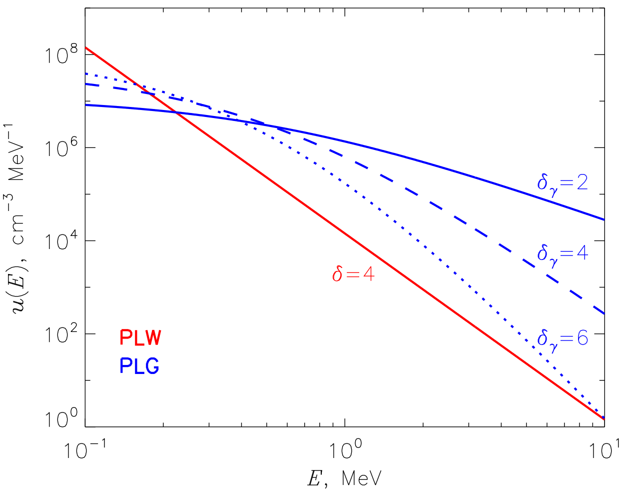

Figure 6: Power-law electron distribution over Lorentz factor (for $n_b = 3 \times 10^7$ cm^{-3}, $E_{min} = 0.1$ MeV, $E_{max} = 10$ MeV, and different power-law indices δ_γ). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

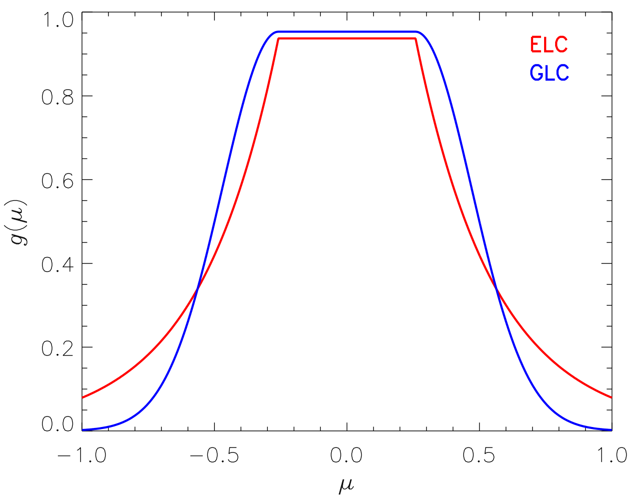

Figure 8: Exponential and gaussian loss-cone distributions (for $\alpha_c = 75°$ and $\Delta\mu = 0.3$).

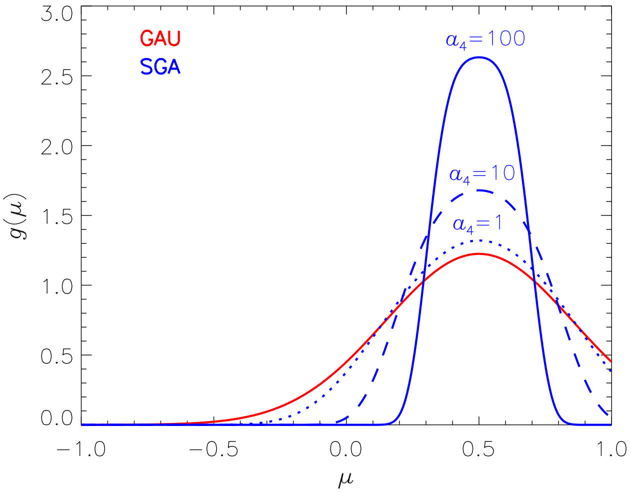

Figure 9: Gaussian distribution (for $\alpha_0 = 60°$ and $\Delta\mu = 0.5$) and supergaussian distribution (for the same α_0 and $\Delta\mu$, and different values of the parameter a_4).