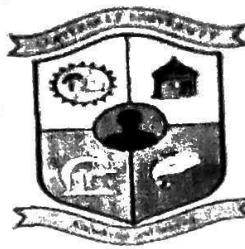


# COPPERBELT UNIVERSITY



## SCHOOL OF MATHEMATICS AND NATURAL SCIENCES DEPT OF PURE & APPLIED MATHEMATICS MA 110 - MATHEMATICAL METHODS I | TEST 1

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.  
2. Indicate clearly your Names, S/N and the Group you belong to.  
3. Duration is 3 Hours Only.

1. a. i.) Let  $A = \{1, 2, 3\}$  and  $B = \{2, 4\}$ , Find  $A \times B$ . 1 Mk  
 ii.) Prove that  $(A^c)^c = A$  by Arbitrary Elementary Method. 4 Mks
- b. If the Operation  $*$  is defined as, "add the first number to 8 times the second number"  
 Find  $(2 * 3) * 5$  ? 2 Mks
- c. Find the value of  $k$  given that when  $2x^3 - 2kx^2 - 3x - 2$  is divided by  $x - 2$ ,  
 the Remainder is 40. 3 Mks
- d. If  $g \circ f(x) = x$  and  $g(x) = \frac{x+1}{x-1}$ ,  
 i.) Find  $f(x)$       ii.) Sketch the Graph of  $f(x)$  and Find the Range of  $f(x)$  3 Mks, 4 Mks
2. a. Prove the De Morgan's Law:  $A^c \cup B^c = (A \cap B)^c$  5 Mks  
 b. Solve the following Equations involving the Absolute value functions:  
 $|8x + 3| = |2x - 21|$  3 Mks

c. Solve the following inequation:

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2}$$

5 Mks

d. Using Synthetic Division, show that both  $x - 2$  and  $x + 3$  are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

4 Mks

Hence, or otherwise Factorize  $f(x)$  completely

3. a. Express the following in the form  $\frac{a}{b}$ , where  $a, b \in \mathbb{Z}$  and  $b \neq 0$ .

i.)  $0.\underline{1}21212\dots$  ii.)  $1.\underline{3}121212\dots$  1.5 Mks, 1.5 Mks

b. Use the fact that  $\sqrt{6}$  is Irrational to prove that  $\sqrt{2} + \sqrt{3}$  is Irrational. 4 Mks

c. Sketch the graphs of:

i.)  $f(x) = -|x + 3| - 4$  ii.)  $f(x) = 3 + \sqrt{3-x}$  2.5 Mks, 2.5 Mks

d. Solve the Polynomial Equation  $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$  5 Mks

4. a. Determine the vertex and Intercepts for the following Quadratic function:

$$f(x) = x^2 - 6x - 16 \quad 2 \text{ Mks}$$

b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4) \quad 5 \text{ Mks}$$

c. Given that the roots of  $x^2 + 3x + 17 = 0$  are  $\alpha$  and  $\beta$  respectively. Find a Quadratic Function

whose roots are  $\alpha^3 + \beta^3$  and  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ . 5 Mks

d. Given that set  $A = \{1, 2, 3\}$  and set  $B = \{2, 4, 6\}$ , Determine whether the Operation;

$$A \circ B = P(A) - P(B).$$

5 Mks

is Binary on the Universal Power set,  $P(E)$ .

5. a. If  $x = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ , find the value of  $8x - x^2$ . 4 Mks

b. Given the Functions,  $f(x) = x^2 + 4$  and  $g(x) = x - 9$ .

Find the value of  $x$  for which  $g[f(x)] = f[g(x)]$  4 Mks

c. Write the Expression  $f(x) = 2x^2 + 12x + 14$  in the form  $f(x) = a(x + h)^2 + k$  where  $a, h, k \in \mathbb{R}$ .

Hence, state the turning point of  $f(x)$ . 4 Mks

d. Calculate the value(s) of  $x$  that are valid for the Equation below.

$$\left| \frac{x-2}{x+3} \right| = 4 \quad 5 \text{ Mks}$$

6. a. Simplify  $-\frac{25}{2} \left[ \frac{1+2i}{3+4i} - \frac{2-5i}{-i} \right]$  3 Mks

b. Solve for  $x$  and  $y$  given that;

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad 5 \text{ Mks}$$

c. Solve the Inequality below and present your answer in Interval Notation:

$$3x^2 + 2x + 2 < 2x^2 + x + 4 \quad 4 \text{ Mks}$$

d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x) = \frac{5x^2 - 2}{1-x} \quad 5 \text{ Mks}$$