COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPT OF PURE & APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I | TEST 1

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.
2. Indicate clearly your Names,SIN and the Group you belong to.
3. Duration is **3 Hours Only.**

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1. ***a.*** *i.)* Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, Find $A \times B$. 1 Mk

ii.) Prove that $(A^c)^c = A$ by Arbitrary Elementary Method. 4 Mks

b. If the Operation $*$ is defined as, " *add the first number* **to** 8 *times the second number*"

Find $(2 * 3) * 5$

2 Mks

c. Find the value of k given that when $2x^3 - 2kx^2 - 3x - 2$ is divided by $x - 2$, the Remainder is **40**. 3 Mks

d. If $gof(x) = x$ and $g(x) = \dfrac{x+1}{x-1}$,

i.) Find $f(x)$ *ii.)* Sketch the Graph of $f(x)$ and Find the Range of $f(x)$ 3 Mks, 4 Mks

2. ***a.*** Prove the De Morgan's Law: $A^c \cup B^c = (A \cap B)^c$ **5 Mks**

b. Solve the following Equations involving the Absolute value functions:

$$|8x + 3| = |2x - 21|$$ **3 Mks**

c. Solve the following inequation: 5 Mks

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2}$$

d. Using Synthetic Division, show that both $x-2$ and $x+3$ are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

Hence, or otherwise Factorize $f(x)$ completely 4 Mks

3. a. Express the following in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

i.) $0.121212\cdots$ ii.) $1.3121212\cdots$ 1.5 Mks, 1.5 Mks

b. Use the fact that $\sqrt{6}$ is Irrational to prove that $\sqrt{2}+\sqrt{3}$ is Irrational. 4 Mks

c. Sketch the graphs of:

i.) $f(x) = -|x+3| - 4$ ii.) $f(x) = 3 + \sqrt{3-x}$ 2.5 Mks, 2.5 Mks

d. Solve the Polynormial Equation $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ 5 Mks

4. a. Determine the vertex and Intercepts for the following Quadratic function:

$$f(x) = x^2 - 6x - 16$$ 2 Mks

b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4)$$ 5 Mks

c. Given that the roots of $x^2 + 3x + 17 = 0$ are α and β respectively. Find a Quadratic Function whose roots are $\alpha^3 + \beta^3$ and $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. 5 Mks

d. Given that set $A = \{1, 2, 3\}$ and set $B = \{2, 4, 6\}$, Determine whether the Operation;

$$A \circ B = P(A) - P(B).$$

is Binary on the Universal Power set, $P(E)$. 5 Mks

5. ***a.*** If $x=\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, find the value of $8x-x^2$. 4 Mks

b. Given the Functions, $f(x)=x^2+4$ and $g(x)=x-9$.

Find the value of x for which $g[f(x)]=f[g(x)]$ 4 Mks

c. Write the Expression $f(x)=2x^2+12x+14$ in the form $f(x)=a(x+h)^2+k$ where $a, h, k \in \mathbb{R}$.

Hence, state the turning point of $f(x)$. 4 Mks

d. Calculate the value(s) of x that are valid for the Equation below.

$$\left|\frac{x-2}{x+3}\right|=4$$ 5 Mks

6. ***a.*** Simplify $-\dfrac{25}{2}\left[\dfrac{1+2i}{3+4i}-\dfrac{2-5i}{-i}\right]$ 3 Mks

b. Solve for x and y given that;

$$\frac{x}{1+i}-\frac{y}{2-i}=\frac{1-5i}{3-2i}$$ 5 Mks

c. Solve the Inequality below and present your answer in Interval Notation:

$$3x^2+2x+2<2x^2+x+4$$ 4 Mks

d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x)=\frac{5x^2-2}{1-x}$$ 5 Mks

..... THE END.....