第二周作业

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4 令 $D = \{(x, y) | x^2 + y^2 \le 4\}$

$$
\begin{aligned}
&\iiint\limits_{\Omega} z \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&= \iint\limits_{D} \mathrm{d}x\mathrm{d}y \int_{0}^{x^2+y^2} z\mathrm{d}z \\
&= \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{2} \frac{r^5}{2}\mathrm{d}r \\
&= \int_{0}^{2\pi} \frac{16}{3}\mathrm{d}\theta \\
&= \frac{32\pi}{3}
\end{aligned}
$$

6 令 $D = \{(x, y)|x^2 + y^2 \leq 1\}$

$$\iiint\limits_{\Omega} \mathrm{d}V$$

$$= \iint\limits_{D} \mathrm{d}r\mathrm{d}\theta \int_{3r}^{3} r^3 \mathrm{d}z$$

$$= \iint\limits_{D} r^3(3 - 3r)\mathrm{d}r\mathrm{d}\theta$$

$$= \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{1} (3r^3 - 3r^4)\mathrm{d}r$$

$$= \int_{0}^{2\pi} \frac{3}{20}\mathrm{d}\theta$$

$$= \frac{3\pi}{10}$$

8 令 $D = \{(x, y)|x^2 + y^2 \leq 1\}$

$$\iiint\limits_{\Omega} (x^2 + z^2)\mathrm{d}V$$

$$= \iint\limits_{D} \mathrm{d}x\mathrm{d}y \int_{x^2+y^2}^{1} (x^2 + z^2)\mathrm{d}z$$

$$= \iint\limits_{D} \left((x^2 + \frac{1}{3}) - [x^2(x^2 + y^2) + \frac{(x^2 + y^2)^3}{3}] \right) \mathrm{d}x\mathrm{d}y$$

$$= \iint\limits_{D} r(r^2cos^2\theta - r^4cos^2\theta - \frac{r^6}{3} + \frac{1}{3})\mathrm{d}r\mathrm{d}\theta$$

$$= \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{1} \left(r^3cos^2\theta - r^5cos^2\theta - \frac{r^7}{3} + \frac{r}{3} \right) \mathrm{d}r$$

$$= \frac{1}{3} \int_{0}^{2\pi} (\frac{cos^2\theta}{4} + \frac{3}{8})\mathrm{d}\theta$$

$$= \frac{\pi}{3}$$