

第八周作业

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(1) 收敛, 由比值判别法:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{(n+1)^5}{(n+1)!} \bigg/ \frac{n^5}{n!} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1)^4}{n^5} = 0 < 1 \end{aligned}$$

(2) 发散, 由比值判别法:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{(n+1)!}{3(n+1)^2} \bigg/ \frac{n!}{3n^2} \\ &= \lim_{n \rightarrow \infty} \frac{n^2}{n+1} = +\infty > 1 \end{aligned}$$

(7) 收敛, 由比值判别法:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1000^{n+1}}{(n+1)!} \bigg/ \frac{1000^n}{n!} \\ &= \lim_{n \rightarrow \infty} \frac{1000}{n+1} = 0 < 1 \end{aligned}$$

(9) 收敛, 由比值判别法:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{3^{n+1}} \left(\frac{n+2}{n+1} \right)^{(n+1)^2} \bigg/ \frac{1}{3^n} \left(\frac{n+1}{n} \right)^{n^2} \\ &= \lim_{n \rightarrow \infty} \frac{1}{3} \left[\frac{(n+1)n}{(n+1)^2} \right]^{n^2} \left(\frac{n+2}{n+1} \right)^{2n+1} \\ &= \frac{1}{3} e^{-1} e^2 = \frac{e}{3} < 1 \end{aligned}$$