

第六周作业

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(2)

$$a \left(x \frac{dy}{dx} + 2y \right) = xy \frac{dy}{dx}$$

若 $xy \neq 0$, 则

$$a \left(\frac{dy}{y} + 2 \frac{dx}{x} \right) = dy$$

$$\ln |x| = \frac{y}{2a} - \frac{1}{2} \ln |y| + C_1, C_1 \in R$$

$$|x||y|^{1/2} = e^{C_1} e^{\frac{y}{2a}}$$

$$x^2|y| = e^{2C_1} e^{\frac{y}{a}} \text{ 即 } x^2y = Ce^{\frac{y}{a}}, C \neq 0$$

检验 $y = 0$ 是原方程的一个特解, 故 $x^2y = Ce^{\frac{y}{a}}, C \in R$

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$$\sqrt{1+x^2}dy - \sqrt{1-y^2}dx = 0$$

$$\frac{dx}{\sqrt{1+x^2}} = \frac{dy}{\sqrt{1-y^2}}$$

$$\ln(x + \sqrt{x^2+1}) = \arcsin y + C$$

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$$(3x + 5y)dx + (4x + 6y)dy = 0$$

$$\frac{dy}{dx} = -\frac{3x + 5y}{4x + 6y}$$

令 $t = \frac{y}{x}$ 则 $y' = xt' + t$ 则

$$\frac{dx}{x} = \frac{4 + 6t}{-6t^2 - 9t - 3} dt$$

$$\frac{dx}{x} = -\frac{2}{3} \left(\frac{1}{2t + 1} + \frac{1}{t + 1} \right) dt$$

$$\ln |x| + C_1 = -\frac{2}{3} \left(\frac{1}{2} \ln |2t + 1| + \ln |t + 1| \right)$$

带入 $t = \frac{y}{x}$

$$\ln |x| + C_1 = -\frac{2}{3} \left(\frac{1}{2} \ln \left| \frac{2y}{x} + 1 \right| + \ln \left| \frac{y}{x} + 1 \right| \right)$$

也即 $(x + 2y)(x + y)^2 = C$

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$$y' = (x + y + 2)^2$$

令 $t = x + y + 2$, 则 $t't' = 1 + y'$, 则

$$t' - 1 = t^2 \frac{dt}{t^2 + 1} = dx$$

$$\arctan t = x + C$$

$$\arctan(x + y + 2) = x + C$$

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$$\begin{aligned}\frac{dx}{y} + \frac{4dy}{x} &= 0 \\ xdx + 4ydy &= 0 \\ \frac{x^2}{2} + 2y^2 &= C\end{aligned}$$

代入 $y(4) = 2$ 得 $C = 16$, 故解为:

$$y = \frac{1}{2}\sqrt{32 - x^2}$$

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$$\begin{aligned}\sqrt{1+x^2}\frac{dy}{dx} &= xy^3 \\ \frac{xdx}{\sqrt{1+x^2}} &= \frac{dy}{y^3} \\ \sqrt{1+x^2} &= -\frac{1}{2y^2} + C\end{aligned}$$

代入初值得 $C = \frac{3}{2}$, 故解为:

$$y^{-2} + 2(1+x^2)^{1/2} = 3$$

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$$\begin{aligned}xy' - y &= (x-1)e^x \\ y' - \frac{y}{x} &= \frac{x-1}{y}e^x\end{aligned}$$

齐次通解:

$$y^* = Cx$$

令 $y = u(x)x$, 则:

$$\begin{aligned}u'(x) &= \frac{(x-1)e^x}{x^2} \\ u(x) &= \frac{e^x}{x} + C\end{aligned}$$

故通解为 $y = e^x + Cx$

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$$y' + 2y = xe^{-x}$$

齐次通解:

$$y^* = Ce^{-2x}$$

令 $y = u(x)e^{-2x}$, 则

$$u'(x)e^{-2x} = xe^{-x}$$

$$u'(x) = xe^x$$

$$u(x) = (x-1)e^x + C$$

$$\text{即 } y = (x-1)e^{-x} + Ce^{-2x}$$

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$$\frac{\partial e^y}{\partial y} = e^y$$

$$\frac{\partial (xe^y - y^2)}{\partial x} = e^y$$

是全微分方程, $d(xe^y - y^2) = e^y dx + (xe^y - 2y)dy$

故通积分为:

$$xe^y - y^2 = C$$

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$$\frac{\partial (x+2y)}{\partial y} = 2$$

$$\frac{\partial (2x+3y)}{\partial x} = 2$$

是全微分方程, 且

$$d\left(\frac{x^2}{2} + \frac{3y^2}{2} + 2xy\right) = (x+2y)dx + (2x+3y)dy$$

通积分为

$$\frac{x^2}{2} + \frac{3y^2}{2} + 2xy = C$$

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$$\frac{\partial(3x^2y + 2xy + y^3)}{\partial y} - \left(\frac{\partial(x^2 + y^2)}{\partial x} \right) / (x^2 + y^2) = 3$$

故得积分因子 $\mu(x) = e^{3x}$

从而有通积分

$$e^{3x}(3x^2y + y^3) = C$$