第五周作业

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(1)

$$\begin{aligned}
&\iint_S x^2\mathrm{d}y\mathrm{d}z + y^2\mathrm{d}z\mathrm{d}x + z^2\mathrm{d}x\mathrm{d}y \\
=&\iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)\mathrm{d}V \\
=&2\iiint_V (x+y+z)\mathrm{d}V \\
=&2\int_0^h \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta \int_0^a r(r\sin\theta + r\cos\theta + z)\mathrm{d}r \\
=&\pi a^2 h^2
\end{aligned}$$

(2)

$$\begin{aligned}
&\iint_S x^2\mathrm{d}y\mathrm{d}z + y^2\mathrm{d}z\mathrm{d}x + z^2\mathrm{d}x\mathrm{d}y \\
=&2\iiint_V (x+y+z)\mathrm{d}V \\
=&2\int_0^b \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta \int_0^{az/b} r(r\sin\theta + r\cos\theta + z)\mathrm{d}r \\
=&2\frac{a^2 z^4 \pi}{4b^2}\Big|_0^b = \frac{1}{2}\pi a^2 b^2
\end{aligned}$$

2

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\begin{aligned}
&\iint_{S^+} xz^2\mathrm{d}y\mathrm{d}z + (x^2y - z^3)\mathrm{d}z\mathrm{d}x + (2xy + y^2z)\mathrm{d}x\mathrm{d}y \\
=&\iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)\mathrm{d}V \\
=&\iiint_V x^2 + y^2 + z^2\mathrm{d}V \\
=&\iiint_V r^2\sin\varphi\mathrm{d}V \\
=&\int_0^{2\pi}\mathrm{d}\theta\int_0^{\pi/2}\mathrm{d}\varphi\int_0^a r^4\sin\varphi\mathrm{d}r \\
=&\frac{2\pi a^5}{5}
\end{aligned}
$$

3

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\begin{aligned}
&\iint_S x^2\mathrm{d}y\mathrm{d}z + y^2\mathrm{d}z\mathrm{d}x + z^2\mathrm{d}x\mathrm{d}y \\
=&\iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)\mathrm{d}V \\
=&2\iiint_V (x + y + z)\mathrm{d}V \\
=&2\int_0^a \mathrm{d}z\int_0^a \mathrm{d}y\int_0^a (x + y + z)\mathrm{d}x \\
=&3a^4
\end{aligned}
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4

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\begin{aligned}
&\iint_S x^3 \mathrm{d}y\mathrm{d}z + y^3 \mathrm{d}z\mathrm{d}x + z^3 \mathrm{d}x\mathrm{d}y \\
=&\iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) \mathrm{d}V \\
=&3\iiint_V (x^2 + y^2 + z^2)\mathrm{d}V \\
=&3\int_0^{2\pi} \mathrm{d}\theta \int_0^{\pi} \mathrm{d}\varphi \int_0^{a} r^4 \sin\varphi \mathrm{d}r \\
=&\frac{12\pi a^5}{5}
\end{aligned}
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5

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\begin{aligned}
&\iint y\mathrm{d}y\mathrm{d}z + xy\mathrm{d}z\mathrm{d}x - z^3\mathrm{d}x\mathrm{d}y \\
=&\iiint_V (x-1)\mathrm{d}V \\
=&\iint_D \mathrm{d}\sigma \int_0^{x^2+y^2} (x-1)\mathrm{d}z \\
=&\int_0^2 \mathrm{d}r \int_0^{2\pi} -2\pi r^3 \mathrm{d}r \\
=&-8\pi
\end{aligned}
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