第一周作业

邓贤杰

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3

$$
\begin{aligned}
&\iint\limits_D ydxdy \\
=&\int_0^{\pi} dx\int_0^{sinx} ydy \\
=&\int_0^{\pi} \frac{sin^2x}{2}dx \\
=&(\frac{x}{4}-\frac{sin2x}{8})|_0^{\pi} \\
=&\frac{\pi}{4}
\end{aligned}
$$

4

$$
\begin{aligned}
&\iint\limits_D xy^2dxdy \\
=&\int_{-2}^{2} dy\int_{\frac{y^2}{4}}^{1} xy^2dx \\
=&\int_{-2}^{2} \frac{y^2}{2}-\frac{y^6}{32}dy \\
=&\frac{32}{21}
\end{aligned}
$$

5

$$\iint\limits_D e^{\frac{x}{y}}dxdy$$
$$=\int_0^1 dy\int_0^{y^2} e^{\frac{x}{y}}dx$$
$$=\int_0^1 (ye^y-y)dy$$
$$=ye^y\Big|_0^1-\int_0^1 e^y dy-\frac{y^2}{2}\Big|_0^1$$
$$=\frac{1}{2}$$

6

$$\int_0^1 dy\int_{y^{\frac{1}{3}}}^1 \sqrt{1-x^4}dx$$
$$=\int_0^1 dx\int_0^{x^3} \sqrt{1-x^4}dy$$
$$=-\frac{1}{4}\int_0^1 \sqrt{1-x^4}\mathrm{d}(1-x^4)$$
$$=-\frac{1}{6}(1-x^4)^{\frac{3}{2}}\Big|_0^1$$
$$=\frac{1}{6}$$

8

$$\int_0^\pi \mathrm{d}x\int_x^\pi \frac{siny}{y}\mathrm{d}y$$
$$=\int_0^\pi \mathrm{d}y\int_0^y \frac{siny}{y}\mathrm{d}x$$
$$=\int_0^\pi siny\mathrm{d}y$$
$$=2$$

9

$$
\begin{aligned}
&\int_0^2 \mathrm{d}x \int_x^2 2y^2 sin(xy)\mathrm{d}y \\
=&\int_0^2 \mathrm{d}y \int_0^y 2y^2 sin(xy)\mathrm{d}x \\
=&\int_0^2 \mathrm{d}y(-\frac{cos(xy)}{y}|_0^y) \\
=&\int_0^2 (1-cos(y^2))\mathrm{d}(y^2) \\
=&4-sin4
\end{aligned}
$$

10

$$
\begin{aligned}
&\iint\limits_D y^2\sqrt{1-x^2}\mathrm{d}\sigma \\
=&\int_{-1}^1 \mathrm{d}x \int_{\sqrt{1-x^2}}^{-\sqrt{1-x^2}} y^2\sqrt{1-x^2}\mathrm{d}y \\
=&\frac{2}{3}\int_{-1}^1 (1-x^2)^2\mathrm{d}x \\
=&\frac{32}{45}
\end{aligned}
$$

11 按照 x 的正负将 D 划分为 D_1 和 D_2

$$
\begin{aligned}
&\iint\limits_{D}(|x|+y)\mathrm{d}\sigma \\
=&\iint\limits_{D_1}(x+y)\mathrm{d}\sigma+\iint\limits_{D_2}(y-x)\mathrm{d}\sigma \\
&\iint\limits_{D_1}(x+y)\mathrm{d}\sigma \\
=&\int_0^1\mathrm{d}x\int_{x-1}^{1-x}(x+y)\mathrm{d}y \\
=&\int_0^1 2x(1-x)\mathrm{d}x \\
=&\frac{1}{3} \\
&\iint\limits_{D_2}(y-x)\mathrm{d}\sigma \\
=&\int_{-1}^0\mathrm{d}x\int_{-x-1}^{x+1}(y-x)\mathrm{d}y \\
=&\int_{-1}^0 -2x(x+1)\mathrm{d}x \\
=&\frac{1}{3}
\end{aligned}
$$

故所求为 $\frac{2}{3}$

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$$\int_0^1\mathrm{d}x\int_{\sqrt{1-x^2}}^0(x^2+y^2)\mathrm{d}y$$

15

$$\int_0^2\mathrm{d}x\int_0^{\sqrt{1-(x-1)^2}}3xy\mathrm{d}y$$

18

$$\iint\limits_{D}r\mathrm{d}\sigma$$

19 证:

$$\begin{aligned} S =& \iint\limits_D \mathrm{d}x\mathrm{d}y \\ =& \iint\limits_D r\mathrm{d}r\mathrm{d}\theta \\ =& \int_\alpha^\beta \int_0^{r(\theta)} r\mathrm{d}r \\ =& \frac{1}{2}\int_\alpha^\beta [r(\theta)]^2\mathrm{d}\theta \end{aligned}$$

20 设所求面积为 S

$$\begin{aligned} \frac{1}{2}S =& \iint\limits_D \mathrm{d}x\mathrm{d}y \\ =& \iint\limits_D rd\mathrm{d}r\mathrm{d}\theta \\ =& \int_0^\pi \mathrm{d}\theta \int_0^{a(1+cos\theta)} r\mathrm{d}r \end{aligned}$$