第四周作业

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$$z_x = -\frac{x}{\sqrt{a^2-x^2-y^2}}$$
$$z_y = -\frac{y}{\sqrt{a^2-x^2-y^2}}$$

故

$$\begin{aligned}\mathrm{d}S &= \sqrt{1+z_x^2+z_y^2}\mathrm{d}\sigma \\ &= \sqrt{\frac{a^2}{a^2-x^2-y^2}}\end{aligned}$$

故令 $x = r\cos\theta, y = r\sin\theta, D = \{(r,\theta)|r \le a\}$，有

$$\begin{aligned}&\iint\limits_S (x+y+z)\mathrm{d}S \\ =&\iint\limits_D ar\left(\frac{r(\sin\theta+\cos\theta)}{\sqrt{a^2-r^2}}+1\right)\mathrm{d}r\mathrm{d}\theta \\ =&\int_0^a \mathrm{d}r\int_0^{2\pi} ar\left(\frac{r(\sin\theta+\cos\theta)}{\sqrt{a^2-r^2}}+1\right)\mathrm{d}\theta \\ =&\int_0^a 2a\pi r\mathrm{d}r \\ =&\pi a^3\end{aligned}$$

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\begin{aligned}
x_u =& \cos v, x_v = -u \sin v \\
y_u =& \sin v, y_v = u \cos v \\
z_u =& 0, z_v = 1 \\
E =& x_u^2 + y_u^2 + z_u^2 = 1 \\
F =& x_u x_v + y_u y_v + z_u z_v = 0 \\
G =& x_v^2 + y_v^2 + z_v^2 = u^2 + 1 \\
\mathrm{d}S =& \sqrt{EG - F^2} \mathrm{d}u \mathrm{d}v = \sqrt{u^2+1} \mathrm{d}u \mathrm{d}v
\end{aligned}
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故令 $D = \{(u,v) | 0 \le u \le a, 0 \le v \le 2\pi\}$，有

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\begin{aligned}
& \iint\limits_S yz \mathrm{d}S \\
=& \iint\limits_D uv \sin v \sqrt{u^2+1} \mathrm{d}u \mathrm{d}v \\
=& \int_0^a \mathrm{d}u \int_0^{2\pi} uv \sin v \sqrt{u^2+1} \mathrm{d}v \\
=& \int_0^a u\sqrt{u^2+1} \mathrm{d}u (\int_0^{2\pi} \cos v - v \cos v \Big|_0^{2\pi}) \\
=& -\pi \int_0^a \sqrt{u^2+1} \mathrm{d}(u^2+1) \\
=& \frac{2\pi}{3} [1 - (a^2+1)^{\frac{3}{2}}]
\end{aligned}
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