第十周作业

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P271(2) 设和函数 $S(x)$，$S(x)$ 在 $(-1,1)$ 上可积，逐项求积得

$$\int_0^x S(x)\mathrm{d}x = \sum_{i=1}^{\infty}(-1)^{n-1}x^{2n-1} = \frac{x}{1+x^2}$$

故

$$S(x) = \frac{1-x^2}{(1+x^2)^2}, -1 < x < 1$$

P271(4) 设和函数 $S(x)$，$S(x)$ 在 $(-1,1)$ 上可导，逐项求导后仍然是幂级数，故逐项求二阶导得

$$S''(x) = \sum_{i=1}^{\infty} 2(-1)^{n-1}x^{2n-2} = \frac{2}{1+x^2}$$

故

$$\begin{aligned} S'(x) &= 2\arctan x \\ S(x) &= 2x\arctan x - \ln(1+x^2), -1 \le x \le 1 \end{aligned}$$

（上册）P297 1.

$$\begin{aligned} f_l = f_x\cos\theta + f_y\sin\theta \\ f_x = 2x - y = 3 \\ f_y = 2y - x = 3\sqrt{3} \end{aligned}$$

(1) 方向导数最大时，方向向量 (f_x, f_y)

$$\theta = \arctan\frac{f_y}{f_x} = \frac{\pi}{3}$$

(2) 方向导数最小时，方向向量 $(-f_x, -f_y)$

$$\theta = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

(3) 方向导数为 0 时，方向向量 $(f_y, -f_x), (-f_y, f_x)$

$$\theta = \arctan\frac{-f_x}{f_y} = \frac{5\pi}{6}\text{或}\frac{11\pi}{6}$$

2 方向向量 $(3,4)$，故 $\sin\varphi = 0.8, \cos\varphi = 0.6$

$$\begin{aligned} f_x &= 3x^2 - 6xy + 3y^2 = 12 \\ f_y &= -3x^2 + 6xy = -9 \\ f_l &= f_x\cos\varphi + f_y\sin\varphi = 0 \end{aligned}$$

4 由题意可知，$\cos\alpha = \cos\beta = \cos\gamma = 1/\sqrt{3}$

$$\begin{aligned} \nabla u = (y+z, x+z, x+y), \nabla u(2,1,3) &= (4,5,3) \\ \frac{\partial u}{\partial l} &= 12/\sqrt{3} = 4\sqrt{3} \end{aligned}$$

考虑两个方向，得结果为 $\pm 4\sqrt{3}$

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$$\begin{aligned} \nabla f &= (2x - 2y + 2xy^2, -2x + 2x^2y) \\ \nabla f(1,1) &= (2,0) \\ f_l &= 2cos\alpha \end{aligned}$$

最大的方向导数：2，最小的：-2，最大/最小方向：沿着 x 轴正方向/沿着 x 轴负方向

（上册）P313 1

(1)

$$\begin{aligned} 3x^2z + (x^3 + 3z^2x - 2y)\frac{\partial z}{\partial x} + z^3 &= 0 \\ \text{也即}\frac{\partial z}{\partial x} &= -\frac{3x^2z + z^3}{x^3 + 3z^2x - 2y} \end{aligned}$$

$$
\begin{aligned}
(x^3+3z^2x-2y)\frac{\partial z}{\partial y} &= 2z \\
\text{也即}\frac{\partial z}{\partial y} &= \frac{2z}{x^3+3z^2x-2y}
\end{aligned}
$$

(3)

$$
\begin{aligned}
1+\frac{\partial z}{\partial x}-\varepsilon\cos z\frac{\partial z}{\partial x} &= 0 \\
\text{也即}\frac{\partial z}{\partial x} &= \frac{1}{\varepsilon\cos z-1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y}-\varepsilon\cos z\frac{\partial z}{\partial y} &= 1 \\
\text{也即}\frac{\partial z}{\partial y} &= \frac{1}{1-\varepsilon\cos z}
\end{aligned}
$$

(3)

$$
\begin{aligned}
(\cos x-y\sin z)\frac{\partial z}{\partial x}+(\cos y-z\sin x) &= 0 \\
\text{也即}\frac{\partial z}{\partial x} &= \frac{z\sin x-\cos y}{\cos x-y\sin z} \\
(\cos x-y\sin z)\frac{\partial z}{\partial y}+(\cos z-x\sin y) &= 0 \\
\text{也即}\frac{\partial z}{\partial y} &= \frac{x\sin y-\cos z}{\cos x-y\sin z}
\end{aligned}
$$

3

$$
\begin{aligned}
\frac{\partial z}{\partial x}-y\sin xy &= e^z\frac{\partial z}{\partial x} \\
\frac{\partial z}{\partial x} &= \frac{y\sin xy}{1-e^z}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 z}{\partial x^2} &= \frac{y^2\cos(xy)(1-e^z)+y\sin(xy)e^z\frac{\partial z}{\partial x}}{(1-e^z)^2} \\
&= \frac{y^2\cos(xy)(1-e^z)^2+y^2\sin^2(xy)e^z}{(1-e^z)^3}
\end{aligned}
$$

10 将参数方程表示的二元隐函数表示为一般二元隐函数有 $x^2 + y^2 + z^2 = 1$，则

$$\frac{\partial z}{\partial x} = -\frac{x}{z}$$