第三周作业

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P60.4

$$
\begin{aligned}
&\int_L \frac{1}{x^2+y^2+z^2}\mathrm{d}s\\
=&\int_0^{2\pi}\frac{1}{a^2cos^2t+a^2sin^2t+b^2t^2}\sqrt{a^2sin^2+a^2cos^2+b^2}\mathrm{d}t\\
=&\int_0^{2\pi}\frac{1}{a^2+b^2t^2}\sqrt{a^2+b^2}\mathrm{d}t\\
=&\frac{\sqrt{a^2+b^2}}{ab}\int_0^{2\pi}\frac{1}{1+\frac{b^2t^2}{a^2}}\mathrm{d}\left(\frac{bt}{a}\right)\\
=&\frac{\sqrt{a^2+b^2}}{ab}arctan\frac{bt}{a}\Big|_0^{2\pi}\\
=&\frac{\sqrt{a^2+b^2}}{ab}arctan\frac{2b\pi}{a}
\end{aligned}
$$

P60.6 令

$$
\begin{cases}
x=acost\\
y=bsint
\end{cases}
\tag{1}
$$

则

$$
\int_L xy\mathrm{d}s
$$

P60.7 由

$$
\begin{aligned}
ds=&\sqrt{x'(t)^2+y'(t)^2}dt\\
=&|a|t\mathrm{d}t
\end{aligned}
$$

有

$$\begin{aligned}&\int_L \sqrt{x^2+y^2}\mathrm{d}s\\=&\int_0^{2\pi} a^2\sqrt{1+t^2}t\mathrm{d}t\\=&\int_0^{2\pi} \frac{a^2}{2}\sqrt{1+t^2}\mathrm{d}(1+t^2)\\=&\frac{a^2}{3}(1+4\pi^2)^{\frac{3}{2}}-\frac{a^2}{3}\end{aligned}$$

P60.8

$$\begin{aligned}&\int_L (x+\sqrt{y}-z^5)\mathrm{d}s\\=&\int_{L1} 2x\mathrm{d}s+\int_{L2} 2-z^5\mathrm{d}s\end{aligned}$$

其中

$$\begin{aligned}&\int_{L1} 2x\mathrm{d}s\\=&\int_0^1 2x\sqrt{1+(2x)^2}\mathrm{d}x\\=&\frac{1}{4}\int_0^1 \sqrt{4x^2+1}\mathrm{d}(x^2)\\=&\frac{1}{6}(4x^2+1)^{\frac{3}{2}}\Big|_0^1\\=&\frac{5\sqrt{5}-1}{6}\end{aligned}$$

$$\begin{aligned}&\int_{L2} 2-z^5\mathrm{d}s\\=&\int_0^1 (2-z^5)\mathrm{d}z\\=&(2z-\frac{z^6}{6})\Big|_0^1\\=&\frac{11}{6}\end{aligned}$$

所以所求为

$$\frac{5\sqrt{5}+10}{6}$$

P70.1

(1) 有

$$L: y = \frac{x}{2}$$

故

$$\begin{aligned}
&\int_L 2xy\mathrm{d}x - x^2\mathrm{d}y \\
=&\int_0^2 \frac{x^2}{2}\mathrm{d}x \\
=&\frac{4}{3}
\end{aligned}$$

(2) 有

$$L: y = \frac{x^2}{4}$$

故

$$\begin{aligned}
&\int_L 2xy\mathrm{d}x - x^2\mathrm{d}y \\
&= \int_0^2 (2x \cdot \frac{x^2}{4}\mathrm{d}x - x^2 \cdot \frac{x}{2}\mathrm{d}x) \\
&= 0
\end{aligned}$$

(3) 有

$$\begin{cases} L_1: y = 0 \\ L_2: x = 2 \end{cases}$$

故

$$\begin{aligned}&\int_L 2xy\mathrm{d}x - x^2\mathrm{d}y\\=&\int_{L1} 2xy\mathrm{d}x - x^2\mathrm{d}y + \int_{L2} 2xy\mathrm{d}x - x^2\mathrm{d}y\\=&\int_0^1 -4y\mathrm{d}y\\=&-4\end{aligned}$$

(4) 有

$$\begin{cases}L_1 : x = 0\\L_2 : y = 1\end{cases} \tag{2}$$

故

$$\begin{aligned}&\int_L 2xy\mathrm{d}x - x^2\mathrm{d}y\\=&\int_{L1} 2xy\mathrm{d}x - x^2\mathrm{d}y + \int_{L2} 2xy\mathrm{d}x - x^2\mathrm{d}y\\=&\int_0^2 2x\mathrm{d}x\\=&4\end{aligned}$$

P71.4

(1)

$$\begin{aligned}&\int_L (x^2 - 2xy)\mathrm{d}x + (y^2 - 2xy)\mathrm{d}y\\=&\int_{-1}^1 x^2 - 2x^5 + 4x^{11} - 8x^8\\=&-\frac{10}{9}\end{aligned}$$

(2)

$$
\begin{aligned}
&\int_L (x^2-2xy)\mathrm{d}x+(y^2-2xy)\mathrm{d}y\\
=&\int_{-1}^{1}(x^2-2x)\mathrm{d}x\\
=&\frac{2}{3}
\end{aligned}
$$

P71.5

$$
\begin{aligned}
&\oint_L y\mathrm{d}x+z\mathrm{d}y+x\mathrm{d}z\\
=&\int_0^{2\pi}(-a^2sin^2t+abtcost+abcost)\mathrm{d}t\\
&\int_0^{2\pi}-a^2sin^2t\mathrm{d}t\\
=&\frac{a^2}{4}(sin2t-2t)\Big|_0^{2\pi}=-a^2\pi\\
&\int_0^{2\pi}abtcost\mathrm{d}t\\
=&abtsint\Big|_0^{2\pi}-ab\int_0^{2\pi}sint\mathrm{d}t\\
=&abcost\Big|_0^{2\pi}\\
=&0\\
&\int_0^{2\pi}abcost\mathrm{d}t\\
=&absint\Big|_0^{2\pi}=0
\end{aligned}
$$

故所求为 $-\pi a^2$

P71.6 令

$$
\begin{cases}
L_1: y=-x, x\in[-1,0]\\
L_2: y=x, x\in(0,2]
\end{cases}
$$

故

$$
\begin{aligned}
&\int_L (x^2+y^2)\mathrm{d}x+(x^2-y)\mathrm{d}y\\
=&\int_{L_1}(x^2+y^2)\mathrm{d}x+(x^2-y)\mathrm{d}y+\int_{L_2}(x^2+y^2)\mathrm{d}x+(x^2-y)\mathrm{d}y\\
=&\int_{-1}^{0}(x^2-x)\mathrm{d}x+\int_0^2(3x^3-x)\mathrm{d}x\\
=&\frac{41}{6}
\end{aligned}
$$

P71.8

$$
\begin{aligned}
&\int_L (x^4-z^2)\mathrm{d}x+2xy^2\mathrm{d}y-y\mathrm{d}z\\
=&\int_0^1((t^4-t^6)\mathrm{d}t+4t^6\mathrm{d}t-3t^4\mathrm{d}t)\\
=&\int_0^1(-2t^4+3t^6)\mathrm{d}t\\
=&\frac{1}{35}
\end{aligned}
$$