Math 76 Exercises – 3.4B Partial Fractions

1. Write out the partial fraction decomposition of each function.

(a) $f(x) = \dfrac{x-1}{x^2+5x+6}$ $= \dfrac{x-1}{(x+2)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{x+3}$

$A(x+3) + B(x+2) = x - 1$

$x = -3$: $0 + B(-1) = -3 - 1 \Rightarrow -B = -4 \Rightarrow B = 4$.

$x = -2$: $A(1) + 0 = -2 - 1 \Rightarrow A = -3$.

So $f(x) = \boxed{-\dfrac{3}{x+2} + \dfrac{4}{x+3}}$

(b) $g(x) = \dfrac{x^2+3x-1}{x^3-x}$ $= \dfrac{x^2+3x-1}{x(x^2-1)} = \dfrac{x^2+3x-1}{x(x+1)(x-1)} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{x-1}$

$A(x+1)(x-1) + Bx(x-1) + Cx(x+1) = x^2 + 3x - 1$

$x = -1$: $0 + B(-1)(-2) + 0 = 1 - 3 - 1 = -3 \Rightarrow 2B = -3$, $B = -\frac{3}{2}$

$x = 0$: $A(1)(-1) + 0 + 0 = -1 \Rightarrow A = 1$

$x = 1$: $0 + 0 + C(1)(2) = 1 + 3 - 1 = 3 \Rightarrow 2C = 3 \Rightarrow C = \frac{3}{2}$

So $g(x) = \boxed{\dfrac{1}{x} - \dfrac{3}{2}\cdot\dfrac{1}{x+1} + \dfrac{3}{2}\cdot\dfrac{1}{x-1}}$

(c) $h(x) = \dfrac{8x^2-7x+4}{x^3-x^2}$ $= \dfrac{8x^2-7x+4}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$

$Ax(x-1) + B(x-1) + Cx^2 = 8x^2 - 7x + 4$

$x = 0$: $0 + B(-1) + 0 = 4 \Rightarrow B = -4$

$x = 1$: $0 + 0 + C(1) = 8 - 7 + 4 = 5 \Rightarrow C = 5$

Now put in these values to find A:

$Ax(x-1) - 4(x-1) + 5x^2 = 8x^2 - 7x + 4$

$Ax^2 - Ax - 4x + 4 + 5x^2 = 8x^2 - 7x + 4$

$(A+5)x^2 + (-A-4)x + 4 = 8x^2 - 7x + 4 \Rightarrow A + 5 = 8 \Rightarrow A = 3$

So $h(x) = \boxed{\dfrac{3}{x} - \dfrac{4}{x^2} + \dfrac{5}{x-1}}$

(d) $j(x) = \dfrac{2x^2 - 10x + 15}{(x+1)(x-2)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$

$A(x-2)^2 + B(x+1)(x-2) + C(x+1) = 2x^2 - 10x + 15$

$x = 2$: $0 + 0 + C \cdot 3 = 2 \cdot 2^2 - 10 \cdot 2 + 15 = 3 \Rightarrow C = 1$

$x = -1$: $A(-3)^2 + 0 + 0 = 2 + 10 + 15 = 27 \Rightarrow A = 3$

Now put in these values to find B:

$3(x-2)^2 + B(x+1)(x-2) + x + 1 = 2x^2 - 10x + 15$

Match coefficients of x^2 terms: $3x^2 + Bx^2 = 2x^2$

$3 + B = 2$

$B = -1$

So $j(x) = \boxed{\dfrac{3}{x+1} - \dfrac{1}{x-2} + \dfrac{1}{(x-2)^2}}$

2. Evaluate each integral.

(a) $\displaystyle\int \frac{x-1}{x^2+5x+6}\,dx = \int -\frac{3}{x+2} + \frac{4}{x+3}\,dx$ (by #1(a))

$= \boxed{-3\ln|x+2| + 4\ln|x+3| + C}$

(b) $\displaystyle\int \frac{x^2+3x-1}{x^3-x}\,dx = \int \frac{1}{x} - \frac{3}{2}\cdot\frac{1}{x+1} + \frac{3}{2}\frac{1}{x-1}\,dx$ (by #1(b))

$= \boxed{\ln|x| - \frac{3}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C}$

(c) $\int \frac{8x^2-7x+4}{x^3-x^2}\,dx$ $= \int \frac{3}{x} - \frac{4}{x^2} + \frac{5}{x-1}\,dx$ (by #1(c))

$= \boxed{3\ln|x| + \frac{4}{x} + 5\ln|x-1| + C}$

(d) $\int \frac{2x^2-10x+15}{(x+1)(x-2)^2}\,dx$ $= \int \frac{3}{x+1} - \frac{1}{x-2} + \frac{1}{(x-2)^2}\,dx$ (by #1(d))

$= \boxed{3\ln|x+1| - \ln|x-2| - \frac{1}{x-2} + C}$