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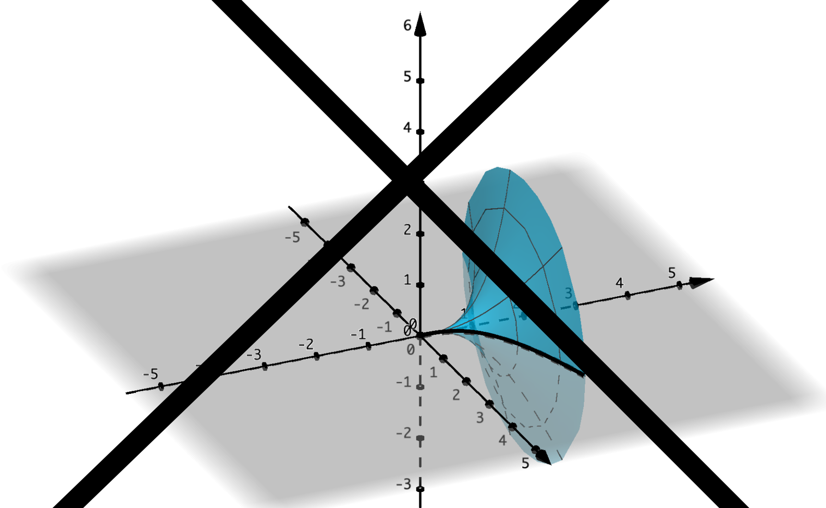
Kelm

Spring 2021 MATH 76

Activity 4

1. SURFACE AREA

The goal of this problem is to compute the area of the surface generated when the curve $y = (3x)^{\frac{1}{3}}$, for $0 \le x \le \dfrac{8}{3}$, is revolved about the $y-$axis. Since $y = (3x)^{\frac{1}{3}}$ then $x = \dfrac{y^3}{3}$ for $0 \le y \le 2$. In class, if the subject was covered, the surface $S = \displaystyle\int_0^2 2\pi x\sqrt{1 + (x')^2}\,dy$ where x' is the first derivative of the function x with respect to y.



Use the step by step approach to evaluate S.

2. PHYSICS APPLICATIONS

Recall that **work** is a measure of the amount of energy transferred when a force moves an object. If the force applied to an object is a constant F over a distance d, the work done is $F \cdot d$. If the force applied to an object at position x is $F(x)$, then the work done to move the object from $x = a$ to $x = b$ is $W = \displaystyle\int_a^b F(x)\,dx$. How much work is required to move an object from $x = 1$ to $x = 3$ (measured in meters) in the presence of a force (in N) given by $F(x) = \dfrac{2}{x^2}$ acting along the $x-$axis.

② $W = \int_a^b F(x)\,dx \quad [1, 3] \quad$ meters

$$W = \int_1^3 \frac{2}{x^2}\,dx$$

$$= 2\int_1^3 \frac{1}{x^2}\,dx$$

$$= 2\left(\frac{1}{x^{2+1}}\right) = 2\left(-\frac{1}{x}\right) = -\frac{2}{x}\Big|_1^3$$

$$3$$

$$= \left(-\frac{2}{3}\right) - \left(-2\right)$$

$$= -\frac{2}{3} + 2 \quad \boxed{= \frac{4}{3}} \checkmark$$

3. INTEGRALS

(a) SUBSTITUTION METHOD
The following integrals can be solved by finding a suitable function u to substitute.

i. $\displaystyle\int \frac{e^{2\sqrt{x}+1}}{\sqrt{x}}dx$

ii. $\displaystyle\int \frac{e^x}{e^x+1}dx$

iii. $\displaystyle\int_{-5}^{0} \frac{dx}{\sqrt{4-x}}$

(b) SUBSTITUTION METHOD (cont'd)
The following integrals can still be solved by the substitution method but the integrand must first be modified.

i. $\displaystyle\int \sin(x)\sin(2x)dx.$

Hint: $\sin(2x) = 2\sin(x)\cos(x)$

ii. $\displaystyle\int \frac{x}{x^4+2x^2+1}dx.$

Hint: $x^4+2x^2+1 = (x^2+1)^2$.

(c) NO SUBSTITUTION NEEDED
The following integrals do not need the substitution method but the integrand needs to be modified.

i. $\displaystyle\int_{0}^{\pi/2} \sqrt{1+\cos(2x)}dx$

Hint: $\cos(2x) = 2\cos^2(x) - 1$

ii. $\displaystyle\int_{4}^{9} \frac{x^{5/2}-x^{1/2}}{x^{3/2}}dx$

3 a I

The following inte

i. $\int \frac{e^{2\sqrt{x}+1}}{\sqrt{x}}dx$ $= \int \frac{e^{2\sqrt{x}+1}}{\sqrt{x}} dx$

$u = 2\sqrt{x}+1$ $du = \frac{1}{\sqrt{x}} dx$

$= 2 \cdot \frac{1}{2} x^{\frac{1}{2}-1}$

$= \frac{1}{\sqrt{x}}$

$= \int e^{u} du$

$= e^{u} + c$

$\boxed{= e^{2\sqrt{x}+1} + C}$ ✓

3 a II

ii. $\int \frac{e^x}{e^x+1}dx$ $= \int \frac{e^x}{e^x+1} dx$

$u = e^x + 1$ $du = e^x dx$

$= \int \frac{1}{u} du$

$= \ln|u| + c$

$\boxed{= \ln|e^x+1| + c}$ ✓

3 a III

iii. $\int_{-5}^{0} \frac{dx}{\sqrt{4-x}}$ $= \int_{-5}^{0} \frac{1}{\sqrt{4-x}} dx$

$u = 4-x$ $du = -1\,dx$

$= -\int_{-5}^{0} \frac{1}{\sqrt{u}} du = -\int_{-5}^{0} \frac{1}{u^{\frac{1}{2}+\frac{2}{2}}} = -2\sqrt{u} + c = -2\sqrt{4-x}\Big|$

$\frac{2}{2}$

$= (\overset{0}{-4}) - (\overset{-5}{6}) = \boxed{2}$ ✓

③ b $\boxed{\text{I}}$ i. $\int \sin(x)\sin(2x)dx$.

Hint: $\sin(2x) = 2\sin(x)\cos(x)$

$$= \int \sin(x)\sin(2x)\,dx$$

$$= \int \sin(x)\, 2\sin(x)\cos(x)\,dx$$

$$= 2\int \sin(x)\cos(x)\sin(x)\,dx$$

$U = \sin(x)$

$dU = \cos(x)dx$

$$= 2\int U^2\,dU$$

$$= \frac{2U^3}{3} + C = \boxed{\frac{2\sin^3(x)}{3} + C}$$

③ b $\boxed{\text{II}}$ ii. $\int \frac{x}{x^4+2x^2+1}dx$.

Hint: $x^4 + 2x^2 + 1 = (x^2+1)^2$.

$$= \int \frac{x}{x^4+2x^2+1}\,dx$$

$$= \int \frac{x}{(x^2+1)^2}\,dx$$

$U = x^2+1$

$dU = 2x\,dx$

$$= \frac{1}{2}\int \frac{1}{(U)^2}\,dU$$

$$= \frac{1}{2}\left(-\frac{1}{U}\right) + C$$

$$\boxed{= -\frac{1}{2x^2+2} + C}$$ ✓

3 c I

i. $\int_0^{\pi/2} \sqrt{1+\cos(2x)}dx$

Hint: $\cos(2x) = 2\cos^2(x) - 1$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+\cos(2x)}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+2\cos^2(x)-1}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2(x)}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos(x)\, dx$$

$$= \sqrt{2}\left(\sin(x)\right)\Big|_0^{\frac{\pi}{2}} = (\overset{0}{0}) - (-\sqrt{2})$$

$$\boxed{= \sqrt{2}}$$ ✓

3 c II

ii. $\int_4^9 \frac{x^{5/2} - x^{1/2}}{x^{3/2}}dx$

$$= \int_4^9 \frac{x^{\frac{5}{2}} - x^{\frac{1}{2}}}{x^{\frac{3}{2}}}\, dx$$

$$= \int_4^9 \frac{x^{\frac{5}{2}}}{x^{\frac{3}{2}}} - \frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}\, dx$$

$$= \int_4^9 x - \frac{1}{x}\, dx$$

$$= \frac{x^2}{2} - \ln|x| \Big|_4^9$$

$$= \frac{\overset{9}{81}}{2} - \ln|9| - 8 + \ln|4|$$

$$\boxed{= 32.5 - \ln|9| + \ln|4|}$$

(d) INTEGRATION BY PARTS

Given that u and v are functions and du, dv are their corresponding derivatives, the integration by parts formula is $\int udv = uv - \int vdu$. Choose appropriately u and dv in the following integrals. Compute also du and v.

i. $\int xe^x dx$ $\qquad u =$ $\qquad dv =$

$du =$ $\qquad v =$

ii. $\int x\sin(x)dx$ $\qquad u =$ $\qquad dv =$

$du =$ $\qquad v =$

iii. $\int \tan^{-1}(x)dx$ $\qquad u =$ $\qquad dv =$

$du =$ $\qquad v =$

iv. $\int x^2 e^{-3x} dx$ $\qquad u =$ $\qquad dv =$

$du =$ $\qquad v =$

v. $\int x^5 \ln(x)dx$ $\qquad u =$ $\qquad dv =$

$du =$ $\qquad v =$

i. $\int xe^x dx$ $u = x$ $dv = e^x dx$

$du = 1 dx$ $v = e^x$

U= LIATE

ii. $\int x \sin(x) dx$ $u = x$ $dv = \sin(x) dx$

$du = 1 dx$ $v = \cos(x)$

U= LIATE

iii. $\int \tan^{-1}(x) dx$ $u = \text{Artan}(x)$ $dv = 1 dx$

$du = \frac{1}{1+x^2} dx$ $v = x$

U= LIATE

iv. $\int x^2 e^{-3x} dx$ $u = x^2$ $dv = e^{-3x} dx$

$du = 2x dx$ $v = -\frac{e^{-3x}}{3}$

U= LIATE

v. $\int x^5 \ln(x) dx$ $u = \ln x$ $dv = x^5 dx$

$du = \frac{1}{x} dx$ $v = 5x^4$

U= LIATE