Math 76 Exercises – 5.1A **Intro to Sequences and Series**

1. Write out the first six terms of each sequence.

(a) $\{3n+1\}_{n=0}^{\infty}$

$a_0 = 3\cdot 0 + 1 = 1$, etc.

So we have

$\{1, 4, 7, 10, 13, 16, \dots\}$

(b) $\{n^2\}_{n=1}^{\infty}$

$a_1 = 1^2 = 1$, etc.

So we have

$\{1, 4, 9, 16, 25, 36, \dots\}$

(c) $\left\{\frac{1}{n-2}\right\}_{n=4}^{\infty}$

$a_4 = \frac{1}{4-2} = \frac{1}{2}$, etc. So we have $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \dots\}$

(d) $\{a_n\}_{n=1}^{\infty}$ where $a_1 = 3$ and $a_k = 4a_{k-1} - 1$ for $k \geq 2$.

$a_1 = 3$, $a_2 = 4a_1 = 4\cdot 3 = 12$

$a_3 = 4a_2 = 4\cdot 12 = 48$, etc.

So we have $\{3, 12, 48, 192, 768, 3072, \dots\}$

2. Given the sequence $\{a_n\}$, find a formula for a_n. Be sure to say what the starting n is.

(a) $-1, 1, 3, 5, 7, \dots$

If $a_1 = -1$ then $a_n = 2n-3$.

If $a_0 = -1$ then $a_n = 2n-1$, etc.

(b) $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \dots$

Numerators:

$1, 3, 5, 7, \dots, 2n-1$

where first term is for $n=1$.

Denominators:

$2, 4, 8, 16, \dots, 2^n$

where first term is for $n=1$.

So we have, given $a_1 = \frac{1}{2}$,

$a_n = \boxed{\frac{2n-1}{2^n}}$

(c) $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \dots$

$a_n = \frac{(-1)^n}{n}$ where

$a_1 = \frac{(-1)^1}{1}$, $a_2 = \frac{(-1)^2}{2}$, etc.

(d) $\{a_n\}_{n=1}^{\infty}$ defined by $a_1 = 3$ and $a_{k+1} = 2a_k$ for $k \geq 2$.

$a_1 = 3$

$a_2 = 2a_1 = 2\cdot 3 = 6$

$a_3 = 2a_2 = 2\cdot 2\cdot 3 = 2^2\cdot 3$

$a_4 = 2a_3 = 2\cdot 2^2\cdot 3 = 2^3\cdot 3$

$a_5 = 2a_4 = 2^4\cdot 3$

$a_6 = 2^5\cdot 3$, ...

Based on this pattern, we get $a_n = \boxed{2^{n-1}\cdot 3}$

3. For each series, find the third partial sum.

(a) $\sum_{n=0}^{\infty} \frac{2}{3^n}$

$S_2 = \frac{2}{3^0} + \frac{2}{3^1} + \frac{2}{3^2}$

$= 2 + \frac{2}{3} + \frac{2}{9} = \frac{18+6+2}{9} = \boxed{\frac{26}{9}}$

(b) $\sum_{n=3}^{\infty} \frac{1}{n(n-2)}$

$S_5 = \frac{1}{3(3-2)} + \frac{1}{4(4-2)} + \frac{1}{5(5-2)}$

$= \frac{1}{3} + \frac{1}{8} + \frac{1}{15} = \frac{40+15+8}{120} = \frac{63}{120} = \boxed{\frac{21}{40}}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

$S_3 = -1 + \frac{1}{2} - \frac{1}{3} = \frac{-6+3-2}{6}$

$= \boxed{-\frac{5}{6}}$

(d) $\sum_{n=2}^{\infty} \frac{5}{8}$

$S_4 = \frac{5}{8} + \frac{5}{8} + \frac{5}{8} = \boxed{\frac{15}{8}}$

4. Find the first six partial sums of the series $\sum_{n=0}^{\infty} \frac{2}{3^n}$. Based on your answers, do you think the series converges? If so, what does the sum of the series appear to be?

$S_0 = \frac{2}{3^0} = 2$

$S_1 = \frac{2}{3^0} + \frac{2}{3^1} = 2 + \frac{2}{3} = \frac{5}{3} = 2.\overline{6}$

$S_2 = S_1 + \frac{2}{3^2} = 2.\overline{8}$

$S_3 = S_2 + \frac{2}{3^3} = 2.\overline{962}$

$S_4 = S_3 + \frac{2}{3^4} \approx 2.988$

$S_5 = S_4 + \frac{2}{3^5} \approx 2.999.$

$\sum_{n=0}^{\infty} \frac{2}{3^n} = \lim_{n\to\infty} S_n$ which appears to be $\boxed{3}$

(so the series converges to 3).