Choose an n to begin with. I'll pick $n = 0$. Then adjust everything so that plugging in $n = 0$ gives $a_0 = \frac{1}{3}$:

$$a_n = (-1)^n \frac{4n+1}{3^{n+1}}.$$

Alternate solution: start with $n = 1$. Then the sequence is

$$\left\{ (-1)^{n-1} \frac{4n-3}{3^n} \right\}_{n=1}^{\infty}$$

Notice the following convenient trick:

Convenient Trick. To start the sequence from $n = 1$ instead of $n = 0$, I replaced all the n's in the n-th term formula by $n - 1$. Similarly, if I had wanted to begin with $n = -10$, I could have replaced all the n's in the n-th term formula by $n + 10$:

$$\left\{ (-1)^{n+10} \frac{4n+41}{3^{n+11}} \right\}_{n=-10}^{\infty}$$

Practice Problems.

For each problem, find a formula for a_n. If your first term is not a_1, be sure to make it clear what your first term is by writing the sequence in the form $\{a_n\}_{n=?}^{\infty}$.

(1) $\{1 \cdot 4,\ 4 \cdot 8,\ 7 \cdot 16,\ 10 \cdot 32,\ \ldots\}$

$$\left\{ (3n-2) \cdot 2^{n+1} \right\}_{n=1}^{\infty} = \left\{ (3n+1) \cdot 2^{n+2} \right\}_{n=0}^{\infty}$$

(2) $\{-8, -1, 0, 1, 8, 27, 64, \ldots\}$

$$\left\{ n^3 \right\}_{n=-2}^{\infty} = \left\{ (n-3)^3 \right\}_{n=1}^{\infty}$$

There are many correct answers

(3) $\left\{ -\frac{1}{6}, \frac{4}{7}, -2, \frac{64}{9}, -\frac{256}{10}, \ldots \right\}$

$-\frac{16}{8}$

“$\dfrac{(-1)^{\blacktriangle}\, 4^{*}}{n \pm ☺}$”

$$\left\{ \frac{(-1)^n 4^{n-1}}{n+5} \right\}_{n=1}^{\infty} = \left\{ \frac{(-1)^{n+1} 4^n}{n+6} \right\}_{n=0}^{\infty}$$