Math 76 Assessment 7 §3.3 Trigonometric Substitution

Things to remember:

1. The problem to complete is shown below. Write your name and solution on the next page where instructed.

2. Please make sure your full name is written neatly in the box.

3. Your score will be determined by **Mechanics** (2 points) and by **Content** (3 points).

4. The following rubric will be used for **Mechanics**:

Clear neat work, steps in order and easily followed, proper use of notation	2
Mostly clear work; minor errors in notation or skipped steps	1.5
Steps/handwriting hard to follow/read; major errors in notation	1
No discernible or relevant work, or work impossible to read/follow	0

5. You are not allowed to consult outside sources, including notes, books, the internet, or other people, while taking this assessment. Calculators are allowed only for basic numerical or scientific computations, not for graphing or algebra.

6. If you need more room, you may finish on a plain piece of paper or blank document.

7. When you are finished, create a legible, well-lit **.pdf file** of your work and upload it to Assessment 7 on Gradescope. If prompted, follow the directions to assign the page(s) of your submission that contain your work for the question. More info about submitting to Gradescope:

`http://bit.ly/gradescope-help`

Use trigonometric substitution to evaluate the integral on the next page.

Your solution should include:

- (0.5 point) Correct trigonometric substitution strategy;
- (1 point) Explanation and result of integral correctly rewritten in terms of θ;
- (0.5 point) Correct evaluation of the integral in θ;
- (0.5 point) Explanation and result of correct back substitution;
- (0.5 point; 0 if no relevant work/explanation) Correct final answer.

You may or may not find the following identities useful:

- $\sin^2\theta + \cos^2\theta = 1$
- $\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$

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Version D

Follow the directions on the previous page to evaluate the integral

$$\int \frac{x^3}{\sqrt{x^2-25}}\,dx.$$

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$\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

$x = 5^3\sec^3\theta$

$\tan^2\theta = \sec^2\theta - 1$

$$= \int \frac{x^3}{\sqrt{x^2-5^2}}\,dx$$

$$= \int \frac{5^3\sec^3\theta}{\sqrt{5^2\sec^2\theta-5^2}}\,d\theta$$

$$= 25\int \frac{\sec^3\theta}{\sqrt{(\sec^2\theta-1)}}\,d\theta \quad = 25\int \frac{\sec^3\theta}{\sqrt{\tan^2\theta}}\,d\theta$$

$$= 25\int \frac{\sec^3\theta}{\tan\theta}\,d\theta \quad = 25\int \frac{\sec^3\theta}{\tan\theta}\,d\theta$$

$$= 25\int \frac{\sec^2\theta\sec\theta}{\tan\theta}\,d\theta = 25\int \frac{(\tan^2\theta+1)\sec\theta}{\tan\theta}\,d\theta$$

$\sec x = \frac{1}{\cos x}$

$\tan x = \frac{\sin x}{\cos x}$

$$= 25\int \frac{1}{\cos\theta}\left(\frac{\tan^2\theta+1}{\frac{\sin\theta}{\cos\theta}}\right)d\theta = 25\int \left(\frac{\cancel{\cos\theta}}{\sin\theta}\right)\left(\frac{1}{\cancel{\cos\theta}}\right)\left(\frac{1}{\frac{\cancel{\cos\theta}}{\sin\theta}}\right)\left(\frac{\tan^2\theta+1}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right)d\theta$$

$$= 25\int \frac{\tan^2\theta+1}{\sin\theta}\,d\theta = 25\int \frac{\sec^2\theta}{\sin\theta}\,d\theta \quad \frac{1}{\sin\theta} = \csc\theta$$

Not sure what to do Next.