1. An air conditioning duct has cross-sectional area (perpendicular to the x-axis) given by $A(x)$ at a distance of x cm from the vent ($0 \le x \le 30$). For each $A(x)$, find the volume of the duct.

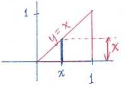
(a) $A(x) = 3x^2$

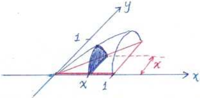
$$V = \int_0^{30} A(x)\,dx = \int_0^{30} 3x^2\,dx = x^3 \Big|_0^{30} = 30^3 - 0^3 = \boxed{27000 \text{ cm}^3}$$

(b) $A(x) = \sqrt[3]{x} + 8$

$$V = \int_0^{30} A(x)\,dx = \int_0^{30} \left(\sqrt[3]{x} + 8\right) dx = \int_0^{30} \left(x^{\frac{1}{3}} + 8\right) dx = \frac{3}{4}x^{4/3} + 8x \Big|_0^{30} = \frac{3}{4}(30)^{4/3} + 8 \cdot 30 - 0 = \boxed{\frac{3}{4}(30)^{4/3} + 240 \text{ cm}^3}$$

2. (**) Find the volume of the solid whose base is a triangle with vertices $(0,0)$, $(1,0)$, and $(1,1)$ and whose cross sections (perpendicular to the base and parallel to the y-axis) are semicircles.

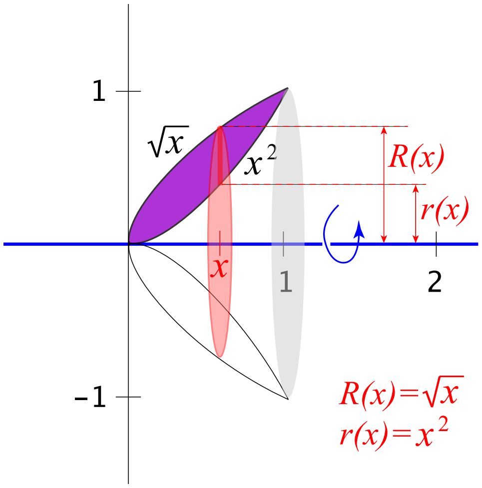




Cross section at x is a semicircle with radius $\frac{x}{2}$, so area is $A(x) = \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{8}\pi x^2$.

$$\text{Volume} = \int_0^1 A(x)\,dx = \int_0^1 \frac{1}{8}\pi x^2\,dx = \frac{1}{8}\pi \cdot \frac{1}{3}x^3 \Big|_0^1 = \frac{\pi}{24}(1^3 - 0^3) = \boxed{\frac{\pi}{24}}$$

3. Find the volume of the solid formed by rotating the region shown about the x-axis.



$R = \sqrt{x}$, $r = x^2$

$$V = \pi \int_0^1 (R^2 - r^2)\,dx = \pi \int_0^1 (x - x^4)\,dx = \pi\left(\tfrac{1}{2}x^2 - \tfrac{1}{5}x^5\right)\Big|_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{5} - (0-0)\right) = \boxed{\frac{3\pi}{10}}$$

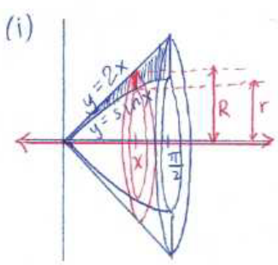
4. For each problem, **sketch the solid** formed by rotating the given region

   (i) about the x-axis;

   (ii) about the line $y = -3$,

   and **set up** an integral for the volume of the solid.

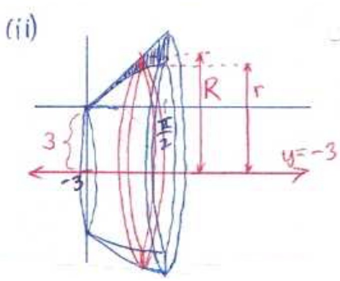
   (a) The region enclosed by the curves $y = \sin x$, $y = 2x$, $x = \dfrac{\pi}{2}$



$R = 2x$

$r = \sin x$

$$V = \pi \int_0^{\frac{\pi}{2}} (R^2 - r^2)\,dx = \pi \int_0^{\frac{\pi}{2}} \left((2x)^2 - (\sin x)^2\right) dx = \boxed{\pi \int_0^{\frac{\pi}{2}} (4x^2 - \sin^2 x)\,dx}$$



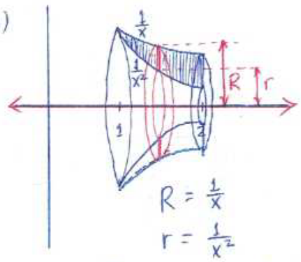
$R = 2x + 3$

$r = \sin x + 3$

$$V = \boxed{\pi \int_0^{\frac{\pi}{2}} \left((2x+3)^2 - (\sin x + 3)^2\right) dx}$$

(b) **(**)** The region enclosed by the curves $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 2$

(i)



$R = \frac{1}{x}$

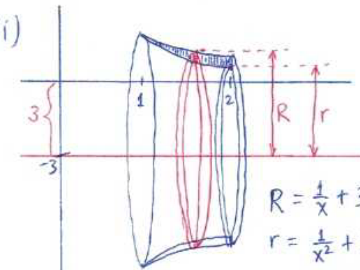
$r = \frac{1}{x^2}$

$$V = \pi \int_1^2 \left(\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x^2}\right)^2 \right) dx$$

$$= \boxed{\pi \int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^4} \right) dx}$$

$\frac{1}{x}$ and $\frac{1}{x^2}$ intersect at $x = 1$.

(ii)



$R = \frac{1}{x} + 3$

$r = \frac{1}{x^2} + 3$

$$V = \boxed{\pi \int_1^2 \left(\left(\frac{1}{x} + 3\right)^2 - \left(\frac{1}{x^2} + 3\right)^2 \right) dx}$$

5. For each problem, **sketch the solid** formed by rotating the given region

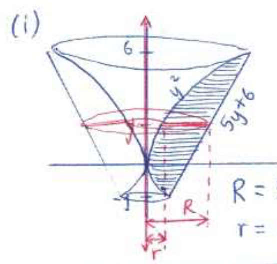
   (i) about the y-axis;

   (ii) about the line $x = 36$,

   and **set up** an integral for the volume of the solid.

   (a) The region enclosed by the curves $x = y^2$, $x = 5y + 6$

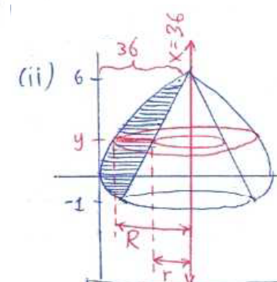
$$\left.\begin{aligned} y^2 &= 5y+6 \\ 5y-6 &= 0 \\ 6)(y+1) &= 0 \end{aligned}\right\} \text{ y^2 and $5y+6$ intersect at $y=6$ and $y=-1$}$$



$R = 5y+6$

$r = y^2$

$$\boxed{V = \pi \int_{-1}^{6} \left((5y+6)^2 - (y^2)^2 \right) dy}$$



$R = 36 - y^2$

$r = 36 - (5y+6)$

$$\boxed{V = \pi \int_{-1}^{6} \left((36-y^2)^2 - (36-(5y+6))^2 \right) dy}$$

   (b) **(**)** The region enclosed by the curves $\dfrac{x}{3} = y^2$, $y = -\dfrac{1}{3}x + 2$

   This is a presentation problem. See me for help, if you want!