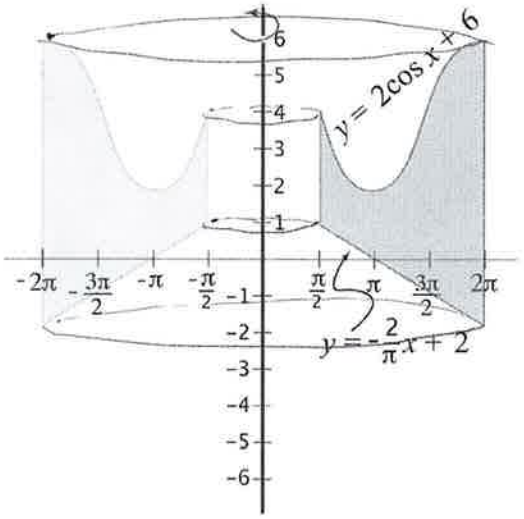
Set up an integral for the volume of the solid formed by rotating the region shown about the y-axis. (For extra practice later, evaluate the integral.)



$$V = 2\pi \int_{\frac{\pi}{2}}^{2\pi} x\left(2\cos x + 6 - \left(-\tfrac{2}{\pi}x + 2\right)\right) dx$$

For each problem, **sketch the solid** formed by rotating the region enclosed by the curves whose equations are given

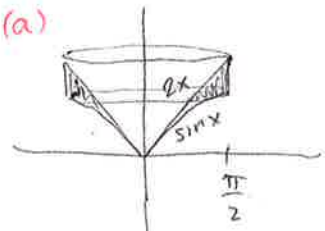
(a) about the y-axis;

(b) about the line $x = -3$,

and **set up** an integral for the volume of the solid. (For extra practice later, evaluate the integrals, if you can.)

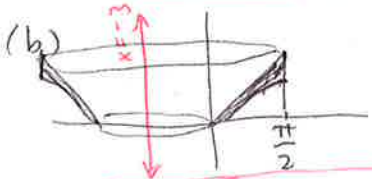
1. $y = \sin x, \quad y = 2x, \quad x = \frac{\pi}{2}$

(a)



$$V = 2\pi \int_0^{\frac{\pi}{2}} x\,(2x - \sin x)\,dx$$

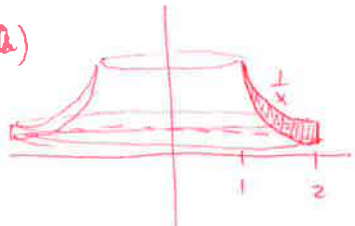
(b)



$$V = 2\pi \int_0^{\frac{\pi}{2}} (x+3)(2x - \sin x)\,dx$$

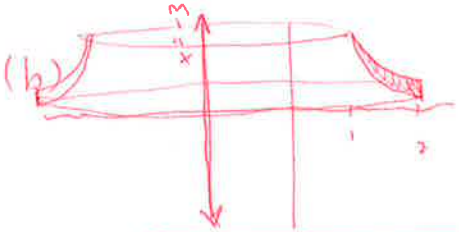
2. $y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2$

(a)



$$V = 2\pi \int_1^2 x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

(b)



$$V = 2\pi \int_1^2 (x+3)\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

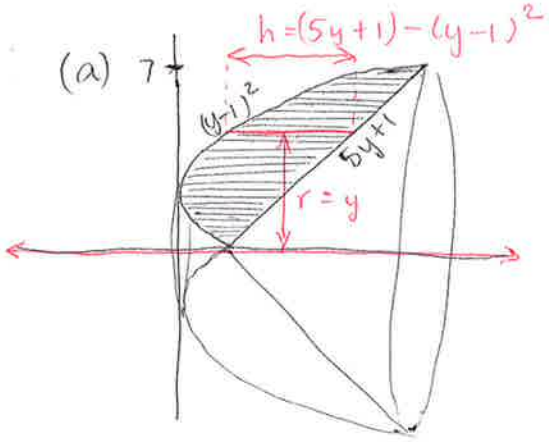
For each problem, **sketch the solid** formed by rotating the region enclosed by the curves whose equations are given

(a) about the x-axis;

(b) about the line $y = 8$,

and **set up** an integral for the volume of the solid. (For extra practice later, evaluate the integrals, if you can.)

1. $x = (y-1)^2, \quad x = 5y+1$



$(y-1)^2 = 5y+1$

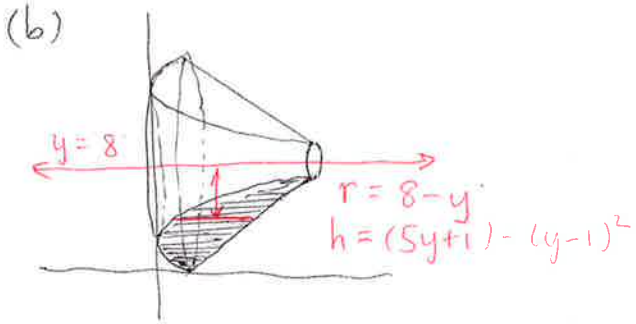
$y^2 - 2y + 1 = 5y + 1$

$y^2 - 7y = 0$

$y(y-7) = 0$

$y = 0, \; y = 7.$

$$V = 2\pi \int_0^7 y\left((5y+1) - (y-1)^2\right) dy$$



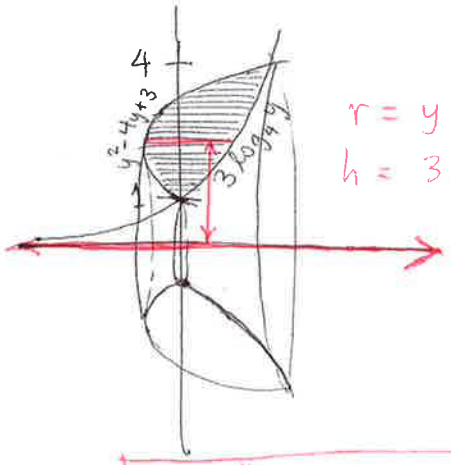
$$V = 2\pi \int_0^7 (8-y)\left((5y+1) - (y-1)^2\right) dy$$

2. $x = 3\log_4 y, \quad x = y^2 - 4y + 3$

Hint: these curves intersect at $y = 1$ and $y = 4$.

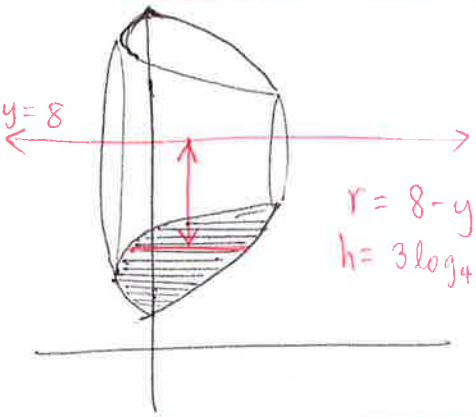
Note: $x = 3\log_4 y$ is the same curve as $x = \log_4(y^3)$ which is $4^x = y^3$, so $y = 4^{x/3}$ ... This helps us graph...

(a)



$$V = 2\pi \int_1^4 y\left(3\log_4 y - (y^2 - 4y + 3)\right) dy$$

(b)



$$V = 2\pi \int_1^4 (8-y)\left(3\log_4 y - (y^2 - 4y + 3)\right) dy$$