Math 76 Exercises – 7.3A Polar Coordinates

1. Graph and label each polar point. You can plot them all on one set of (**large!**) axes.

(a) $(1, \pi)$

(b) $\left(2, \frac{\pi}{4}\right)$

(c) $\left(0, \frac{\pi}{3}\right)$

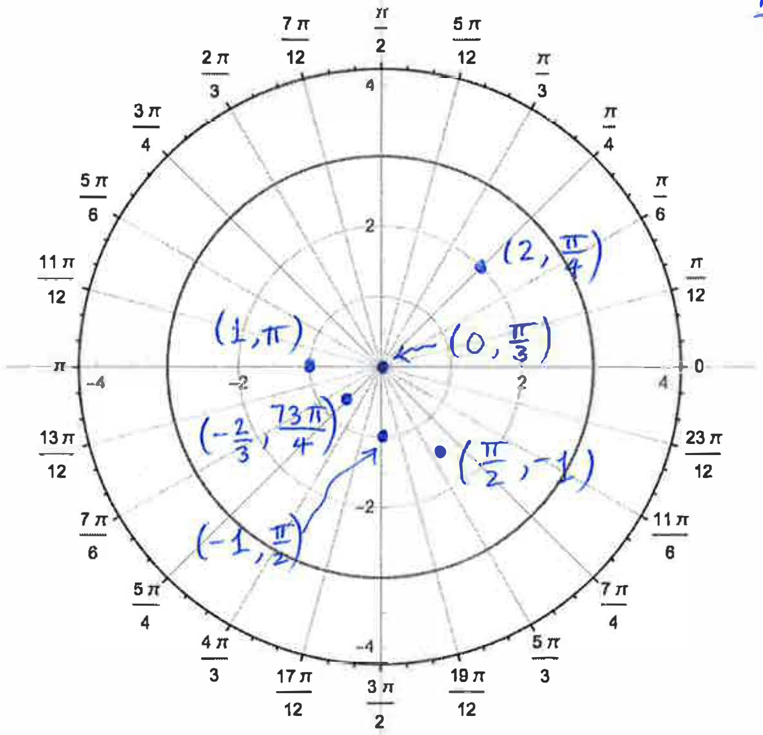
(d) $\left(-1, \frac{\pi}{2}\right)$

(e) $\left(\frac{\pi}{2}, -1\right)$ *(Careful!)*

(f) $\left(-\frac{2}{3}, \frac{73\pi}{4}\right)$

$$\frac{73\pi}{4} = \frac{72\pi}{4} + \frac{\pi}{4}$$

$$= 18\pi + \frac{\pi}{4}$$



2. Convert each polar point above to rectangular (Cartesian) coordinates.

(a) $(-1, 0)$

(c) $(0, 0)$

(e) $x = \frac{\pi}{2}\cos(-1)$

≈ 0.8487

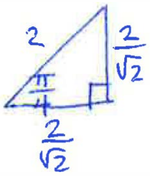
$y = \frac{\pi}{2}\sin(-1)$

≈ -1.3218

So $\left(\frac{\pi}{2}\cos(-1), \frac{\pi}{2}\sin(-1)\right)$

$\approx \boxed{(0.8487, -1.3218)}$

(b)



$\frac{2}{\sqrt{2}} = \sqrt{2}$.

So $\boxed{(\sqrt{2}, \sqrt{2})}$

(d) $(0, -1)$

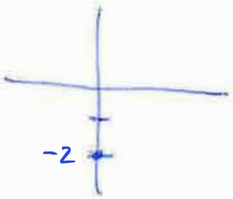
(f) $x = -\frac{2}{3}\cos\left(\frac{73\pi}{4}\right) = -\frac{2}{3}\cdot\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{3}$

$y = -\frac{2}{3}\sin\left(\frac{73\pi}{4}\right) = -\frac{2}{3}\cdot\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{3}$

So $\boxed{\left(-\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)}$

3. For each Cartesian point, find two equivalent polar points (r, θ), one with $r > 0$ and one with $r < 0$.

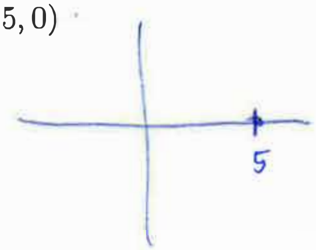
(a) $(0, -2)$



$\left(2, \frac{3\pi}{2}\right)$

$= \left(-2, \frac{\pi}{2}\right)$

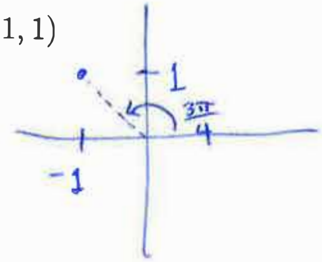
(b) $(5, 0)$



$(5, 0)$ (same in polar!)

$= (-5, \pi)$

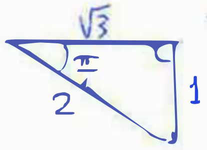
(c) $(-1, 1)$

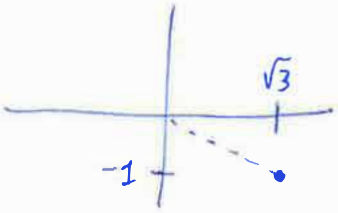


$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

$= \left(-\sqrt{2}, \frac{7\pi}{4}\right)$

(d) $\left(\sqrt{3}, -1\right)$

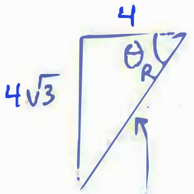


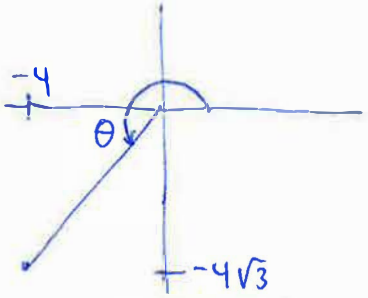


$\left(2, -\frac{\pi}{6}\right)$

$= \left(-2, \frac{5\pi}{6}\right)$

(e) $\left(-4, -4\sqrt{3}\right)$





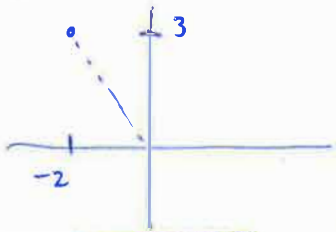
$|r| = \sqrt{4^2 + (4\sqrt{3})^2} = 4\sqrt{1+3}$
$= 8$

$\tan \theta_R = \frac{4\sqrt{3}}{4} = \sqrt{3}$

$\theta_R = \frac{\pi}{3}$. So $\theta = \frac{4\pi}{3}$ (one choice).

So $\left(8, \frac{4\pi}{3}\right) = \left(-8, \frac{\pi}{3}\right)$

(f) $(-2, 3)$



$|r| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$.

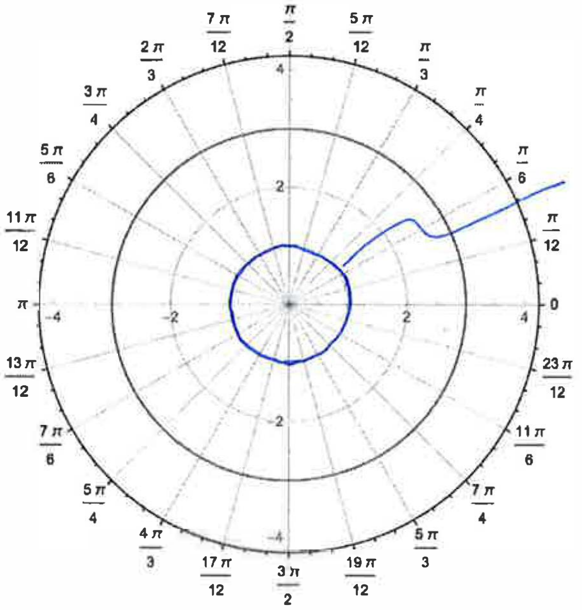
$\tan \theta = -\frac{3}{2}$. $\theta_R = \tan^{-1}\left(\frac{3}{2}\right)$

So one choice for θ is $\pi - \tan^{-1}\left(\frac{3}{2}\right)$.

$\left(\sqrt{13}, \pi - \tan^{-1}\left(\frac{3}{2}\right)\right) = \left(-\sqrt{13}, \tan^{-1}\left(-\frac{3}{2}\right)\right)$

4. Sketch the curve with the given polar equation.

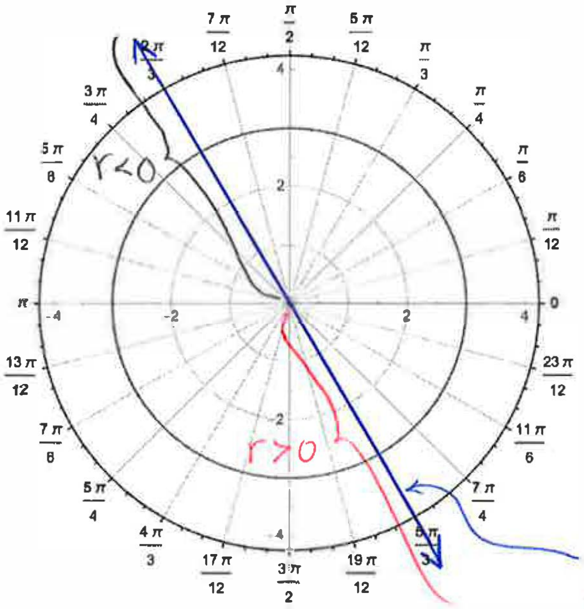
(a) $r = -1$



circle of radius 1 centered at the origin.

$x^2+y^2=1$

(b) $\theta = -\frac{\pi}{3}$



line of slope $-\sqrt{3}$ passing through $(0,0)$

$y = -\sqrt{3}\,x$

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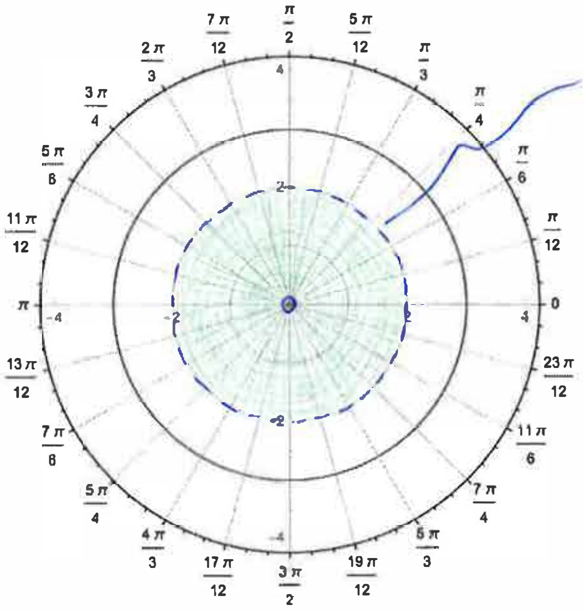
5. Sketch and shade the region of points that satisfy each polar inequality.

(a) $0 < r < 2$



Disk of radius 2 centered at (0,0), but with the boundary circle and origin not included.

(b) $\dfrac{7\pi}{2} \leq \theta < 4\pi$