Newton's Laws

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1 Review

(under construction)

2 Newton's Laws

2.1 Newton's First Law

But, Newton based much of his new theories on relativistic motion, maintaining that whatever physics we devise in one frame of motion, they should also apply in any frame of motion.

This meant that if an observer was to ride alongside an object that was moving at constant velocity (that is, at a constant speed and direction) then the observer would note that, in his or her inertial reference frame, the object was not moving at all. Therefore, the "force of propulsion" would disappear. (The term "inertial" simply means *non-accelerating*.)

Newton surmised (correctly) that the easiest way to alleviate this inconsistency was to abandon the idea of a "force of propulsion" altogether. In his view, an object doesn't move because there is necessarily something pushing or pulling on it. Rather, the object moves because the object inherently does not want to change its motion.

- If the object was moving at 56 mph, it would stay moving at 56 mph if unmolested.
- If the object was moving north, it would stay moving north if unmolested.
- If the object was spinning at a certain rate, it would remain spinning at that rate if unmolested.

This viewpoint meshed perfectly with Newton's insistence that the physics of a problem remain identical if one chose a different inertial reference system to view the motion. If an object was moving in a certain direction at a certain speed, an observer could always move along with the object, freezing it in his/her reference frame. In that frame, the object would be motionless.

Therefore, another way of stating this claim is that an object always wants to remain at rest, and we can always make an object remain at rest by moving in an inertial reference frame alongside the object. However, because we can always choose an inertial reference frame such that the object is not at rest, in which case the object would move at a constant velocity, then we have to consider that it is perfectly natural for an object to move at a constant velocity as well.

This view gradually became accepted by the physics community as a whole, therefore becoming a law called Newton's First Law of Motion:

> In any inertial (non-accelerating) reference frame, an object's natural tendency is to either remains at rest or move at a constant velocity.

More colloquially, this is often stated as "An object at rest wants to remain at rest; an object moving at a certain velocity wants to remain moving at that velocity."

2.2 Newton's Second Law

Newton's First Law alone is insufficient to explain what we see around us. Objects are constantly moving. For some of them, we can move in an inertial reference frame and freeze their motion, at least in our frame of view. But when an object accelerates, that is no longer possible. If we look around, we see objects accelerating all the time. Clearly, Sir Isaac Newton had a problem. He solved it.

Newton's First Law describes objects moving at a constant velocity. But what if we don't want the object to move at a constant velocity? In other words, what if we want to *accelerate* it?

To accelerate an object, Newton devised another law, which states that a net external force is required. In this guise, a force is a push or pull exerted by something external to the object. For example, a nearby planet could pull on the object with its gravitational force, or a person could push on an object with an applied force. Quantitatively, a force is measured in newtons in SI units, with 1 newton about the force needed to hold up an apple or a hamburger. (In the British system, a force is measured in the familiar pounds, so it takes about a quarter-pound to hold up a hamburger, hence the nickname "quarter-pounder.")

How much external force is required to accelerate an object? The answer to that question depended on two things:

1. How much we want to accelerate the object.

2. How resistant the object is to changes in its motion.

Newton encapsulated both concepts into a single equation, which is perhaps the most important equation ever devised in science. We call this equation *Newton's Second Law.*

$$\vec{a} = \frac{\Sigma \vec{F}}{m} \quad \longrightarrow \quad \Sigma \vec{F} = m\vec{a} \tag{1}$$

(Notice that the equation on the right – which is the more familiar form – is the same as on the left. It makes no difference which form we use.)

We should note a few things about Newton's Second Law:

1. The equation is *not* expressed as merely $F = ma$, as it is written in many physics textbooks. This equation only applies to the simplest of systems and is generally incorrect.

2. The force dependence in the equation is a *vector sum of all* forces acting on the body. This vector sum itself is *not* a force – it is the net effect of all forces.

3. For this reason, Newton's second law is often expressed as $\vec{F}_{\text{net}} = m\vec{a}$, where $\Sigma\vec{F} = \vec{F}_{\text{net}}$. The terms *net force* and *vector sum of all forces* are synonymous.

4. If there is a net force acting on a body (that is, if the vector sum of all forces is nonzero), then the body *must* accelerate. The direction of this acceleration is in the same direction as the net force.

5. If an object is accelerating, then there must be a net force acting on it. This net force must be pointing in the same direction as the acceleration.

6. If an object is not accelerating, then all forces acting on the object must cancel.

7. If all the forces acting on an object cancel, then the object cannot accelerate.

8. The fact that the sum of forces in Newton's Second Law is, in fact, a *vector* sum means that forces can cancel each other. So it is perfectly possible for a body to be acted on by numerous forces and still not accelerate if the forces cancel.

Note in the above the importance of distinguishing between acceleration and " moving." If an object is moving at constant velocity, it is not accelerating. In such a case, the vector sum of all forces acting on the body must be 0.

Before moving on, I want to express Newton's Second Law more pedantically, which will help us greatly when we turn our attention toward systems involving multiple bodies. In words, we can say that

> The vector sum of all forces acting on a system equals the mass of that system times the acceleration of that system.

In equation form:

$$\Sigma\vec{F}_{\text{acting on a system}} = m_{\text{of that system}}\vec{a}_{\text{of that system}}. \tag{2}$$

2.3 Mass

When we described Newton's second law, we stated that the net force needed to accelerate an object depended on the object's natural willingness to change its motion. That is, some objects are harder to accelerate than others. As extreme examples, an oil tanker is relatively tough to accelerate – simply pushing on it with one's hand does little to make it budge. On the other hand, a Cheez-it cracker is relatively easy to acceleration – blowing on it is often all that is needed.

This natural tendency of an object to resist changes in its motion is called *inertia*. Objects with lots of inertia are highly resistant to changes in their motion, and so forth.

Inertia is a conceptual term. In science, we need quantitative terms so that we can assign numerical values to signify magnitude. We call the quantitative measure of inertia *mass* and measure it in grams, with the understanding that 1,000 grams is a kilogram.

A Cheez-it cracker has a mass of roughly 1 gram. However, the kilogram is the SI unit of mass and is the unit we will rely on in this course to produce results in SI units. Your textbook has a mass of roughly 1 kilogram. A 100 kilogram man is fairly good size, like a running back on a football team. A small car has a mass of roughly 1,000 kilograms.

If we go back to Newton's Second Law, we can deduce the impact of mass on acceleration:

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

According to Newton's Second Law, the larger the mass, the less the acceleration. But this makes sense given that mass measures an object's unwillingness to accelerate.

Hopefully, this all makes so far. Now, we have to turn our attention to examples to flesh out the meaning of Newton's Second Law.

2.3.1 Example 1 – Simple body

Figure 1 shows a $m = 2$ kg body acted on by two forces, one of $\vec{F}_1 = 8$ newtons pushing to the left and one of $\vec{F}_2 = 6$ newtons pushing to the right.

The source of these forces is ambiguous but we know they are there because our problem explicitly states their existence. We call such forces *applied forces*, which we usually denote as $\vec{F}_{\mathrm{A}}$. In this example, we have instead denoted them as $\vec{F}_1$ and $\vec{F}_2$.

We will assume that the object is located in outer space far from any noticeable gravitational effects. If we decide that "toward the right" is the positive direction, then Newton's Second Law states that the acceleration of the body will be

$$\vec{a}_{\mathrm{x}} = \frac{\vec{F}_1 - \vec{F}_2}{m} = \frac{6-8}{2} = -1\mathrm{m/s}^2.$$

What about the negative sign? Because Newton's Second Law is a vector equation, then the polarity of our answers (that is, positive or negative) has meaning. Here, it means "to the left." Therefore, this object is accelerating toward the left at 1 $\mathrm{m/s}^2$.

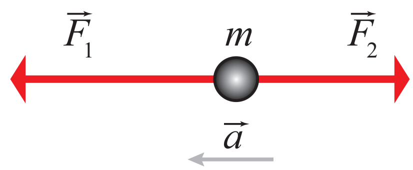

Figure 1: A simple problem for illustrating how to extract the acceleration of an object using Newton's Second Law.

Notice that in the above I did not substitute units of measure into the equation. I can do that because the SI unit of force is the newton and the SI unit of mass is the kilogram. As long as I have expressed all of my units in SI units, then I am guaranteed that (1) the equations will provide a numerically accurate answer and (2) the answer will be in SI units. Therefore, the "1" I received for my answer is actually 1 $\mathrm{m/s}^2$, the natural SI unit of acceleration.

2.4 Force Inventory

If we are to use Newton's Second Law to extract the acceleration of a body, then we need to know which forces act on the body, how strong they are (that is, their magnitude), and in which direction they point.

Although a body can be acted upon by any number of forces, there are relatively few types of forces that we need to consider in this class. The following table illustrates the most important ones that will arise.

(table under construction)

From this table, we see that gravity acts on a body by applying a force of magnitude mg, where m is the mass of the body and g is the usual gravitational acceleration constant. We also know the direction this force acts – always straight down.

There is also another force listed called the normal force $\vec{F}_N$. It is not obvious where this force arises, but a simple example will hopefully make it clear.

2.4.1 Example 2 – Gravity and normal force

We are going to take the simple problem we had in Example 1 and remove the stipulation that the body was in far outer space. In fact, we will have the body placed on a surface. Although our original forces $\vec{F}_1$ and $\vec{F}_2$ will still act on the body, the fact that the object is now in the Earth's gravitational field means it has weight and, as a consequence, wants to accelerate downward.

But it doesn't because the table is holding it up. Because the object is not accelerating downwards, then we know the vector sum of all forces acting vertically must cancel. This can only happen if there is a force pointing upward counteracting the force of gravity. And there is – we call it the *normal force*.

We can now see why it is so important to realize that if an equation is expressed as a vector equation, we can express it for any direction we choose. Here, we can say that, if $\vec{a} = \frac{\Sigma\vec{F}}{m}$ then

$$\vec{a}_{\text{x}} = \frac{\Sigma\vec{F}_{\text{x}}}{m},$$

$$\vec{a}_{\text{x}} = \frac{\Sigma\vec{F}_{\text{y}}}{m},$$

where we can denote the x-direction as horizontal and the y-direction as vertical.

Newton's Second Law for the x-direction is nothing more than our previous result:

$$\vec{a}_{\text{x}} = \frac{\vec{F}_1 - \vec{F}_2}{m} = \frac{6-8}{2} = -1\text{m/s}^2$$

. However, for the vertical direction, we note that the object is not acceleration along this direction.

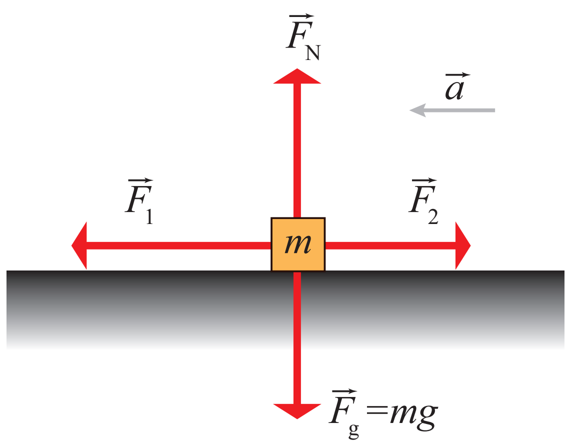

Figure 2: We revise the simple problem in Figure 1 to account for the gravitational and normal force. In this problem, the object has been placed on top of a table.

(The only way it could accelerate up or down is to either crash through the table or hop off the table.) Therefore

$$\vec{a}_{\text{x}} = \frac{\Sigma\vec{F}_{\text{y}}}{m} \quad \longrightarrow \quad 0 = \frac{\vec{F}_N - mg}{m}.$$

This means that the normal force $\vec{F}_N$ *in this problem* offsets the force of gravity.

If I had pressed down on the block, the normal force would not only need to offset the force of gravity acting on the block, it would also have to offset my applied force as well. Therefore, it is a mistake to assume that $\vec{F}_N = mg$ – *this is only true in certain instances.*

The normal force is provided by the molecular bonds between molecules in the surface. We can liken the normal force as a support force provided by a surface to prevent objects from crashing through. The normal force provided by a surface will rise to keep the acceleration 0. However, every surface is limited in the maximum normal force it can provide. Beyond that, the surface breaks and whatever object is in contact with the surface will accelerate through it.

Before moving on, let us discuss the direction of the normal force. This one is easy. *The normal force provided by a surface on an object always points in a direction perpendicular to the surface and toward the object.* For example, the block on the plane in Figure 3 does not point straight up, but rather points perpendicular to the plane.

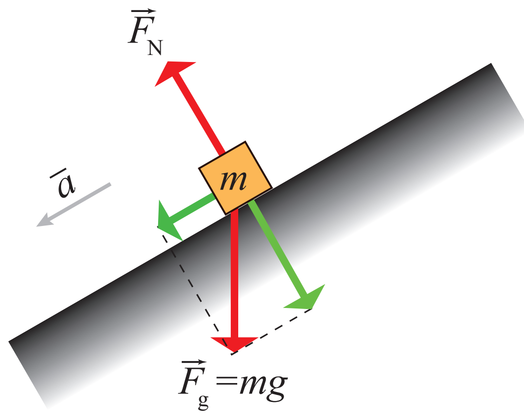

Figure 3: We revise the simple problem in Figure 2 by placing the object on an inclined plane rather than a flat surface. In this case, the force of gravity still acts downward. However, the normal force, which always points perpendicular to the surface providing the normal force, does not point down. Here, the force of gravity has two "good vector" components shown in green – one that points along the ramp and one perpendicular to the ramp. The component along the ramp is unbalanced, therefore the object will accelerate down the ramp.

Finally, I would like to note that floor scales (that is, the scales that one stands on to measure their weight) do not in fact measure weight at all. They measure the normal force they must exert to support the object resting on them. As such, weight scales are "normal force o'meters."

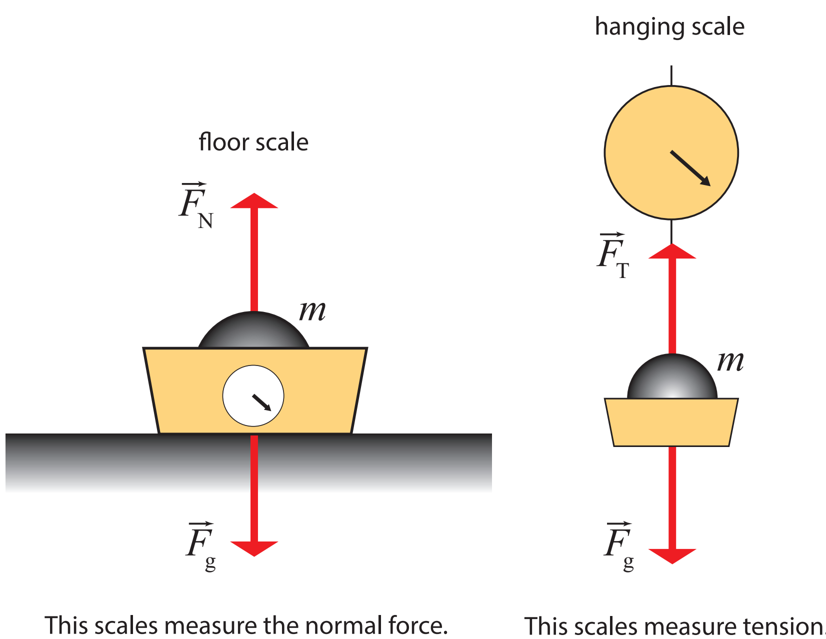

Figure 4: A floor scale (for example, a bathroom scale) does not measure weight; it measures the normal force it must exert to prevent the object on it from accelerating through it. Similarly, a a hanging scale measures tension, not weight.

2.5 Tension force

We know from the previous section that a surface can exert a force on an object. But so can a rope. Typically, ropes always *pull* on objects, as pushing on a rope does little good.

We use the term *tension force* to describe the force exerted by chains, ropes, strings, and so on. We denote this force as $\vec{F}_{\mathrm{T}}$. Because we can always pull on a rope with variable amounts of force, we cannot assign a magnitude to it – the amount of tension force exerted by a rope depends on the other forces acting on the body. However, we can assign a direction. Because we cannot push on a rope, the direction of a tension force is always along the rope but *away* from the body.

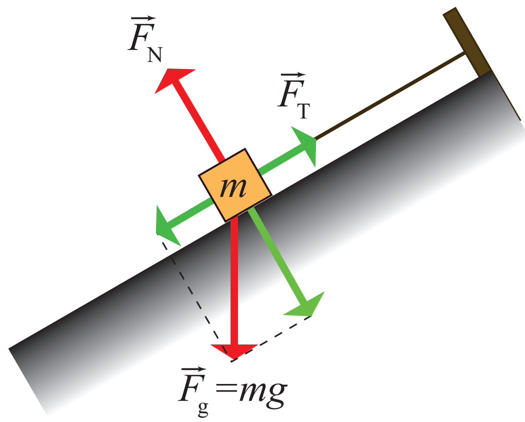

Figure 5: We revise the problem in Figure 3 by attaching the mass to a rope, which is tied to a post at the top of the ramp. The tension force provided by the rope prevents the body from accelerating down the ramp. In this case, the acceleration $\vec{a} = 0$, which means it is 0 in all directions. As such, the tension force, which always points along the rope and away from the object, must be of such magnitude that it counteracts the component of the gravitational force pointing down the ramp.

Hanging scales (the kind you see in a fruit market for weighing tomatoes) do not measure weight no more than a floor scale does. They measure the tension necessary to prevent the object hanging on the scale from accelerating downwards. As such, we can call a hanging scale a "tension force o' meter." (See 4 above.)

2.6 Friction

(under construction)