Lab Activity 1

① $y = 3x$ $\qquad$ $y = -2x^2 + 7x$ $\qquad$ $[0, 2]$

$$\int_0^2 (-2x^2 + 7x) - (3x)\, dx$$

$$-2x^2 + 7x - 3x$$

$$\int_0^2 -2x^2 + 4x\, dx$$

$$-\frac{2x^3}{3} + 2x^2 \Big|_0^2 \quad \rightarrow \quad \frac{8}{3} - 0 \quad \rightarrow \quad \boxed{\frac{8}{3}}$$

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② $y = x^2 - 1$ $\qquad$ $y = x^3 - 1$ $\qquad$ $[-1, 1]$

$$\int_{-1}^0 (x^2 - 1) - (x^3 - 1) + \int_0^1 (x^2 - 1) - (x^3 - 1)$$

$$x^2 \cancel{-1} - x^3 \cancel{+1}$$

$$\int_{-1}^0 -x^3 + x^2\, dx + \int_0^1 -x^3 + x^2$$

$$\int_{-1}^0 -\frac{x^4}{4} + \frac{x^3}{3}\, dx + \int_0^1 -\frac{x^4}{4} + \frac{x^3}{3}$$

$$\overset{0}{(0)} - \overset{-1}{\left(+\left(\frac{1}{4}\right) - \frac{1}{3}\right)} + \overset{1}{\left(-\frac{1}{4} + \frac{1}{3}\right)} - (0)$$

$$-\frac{7}{12} \quad + \quad \frac{1}{12}$$

$$\boxed{\frac{2}{3}}$$

③ $y = 7x$ $y = x(x^2 - 9)$ $[-4, 0]$ $[0, 4]$

$$\int_{-4}^{0} (x(x^2-9)) - (7x)\, dx + \int_{0}^{4} (7x) - (x(x^2-9))\, dx$$

$$\int_{-4}^{0} x^3 - 9x - 7x\, dx + \int_{0}^{4} 7x - x^3 + 9x\, dx$$

$$\int_{-4}^{0} x^3 - 16x\, dx + \int_{0}^{4} -x^3 + 16x\, dx$$

$$\frac{x^4}{4} - \frac{16x^2}{2}\Big|_{-4}^{0} + \frac{-x^4}{4} + \frac{16x^2}{2}\Big|_{0}^{4}$$

$$\frac{x^4}{4} - 8x^2\Big|_{-4}^{0} + \frac{-x^4}{4} + 8x^2\Big|_{0}^{4}$$

$$(0) - (-64 - 128) + \left(\frac{-256}{4} + 128\right) - (0)$$

$$\frac{256}{4} - 128$$

$$64 \qquad + \qquad 64$$

$$\boxed{128}$$

②

Lab Activity 1

$y = \sqrt{\frac{x}{2} + 1}$ $y = \sqrt{1-x}$ $[-2, 0]$ $[0, 1)$

a.I $\int_{-2}^{0} \left(\sqrt{\frac{x}{2}+1}\right) dx + \int_{0}^{1} \left(\sqrt{1-x}\right) dx$

$-\left(\frac{2(u)^{\frac{3}{2}}}{3}\right)\Big|_{1}^{0}$

$2\left(\frac{2}{3}\right) - (0) + (0) - \left(-\frac{2}{3}\right)$

$U = \frac{x}{2} + 1$ $U = 1-x$

$du = \frac{1}{2} dx$ $+$ $du = -1 dx$

$\frac{4}{3} + \frac{2}{3}$

$2\int_{0}^{1} \sqrt{U} \, du$ $-\int_{1}^{0} \sqrt{U}$

$\boxed{2}$

$\frac{2(U)^{\frac{3}{2}}}{3}\Big|_{0}^{1} +$ $-\frac{2(U)^{\frac{3}{2}}}{3}\Big|_{1}^{0}$

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b.I $y = \sqrt{\frac{x}{2} + 1}$ $y = \sqrt{1-x}$

$x = -y^2 + 1$

$\boxed{\begin{array}{l} x = 2y^2 - 2 \\ x = -y^2 + 1 \end{array}}$

b.II $\int_{0}^{1} (2y^2 - 2) - (-y^2 + 1) \, dy$

$2y^2 - 2 + y^2 - 1$

$\int_{0}^{1} 3y^2 - 3 \, dy$

$\frac{3y^3}{3} - 3x \Big|_{0}^{1} \longrightarrow y^3 - 3x \Big|_{0}^{1}$

$(1-3) - (0)$

$\boxed{2}$

b.III

2

C

The drawback to keeping everything in terms of x is that you need to use u substitution. The Advantage of changing everything in terms of y is that you don't need to use u substitution. the integral is much easier to calculate in terms of y in this case