Math 76 Exercises –3.1A Integration by Parts

Evaluate each integral. Check by differentiating.

1. $\int x \ln x \, dx$

$u = \ln x \qquad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx \qquad dv = x\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{x^2}{x}\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx = \boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

2. $\int (\ln x)^2 \, dx$

$u = (\ln x)^2 \qquad v = x$

$du = 2\ln x \cdot \frac{1}{x}dx \qquad dv = dx$

$= x(\ln x)^2 - 2\int \ln x\,dx$

$u = \ln x \qquad v = x$

$du = \frac{1}{x}dx \qquad dv = dx$

$= x(\ln x)^2 - 2\left(x\ln x - \int 1\,dx\right)$

$= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C}$

3. $\int (x-3)\sin(5x+1)\,dx$

$u = x-3 \qquad v = -\frac{1}{5}\cos(5x+1)$

$du = dx \qquad dv = \sin(5x+1)\,dx$

$= -\frac{1}{5}(x-3)\cos(5x+1) + \frac{1}{5}\int \cos(5x+1)\,dx$

$= -\frac{1}{5}(x-3)\cos(5x+1) + \frac{1}{5}\cdot\frac{1}{5}\sin(5x+1) + C$

$= \boxed{-\frac{1}{5}(x-3)\cos(5x+1) + \frac{1}{25}\sin(5x+1) + C}$

4. $\int (x^2+5)e^{2x}\,dx$

$u = x^2+5 \qquad v = \frac{1}{2}e^{2x}$

$du = 2x\,dx \qquad dv = e^{2x}\,dx$

$= \frac{1}{2}(x^2+5)e^{2x} - \int x e^{2x}\,dx$

$u = x \qquad v = \frac{1}{2}e^{2x}$

$du = dx \qquad dv = e^{2x}\,dx$

$= \frac{1}{2}(x^2+5)e^{2x} - \left(\frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x}\,dx\right)$

$= \boxed{\frac{1}{2}(x^2+5)e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C}$

5. $\int x^3 \cos(4x^2-2)\,dx$

Note: If we try $dv = \cos(4x^2-2)\,dx$, this is hard to integrate!

$u = x^2 \qquad v = \frac{1}{8}\sin(4x^2-2)$

$du = 2x\,dx \qquad dv = x\cos(4x^2-2)\,dx$

$= \frac{1}{8}x^2\sin(4x^2-2) - \frac{1}{4}\int x\sin(4x^2-2)\,dx$

$= \frac{1}{8}x^2\sin(4x^2-2) - \frac{1}{4}\cdot\frac{1}{8}(-\cos(4x^2-2)) + C$

$= \boxed{\frac{1}{8}x^2\sin(4x^2-2) + \frac{1}{32}\cos(4x^2-2) + C}$

6. $\int e^{\sqrt{x}}\,dx$

$= 2\int \frac{\sqrt{x}\,e^{\sqrt{x}}}{2\sqrt{x}}\,dx \qquad t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\,dx$

$= 2\int t e^t\,dt \qquad u = t \qquad v = e^t$

$du = dt \qquad dv = e^t\,dt$

$= 2\left(te^t - \int e^t\,dt\right)$

$= 2te^t - 2e^t + C$

$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = \boxed{2e^{\sqrt{x}}(\sqrt{x}-1) + C}$