Math 76 Assessment 14 §5.4 Comparison Tests

Things to remember:

1. The problem to complete is shown below. Write your name and solution on the next page where instructed.

2. Please make sure your full name is written neatly in the box.

3. Your score will be determined by **Mechanics** (2 points) and by **Content** (3 points).

4. The following rubric will be used for **Mechanics**:

Clear neat work, steps in order and easily followed, proper use of notation	2
Mostly clear work; minor errors in notation or skipped steps	1.5
Steps/handwriting hard to follow/read; major errors in notation	1
No discernible or relevant work, or work impossible to read/follow	0

5. You are not allowed to consult outside sources, including notes, books, the internet, or other people, while taking this assessment. Calculators are allowed only for basic numerical or scientific computations, not for graphing or algebra.

6. If you need more room, you may finish on a plain piece of paper or blank document. If you do all your work on separate sheets, please **copy the problem** and make sure to write **Version C** at the top of the first page.

7. When you are finished, create a legible, well-lit **.pdf file** of your work and upload it to Assessment 14 on Gradescope. Please follow the directions to **assign the page(s)** of your submission that contain your work for the question. More info about submitting to Gradescope:

   `http://bit.ly/gradescope-help`

Use either the Direct Comparison Test or the Limit Comparison Test to evaluate the series $\sum a_n$ on the next page.

Your solution should include:

- (0.5 point) Statement of the test to be used and an explanation of why the test is valid for the series;
- (0.5 point) Identification of a valid comparison series $\sum b_n$;
- (1 point) Explanation of why $\sum b_n$ is comparable to the series in the problem;
- (0.5 point) Explanation of why $\sum b_n$ converges or diverges;
- (0.5 point) Correct conclusion ($\sum a_n$ converges/diverges), with explanation.

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Version C

Follow the directions on the previous page. Let

$$a_n = \frac{2n-7}{4n^{5/3}+n+1}.$$

Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

$a_n = \frac{2n-7}{4n^{5/3}+n+1}$.

Comparison test

$a_n \leq b_n$ for all n

$a_n \leq b_n$ ✗

So we cannot use the Comparison test because b_n is Less than a_n.

$b_n = \frac{2n}{4n^{\frac{5}{3}}}$

$b_n = \frac{1}{2n^{\frac{2}{3}}}$

Limit comparison test

$a_n = \frac{2n-7}{4n^{5/3}+n+1}$.

$a_n \geq 0$ for all n ✓ $\quad 0 < C < \infty$

$b_n \geq 0$

$b_n = \frac{1}{2n^{\frac{2}{3}}}$

$$C = \lim_{n\to\infty} \frac{a_n}{b_n}$$

$$= \lim_{n\to\infty} \frac{2n-7}{4n^{\frac{5}{3}}+n+1} \cdot \frac{2n^{\frac{2}{3}}}{1} = \lim_{n\to\infty} \frac{4n^{\frac{5}{3}} - 14n^{\frac{2}{3}}}{4n^{\frac{5}{3}}+n+1}$$

$$= \lim_{n\to\infty} \frac{4n^{\frac{5}{3}}}{4n^{\frac{5}{3}}} \to \boxed{1}$$

because $C = 1$ and $0 < 1 < \infty$ so by the Limit Comparison test both series have the same out come. Both series Converge or diverge.

$\lim_{n\to\infty} \frac{1}{2n^{\frac{2}{3}}}$

based on the p series test $p > 1$

$p = \frac{2}{3}$, $p < 1$ This series diverges. ✓

both our series would diverge