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Spring 2021 MATH 76
Activity 3

1. VOLUMES

   (a) Find the volume of a solid of revolution formed by revolving the region bounded above by the graph of $f(x) = x + 2$ and below by the $x-$axis over the interval $[0,\ 3]$ around the line $y = -1$.

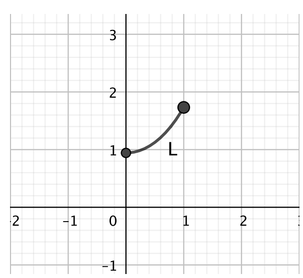
   (b) Define R as the region bounded above by the graph of $f(x) = x^2$ and below by the x-axis over the interval $[0,\ 1]$. Find the volume of the solid of revolution formed by revolving R around the line $x = -2$.

   (c) Define R as the region bounded above by the graph of $f(x) = x$ and below by the graph of $g(x) = x^2$ over the interval $[0,\ 1]$. Find the volume of the solid of revolution formed by revolving R around the $y-$axis.

   (d) Select the best method to find the volume of a solid of revolution generated by revolving the given region around the x-axis, and set up the integral to find the volume (do not evaluate the integral): the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$.

2. ARC LENGTH

   (a) The goal of this problem is to compute the arc length of the curve $y = \dfrac{(x^2+2)^{3/2}}{3}$ when $0 \leq x \leq 1$. In class you were taught that $L = \displaystyle\int_0^1 \sqrt{1 + (y')^2}dx$ where y' is the first derivative of y. It is often not trivial to compute that integral and using a step by step approach may help avoid mistakes.



      i. Determine y'.

      ii. Compute and simplify if possible, the expression $\sqrt{1 + (y')^2}$.

      iii. Compute $L = \displaystyle\int_0^1 \sqrt{1 + (y')^2}dx$

   (b) Let $f(x) = \sin x$. Calculate the arc length of the graph of $f(x)$ over the interval $[0,\ \pi]$. Use a computer or calculator to approximate the value of the integral.

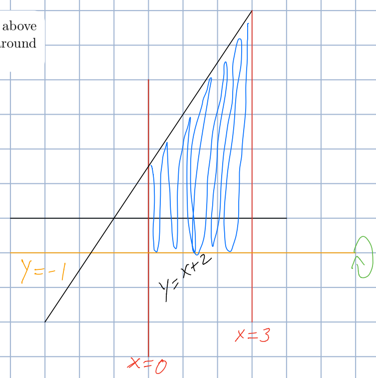
   (c) Let $x = g(y) = \dfrac{1}{y}$. Calculate the arc length of the graph of $x = g(y)$ over the interval $[1,\ 4]$. Use a computer or calculator to approximate the value of the integral.

(a) Find the volume of a solid of revolution formed by revolving the region bounded above by the graph of $f(x) = x + 2$ and below by the x−axis over the interval [0, 3] around the line $y = -1$.

$$\pi\int_0^3 \left((x+3)^2 - (1)^2\right)dx \qquad x+2-(-1)$$

$$\pi\left(\frac{(x+3)^3}{3} - x\right)\Big|_0^3 \qquad 0-(-1)$$

$$\pi(72-3-9-\emptyset) = \boxed{60\pi}$$



60π

(b) Define R as the region bounded above by the graph of $f(x) = x^2$ and below by the x-axis over the interval [0, 1]. Find the volume of the solid of revolution formed by revolving R around the line $x = -2$.

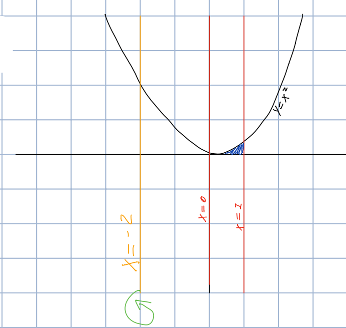
$$0-(-2) = 2$$

$$x^2-(-2) = x^2+2$$

$$2\pi\int_0^1 (x+2)(x^2)\,dx$$

$$2\pi\int_0^1 x^3+2x^2$$

$$2\pi\left(\frac{x^4}{4}+\frac{2}{3}x^3\right)\Big|_0^1 = 2\pi\left(\frac{1}{4}+\frac{2}{3}\right) = 2\pi\left(\frac{11}{12}\right) = \boxed{\frac{11\pi}{6}}\ \checkmark$$

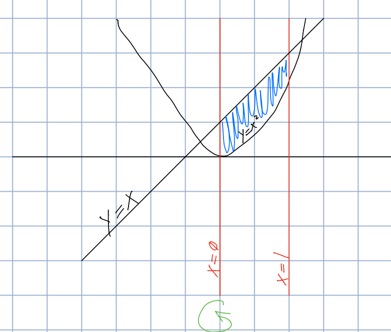


(c) Define R as the region bounded above by the graph of $f(x) = x$ and below by the graph of $g(x) = x^2$ over the interval [0, 1]. Find the volume of the solid of revolution formed by revolving R around the y−axis.

$$2\pi\int_0^1 x\left((x)-(x^2)\right)dx$$

$$2\pi\int_0^1 x^2-x^3\,dx$$

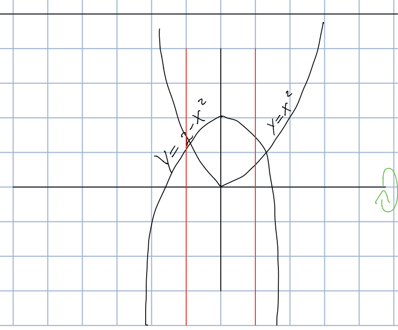
$$2\pi\left(\frac{x^3}{3}-\frac{x^4}{4}\right)\Big|_0^1 = 2\pi\left(\frac{1}{3}-\frac{1}{4}\right) = \boxed{\frac{\pi}{6}}\ \checkmark$$



(d) Select the best method to find the volume of a solid of revolution generated by revolving the given region around the x-axis, and set up the integral to find the volume (do not evaluate the integral): the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$.

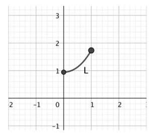
The disc method would be the best

$$\boxed{\pi\int_0^1 x\left((2-x^2)^2-(x^2)^2\right)dx}$$



x=-1 x=1

(a) The goal of this problem is to compute the arc length of the curve $y = \frac{(x^2+2)^{3/2}}{3}$ when $0 \le x \le 1$. In class you were taught that $L = \int_0^1 \sqrt{1+(y')^2}\,dx$ where y' is the first derivative of y. It is often not trivial to compute that integral and using a step by step approach may help avoid mistakes.



i. Determine y'.

ii. Compute and simplify if possible, the expression $\sqrt{1+(y')^2}$.

iii. Compute $L = \int_0^1 \sqrt{1+(y')^2}\,dx$

$$y = \frac{(x^2+2)^{\frac{3}{2}}}{3} \qquad [0,1]$$

I

$$\frac{1}{3} \cdot (x^2+2)^{\frac{3}{2}}$$

$$= \frac{\frac{3}{2}(x^2+2)^{\frac{3}{2}-\frac{2}{2}} \cdot (x^2+2)}{3}$$

$$= \frac{(x^2+2)\sqrt{x^2+2}}{2}$$

$$\boxed{y' = x\sqrt{x^2+2}}\ \checkmark$$

II

$$\sqrt{1+(x\sqrt{x^2+2})^2} = \sqrt{1+x^2(x^2+2)}$$

$$= \sqrt{x^4+2x^2+1} = \sqrt{(x^2+1)^2} = \boxed{x^2+1}\ \checkmark$$

III

$$\int_0^1 \sqrt{x^4+2x^2+1}\,dx$$

$$x^4+2x^2+1$$

$$(x^2+1)^2$$

$$= \int_0^1 \sqrt{(x^2+1)^2} = \int_0^1 (x^2+1)\,dx = \frac{x^3}{3}+x\Big|_0^1 = \frac{1}{3}+1 = \boxed{\frac{4}{3}}\ \checkmark$$

(b) Let $f(x) = \sin x$. Calculate the arc length of the graph of $f(x)$ over the interval $[0, \pi]$. Use a computer or calculator to approximate the value of the integral.

$$L = \int_0^{\pi} \sqrt{1+(\cos x)^2}\,dx \qquad y = \sin x$$

$$y' = \cos x$$

$$= \int_0^{\pi} \sqrt{1+\cos^2 x}\,dx$$

$$\boxed{= 3.82019779}\ \checkmark$$

(c) Let $x = g(y) = \frac{1}{y}$. Calculate the arc length of the graph of $x = g(y)$ over the interval [1, 4]. Use a computer or calculator to approximate the value of the integral.

$$= \int_1^4 \sqrt{1 + \left(-\frac{1}{y^2}\right)^2}\, dy$$

$$x = \frac{1}{y}$$

$$x' = -\frac{1}{y^2}$$

$$= \int_1^4 \sqrt{1 + y^{-4}}\, dy$$

$$\boxed{= 3.15018387}$$