Evaluate each integral. Check by differentiating.

1. $\int \underbrace{\sin^{-1}(x)}_{u} \, \underbrace{dx}_{dv}$

$u = \sin^{-1} x \qquad v = x$

$du = \dfrac{1}{\sqrt{1-x^2}}\, dx \qquad dv = dx$

$= x \sin^{-1} x - \int \dfrac{x}{\sqrt{1-x^2}}\, dx$

$= x \sin^{-1} x - \left(-\frac{1}{2}\right) \int \dfrac{-2x}{\sqrt{1-x^2}}\, dx$

$u = 1 - x^2$
$du = -2x\, dx$

$= x \sin^{-1} x + \frac{1}{2} 2 \int \dfrac{1}{2\sqrt{u}}\, du$

$= x \sin^{-1} x + \sqrt{u} + C = \boxed{x \sin^{-1} x + \sqrt{1-x^2} + C}$

2. $\int x^2 \sin^{-1}(5x^3 - 4)\, dx$

$\frac{1}{15} \int 15x^2 \sin^{-1}(5x^3 - 4)\, dx$

$u = 5x^3 - 4$
$du = 15x^2\, dx$

$= \frac{1}{15} \int \sin^{-1}(u)\, du$

$= \frac{1}{15}\left(u \sin^{-1} u + \sqrt{1-u^2}\right) + C$ ... from #1

$= \boxed{\frac{1}{15}\left((5x^3-4)\sin^{-1}(5x^3-4) + \sqrt{1-(5x^3-4)^2}\right) + C}$

3. (*) $\displaystyle\int_1^{\sqrt[4]{3}} x\tan^{-1}(x^2)\,dx$

$$= \frac{1}{2}\int_1^{\sqrt[4]{3}} 2x\tan^{-1}(x^2)\,dx$$

$$t = x^2 \qquad x=\sqrt[4]{3} : t=\sqrt{3}$$
$$dt = 2x\,dx \qquad x=1 : t=1^2=1$$

$$= \frac{1}{2}\int_1^{\sqrt{3}} \tan^{-1}(t)\,dt$$

$$u = \tan^{-1}t \qquad v = t$$
$$du = \frac{1}{1+t^2}dt \qquad dv = dt$$

$$= \frac{1}{2}\left(t\tan^{-1}t\,\Big|_1^{\sqrt{3}} - \frac{1}{2}\int_1^{\sqrt{3}}\frac{2t}{1+t^2}\,dt\right)$$

$$= \frac{1}{2}\left(t\tan^{-1}t - \frac{1}{2}\ln|1+t^2|\right)\Big|_1^{\sqrt{3}}$$

$$= \frac{1}{2}\left[\left(\sqrt{3}\tan^{-1}(\sqrt{3}) - \frac{1}{2}\ln(1+3)\right) - \left(1\cdot\tan^{-1}(1) - \frac{1}{2}\ln(1+1)\right)\right]$$

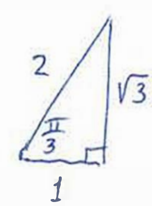
$$= \frac{1}{2}\left[\sqrt{3}\cdot\overset{①}{\frac{\pi}{3}} - \frac{1}{2}\ln(4) - \overset{②}{\frac{\pi}{4}} + \frac{1}{2}\ln(2)\right]$$

$$= \frac{1}{2}\left[\frac{\pi\sqrt{3}}{3} - \frac{1}{\cancel{2}}\cdot\overset{③}{\cancel{2}}\ln 2 - \frac{\pi}{4} + \frac{1}{2}\ln(2)\right]$$

$$= \frac{1}{2}\left[\frac{\pi\sqrt{3}}{3} - \frac{\pi}{4} - \frac{\ln(2)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{(4\sqrt{3}-3)\pi}{12} - \frac{\ln(2)}{2}\right]$$

$$= \boxed{\frac{(4\sqrt{3}-3)\pi - 6\ln(2)}{24}}$$

① 

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

② 

$\tan^{-1}(1) = \frac{\pi}{4}$

③ $\ln(4) = \ln(2^2) = 2\ln 2$

4. $\int e^x \cos 2x \, dx$

Hint: use parts twice and solve for the integral.

$u = e^x \qquad v = \frac{1}{2}\sin(2x)$

$du = e^x dx \qquad dv = \cos(2x)\,dx$

$$\int e^x \cos(2x)dx$$

$$= \frac{1}{2}e^x \sin(2x) - \frac{1}{2}\int e^x \sin(2x)\,dx$$

$u = e^x \qquad v = -\frac{1}{2}\cos(2x)$

$du = e^x dx \qquad dv = \sin(2x)dx$

$$= \frac{1}{2}e^x \sin(2x) - \frac{1}{2}\left(-\frac{1}{2}e^x \cos(2x) + \frac{1}{2}\int e^x \cos(2x)dx\right)$$

$$= \frac{1}{2}e^x \sin(2x) + \frac{1}{4}e^x \cos(2x) - \frac{1}{4}\int e^x \cos(2x)\,dx$$

Therefore

$$\frac{5}{4}\int e^x \cos(2x)\,dx = \frac{1}{2}e^x \sin(2x) + \frac{1}{4}e^x \cos(2x) + C$$

So

$$\int e^x \cos(2x)dx = \boxed{\frac{2}{5}e^x \sin(2x) + \frac{1}{5}e^x \cos(2x) + C}$$

5. $\int x^4 \cos(3x)\, dx$

Hint: Try the tabular ("tic-tac-toe") method for doing parts multiple times.

Term	Sign	Derivative	Integral
1	+	x^4	$\cos(3x)$
2	−	$4x^3$	$\frac{1}{3}\sin(3x)$
3	+	$12x^2$	$-\frac{1}{9}\cos(3x)$
4	−	$24x$	$-\frac{1}{27}\sin(3x)$
5	+	24	$\frac{1}{81}\cos(3x)$
	−	0	$\frac{1}{243}\sin(3x)$

$$\int x^4 \cos(3x)\, dx = 1 + 2 + 3 + 4 + 5 + C$$

$$= \boxed{\underbrace{\frac{1}{3}x^4 \sin(3x)}_{1} + \underbrace{\frac{4}{9}x^3\cos(3x)}_{2} - \underbrace{\frac{12}{27}x^2\sin(3x)}_{3} - \underbrace{\frac{24}{81}x\cos(3x)}_{4} + \underbrace{\frac{24}{243}\sin(3x)}_{5} + C}$$

Verify yourself that this works! To get started, note that if we let $u = x^4$, $dv = \cos(3x)dx$, then $du = 4x^3 dx$, $v = \frac{1}{3}\sin(3x)$, so

$$\int x^4 \cos(3x)\, dx = \underbrace{\frac{1}{3}x^4\sin(3x)}_{1} - \frac{4}{3}\int x^3 \sin(3x)\, dx$$

$u = 4x^3$ $\quad v = -\frac{1}{9}\cos(3x$

$du = 12x^2 dx$ $\quad dv = \frac{1}{3}\sin(3x$

$$= \underbrace{\frac{1}{3}x^4\sin(3x)}_{1} + \underbrace{\frac{4}{9}x^3\cos(3x)}_{2} + \frac{12}{27}\int x^2\cos(3x)\, dx \ldots \text{(etc.)}$$