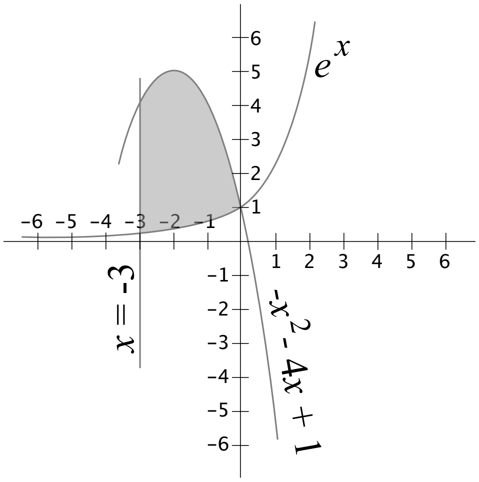
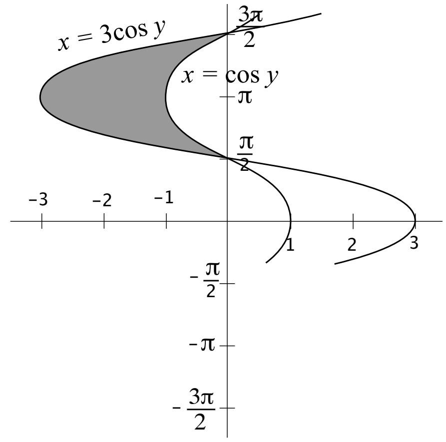
1. Find the area of each shaded region.





First picture:

$y = -x^2 - 4x + 1$ is above $y = e^x$ for the region. The x-values of the region range from -3 to 0, so the area is

$$\begin{aligned}\int_{-3}^{0} (-x^2 - 4x+) - e^x \; dx &= -\frac{1}{3}x^3 - 2x^2 + x - e^x \Big|_{-3}^{0} \\ &= (-0 - 0 + 0 - e^0) - \left(-\frac{1}{3}(-27) - 2 \cdot 9 - 3 - e^{-3}\right) \\ &= \boxed{11 + \frac{1}{e^3}}\end{aligned}$$

Second picture:

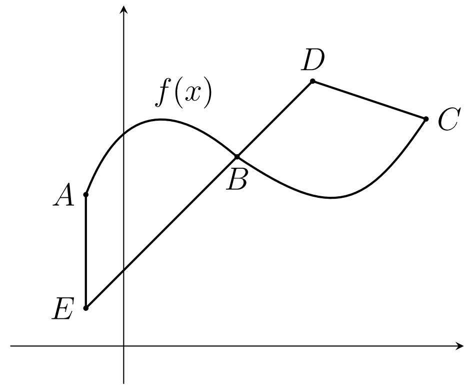
$x = \cos y$ is to the right of $x = 3\cos y$ for the region. The y-values of the region range from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$, so the area is

$$\begin{aligned}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\cos y - 3\cos y) \; dy &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -2\cos y \; dy \\ &= -2\sin y \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \\ &= -2\left(\sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right) \\ &= -2(-1-1) = \boxed{4}\end{aligned}$$

2. **(**)** Consider the curves shown. All curves are straight lines except for the one labeled $f(x)$ passing through the points A, B, and C. The figure is not necessarily to scale. Find the total area of the regions enclosed by the curves. Your answer should be expressed as an integral or sum of integrals in terms of $f(x)$.

   The points labeled in the figure are as follows:

   $$\begin{aligned} A &= (-1, 4) \\ B &= (3, 5) \\ C &= (8, 6) \\ D &= (5, 7) \\ E &= (-1, 1) \end{aligned}$$



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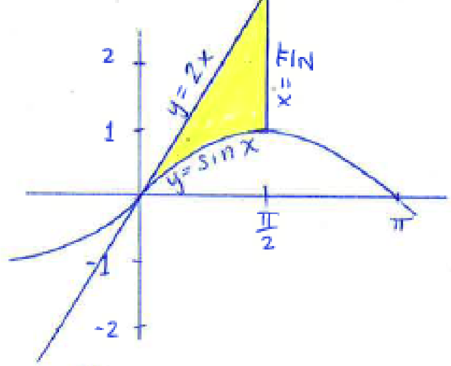
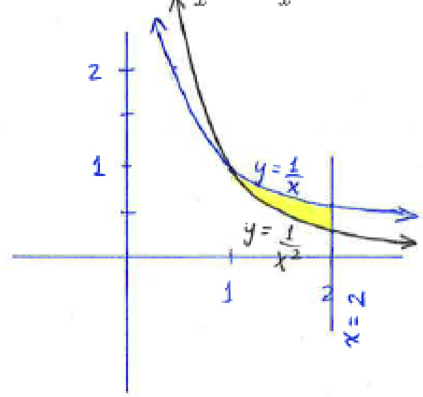
3. For each problem, sketch the region enclosed by the curves whose equations are given, and **set up** an integral for the area of the region. (For extra practice later, evaluate the integrals, if you can.)

(a) $y = \sin x$, $y = 2x$, $x = \frac{\pi}{2}$



$$A = \int_0^{\frac{\pi}{2}} (2x - \sin x)\, dx$$

(b) $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $x = 2$



$$\frac{1}{x} \overset{\text{set}}{=} \frac{1}{x^2}$$

$$x = 1.$$

$$A = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

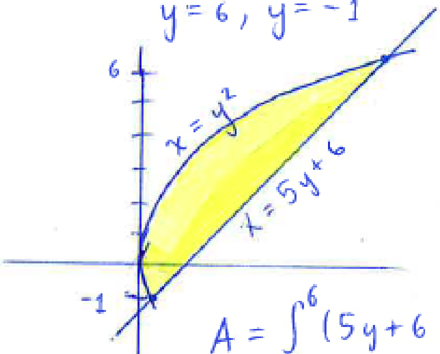
(c) $x = y^2$, $x = 5y + 6$

$$y^2 \overset{\text{set}}{=} 5y + 6$$

$$y^2 - 5y - 6 = 0$$

$$(y-6)(y+1) = 0$$

$$y = 6,\ y = -1$$



$$A = \int_{-1}^{6} (5y + 6 - y^2)\, dy$$

(d) $\frac{x}{3} = y^2$, $y = -\frac{1}{3}x + 2$

$$x = 3y^2 \ ; \quad \tfrac{1}{3}x = -y + 2$$

$$x = -3y + 6$$

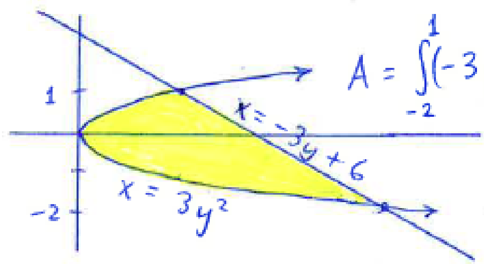
$$3y^2 \overset{\text{set}}{=} -3y + 6$$

$$3y^2 + 3y - 6 = 0$$

$$y^2 + y - 2 = 0$$

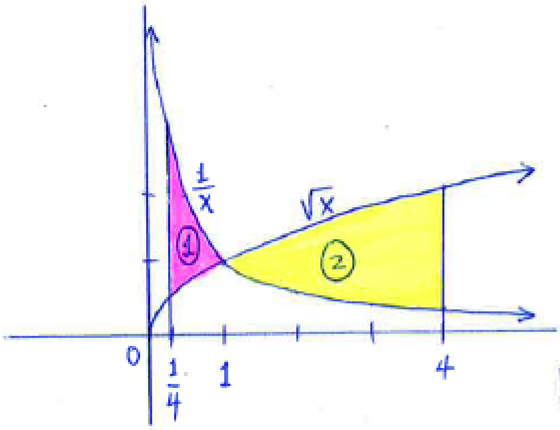
$$(y+2)(y-1) = 0$$

$$y = -2,\ y = 1$$



$$A = \int_{-2}^{1} (-3y + 6 - 3y^2)\, dy$$

4. **(*)** Find the total area between the curves $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x}$ from $x = \dfrac{1}{4}$ to $x = 4$.



From the graph, we can see that

$$A = \int_{\frac{1}{4}}^{1} \frac{1}{x} - \sqrt{x}\, dx + \int_{1}^{4} \sqrt{x} - \frac{1}{x}\, dx.$$

Let $A(x) = \int \frac{1}{x} - \sqrt{x}\, dx = \ln|x| - \frac{2}{3}x^{3/2}$.

$A(\frac{1}{4}) = \ln(\frac{1}{4}) - \frac{2}{3}(\frac{1}{4})^{3/2} = \ln 1 - \ln 4 - \frac{2}{3}(\frac{1}{8}) = -\ln 4 - \frac{1}{12}$;

$A(1) = \ln(1) - \frac{2}{3} = -\frac{2}{3}$; $A(4) = \ln(4) - \frac{2}{3} \cdot 4^{3/2} = \ln 4 - \frac{16}{3}$.

$$A = \left|A(1) - A(\tfrac{1}{4})\right| + \left|A(4) - A(1)\right| = -\frac{2}{3} + \cancel{\ln 4} + \frac{1}{12} + -\frac{2}{3} - \cancel{\ln 4} + \frac{16}{3} = \boxed{\frac{49}{12}}$$

5. **(**)** Find the total area between the curves $f(y) = y^2 - 4$ and $g(y) = 2y - y^2$ from $y = -2$ to $y = 4$.

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