Spring 2021 MATH 76

Activity 0

1. Consider the following integrals. Write out the expression you will use as u in a u–substitution. Indicate also du. Then complete the u–substitution.

(a) $\int \sin^3 x \cos x dx$ $\quad u = \ldots$ Sin x $\ldots \quad du = \ldots$ Cos x $\ldots$

$\int u^3 du$

$\downarrow$

$\frac{1}{4} u^4 + C$

$\boxed{\frac{1}{4}(\sin x)^4 + c}$

(b) $\int \frac{2x^2}{\sqrt{1-4x^3}} dx$ $\quad u = \ldots 1-4x^3 \ldots \quad du = \ldots -12x^2 \, dx \ldots$

$du = -6(2x^2)dx$

$2\int \frac{x^2}{(u)^{\frac{1}{2}}} \to 2 \cdot \frac{1}{-12} \int \frac{1}{(u)^{\frac{1}{2}}} du \to 2 \cdot \frac{1}{-12} \int u^{-\frac{1}{2}} du$

$2 \cdot -\frac{1}{12}\left[\frac{u^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}}\right] \to 2 \cdot -\frac{1}{12}\left[\frac{(1-4x^3)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right] \to \boxed{-\frac{1}{3}\sqrt{1-4x^3} + c}$

(c) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ $\quad u = \ldots e^x + e^{-x} \ldots \quad du = \ldots e^x - e^{-x} \ldots$

$\frac{1}{x} = \ln x$

$\int \frac{du}{u} \to \int \frac{1}{u} du \to \ln u + c = \boxed{\ln(e^x + e^{-x}) + c}$

2. Evaluate the following integrals

(a) $\int \frac{\sin x}{\cos x} dx$

$\int \tan(x)\, dx \to \boxed{\ln|\sec(x)| + C}$

(b) $\int x^2 e^{x^3+1} dx$

$U = x^3 + 1$ $\quad \frac{du}{3} = 3x^2 dx$

$\int x^2 e^U dx \to \int \frac{e^U du}{3} \to \frac{e^U}{3} \to \boxed{\frac{e^{(x^3+1)}}{3} + C}$

(c) $\int \frac{\ln x}{x} dx$ $\quad U = \ln x$ $\quad du = \frac{1}{x} dx$ or $\frac{dx}{x}$

$\int \frac{U dx}{x} \to \int U du \to \frac{U^2}{2} \to \boxed{\frac{\ln x^2}{2} + C}$

(d) $\int_0^1 2x(4 - x^2) dx$

$\int_0^1 8x - 2x^3 dx$

$\frac{8x^2}{2} - \frac{2x^4}{4}$

$\frac{8x^2}{2} - \frac{1x^4}{2}$

$4x^2 - \frac{x^4}{2}$

$1(\frac{7}{2}) - (0)$

$\boxed{\frac{7}{2}}$

(e) $\int_0^2 \frac{2x}{(x^2+1)^2}dx$ $\quad U = x^2+1 \quad du = 2x\,dx$

$\int_0^2 \frac{du}{U^2} \to \int_0^2 U^{-2}du \to -U^{-1} \to \underline{-(x^2+1)^{-1}}$

$-(2^2+1)^{-1} + (\cancel{0}^2-1)^{-1}$

$-(5)^{-1} \quad (-1)^{-1}$

$-\frac{1}{5} + -1 = \boxed{-\frac{6}{5}} \leftarrow$

(f) $\int_0^{\pi/2} \sin^2\theta\cos\theta\,d\theta$

$U = \sin\theta$

$du = \cos\theta\,d\theta \to \int_0^{\frac{\pi}{2}} U^2du \to \frac{U^3}{3}\Big|_0^{\frac{\pi}{2}} \to \frac{\sin\theta^3}{3} \quad \left(\frac{\sin(\frac{\pi}{2})}{3}\right) - \left(\frac{\sin(0)}{3}\right) \boxed{= 0.333...}$

(g) $\int_0^2 x^3\sqrt{16-x^4}dx$

$U = 16-x^4$

$du = 4x^3dx$

$\frac{du}{-4} = x^3dx$

$\int_0^2 \frac{\sqrt{U}du}{-4} \to \int_0^2 \frac{(U)^{\frac{1}{2}}du}{-4} \to \frac{\frac{2U^{\frac{3}{2}}}{3}}{4} \to \frac{\frac{2(16-x^4)^{\frac{3}{2}}}{3}}{4}$

$(\ 0\ ) - \left(\frac{32}{3}\right) = \boxed{-\frac{32}{3}}$

(h) $\int_2^3 \frac{x}{\sqrt[3]{x^2-1}}dx$

$U = x^2-1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int_2^3 \frac{1}{2\sqrt[3]{U}}du \to \int_2^3 \frac{\frac{1}{\sqrt[3]{U}}du}{2} \to \int_2^3 \frac{U^{-\frac{1}{3}}}{2} \to \frac{\frac{U^{-\frac{1}{3}+\frac{3}{3}}}{-\frac{1}{3}+\frac{3}{3}}}{2} \to \frac{\frac{3U^{\frac{2}{3}}}{2}}{2}$

$\frac{\frac{3(x^2-1)^{\frac{2}{3}}}{2}}{2}$

$(\ 3\ ) - (1.56) = \boxed{1.44}$