Math 76 Midterm 1 – Version D
Spring 2021
Chapter 2

Things to remember:

1. Your camera must be on for the duration of the exam, positioned in such a way that I can see you and your work space, but so that your answers are not visible on camera. Your microphone should be muted. Please use your desktop or laptop to connect via Zoom, and keep your phone turned off and out of sight.

2. You are not allowed to consult outside sources, including notes, books, the internet, or other people, while taking this exam. Calculators are allowed only for basic numerical or scientific computations, not for graphing or algebra. Private chat will be disabled. If you have a question or need help, please put it in the chat **to me only**.

3. All answers must be recorded and submitted on the Google Form **Math 76 Midterm 1 Answer Sheet**, version **D**.

4. Each correct answer is worth 5 points. There is no penalty for guessing.

5. On the Answer Sheet there are some extra questions at the end. There is one additional point available for a reflective question, and some optional comment questions. There is a total of $11 \cdot 5 + 1 = 56$ points available on this exam.

6. The exam is designed to be completed in 50 minutes, but if you need more time I will be available in the Zoom meeting for up to two hours. Your Google form must be submitted no later than 5pm PT, unless otherwise instructed. **You are only allowed to submit your form once**, so please take care not to hit 'Submit' until you have double-checked your responses!

7. **If something goes wrong** (internet goes out, you accidentally hit 'submit' before you finish, computer hangs while you are downloading, etc.), *please don't stress!* Contact me as soon as you can, and we will see what can be done.

   Direct email: `kbyler@csufresno.edu`.

   I am rooting for you to do your best and will do everything I can to make sure you are able to show what you know!

1. A lightning rod of length 2 meters has mass density $7x^3$ kg/m at a distance of x meters from the tip of the rod. The total mass of the rod is ...

   (a) 56 kg

   (b) 28 kg

   (c) $\frac{35}{4}$ kg

   (d) $\frac{63}{4}$ kg

   (e) 42 kg

2. The force required to keep a certain spring stretched x feet beyond its natural length is $F(x) = 40x$ pounds. How much work is done in stretching the spring from its natural length to 3 ft. beyond the natural length?

   (a) 100 ft.-lb.

   (b) 160 ft.-lb.

   (c) 180 ft.-lb.

   (d) 140 ft.-lb.

   (e) 120 ft.-lb.

3. Consider the region B bounded by the curves $x = y^2$ and $x = 4y$. The area of region B is ...

(a) $\dfrac{32}{3}$

(b) $\dfrac{48}{5}$

(c) 36

(d) $\dfrac{45}{2}$

(e) 28

4. Consider the region B bounded by the curves $x = y^2$ and $x = 4y$. The volume of the solid formed by rotating B about the y-axis is ...

(a) $\pi \displaystyle\int_0^4 16y^2 - y^4 \, dy$

(b) $2\pi \displaystyle\int_0^1 y^4 - 16y^2 \, dy$

(c) $\pi \displaystyle\int_1^4 (4y - y^2)^2 \, dy$

(d) $\pi \displaystyle\int_0^4 4y^2 - y^3 \, dy$

(e) $2\pi \displaystyle\int_0^8 y^3 - 4y^2 \, dy$

5. Consider the region B bounded by the curves $x = y^2$ and $x = 4y$. The volume of the solid formed by rotating B about the line $y = -1$ is ...

(a) $2\pi \displaystyle\int_0^1 y(y^2 - 4y + 1)\, dy$

(b) $\pi \displaystyle\int_1^4 y^4 - 16y^2 + 1\, dy$

(c) $2\pi \displaystyle\int_{-1}^0 (1 - y)(4y - y^2)^2\, dy$

(d) $\pi \displaystyle\int_0^4 (4y + 1)^2 - (y^2 + 1)^2\, dy$

(e) $2\pi \displaystyle\int_0^4 (y + 1)(4y - y^2)\, dy$

6. The length of the curve $x = 6y^{3/2} - 7$ from $y = 2$ to $y = 5$ is ...

(a) $\displaystyle\int_2^5 \sqrt{1 + 36y^3}\, dy$

(b) $\pi \displaystyle\int_2^5 (6y^{3/2} - 7)^2\, dy$

(c) $\displaystyle\int_2^5 9\sqrt{y}\, dy$

(d) $\displaystyle\int_2^5 \sqrt{1 + 81y}\, dy$

(e) $2\pi \displaystyle\int_2^5 y(6y^{3/2} - 7)\, dy$

7. A cube with sides 2 m long is sitting on the bottom of an aquarium. The aquarium is filled with water (weight density = 9800 N/m^3) to a depth of 7 m. Assuming the origin is placed at the bottom of the aquarium, the hydrostatic force (in Newtons) on one of the vertical sides of the cube is given by ...

(a) $9800 \int_0^2 (y+7)\ dy$

(b) $4900 \int_0^2 (y-7)\ dy$

(c) $19600 \int_0^7 (y+2)\ dy$

(d) $19600 \int_0^2 (7-y)\ dy$

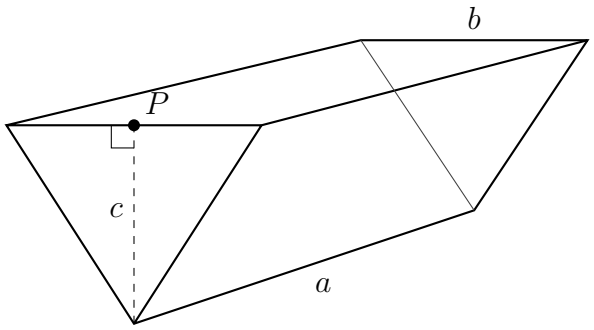
(e) $9800 \int_2^7 (2-y)\ dy$

8. A trough is filled to the top with water (weight density = 62.5 lb./ft.^3). The ends of the trough are vertical triangles and the sides are rectangles, as shown. Find the work required to pump all the water out of the trough. Assume the water will exit at the point P.

   The diagram is not necessarily to scale. The dimensions are as follows:

   $a = 8$ ft.
   $b = 3$ ft.
   $c = 4$ ft.

   Hint. The principle of similar triangles may be helpful.



(a) $62.5 \cdot 16$ ft.-lb.

(b) $62.5 \cdot 48$ ft.-lb.

(c) $62.5 \cdot 72$ lb.

(d) $62.5 \cdot 40$ lb.

(e) $62.5 \cdot 64$ ft.-lb.

9. A solid has cross-sectional area (when slices are cut perpendicular to the x-axis) given by $A(x)$ at a distance of x meters from one end ($0 \le x \le 12$). The volume of the solid is ...

(a) $\displaystyle\int_0^{12} (A(x))^2 \, dx \text{ m}^3$

(b) $\displaystyle\int_0^{12} A(x) \, dx \text{ m}^3$

(c) $\displaystyle\int_0^{12} xA(x) \, dx \text{ m}^3$

(d) $\displaystyle\pi \int_0^{12} A(x) \, dx \text{ m}^3$

(e) $\displaystyle 2\pi \int_0^{12} x^2 A(x) \, dx \text{ m}^3$

10. The shaded area shown is ...

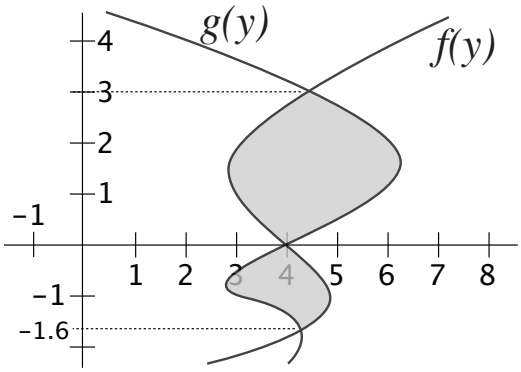
(a) $\displaystyle\int_{-1.6}^{0} g(y) - f(y) \, dy + \int_0^3 f(y) - g(y) \, dy$

(b) $\displaystyle\int_{-1.6}^{0} f(y) + g(y) \, dy - \int_0^3 g(y) + f(y) \, dy$

(c) $\displaystyle\int_{-1.6}^{3} f(y) - g(y) \, dy$

(d) $\displaystyle\int_{-1.6}^{0} f(y) - g(y) \, dy + \int_0^3 g(y) - f(y) \, dy$

(e) $\displaystyle\int_{-1.6}^{3} g(y) - f(y) \, dy$



11. The length of the curve $y = \frac{1}{2}e^{4x} + \frac{1}{32}e^{-4x}$ from $x = 0$ to $x = 1$ is ...

(a) $\frac{1}{2}e^4 + \frac{1}{32}e^{-4} + \frac{17}{32}$

(b) $\frac{1}{2}e^4 + \frac{1}{32}e^{-4} - \frac{3}{16}$

(c) $\frac{1}{2}e^4 - \frac{1}{32}e^{-4} - \frac{15}{32}$

(d) $\frac{1}{2}e^4 - \frac{1}{32}e^{-4} + \frac{5}{16}$

(e) $\frac{1}{2}e^4 + \frac{1}{32}e^{-4} + \frac{7}{8}$