

TABLE OF INTEGRALS

| Substitution Rule | Integration by Parts |
|---|--|
| $\int f(g(x))g'(x) dx = \int f(u) du \quad (u = g(x))$ | $\int u dv = uv - \int v du$ |
| $\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$ | $\int_a^b uv' dx = uv \Big _a^b - \int_a^b vu' dx$ |

Basic Integrals

- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C; n \neq -1$
- $\int \frac{dx}{x} = \ln |x| + C$
- $\int \cos ax dx = \frac{1}{a} \sin ax + C$
- $\int \sin ax dx = -\frac{1}{a} \cos ax + C$
- $\int \tan x dx = \ln |\sec x| + C$
- $\int \cot x dx = \ln |\sin x| + C$
- $\int \sec x dx = \ln |\sec x + \tan x| + C$
- $\int \csc x dx = -\ln |\csc x + \cot x| + C$
- $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
- $\int b^{ax} dx = \frac{1}{a \ln b} b^{ax} + C; b > 0, b \neq 1$
- $\int \ln x dx = x \ln x - x + C$
- $\int \log_b x dx = \frac{1}{\ln b} (x \ln x - x) + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C$
- $\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$
- $\int \cos^{-1} x dx = x \cos^{-1} x - \sqrt{1 - x^2} + C$
- $\int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$
- $\int \sec^{-1} x dx = x \sec^{-1} x - \ln(x + \sqrt{x^2 - 1}) + C$
- $\int \sinh x dx = \cosh x + C$
- $\int \cosh x dx = \sinh x + C$
- $\int \operatorname{csch}^2 x dx = -\coth x + C$
- $\int \operatorname{sech}^2 x dx = \tanh x + C$
- $\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$
- $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$
- $\int \coth x dx = \ln |\sinh x| + C$
- $\int \tanh x dx = \ln \cosh x + C$
- $\int \operatorname{csch} x dx = \ln |\tanh(x/2)| + C$
- $\int \operatorname{sech} x dx = \tan^{-1} \sinh x + C = \sin^{-1} \tanh x + C$

Trigonometric Integrals

- $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$
- $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$
- $\int \sec^2 ax dx = \frac{1}{a} \tan ax + C$
- $\int \csc^2 ax dx = -\frac{1}{a} \cot ax + C$
- $\int \tan^2 x dx = \tan x - x + C$
- $\int \cot^2 x dx = -\cot x - x + C$
- $\int \cos^3 x dx = -\frac{1}{3} \sin^3 x + \sin x + C$
- $\int \sin^3 x dx = \frac{1}{3} \cos^3 x - \cos x + C$

$$38. \int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$

$$40. \int \tan^3 x \, dx = \frac{1}{2} \tan^2 x - \ln |\sec x| + C$$

$$42. \int \sec^n ax \tan ax \, dx = \frac{1}{na} \sec^n ax + C; n \neq 0$$

$$44. \int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$$

$$46. \int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$$

$$48. \int \sin mx \cos nx \, dx = \frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C; m^2 \neq n^2$$

$$49. \int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C; m^2 \neq n^2$$

$$50. \int \cos mx \cos nx \, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C; m^2 \neq n^2$$

$$39. \int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x - \frac{1}{2} \ln |\csc x + \cot x| + C$$

$$41. \int \cot^3 x \, dx = -\frac{1}{2} \cot^2 x - \ln |\sin x| + C$$

$$43. \int \csc^n ax \cot ax \, dx = -\frac{1}{na} \csc^n ax + C; n \neq 0$$

$$45. \int \frac{dx}{1 - \sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$$

$$47. \int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$$

Reduction Formulas for Trigonometric Functions

$$51. \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$53. \int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx; n \neq 1$$

$$55. \int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx; n \neq 1$$

$$57. \int \sin^m x \cos^n x \, dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \, dx; m \neq -n$$

$$58. \int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, dx; m \neq -n$$

$$59. \int x^n \sin ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx; a \neq 0$$

$$52. \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$54. \int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx; n \neq 1$$

$$56. \int \csc^n x \, dx = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx; n \neq 1$$

$$60. \int x^n \cos ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx; a \neq 0$$

Integrals Involving $a^2 - x^2$; $a > 0$

$$61. \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$63. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$65. \int \frac{\sqrt{a^2 - x^2}}{x^2} \, dx = -\frac{1}{x} \sqrt{a^2 - x^2} - \sin^{-1} \frac{x}{a} + C$$

$$67. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$62. \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$64. \int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} + C$$

$$66. \int \frac{x^2}{\sqrt{a^2 - x^2}} \, dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Integrals Involving $x^2 - a^2$; $a > 0$

$$68. \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + C$$

$$70. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$

$$72. \int \frac{\sqrt{x^2 - a^2}}{x^2} \, dx = \ln |x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$$

$$74. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$69. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln |x + \sqrt{x^2 - a^2}| + C$$

$$71. \int x^2 \sqrt{x^2 - a^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 - a^2}| + C$$

$$73. \int \frac{x^2}{\sqrt{x^2 - a^2}} \, dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$$

$$75. \int \frac{dx}{x(x^2 - a^2)} = \frac{1}{2a^2} \ln \left| \frac{x^2 - a^2}{x^2} \right| + C$$

Integrals Involving $a^2 + x^2$; $a > 0$

76. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$
77. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{a^2 + x^2}) + C$
78. $\int \frac{dx}{x\sqrt{a^2 + x^2}} = \frac{1}{a} \ln \left| \frac{a - \sqrt{a^2 + x^2}}{x} \right| + C$
79. $\int \frac{dx}{x^2\sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$
80. $\int x^2 \sqrt{a^2 + x^2} dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$
81. $\int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln|x + \sqrt{a^2 + x^2}| - \frac{\sqrt{a^2 + x^2}}{x} + C$
82. $\int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$
83. $\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
84. $\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} + C$
85. $\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2} \ln \left(\frac{x^2}{a^2 + x^2} \right) + C$

Integrals Involving $ax \pm b$; $a \neq 0, b > 0$

86. $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C; n \neq -1$
87. $\int (\sqrt{ax + b})^n dx = \frac{2(\sqrt{ax + b})^{n+2}}{a(n+2)} + C; n \neq -2$
88. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax + b}{b}} + C; b > 0$
89. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right| + C; b > 0$
90. $\int \frac{x}{ax + b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax + b| + C$
91. $\int \frac{x^2}{ax + b} dx = \frac{1}{2a^3} ((ax + b)^2 - 4b(ax + b) + 2b^2 \ln|ax + b|) + C$
92. $\int \frac{dx}{x^2(ax + b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax + b}{x} \right| + C$
93. $\int x\sqrt{ax + b} dx = \frac{2}{15a^2} (3ax - 2b)(ax + b)^{3/2} + C$
94. $\int \frac{x}{\sqrt{ax + b}} dx = \frac{2}{3a^2} (ax - 2b)\sqrt{ax + b} + C$
95. $\int x(ax + b)^n dx = \frac{(ax + b)^{n+1}}{a^2} \left(\frac{ax + b}{n+2} - \frac{b}{n+1} \right) + C; n \neq -1, -2$
96. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$

Integrals with Exponential and Trigonometric Functions

97. $\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C$
98. $\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C$

Integrals with Exponential and Logarithmic Functions

99. $\int \frac{dx}{x \ln x} = \ln|\ln x| + C$
100. $\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1} \right) + C; n \neq -1$
101. $\int x e^x dx = x e^x - e^x + C$
102. $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx; a \neq 0$
103. $\int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$

Miscellaneous Formulas

104. $\int x^n \cos^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \cos^{-1} x + \int \frac{x^{n+1} dx}{\sqrt{1-x^2}} \right); n \neq -1$
105. $\int x^n \sin^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \sin^{-1} x - \int \frac{x^{n+1} dx}{\sqrt{1-x^2}} \right); n \neq -1$
106. $\int x^n \tan^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \tan^{-1} x - \int \frac{x^{n+1} dx}{x^2 + 1} \right); n \neq -1$
107. $\int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C; a > 0$
108. $\int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left(\frac{x-a}{a} \right) + C; a > 0$