Spring 2021 MATH 76 Activity 8

SUMMATION

The \sum notation.

Consider a list of real numbers named x_1, x_2, x_3, \ldots , up to x_n where n is an integer. Note that x_1 is the first number on the list, x_2 the second number, and so on. The sum of these numbers $x_1 + x_2 + x_3 + \ldots + x_n$ is represented by $\sum_{k=1}^{n} x_k$. The symbol $\sum_{k=1}^{n} x_k$ (greek letter "sigma") represents the sum, k is the index that takes values from 1 (the lower limit) to n (upper limit). The term x_k is the quantity (summand) to be added, and when

$$k=1$$
 $x_k=x_1$ $x_k=x_2$ \vdots \vdots $x_k=x_n$

You should have already seen the \sum notation in some topics such as:

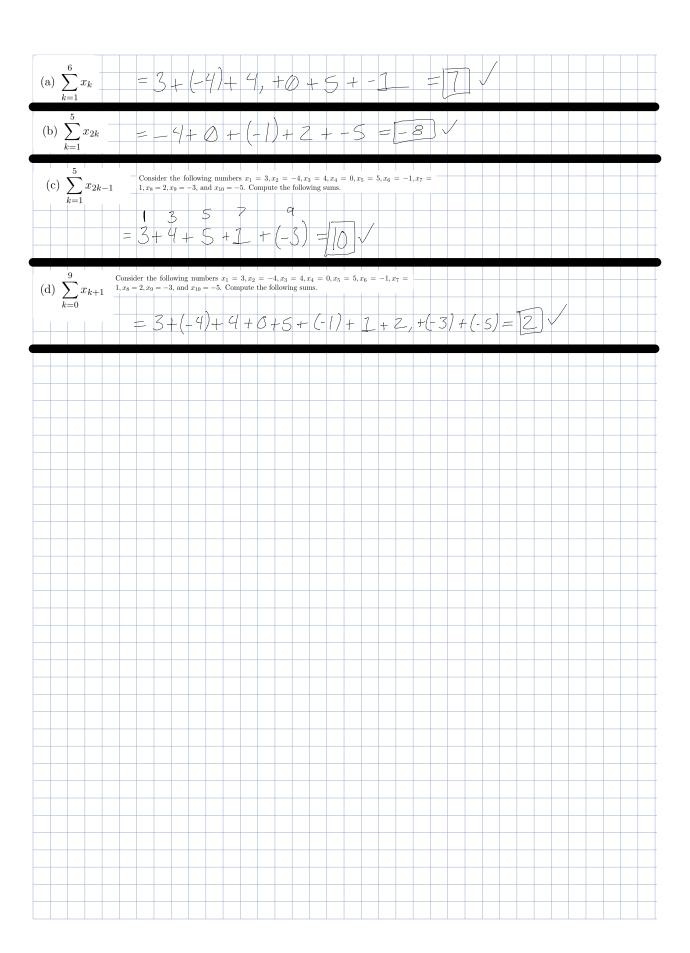
- Riemann sum $R_n = \Delta x \sum_{k=1}^n f(x_k)$.
- Midpoint rule for numerical integration $\int_{x_0}^{x_n} f(x)dx \approx \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$.
- Trapezoid rule for numerical integration $\int_{x_0}^{x_n} f(x) dx \approx \frac{\Delta x}{2} \left(f(x_0) + f(x_n) + 2 \sum_{k=1}^{n-1} f(x_k) \right).$
- 1. Consider the following numbers $x_1 = 3, x_2 = -4, x_3 = 4, x_4 = 0, x_5 = 5, x_6 = -1, x_7 = 1, x_8 = 2, x_9 = -3, \text{ and } x_{10} = -5.$ Compute the following sums.

(a)
$$\sum_{k=1}^{6} x_k$$

(b)
$$\sum_{k=1}^{5} x_{2k}$$

(c)
$$\sum_{k=1}^{5} x_{2k-1}$$

(d)
$$\sum_{k=0}^{9} x_{k+1}$$



2. It is known that:

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

Find the following sums.

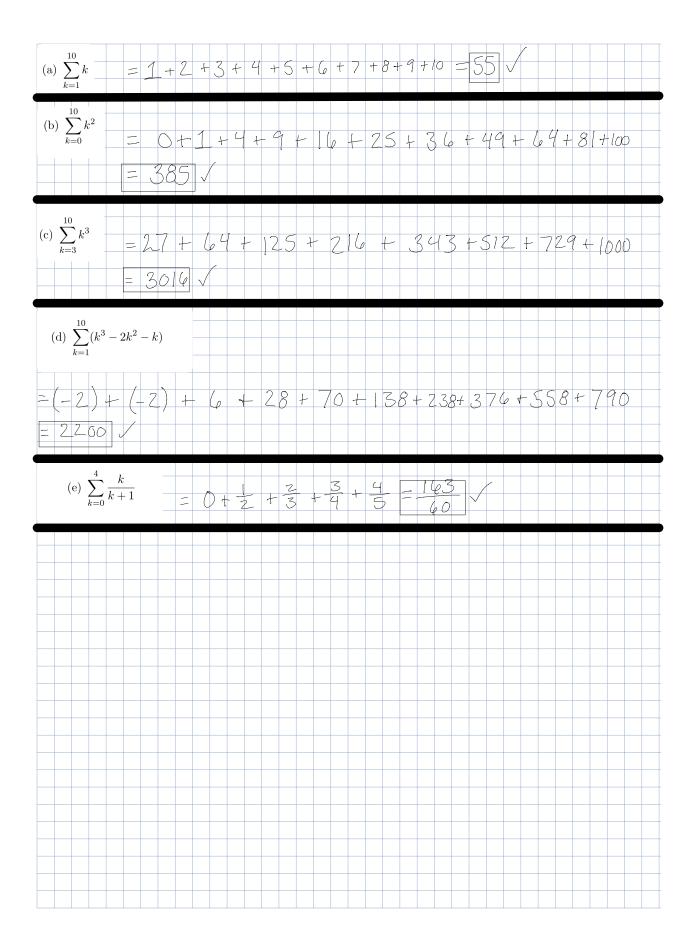
(a)
$$\sum_{k=1}^{10} k$$

(b)
$$\sum_{k=0}^{10} k^2$$

(c)
$$\sum_{k=3}^{10} k^3$$

(d)
$$\sum_{k=1}^{10} (k^3 - 2k^2 - k)$$

(e)
$$\sum_{k=0}^{4} \frac{k}{k+1}$$



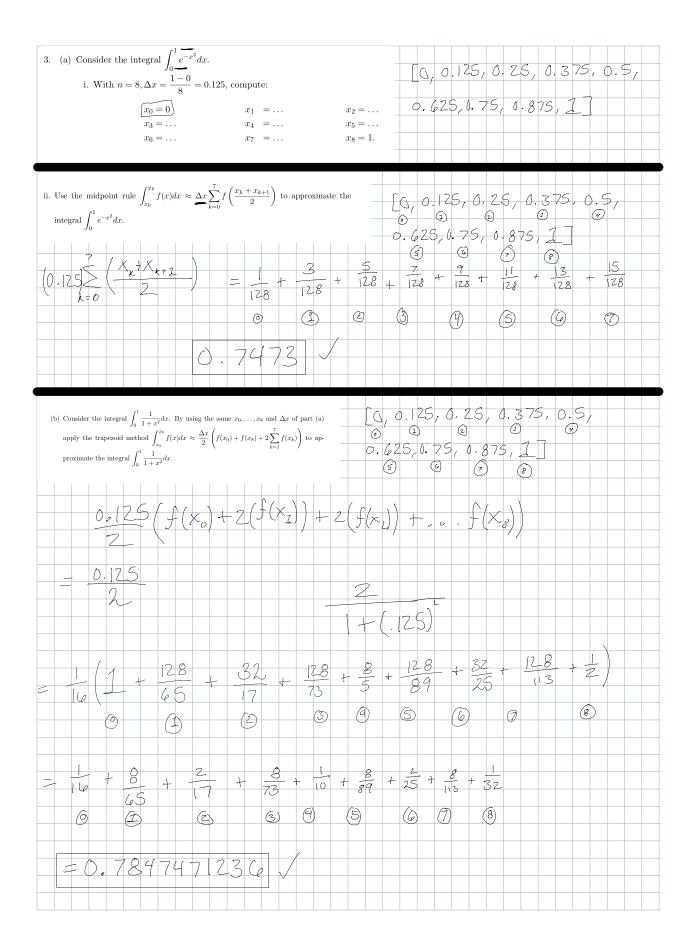
3. (a) Consider the integral
$$\int_0^1 e^{-x^2} dx$$
.

i. With
$$n = 8, \Delta x = \frac{1-0}{8} = 0.125$$
, compute:

$$x_0 = 0$$
 $x_1 = \dots$ $x_2 = \dots$ $x_3 = \dots$ $x_4 = \dots$ $x_5 = \dots$ $x_6 = \dots$ $x_7 = \dots$ $x_8 = 1$.

ii. Use the midpoint rule
$$\int_{x_0}^{x_8} f(x)dx \approx \Delta x \sum_{k=0}^7 f\left(\frac{x_k+x_{k+1}}{2}\right)$$
 to approximate the integral $\int_0^1 e^{-x^2} dx$.

(b) Consider the integral
$$\int_0^1 \frac{1}{1+x^2} dx$$
. By using the same x_0, \dots, x_8 and Δx of part (a) apply the trapezoid method $\int_{x_0}^{x_8} f(x) dx \approx \frac{\Delta x}{2} \left(f(x_0) + f(x_8) + 2 \sum_{k=1}^7 f(x_k) \right)$ to approximate the integral $\int_0^1 \frac{1}{1+x^2} dx$.



- 4. The goal of this problem is to derive a formula for the sum $\sum_{k=0}^{n} \frac{1}{2^k}$.
 - (a) Let $S = \sum_{k=0}^{n} \frac{1}{2^k}$. Write out $\frac{1}{2}S$ as a sum.

(b) Write $S - \frac{1}{2}S$ as a sum.

(c) Using your result in part (b), what is a formula for S?

(d) Compute $\sum_{k=0}^{100} \frac{1}{2^k}.$

