Math 76 Exercises – 3.2A Trigonometric Integrals

Evaluate each integral. Check by differentiating.

1. $\int \cos^3 x \sin^4 x \, dx$

Power of cosine is odd

$= \int \sin^4 x \cdot \cos^2 x \cos x \, dx$

$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^4 (1 - u^2) \, du$

$= \int (u^4 - u^6) \, du$

$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C$

$= \boxed{\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C}$

2. $\int \cos^2(5x) \sin^3(5x) \, dx$

Power of sine is odd

$= \int \cos^2(5x) \sin^2(5x) \sin(5x) \, dx$

$= -\frac{1}{5} \int \cos^2(5x) (1 - \cos^2(5x)) (-5 \sin(5x)) \, dx$

$u = \cos(5x)$

$du = -5 \sin(5x) \, dx$

$= -\frac{1}{5} \int u^2 (1 - u^2) \, du = -\frac{1}{5} \int (u^2 - u^4) \, du$

$= -\frac{1}{5} \left(\frac{1}{3} u^3 - \frac{1}{5} u^5 \right) + C$

$= \boxed{-\frac{1}{15} \cos^3(5x) + \frac{1}{25} \cos^5(5x) + C}$

3. $\int \sin^4 x \cos^2 x \, dx$

Powers of sine and cosine are both even

$\sin x \cos x = \frac{1}{2} \sin(2x)$

$\sin^2 x = \frac{1 - \cos(2x)}{2}$

$= \int (\sin x \cos x)^2 \sin^2 x \, dx$

$= \int \left(\frac{1}{2} \sin(2x) \right)^2 \cdot \frac{1}{2} (1 - \cos(2x)) \, dx$

$= \frac{1}{8} \int \sin^2(2x) \, dx - \frac{1}{8} \int \sin^2(2x) \cos(2x) \, dx$

$u = \sin(2x)$

$du = 2 \cos(2x) \, dx$

$= \frac{1}{8} \int \frac{1}{2} (1 - \cos(4x)) \, dx - \frac{1}{16} \int u^2 \, du$

$= \frac{1}{16} (x - \frac{1}{4} \sin(4x)) - \frac{1}{16} \cdot \frac{1}{3} u^3 + C$

$= \boxed{\frac{1}{16} \left(x - \frac{1}{4} \sin(4x) - \frac{1}{3} \sin^3(2x) \right) + C}$

4. $\int \sec^2 x \tan^4 x \, dx$

Power of secant is even

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^4 \, du$

$= \frac{1}{5} u^5 + C$

$= \boxed{\frac{1}{5} \tan^5 x + C}$

5 $\frac{1}{4}\int 4x \sec^3(2x^2 - 1) \tan^5(2x^2 - 1) \, dx$

Power of tangent is odd

$u = 2x^2 - 1$

$du = 4x \, dx$

$= \frac{1}{4} \int \sec^3(u) \tan^5(u) \, du$

$= \frac{1}{4} \int \sec^2(u) \tan^4(u) \sec(u) \tan(u) \, du$

$= \frac{1}{4} \int \sec^2(u) (\sec^2(u) - 1)^2 \sec u \tan u \, du$

$t = \sec u$

$dt = \sec u \tan u \, du$

$= \frac{1}{4} \int t^2 (t^2 - 1)^2 \, dt = \frac{1}{4} \int t^2 (t^4 - 2t^2 + 1) \, dt$

$= \frac{1}{4} \int t^6 - 2t^4 + t^2 \, dt = \frac{1}{4} \left(\frac{1}{7} t^7 - \frac{2}{5} t^5 + \frac{1}{3} t^3 \right) + C$

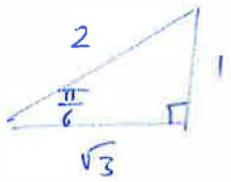
$= \boxed{\frac{1}{28} \sec^7(2x^2 - 1) - \frac{1}{10} \sec^5(2x^2 - 1) + \frac{1}{12} \sec^3(2x^2 - 1) + C}$

6. $\int_0^{\pi/6} \sec^2 x \tan x \, dx$

use guess and check or $u = \tan x$

$= \frac{1}{2}\tan^2 x \Big|_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\tan^2\left(\frac{\pi}{6}\right) - \frac{1}{2}\tan^2(0)$



$= \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{2}\cdot 0$

$= \frac{1}{2}\cdot\frac{1}{3}$

$= \boxed{\frac{1}{6}}$

7. $\int \sec x \, dx$

Hint: Multiply top and bottom by $\sec x + \tan x$.

$= \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}\, dx$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

$= \int \frac{1}{u}\, du$

$= \ln|u| + C$

$= \boxed{\ln|\sec x + \tan x| + C}$

This is a good formula to memorize!