LAB Activity 2

① [a] $y=\sqrt{x}$ $y=2-x$ $y=0$ rotate = X-Axis

$\sqrt{x}^2 = (2-x)^2$

$x = 4-2x-2x+x^2 \quad = 4-4x+x^2 \quad x=4 \quad \boxed{x=1}$ ✓

[b] I $V = \pi\int_0^1 (\sqrt{x})^2 dx + \pi\int_1^2 (2-x)^2 dx$

II $V = \pi\int_0^1 (\sqrt{x})^2 dx + \pi\int_1^2 (2-x)^2 dx$

Part b+c+d

$= \pi\int_0^1 x\,dx + \pi\int_1^2 4-4x+x^2\,dx$

$= \pi\left(\frac{x^2}{2}\right)\Big|_0^1 + \pi\left(4x-2x^2+\frac{x^3}{3}\right)\Big|_1^2$

$= \frac{1}{2}\pi + \left(\pi\left(8-8+\frac{8}{3}\right) - \pi\left(4+2-\frac{1}{3}\right)\right)$

$= \frac{1}{2}\pi + \left(\frac{8}{3}\pi - \frac{7}{3}\pi\right)$

$= \frac{1}{2}\pi + \frac{1}{3}\pi$

$= \boxed{\frac{5}{6}\pi}$ ✓

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② $y=\sqrt{x}$ $y=2-x$ $x=0$ x-Axis $[0,1]$

$$V=\pi\int_0^1 (2-x)^2-(\sqrt{x})^2\,dx=\pi\int_0^1 4-4x+x^2-x\,dx=\pi\int_0^1 4-3x+x^2\,dx$$

$$=\pi\left(4x-\frac{3x^2}{2}+\frac{x^3}{3}\right)\Big|_0^1=\pi\left(4-\frac{3}{2}+\frac{1}{3}\right)=\boxed{\frac{17\pi}{6}}\ \checkmark$$

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③ $y=x^2$ $y=\frac{x}{2}$ rotate = y-Axis $\left[0,\frac{1}{2}\right]$

$$V=2\pi\int_0^{\frac{1}{2}}\frac{x}{2}-x^2\,dx=2\pi\left(\frac{x^2}{4}-\frac{x^3}{3}\right)\Big|_0^{\frac{1}{2}}=2\pi\left(\frac{1}{16}-\frac{1}{24}\right)=\boxed{\frac{\pi}{24}}\ \checkmark$$

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④ [a] $y=x\sqrt{2-x}$ $y=0$ rotate = x-Axis $[0,2]$

$$V=\pi\int_0^2 \left(x\sqrt{2-x}\right)^2dx=\pi\int_0^2 x^2(2-x)\,dx=\pi\int_0^2 2x^2-x^3\,dx$$

$$=\pi\left(\frac{2x^3}{3}-\frac{x^4}{4}\right)\Big|_0^2=\pi\left(\frac{16}{3}-4\right)=\boxed{\frac{4\pi}{3}}\ \checkmark$$

[b] $y=x\sqrt{2-x}$ $y=0$ rotate = y-Axis $[0,2]$

$$V=2\pi\int_0^2 \left(x\sqrt{2-x}\right)dx$$

$$=2\pi\int_0^2 x\sqrt{u}\,dx \qquad u=2-x \quad du=-1\,dx$$

$$=\pi\int\left(u^{\frac{3}{2}}-2\sqrt{u}\right)du=\int u^{\frac{3}{2}}\,du-2\int\sqrt{u}\,du$$

$$=\frac{2(2-x)^{\frac{5}{2}}}{5}-2\left(\frac{2(2-x)^{\frac{3}{2}}}{3}\right)\Big|_2^0$$

$$=\frac{4\pi(2-x)^{\frac{5}{2}}}{5}-\frac{8\pi(2-x)^{\frac{3}{2}}}{3}$$

↳ Continued

Lab Activity 2

$$= \frac{4\pi(-3x-4)(2-x)^{\frac{3}{2}}}{15}\Bigg|_0^2 = \boxed{\frac{2^{\frac{11}{2}}}{15}\pi}$$