Complete each problem. Use an integral table if needed.

1. Evaluate the integral $\int \sin(3x)\cos(8x)\,dx$.

Using the formula $\int \sin(au)\cos(bu)\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

with $a=3$ and $b=8$ (and $u=x$), we have

$$\int \sin(3x)\cos(8x)\,dx = -\frac{\cos(-5x)}{2(-5)} - \frac{\cos(11x)}{2\cdot 11} + C$$

$$= \boxed{\frac{\cos(5x)}{10} - \frac{\cos(11x)}{22} + C}$$

$\cos(-5x) = \cos(5x)$ since cosine is an even function

2. (**) Evaluate the integral $\int \sqrt{e^{2x}+9}\,dx$.

Hmm... $\sqrt{u^2+a^2}$?
$u = e^x$
$du = e^x\,dx$
Need a factor of e^x for this to work!

$$\int \sqrt{e^{2x}+9}\,dx$$

$$= \int \frac{e^x\sqrt{e^{2x}+9}}{e^x}\,dx$$

$$= \int \frac{\sqrt{e^{2x}+9}}{e^x}\,e^x\,dx$$

$$= \int \frac{\sqrt{u^2+9}}{u}\,du$$

Now use trig. substitution or formula
$\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$
with $a=3$

$$= \sqrt{u^2+9} - 3\ln\left|\frac{3+\sqrt{u^2+9}}{u}\right| + C = \sqrt{e^{2x}+9} - 3\ln\left|\frac{3+\sqrt{e^{2x}+9}}{e^x}\right| + C$$

$$= \sqrt{e^{2x}+9} - 3\left(\ln(3+\sqrt{e^{2x}+9}) - \ln(e^x)\right) + C$$

$$= \boxed{\sqrt{e^{2x}+9} - 3\ln(3+\sqrt{e^{2x}+9}) + 3x + C}$$

3. (**) Find the length of the curve $x = 5y^2$ from $y = 0$ to $y = 1$.

$$L = \int_0^1 \sqrt{1+(x')^2}\, dy \qquad x' = 10y$$

$$1+(x')^2 = 1 + 100y^2$$

$$= \int_0^1 \sqrt{1+100y^2}\, dy$$

$$= \frac{1}{10}\int_0^1 \sqrt{1+(10y)^2}\cdot 10\, dy \qquad u = 10y \quad du = 10\, dy \qquad y=1 : u=10, \quad y=0 : u=0$$

$$= \frac{1}{10}\int_0^{10} \sqrt{1+u^2}\, du$$

Use trig. substitution or formula

$$\int \sqrt{a^2+u^2}\, du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) + C$$

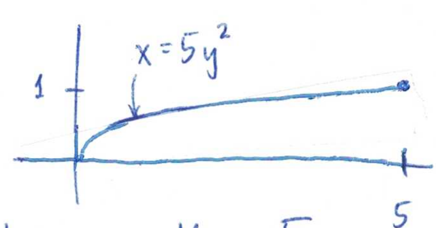
with $a = 1$

$$= \frac{1}{10}\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u+\sqrt{1+u^2}\right)\right]_1^{10}$$

$$= \frac{1}{10}\left[5\sqrt{101} + \frac{1}{2}\ln\left(10+\sqrt{101}\right) - \left(0 + \underbrace{\tfrac{1}{2}\ln(1)}_{=0}\right)\right]$$

$$= \boxed{\frac{\sqrt{101}}{2} + \frac{1}{20}\ln\left(10+\sqrt{101}\right)}$$

$\simeq 5.175$



[Reality check: arc should be slightly more than 5 units long, so our answer is plausible!]

4. (*) Find the area between the curves $f(x) = \dfrac{3}{\sqrt{x^2+4x+1}}$ and $g(x) = -5x$ from $x = 0$ to $x = 4$.

Since $f(x) \geq 0$ and $g(x) \leq 0$ for all x, we know $f(x)$ lies above the graph of $g(x)$. So

$$A = \int_0^4 \left(\frac{3}{\sqrt{x^2+4x+1}} - (-5x) \right) dx = \underbrace{\boxed{\int_0^4 \frac{3}{\sqrt{x^2+4x+1}}\, dx}}_{1} + \underbrace{\boxed{\int_0^4 5x\, dx}}_{2}$$

1 $$3\int_0^4 \frac{1}{\sqrt{x^2+4x+1}}\, dx = 3\int_0^4 \frac{1}{\sqrt{x^2+4x+\underline{4}+1-\underline{4}}}\, dx$$

(completing the square)

$$= 3\int_0^4 \frac{1}{\sqrt{(x+2)^2-3}}\, dx$$

$u = x+2$, $du = dx$; $x = 4: u = 6$; $x = 0: u = 2$

$$= 3\int_2^6 \frac{1}{\sqrt{u^2-3}}\, du$$

Use trig. substitution or formula

$$\int \frac{1}{\sqrt{u^2-a^2}}\, du = \ln\left|u + \sqrt{u^2-a^2}\right| + C$$

with $a = \sqrt{3}$

$$= 3 \ln\left|u + \sqrt{u^2-3}\right| \Big|_2^6 = 3\left[\ln\left|6+\sqrt{33}\right| - \ln|2+1|\right].$$

$$= 3\ln\left(\frac{6+\sqrt{33}}{3}\right).$$

2 $$\int_0^4 5x\, dx = \frac{5}{2}x^2 \Big|_0^4 = \frac{5}{2}\cdot 16 = 40.$$

So $A = 1 + 2 = \boxed{3\ln\left(\frac{6+\sqrt{33}}{3}\right) + 40}$

5. (**) Find the volume of the solid formed by rotating the region bounded by $y = 2x\sqrt{\ln x}$, $y = 0$, and $x = e$ about the x-axis.

$$2x\sqrt{\ln x} \overset{\text{set}}{=} 0$$

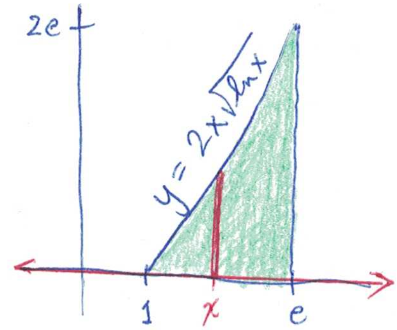
$$\cancel{x = 0} \quad \text{or} \quad \sqrt{\ln x} = 0$$

$$\ln x = 0$$

$$x = 1$$

$\ln x$ is undefined at $x = 0$

$$2e\sqrt{\ln e} = 2e \cdot 1 = 2e.$$



Slice is perpendicular to axis of rotation, so we use the disk method.

$R = 2x\sqrt{\ln x}$

$r = 0$.

$$V = \pi \int_1^e (R^2 - r^2)\,dx$$

$$= \pi \int_1^e 4x^2 \ln x \, dx$$

$$= 4\pi \int_1^e x^2 \ln x \, dx$$

Use parts with $u = \ln x$ and $dv = x^2\,dx$, or use formula

$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

with $n = 2$ (and $u = x$)

$$= 4\pi \cdot \frac{x^3}{9}(3\ln x - 1) \Big|_1^e$$

$$= \frac{4\pi}{9}\Big[e^3(3\underbrace{\ln e}_{=1} - 1) - (3\underbrace{\ln 1}_{=0} - 1)\Big] = \frac{4\pi}{9}\left[e^3 \cdot 2 + 1\right]$$

$$= \boxed{\frac{4\pi}{9}(2e^3 + 1)}$$