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## Spring 2021 MATH 76 Activity 10

## SOME CONVERGENCE TESTS

Strategies for infinite series  $\sum_{k=M}^{\infty} x_k$  for some positive integer M.

- With a quick glance does it look like the series terms don't converge to zero in the limit, i.e. does  $\lim_{k\to\infty} x_k \neq 0$ ? If so, use the **Divergence Test**. Note that you should only use the Divergence Test if a quick glance suggests that the series terms may not converge to zero in the limit.
- Is the series a p-series  $\left(\sum_{k=M}^{\infty} \frac{a}{k^p}\right)$  or a **geometric series**  $\left(\sum_{k=M}^{\infty} ar^k \text{ or } \sum_{k=M}^{\infty} ar^{k-j}\right)$ ? If so use, the fact that a p-series will only converge if p > 1 and a geometric series will only converge if |r| < 1. Remember as well that some algebraic manipulations may be required to get a geometric series into the correct form.
- Does the series  $\sum_{k=M}^{\infty} x_k$  look similar but not equal to a p-series or a geometric series? If so,

then with either a p-series or a geometric series  $\sum_{k=M}^{\infty} y_k$  that you come up with, try:

- 1. the Comparison Test. In this case
  - (a) if  $\sum_{k=M}^{\infty} y_k$  is convergent, you need to justify that  $x_k \leq y_k$
  - (b) if  $\sum_{k=M}^{\infty} y_k$  is divergent, you need to justify that  $y_k \leq x_k$
- 2. or the Limit Comparison Test. In this case
  - (a) if  $\lim_{k\to\infty} \frac{x_k}{y_k} \neq 0$  and a real number,  $\sum_{k=M}^{\infty} x_k$  and  $\sum_{k=M}^{\infty} y_k$  either both converge or both diverge
  - (b) if  $\lim_{k\to\infty}\frac{x_k}{y_k}=0$  and  $\sum_{k=M}^{\infty}y_k$  converges then  $\sum_{k=M}^{\infty}x_k$  converges too
  - (c) if  $\lim_{k\to\infty} \frac{x_k}{y_k} = \infty$  and  $\sum_{k=M}^{\infty} y_k$  diverges then  $\sum_{k=M}^{\infty} x_k$  diverges too.

Remember however, that in order to use the Comparison Test and the Limit Comparison Test the series terms all need to be positive. This is also applicable to a series with rational terms or with terms involving polynomials under radicals (i.e. a fraction involving only polynomials or polynomials under radicals).

• If  $x_k = f(k)$  for some positive, decreasing function f and  $\int_M^\infty f(x)dx$  is easy to evaluate, then the **Integral Test** may work.

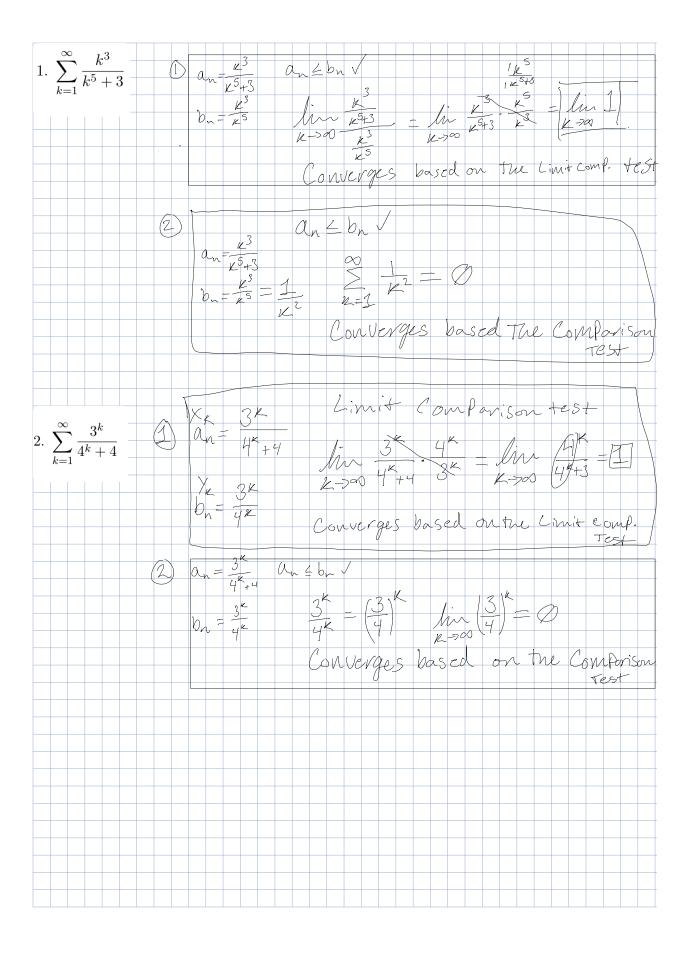
The following table summarizes the tests.

Tests	Series	Convergent if	Divergent if	Comments
Geometric series	$\sum_{k=M}^{\infty} ar^k, \ a \neq 0$	r  < 1	$ r  \ge 1$	$\sum_{k=M}^{\infty} ar^k = \frac{ar^M}{1-r},$ if $ r  < 1$
p-series	$\sum_{k=M}^{\infty} \frac{a}{k^p}$	p > 1	$p \le 1$	
Divergence test	$\sum_{k=M}^{\infty} x_k$	N/A	$\lim_{k \to \infty} x_k \neq 0$	Inconcl. if $\lim_{k \to \infty} x_k = 0$
Integral test	k=M	$\int_{M}^{\infty} f(x)dx \text{ conv.}$	$\int_{M}^{\infty} f(x)dx \text{ div.}$	
	f is continuous, positive, decreasing			
Comparison	$\sum_{k=M}^{\infty} x_k, \ x_k > 0$	$0 < x_k \le y_k$	$0 < y_k \le x_k$	$\sum_{k=M}^{\infty} x_k \text{ is given,}$
		and $\sum_{k=M}^{\infty} y_k$ conv.	and $\sum_{k=M}^{\infty} y_k$ div.	you supply $\sum_{k=M}^{\infty} y_k$
Limit Comparison	$\sum_{k=M}^{\infty} x_k,$	$0 \le \lim_{k \to \infty} \frac{x_k}{y_k} < \infty$	$\lim_{k \to \infty} \frac{x_k}{y_k} > 0$	$\sum_{k=M}^{\infty} x_k \text{ is given,}$
		$\infty$	or $\lim_{k \to \infty} \frac{x_k}{y_k} = \infty$	$\infty$
	$x_k > 0, y_k > 0$	and $\sum_{k=M} y_k$ conv.	and $\sum_{k=M} y_k$ div.	you supply $\sum_{k=M} y_k$

Determine if the following infinite series converges or diverges and explain why. Try several tests for the same series.

1. 
$$\sum_{k=1}^{\infty} \frac{k^3}{k^5 + 3}$$

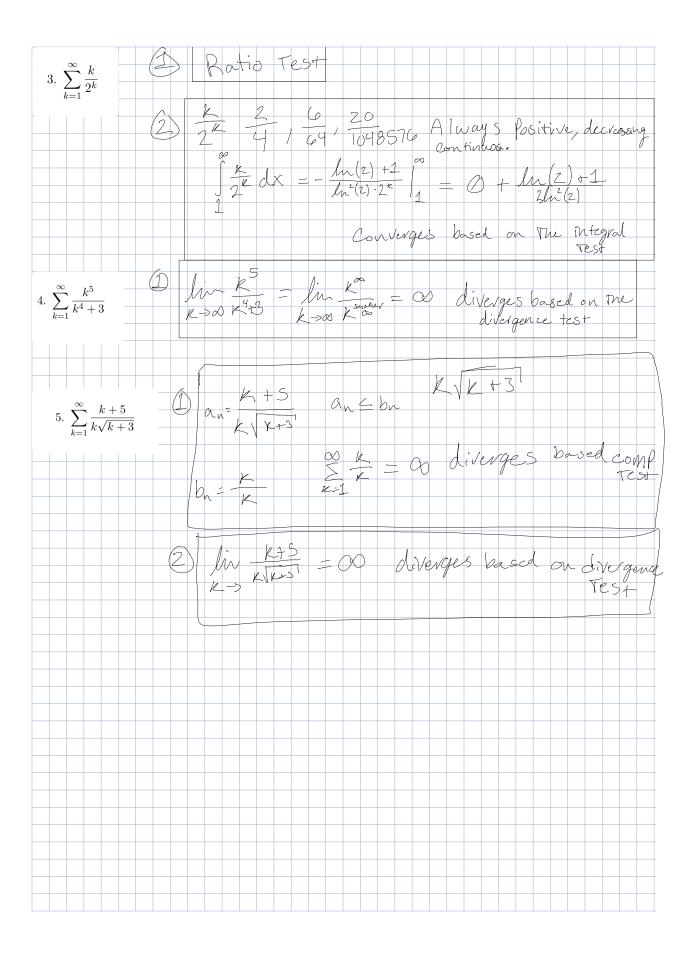
$$2. \sum_{k=1}^{\infty} \frac{3^k}{4^k + 4}$$



$$3. \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$4. \sum_{k=1}^{\infty} \frac{k^5}{k^4 + 3}$$

$$5. \sum_{k=1}^{\infty} \frac{k+5}{k\sqrt{k+3}}$$



$$6. \sum_{k=1}^{\infty} \frac{3 + \cos(k)}{e^k}$$

7. Given that 
$$\sum_{k=0}^{\infty} \frac{1}{k^3 + 1} = 1.6865$$
, determine the value of 
$$\sum_{k=3}^{\infty} \frac{1}{k^3 + 1}$$
.

8. Find the **value** (or limit) of the infinite series  $\sum_{k=1}^{\infty} \frac{1+2^k}{3^{k-1}}$ , if it exists.

