Math 76 Exercises – 7.1 Parametric Equations

1. For each pair of parametric equations,

   (i) Eliminate the parameter to find a Cartesian equation of the curve.

   (ii) Sketch the graph and label at least three points with both Cartesian coordinates and their corresponding t-values. Draw arrows to indicate the direction of increasing t-values.

(a) $x = 5t + 2$
$y = t^2$

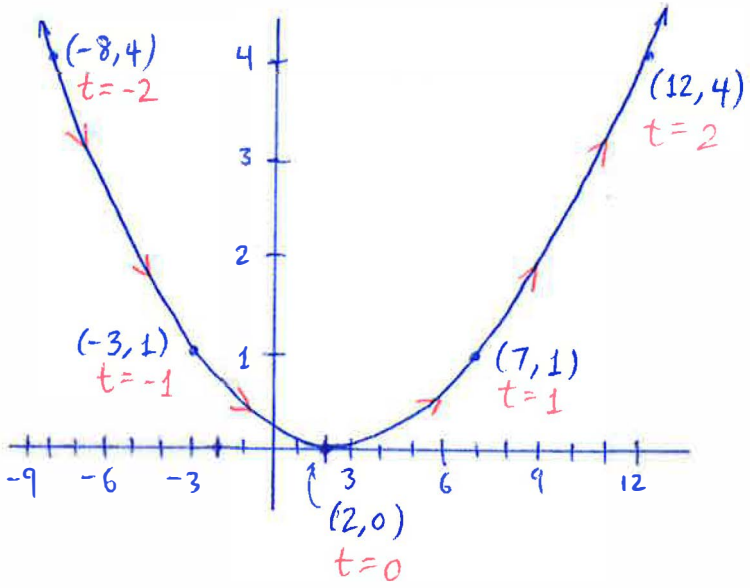
$5t = x - 2$

$t = \frac{x-2}{5}$

$y = \left(\frac{x-2}{5}\right)^2$

$= \frac{(x-2)^2}{25}$

t	x	y
-2	-8	4
-1	-3	1
0	2	0
1	7	1
2	12	4



x^2 shift right 2 $(x-2)^2$ compress vertically to $\frac{1}{25}$ $\frac{1}{25}(x-2)^2$

(b) $x = \sin t$
$y = \cos^2 t \qquad (-2\pi \le t \le 2\pi)$

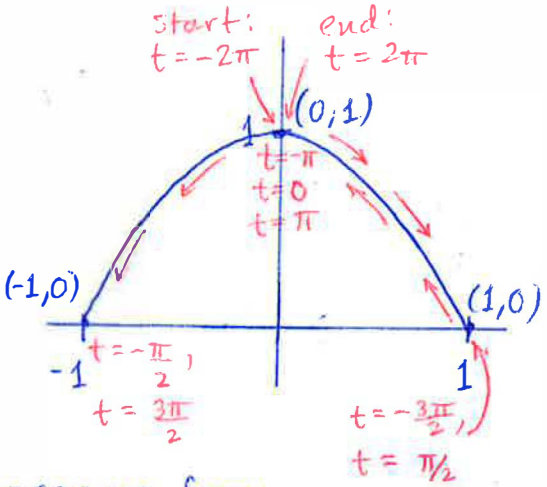
$\sin^2 t + \cos^2 t = 1$

$x^2 + y = 1$

$y = -x^2 + 1$

x^2 reflect about x-axis $-x^2$ shift up 1 $-x^2 + 1$

t	x	y
-2π	0	1
$-3\pi/2$	1	0
$-\pi$	0	1
$-\pi/2$	-1	0
0	0	1
$\pi/2$	1	0
π	0	1
$3\pi/2$	-1	0
2π	0	1



Note that $-1 \le x \le 1$
and $0 \le y \le 1$
for all values of t.

As t increases from -2π to 2π, particle traces the curve back and forth, two complete circuits.

(c) $x = 4\cos t$
$y = 5\sin t$ $\quad (-\frac{\pi}{2} \le t \le \frac{\pi}{2})$

$\frac{x}{4} = \cos t$

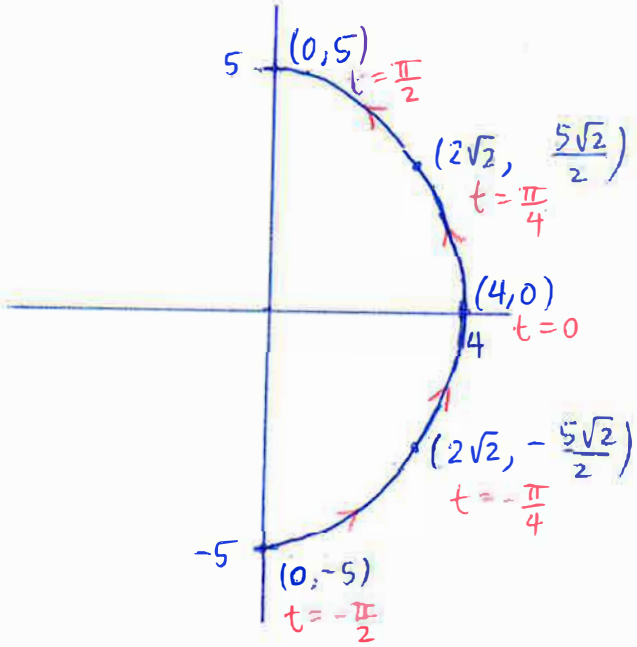
$\frac{y}{5} = \sin t$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$\frac{1}{16}x^2 + \frac{1}{25}y^2 = 1$

Ellipse with minor axis $[-4, 4]$ and major axis $[-5, 5]$.

t	x	y
$-\frac{\pi}{2}$	0	-5
$-\frac{\pi}{4}$	$2\sqrt{2}$	$-\frac{5\sqrt{2}}{2}$
0	4	0
$\frac{\pi}{4}$	$2\sqrt{2}$	$\frac{5\sqrt{2}}{2}$
$\frac{\pi}{2}$	0	5



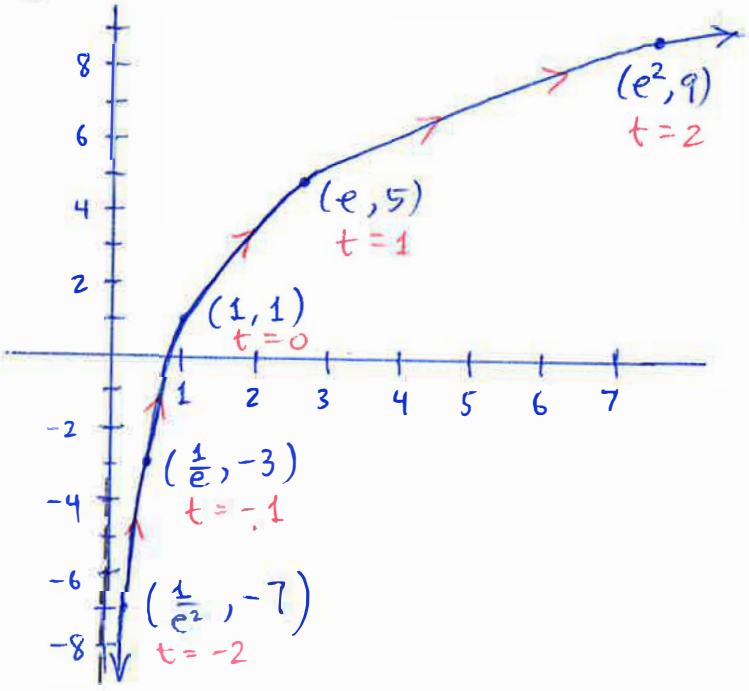
(d) $x = e^t$
$y = 4t + 1$

$t = \ln x$

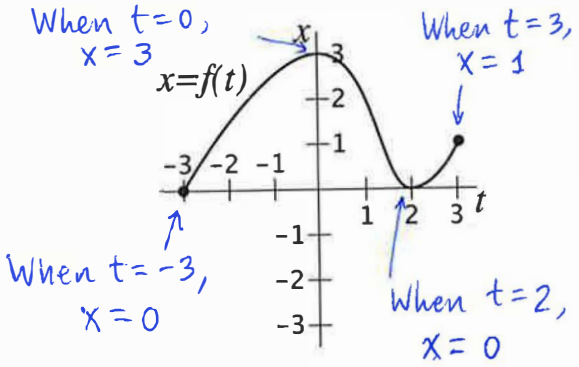
$y = 4\ln x + 1$

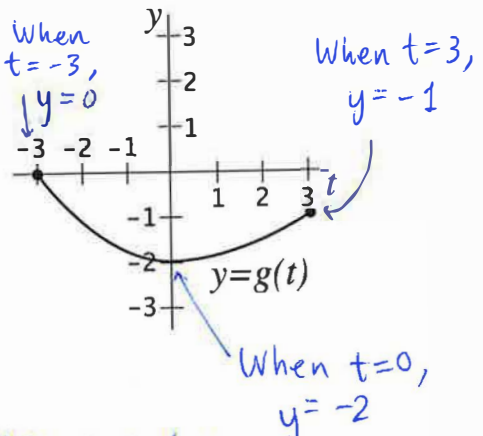
$\ln x \xrightarrow[\text{vertically by 4}]{\text{stretch}} 4\ln x \xrightarrow{\text{shift up 1}} 4\ln x + 1$

t	x	y
-2	$\frac{1}{e^2}$	-7
-1	$\frac{1}{e}$	-3
0	1	1
1	e	5
2	e^2	9



2. Use the graphs of $x = f(t)$ and $y = g(t)$ below to sketch a graph of the parametric curve represented by the equations. Draw arrows to indicate the direction of increasing t-values.





From the graphs, we get the following points:

t	x	y
-3	0	0
•	3	-2
2	0	≈-1.5
3	1	-1

So the graph of the parametric curve looks like: