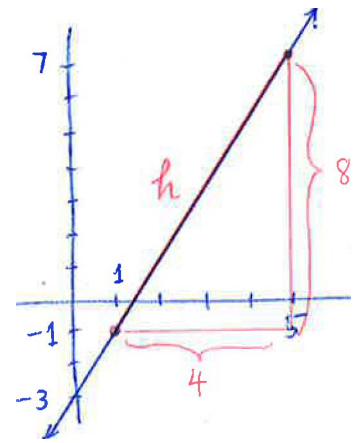
Math 76 Exercises – 2.4A Arc Length – SOLUTIONS

1. Verify that the line $y = 2x - 3$ from $x = 1$ to $x = 5$ has length $4\sqrt{5}$

   (a) using the Pythagorean Theorem directly;

   (b) using the arc length formula.



(a) $h = \sqrt{4^2 + 8^2} = \sqrt{80} = \boxed{4\sqrt{5}}$

(b) $y' = 2$

$1 + (y')^2 = 1 + 2^2 = 5$

$h = \int_1^5 \sqrt{5}\, dx = \sqrt{5}\, x \Big|_1^5$

$= \sqrt{5}(5 - 1)$

$= \boxed{4\sqrt{5}}$

2. **Set up** an integral for the length of each curve. (You can try to evaluate the integrals later, if possible.)

(a) $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$

$y' = \cos x$

$1 + (y')^2 = 1 + \cos^2 x$

$\ell = \int_0^{\frac{\pi}{2}} \sqrt{1 + \cos^2 x}\, dx$

(b) $y = \frac{1}{x}$ from $x = 1$ to $x = 4$

$y' = -\frac{1}{x^2}$

$1 + (y')^2 = 1 + \frac{1}{x^4}$

$\ell = \int_1^4 \sqrt{1 + \frac{1}{x^4}}\, dx$

(c) $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{3}$

$y' = \frac{-\sin x}{\cos x} = -\tan x$

$1 + (y')^2 = 1 + \tan^2 x$

$\ell = \int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x}\, dx$

(d) $x = y^2 + 5$ from $y = -1$ to $y = 3$

$x' = 2y$

$1 + (x')^2 = 1 + 4y^2$

$\ell = \int_{-1}^{3} \sqrt{1 + 4y^2}\, dy$

(e) $x = \sqrt[4]{5y - 1}$ from $y = 2$ to $y = 4$

$x' = \frac{1}{4}(5y-1)^{-3/4} \cdot 5 = \frac{5}{4(5y-1)^{3/4}}$

$1 + (x')^2 = 1 + \frac{25}{16(5y-1)^{3/2}}$

$\ell = \int_2^4 \sqrt{1 + \frac{25}{16(5y-1)^{3/2}}}\, dy$

(f) $x = y \ln y$ from $y = 1$ to $y = 2$

$x' = y \cdot \frac{1}{y} + \ln y = 1 + \ln y$

$1 + (x')^2 = 1 + (1 + \ln y)^2$

$= 1 + 1 + 2\ln y + (\ln y)^2$

$= 2 + 2\ln y + (\ln y)^2$

$\ell = \int_1^2 \sqrt{2 + 2\ln y + (\ln y)^2}\, dy$