Math 76 Assessment 18 §6.3 Taylor's Theorem and Series Approximation

Things to remember:

1. The problem to complete is shown below. Write your name and solution on the next page where instructed.

2. Please make sure your full name is written neatly in the box.

3. Your score will be determined by **Mechanics** (2 points) and by **Content** (3 points).

4. The following rubric will be used for **Mechanics**:

Clear neat work, steps in order and easily followed, proper use of notation	2
Mostly clear work; minor errors in notation or skipped steps	1.5
Steps/handwriting hard to follow/read; major errors in notation	1
No discernible or relevant work, or work impossible to read/follow	0

5. You are not allowed to consult outside sources, including notes, books, the internet, or other people, while taking this assessment. Calculators are allowed only for basic numerical or scientific computations, not for graphing or algebra.

6. If you need more room, you may finish on a plain piece of paper or blank document. If you do all your work on separate sheets, please **copy the problem** and make sure to write **Version B** at the top of the first page.

7. When you are finished, create a legible, well-lit **.pdf file** of your work and upload it to Assessment 18 on Gradescope. Please follow the directions to **assign the page(s)** of your submission that contain your work for the question. More info about submitting to Gradescope:

   `http://bit.ly/gradescope-help`

Recall that by Taylor's Theorem, if

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)(x-c)^k}{k!} = p_n(x) + R_n(x),$$

then $f(x)$ can be approximated by a polynomial $p_n(x)$, and the error is the remainder $R_n(x)$. If we can find a number M such that $|f^{(n+1)}(b)| \leq M$ for all b between x and c, then

$$|R_n(x)| \leq M \frac{|x-c|^{n+1}}{(n+1)!}.$$

Assessment 18 Full Name:

Version B

Let $p_n(x)$ be the degree-n Taylor polynomial of the function $f(x) = \sin(x)$ centered at $c = \dfrac{\pi}{2}$.

(a) Find $p_6(x)$.

(b) Use Taylor's Theorem to find a value of n that will guarantee that $p_n(1.9)$ will approximate $f(1.9)$ with absolute error less than 0.0001.

Your solution should include:

- (1.5 points) Correct $p_6(x)$, with explanation;
- (0.5 point) Correct bound for $R_n(1.9)$, including correct value of M in Taylor's Theorem;
- (1 point) Correct value of n, with explanation.

The information on the previous page may be helpful.