Math 76 Exercises – 7.3B Graphing Polar Curves

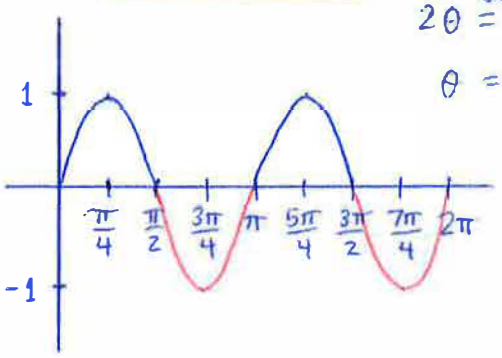
Sketch the curve with the given polar equation. Use r-value analysis when necessary. Identify the type of polar curve (rose, lemniscate, cardioid, spiral, etc.)

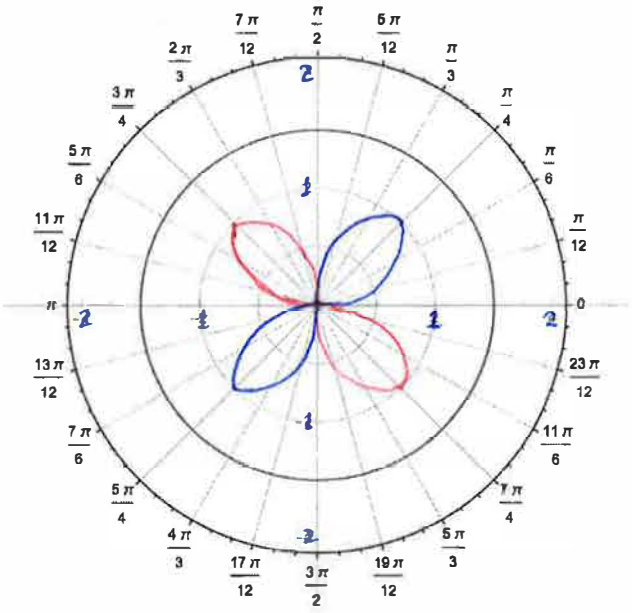
1. $r = \sin(2\theta)$ Period $= \frac{2\pi}{2} = \pi$

4-leaf rose

Set $2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

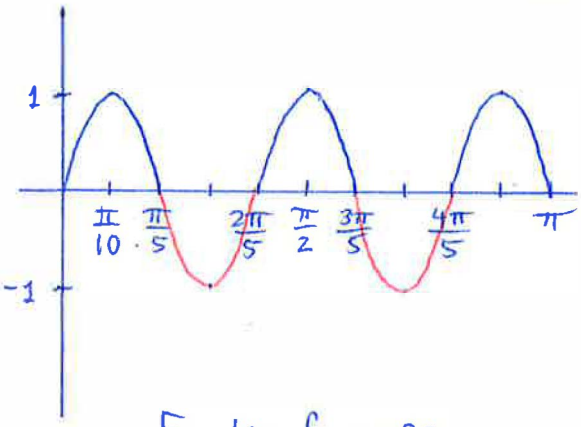




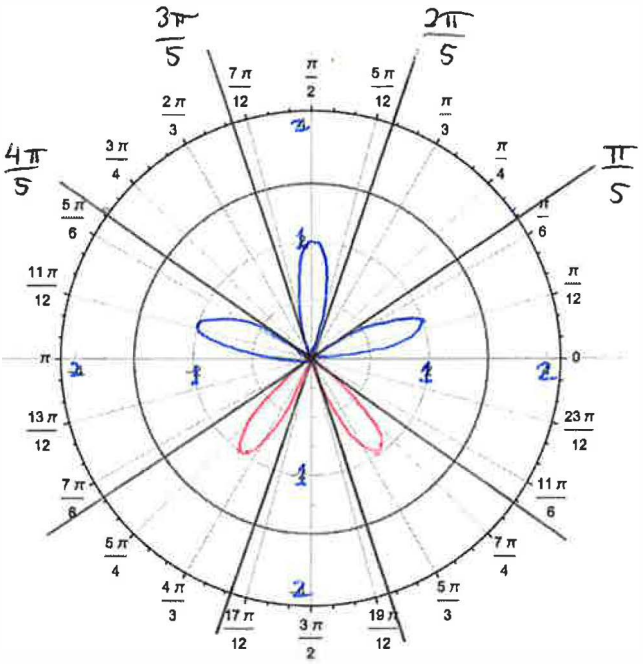
2. $r = \sin(5\theta)$ Period $= \frac{2\pi}{5}$

Set $5\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{10}$



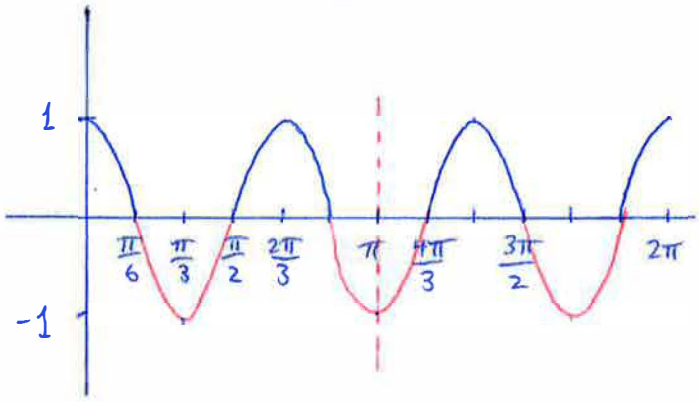
5-leaf rose

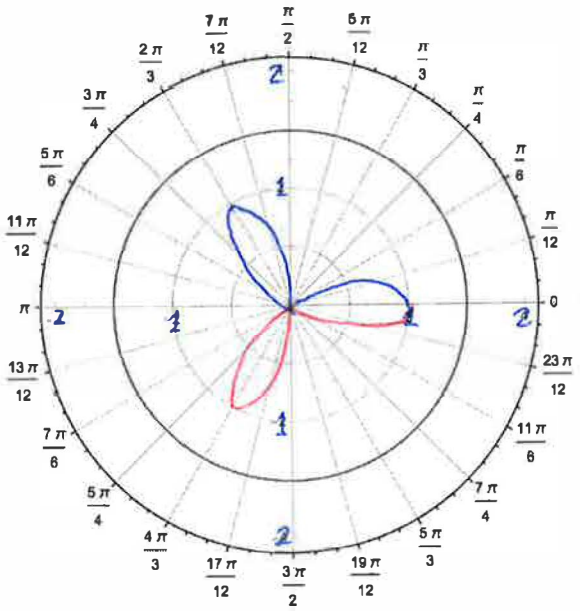


3. $r = \cos(3\theta)$ Period $= \frac{2\pi}{3}$

$3\theta \overset{\text{Set}}{=} \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$



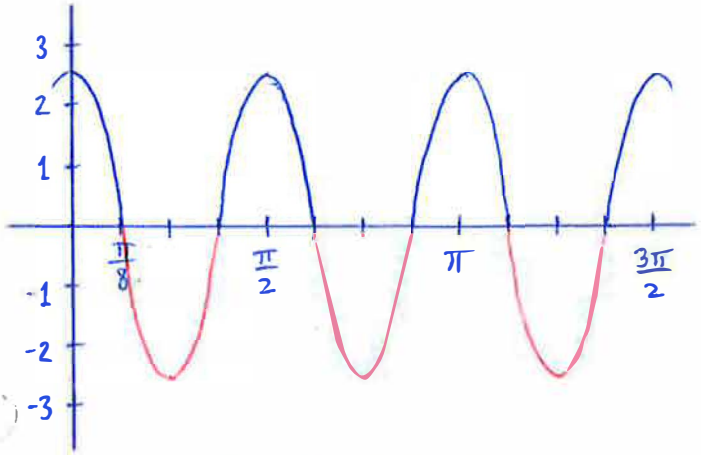


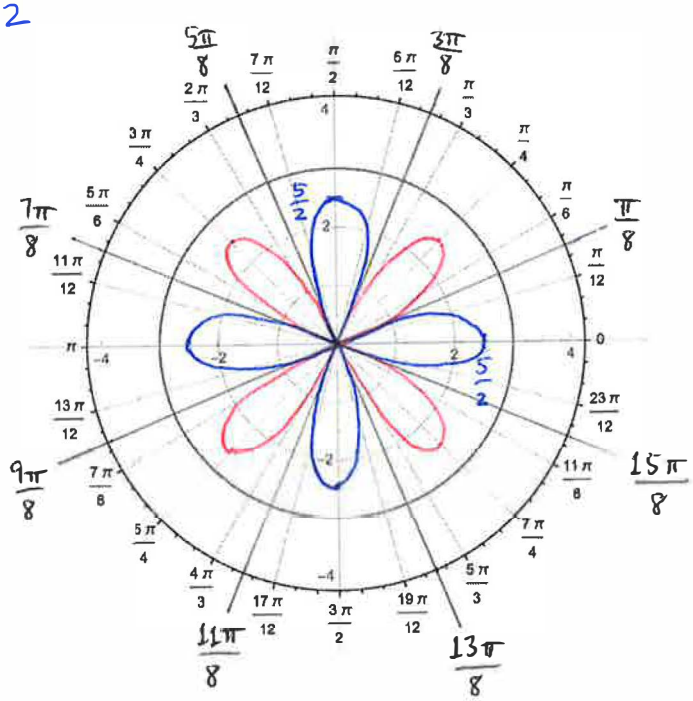
3-leaf rose

4. $r = \frac{5}{2}\cos(4\theta)$ Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$4\theta \overset{\text{Set}}{=} \frac{\pi}{2}$

$\theta = \frac{\pi}{8}$





8-leaf rose

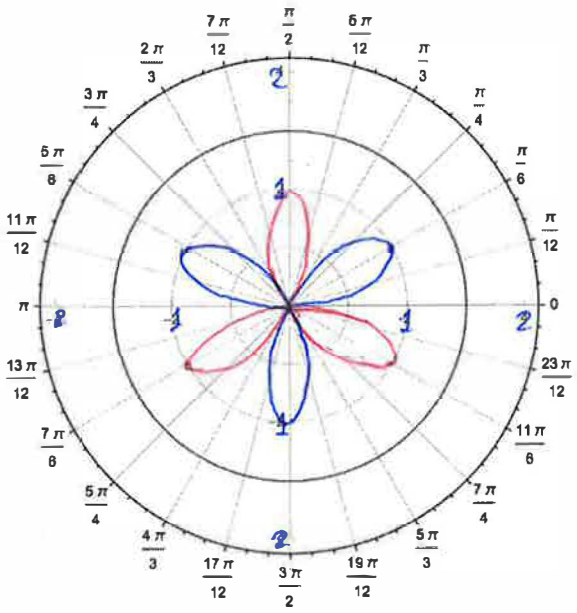
5. $r^2 = \sin(3\theta)$ Period $= \frac{2\pi}{3}$

$3\theta \overset{\text{set}}{=} \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

r<0	r>0	θ
0	0	0
-1	1	$\frac{\pi}{6}$
0	0	$\frac{\pi}{3}$
undef.	undef.	$\frac{\pi}{2}$
0	0	$\frac{2\pi}{3}$
-1	1	$\frac{5\pi}{6}$
0	0	π

r<0	r>0	θ
undef.	undef.	$\frac{7\pi}{6}$
0	0	$\frac{4\pi}{3}$
-1	1	$\frac{3\pi}{2}$
0	0	$\frac{5\pi}{3}$
undef.	undef.	$\frac{11\pi}{6}$
0	0	2π



Lemniscate with 6 leaves

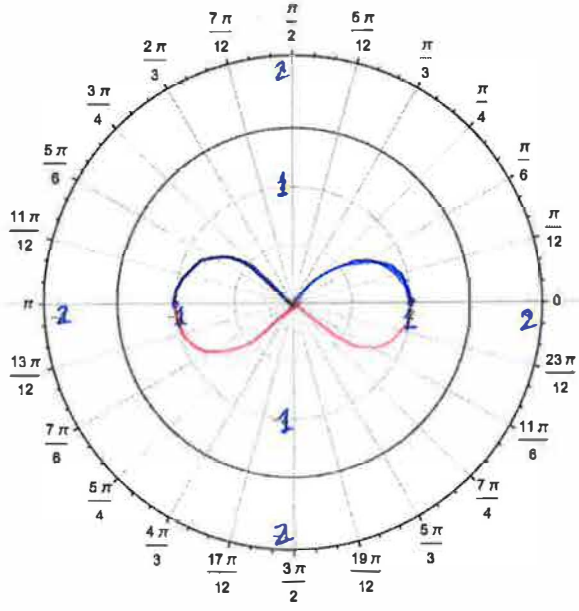
6. $r^2 = \cos(2\theta)$ Period $= \frac{2\pi}{2} = \pi$

$2\theta \overset{\text{set}}{=} \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

r<0	r>0	θ
-1	1	0
0	0	$\frac{\pi}{4}$
undef.	undef.	$\frac{\pi}{2}$
0	0	$\frac{3\pi}{4}$
-1	1	π

r<0	r>0	θ
0	0	$\frac{5\pi}{4}$
undef.	undef.	$\frac{5\pi}{2}$
0	0	$\frac{7\pi}{4}$
-1	1	2π



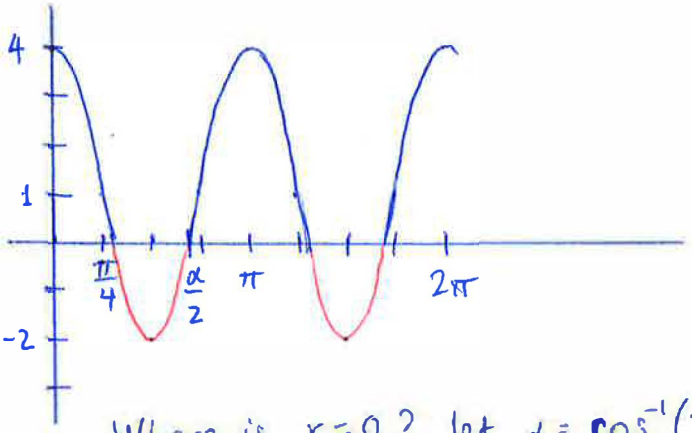
Lemniscate with 2 leaves

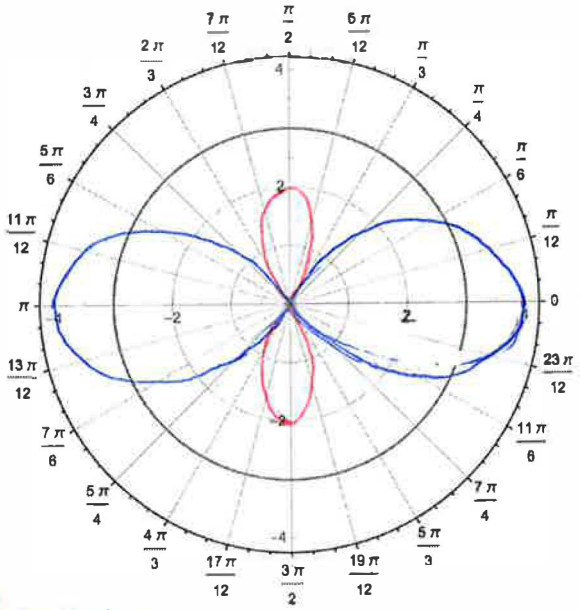
7. $r = 1 + 3\cos(2\theta)$

period $= \frac{2\pi}{2} = \pi$.

$2\theta \overset{\text{set}}{=} \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$





Where is $r = 0$? let $\alpha = \cos^{-1}(-\frac{1}{3}) \approx 1.91$

$1 + 3\cos(2\theta) \overset{\text{set}}{=} 0$

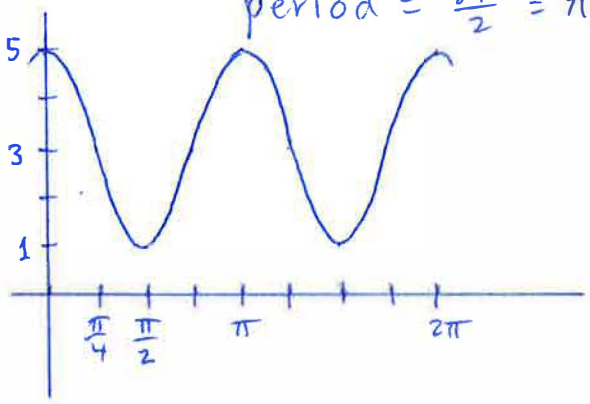
$\cos(2\theta) = -\frac{1}{3}$

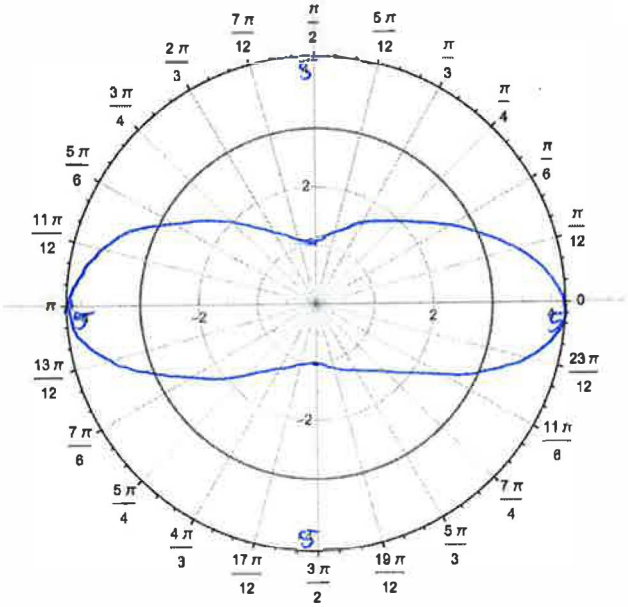
$2\theta = \cos^{-1}(-\frac{1}{3}) = \alpha$ or $\pi - \alpha$ or $2\pi - \alpha$ or $\pi + \alpha$

So $\theta = \frac{\alpha}{2} \approx 0.96$ or $\frac{\pi - \alpha}{2} \approx 0.615$ or $\frac{2\pi - \alpha}{2} \approx 2.19$ or $\frac{\pi + \alpha}{2} \approx 2.53$

8. $r = 3 + 2\cos(2\theta)$

period $= \frac{2\pi}{2} = \pi$



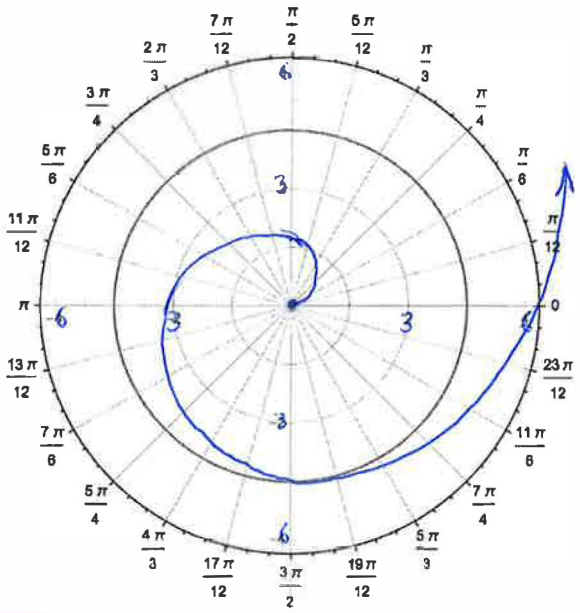


Note that $r > 0$ for all θ.

9. $r = \theta \quad (\theta \geq 0)$

Spiral opening

counter-clockwise

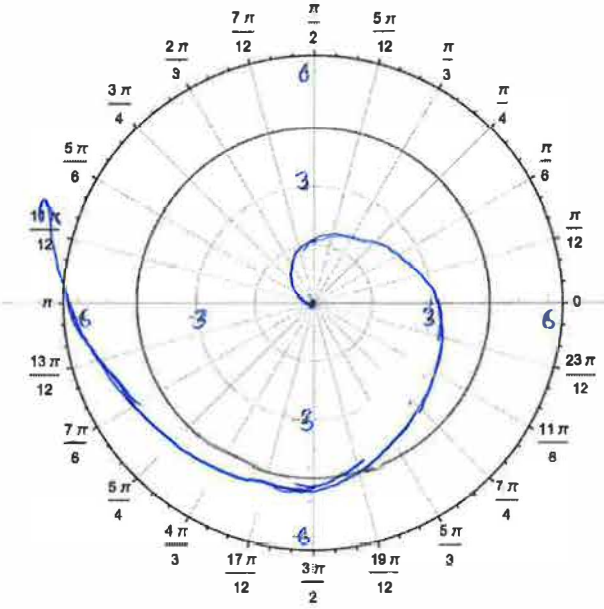


10. $r = \theta \quad (\theta \leq 0)$

r	θ ≤ 0
0	0
-1.5	-1.5
-3	-3
-4.5	-4.5
-6	-6

r	θ
0	0
1.5	-1.5+π
3	-3+π
4.5	-4.5+π
6	-6+π

equivalent polar points



Spiral opening clockwise