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## Spring 2021 MATH 76 Activity 6

## TRIG SUBSTITUTION

When a is a positive fixed number, we choose  $\theta = \sin^{-1}\left(\frac{x}{a}\right)$ , or  $\theta = \tan^{-1}\left(\frac{x}{a}\right)$ ,

or  $\theta = \sec^{-1}\left(\frac{x}{a}\right)$  when it is convenient. The different types of substitutions are highlighted in the table below, along with the values of x and corresponding values of  $\theta$  that make the substitutions valid.

Expression	Trig sub	Trig identities	Simplification
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta$	$\sqrt{a^2 - x^2} = a\cos\theta$ , $-a \le x \le a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$a^2 + a^2 \tan^2 \theta = a^2 \sec^2 \theta$	$\sqrt{a^2 + x^2} = a \sec \theta$ , all $x$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$a^2 \sec^2 \theta - a^2 = a^2 \tan^2 \theta$	$\sqrt{x^2 - a^2} = \begin{cases} a \tan \theta, & x \ge a \text{ and } 0 \le \theta < \frac{\pi}{2} \\ -a \tan \theta, & x \le -a \text{ and } \frac{\pi}{2} < \theta \le \pi \end{cases}$

1. Use a trig substitution of x to simplify the following expressions.

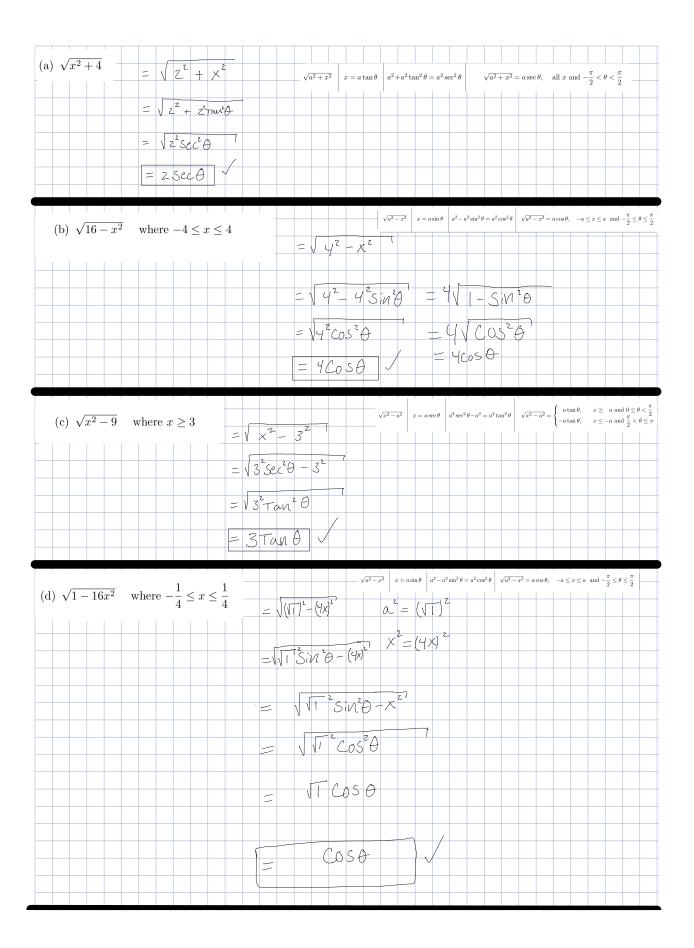
(a) 
$$\sqrt{x^2 + 4}$$

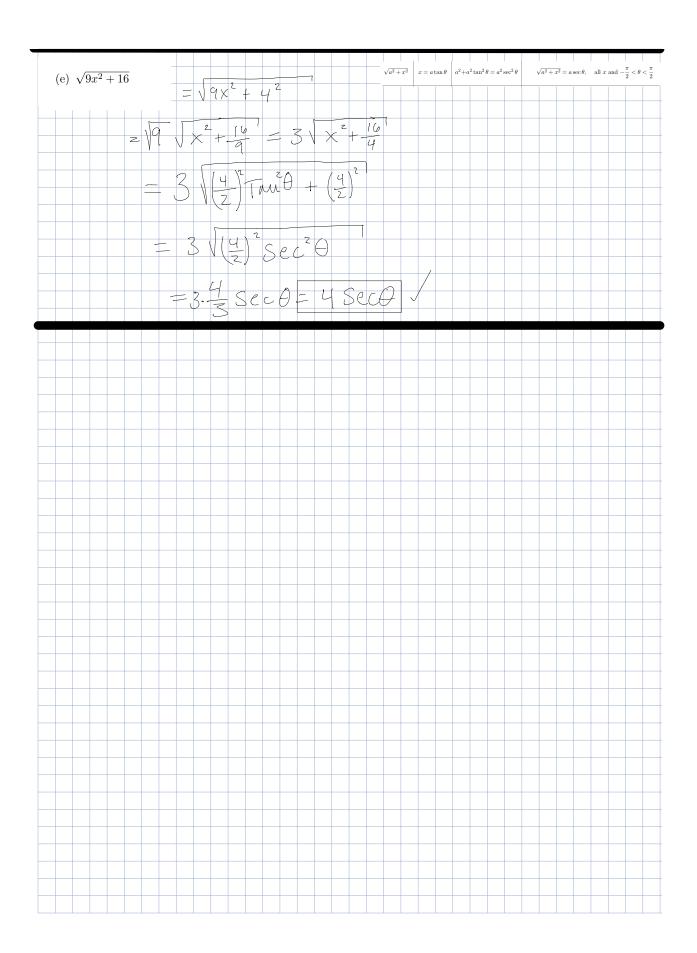
(b) 
$$\sqrt{16 - x^2}$$
 where  $-4 \le x \le 4$ 

(c) 
$$\sqrt{x^2 - 9}$$
 where  $x \ge 3$ 

(d) 
$$\sqrt{1-16x^2}$$
 where  $-\frac{1}{4} \le x \le \frac{1}{4}$ 

(e) 
$$\sqrt{9x^2 + 16}$$





2. Make the appropriate trig substitution of x in the following integrals and simplify. DO NOT FORGET dx. DO NOT INTEGRATE.

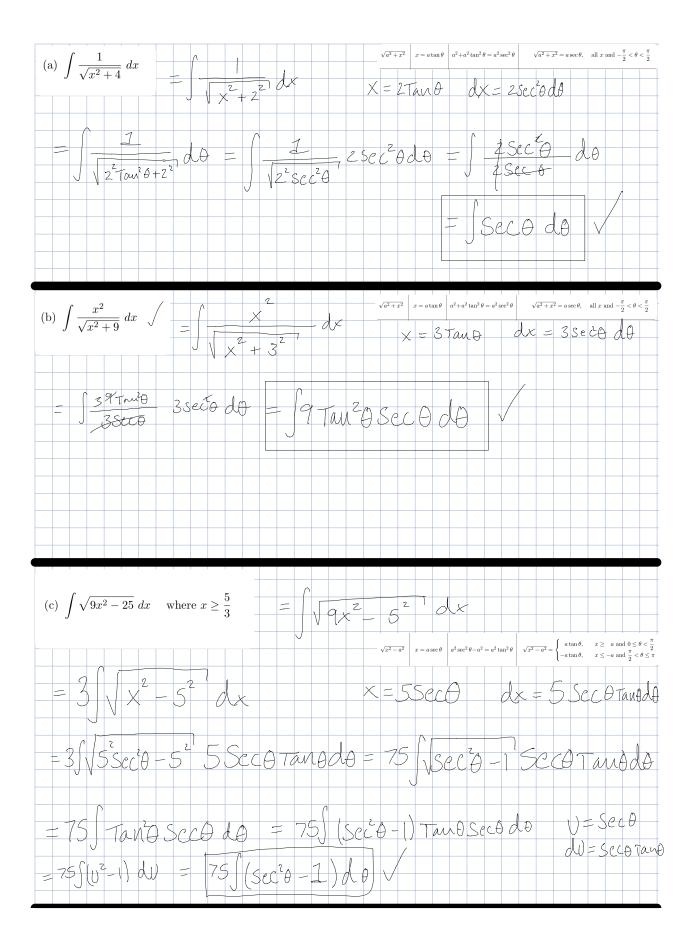
(a) 
$$\int \frac{1}{\sqrt{x^2 + 4}} \, dx$$

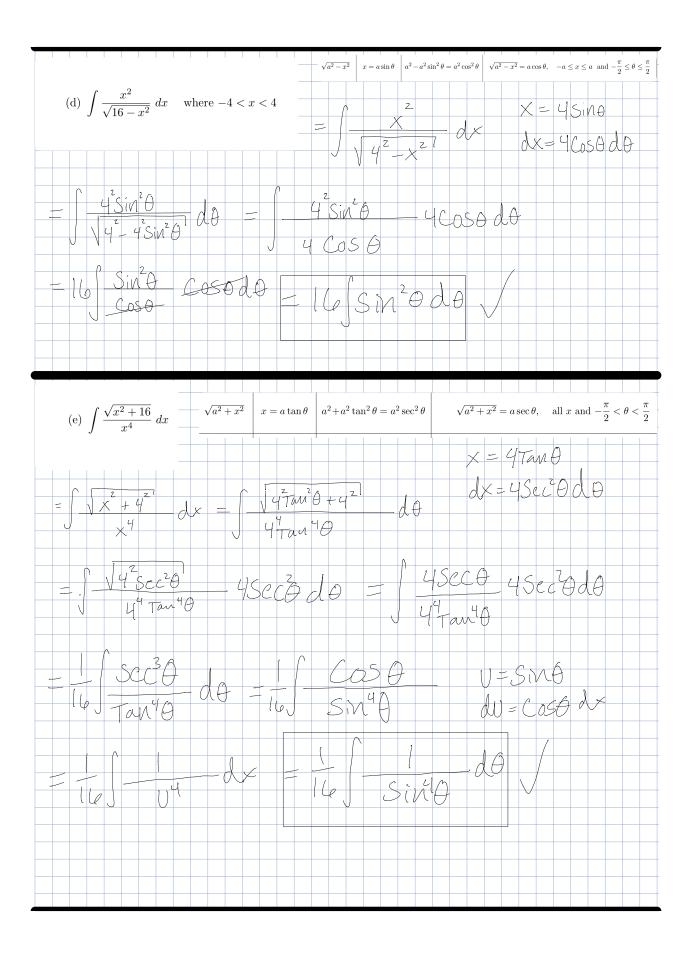
(b) 
$$\int \frac{x^2}{\sqrt{x^2+9}} \ dx$$

(c) 
$$\int \sqrt{9x^2 - 25} \ dx$$
 where  $x \ge \frac{5}{3}$ 

(d) 
$$\int \frac{x^2}{\sqrt{16-x^2}} dx$$
 where  $-4 < x < 4$ 

(e) 
$$\int \frac{\sqrt{x^2 + 16}}{x^4} dx$$





3. Compute the following integrals by using a trig substitution of x. Do not forget dx.

(a) 
$$\int \frac{1}{\sqrt{x^2 - 25}} dx \quad \text{where } x > 5$$

(b) 
$$\int \frac{x^2}{\sqrt{1-x^2}} dx$$
 where  $-1 < x < 1$ 

(c) 
$$\int \sqrt{1+4x^2} dx$$

