Chapter 5

19) The value of p (p*) that maximises the efficiency of slotted ALOHA is:

$E(p) = Np(1 - p)^{N-1}$
$E'(p) = N(1 - p)^{N-1} - Np(N-1)(1 - p)^{N-2}$
$\quad = N(1-p)^{N-2}((1 - p)-p(N- 1))$
$E'(p) = 0 \Rightarrow p^* = 1/N$

Using this value, the max efficiency of slotted ALOHA is;

$E(p^*)=N\ 1/N(1-1/N)^{N-1}= (1-1/N)^{N-1}= (1-1/N)^{N}/(1-1/N)$

$\lim_{(N\to \text{infinity})} (1-1/N) = 1 \qquad \lim_{(N\to \text{infinity})} (1-1/N)^{N} = 1/e$

Thus:

$\lim_{(N\to \text{infinity})} E(p^*)= 1/e$

The value of p (p*) that maximises the efficiency of ALOHA is:

$E(p) = Np(1 - p)^{2(N-1)}$
$E'(p) = N(1 - p)^{2N-2} – Np2(N-1)(1 - p)^{2(N-3)}$
$\quad = N(1-p)^{2(N-3)}((1 - p)-p2(N- 1))$
$E'(p) = 0 \Rightarrow p^* = 1/(2N-1)$

Using this value, the max efficiency of ALOHA is;

$\lim_{(N\to \text{infinity})} E(p^*)= ½ * 1/e =1/2e$

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