Buoyancy

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1 Review

(under construction)

2 Buoyant Force

We have neglected an important force in our study of cause and effect. This force appeared in nearly every problem we encountered, but we neglected it because it was typically too small to worry about. But for some problems, it becomes a dominant force. We are talking about buoyancy.

2.1 Why do objects float?

Consider a block of wood placed on the surface of water and released. Initially, the block of wood accelerates downwards until at some point its speed drops to zero, at which point we say that the object "floats." Once an object is floating, it no longer accelerates. How does this happen?

Clearly, the water must apply some upward form of force to the wood, otherwise the wood would accelerate downwards at $g = 9.8 \text{ m/s}^2$. This force is called the *buoyant force* and denoted $\vec{F}_{\mathrm{B}}$.

Initially, the object accelerates downwards because the force of gravity is stronger than the upward force exerted by the water. As the object sinks further downward, this upward increases to the point where it vector cancels the force of gravity. At this point, the wood is no longer accelerating and has therefore reached equilibrium. Once the object is floating, Newton's second law states:

$$\sum \vec{F} = m\vec{a} \qquad \longrightarrow \qquad \vec{F}_{\mathrm{B}} - mg = 0 \qquad \longrightarrow \qquad F_{\mathrm{B}} = mg.$$

All this states so far is that when the object is floating, the upward buoyant force F_{B} must equal the force of gravity acting on the object.

At what point will this happen? Clearly, the depth at which the wood sinks into the water must play a role in the strength of F_{rmB}. After all, as the wood sinks, nothing else changes. We can easily surmise that as the object sinks, the buoyant force gets larger until it offsets the force of gravity.

Archimedes thought this question over and arrived at Archimedes Principle: *Objects will sink until the weight of the liquid they displace equals their own weight, at which point they float.*

What does this mean? It is perhaps best to build this concept slowly. First, let us review a concept that you have probably seen before in your high school classes. We define the *density* ρ of an object (whether it is a gas, liquid, or solid) as

$$\rho = \frac{m}{V},$$

where m is the mass of the object and V is its volume. Naturally, the units of density are grams per cubic centimeters (g/cm^3) in the CGS system and kilograms per cubic meter (kg/m^3) in the SI system. Water has a density of 1 g/cm^3 in the CGS system and 1000 kg/m^3 in the SI system. This means that a 1 m^3 tank (think about the size of a coffin, which is pretty close a cubic meter) when filled with water would have a mass of 1000 kg (that is, about 1,000 physics textbooks).

Because water is so ubiquitous, we often use it for comparison purposes. The *specific gravity* of an object relates its density to that of water. Therefore, an object with a specific gravity of 2 is twice as dense as water. Objects with a specific gravity greater 1 will sink; those with a specific gravity lower than 1 will float.

Now that we have defined density, let us see if we can examine what happens when an object is placed in a fluid. For now, we will concentrate on solid objects being placed in liquids. We will expand our coverage later.

In Figure 1 we show a block of wood being lowered into water. The block is a perfect cube of $H = 2$ meters on a side (therefore having a volume of 8 m^3). This is a very large block, about the size of a professor's office.[1] The density of the block is 800 kg/m^3, which means it must have a mass of $m = \rho V = (800)(8) = 64,000$ kg. That's a huge amount of mass, but remember this block of wood is also huge.

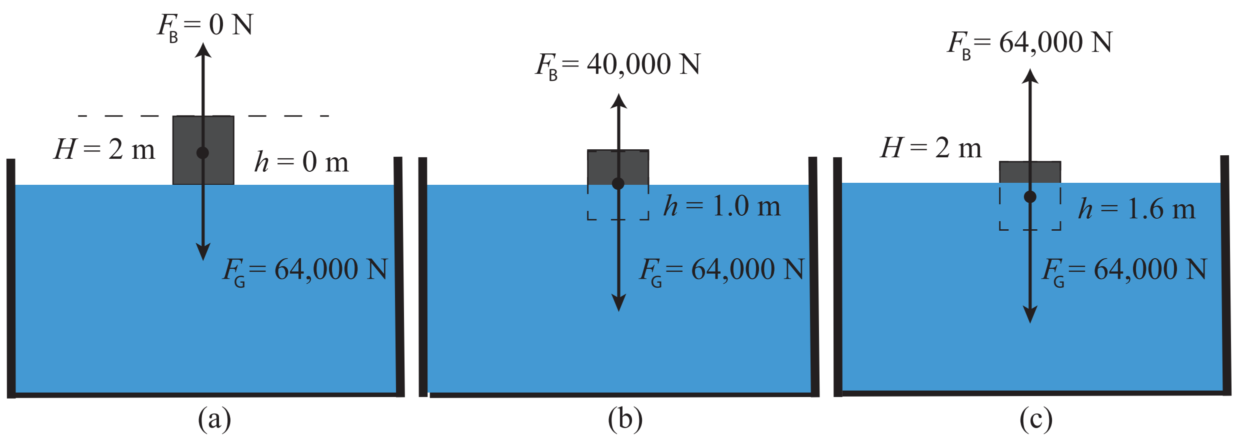

Figure 1: The buoyant force acting on a 2 X 2 X 2 wood block as it is lowered into water. In (c), the wood reaches equilibrium and therefore floats.

When the object is placed initially on the surface, as shown in part (a), the liquid does not apply any buoyant force at all, so the force of gravity is unbalanced and the wood will accelerate downward. In part (b), the object is halfway submerged ($h = 1$ meter) — at this point the upward buoyant force is only $40,000$ newtons, which is less than the force of gravity so the object continues to accelerate downward. Once the object reaches a depth of $h = 1.6$ meters, the buoyant force reaches $64,000$ newtons and therefore vector cancels the force of gravity. At this point, Newton's second law dictates that the object must not accelerate and so the object floats.

But why does the object float at 1.6 meters? To answer that question, we must understand how to calculate the magnitude of this buoyant force. Clearly, it depends on how much of the object is under the surface of the liquid. Also, the density of the liquid must make a difference — after all, if the water was replaced with something more dense, like syrup, we would expect the wood block to rise a little higher in depth.

[1]Not the office of a hot-shot professor, mind you. More like the professor you ended up with teaching your course.

It turns that the magnitude of the buoyant force is given by

$$\vec{F}_{\mathrm{B}} = \rho_{\mathrm{f}} V_{\mathrm{D}} g, \tag{1}$$

where ρ_{f} is the density of the fluid (not the object!), V_{D} is that volume of the object that is under the surface of the fluid, and g is the gravitational acceleration constant. Notice that denser fluids will generate larger buoyant forces. Also notice as the object sinks further into the surrounding fluid, the buoyant force will increase.

We can now answer our question: The object will float once the buoyant force acting on it from the surrounding fluid vector cancels the other forces. If the only other force is gravity, then

$$\rho_{\mathrm{f}} V_{\mathrm{D}} g = mg.$$

This is the essence of Archimedes Principle, which states that an object floats when it displaces its own weight in water.

Let us see if we can elaborate on Archimedes' Principle a bit more. If you examine part (c) of Figure 2 you will notice that the certain portion of the wood is under water. This portion is denoted with the dashed line. If I pull the object out of the water, it would ideally leave a hole in the water. (In real life, the water would rush in to fill the hole, but we will pretend that the water does not move. This hole has a volume V_{D}. If the wood block was all of a sudden turned into a

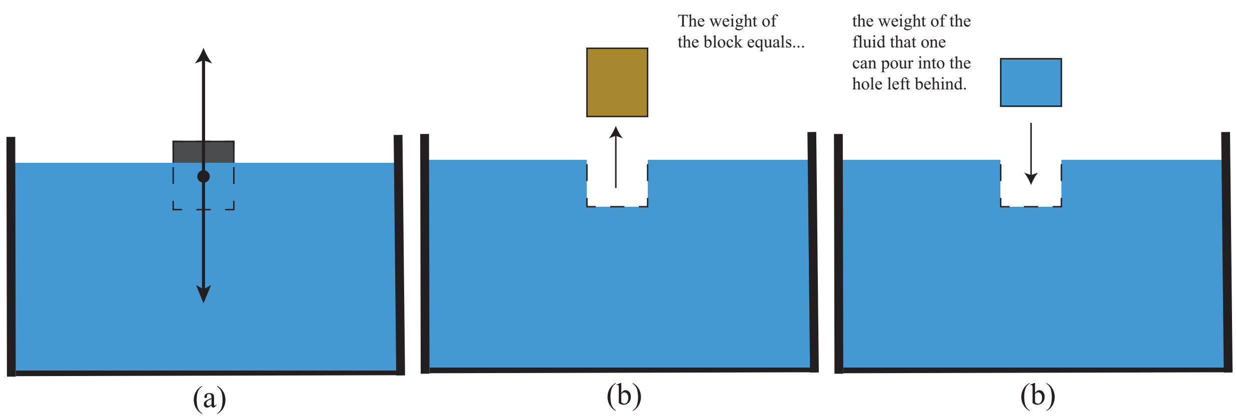

Figure 2: Archimedes Principle states that a floating object displaces its weight in water (or whatever fluid the object is immersed. In (a), the object is floating in equilibrium. In (b), the object is plucked out of the water, magically leaving behind a crater where it once was. In (b), the wood is transformed into a volume of water having the same weight as the original wood. This water is poured into the crater, where it fills up the crater exactly.

block of water that had the same weight as the wood block, it would fill in this hole exactly.

2.2 What happens when objects sink?

Once an object is completely submerged, the volume of water it displaces exactly equals its own volume, therefore $V_{\mathrm{D}} \to V_{\mathrm{obj}}$, where "obj" stands for "object." In this case, the buoyancy force becomes

$$\vec{F}_{\mathrm{B}} = \rho_{\mathrm{f}} V_{\mathrm{D}} g \qquad \to \qquad \vec{F}_{\mathrm{B}} = \rho_{\mathrm{f}} V_{\mathrm{obj}} g$$

Because this buoyant force is less than its weight (otherwise it would float), then the object will continue to accelerate downward. (The friction due to water will lower its acceleration a lot.)

Suppose the object was placed on a hanging scale. What would it read? Well, we know that a hanging scale measures the tension force necessary to keep the object from accelerating, so we need to find this tension force.

An object appears to weigh less when immersed in a fluid. This is an illusion caused by our inability to distinguish between a normal force and weight. In reality, the weight of an object (that is, mg) is fixed; what changes is the support force needed to keep the object from accelerating. A hanging scale (as shown in Figure 3, is a tension force o'meter — it measures tension force. (A floor scale, such as a bathroom scale, is a normal force o'meter.) By how much does the reading

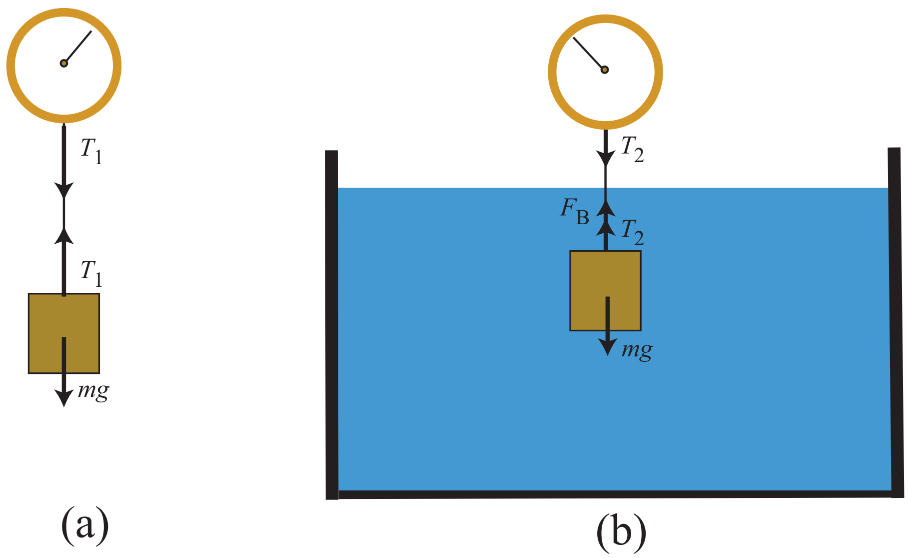

Figure 3: A hanging scale is a tension force o'meter; it measures the tension force needed to prevent an object from accelerating. When there is no surrounding fluid, as in (a), this tension force is equivalent to mg since Newton's second law states that $T_1 - mg = 0$. Once immersed in a fluid, however, this tension force drops to a lower value T_2 because there is a buoyant force helping prevent the object from accelerating downward. Therefore, the reading on a hanging scale drops.

on a hanging scale change when the object is immersed in a fluid. Let us assume the mass of the wood block in Figure 3 is $m = 3$ kg and that the fluid is (say) alcohol with a density of $\rho = 800$ kg/m^3. In part (a) of Figure 3, the tension required to keep the block from accelerating is given by Newton's second law, which dictates that

$$T_1 - mg = 0 \qquad \longrightarrow \qquad T1 = mg.$$

This simply says that in normal circumstances the hanging scale reads a tension that happens to be the same as the weight of the object.

When it is immersed in a fluid, as in Figure 3 (b), Newton's second law states that

$$T_2 + F_\text{B} - mg = 0,$$

which means that $T_2 = mg - F_\text{B}$. So the hanging scale reading in the second scenario is lower than the scale reading in the first scenario by an amount F_B. In other words, the difference between the two scale readings, ΔT, is the amount of buoyant force F_B exerted on the block when it is immersed, that is,

$$F_\text{B} = \Delta T.$$

Our expression for the buoyant force can now be substituted into our relationship,

$$\vec{F}_{\mathrm{B}} = \rho_{\mathrm{f}} V_{\mathrm{obj}} g = \Delta T.$$

(Notice that we use V_{obj} instead of V_{D}; when an object is submerged its volume is equivalent to the volume displaced.)

The above result means that if we "weigh" an object when immersed in a fluid and when not, the difference in the readings can be used to compute the density of the fluid (if one knows the volume of the object) or the volume of the object (if one knows the density of the fluid).

We can go even further and find something even more useful. Suppose we replace the volume of the object by noting that

$$\rho \equiv \frac{m}{V} \qquad \longrightarrow \qquad V = \frac{m}{\rho}.$$

Then our equation above becomes

$$\rho_{\mathrm{f}} V_{\mathrm{obj}} g = \Delta T \qquad \longrightarrow \qquad \rho_{\mathrm{f}} \frac{m_{\mathrm{obj}}}{\rho_{\mathrm{obj}}} V_{\mathrm{obj}} g = \Delta T.$$

This means that if we know the mass of the object (which we already do because of our earlier scale reading), then we can find the density of the object.

What are the applications of this result? A person's mass-body index describes his or her density. So, if we weighed the person outside of water and then while immersed in water, we could find their density ρ_{obj}. The lower the density, the higher their body-mass index.

2.3 Flotsam and Jetsam

One of the age-old physics questions concerns what happens to water level when one throws jetsam[2] overboard. The question can take two different avenues: (1) What happens if the jetsam has a specific gravity (SG) less than 1 and (2) what happens if the jetsam has a specific gravity greater than 1.

2.3.1 SG > 1

When the specific gravity of the jetsam is greater than 1, then the object will sink. We should note right up front that, because the density of the jetsam is greater than that of water, then it takes a greater volume of water to make up the the same weight as the volume of jetsam.

When the jetsam is on board the boat, it presses the boat into the water such that its presence displaces a volume equal to its weight in water. (The boat displaces a certain amount of water due to its own weight, so this displacement is an *additional* displacement above that.)

Once thrown overboard and under water, the jetsam displaces only its own volume.

However, because it takes more water to make up the weight of jetsam than the volume of the jetsam itself — remember, we are considering here the case where the density of the jetsam is greater than that of water — then the jetsam takes up less volume of water when it is under the water than it is in the boat.

For this reason, the water level of the lake will drop when jetsam is thrown overboard.

2.3.2 SG < 1

When the jetsam has a specific gravity less than that of water, it will float once thrown overboard. In both cases — on the boat and in the water — the jetsam will displace its weight in water. Therefore, the water level will not change in this situation.

[2] You may not know what jetsam is. To the Google cave goes you!