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Spring 2021 MATH 76 Activity 12

$$L(x) = f(a) + f'(a)(x - a)$$

TAYLOR POLYNOMIALS

$$Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^{2}$$

1. LINEAR AND QUADRATIC APPROXIMATIONS

Compute the linear approximation centered at a defined by

$$L(x) = f(a) + f'(a)(x - a)$$

and the quadratic approximation centered at a defined by

$$Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$$

for the following functions when available:

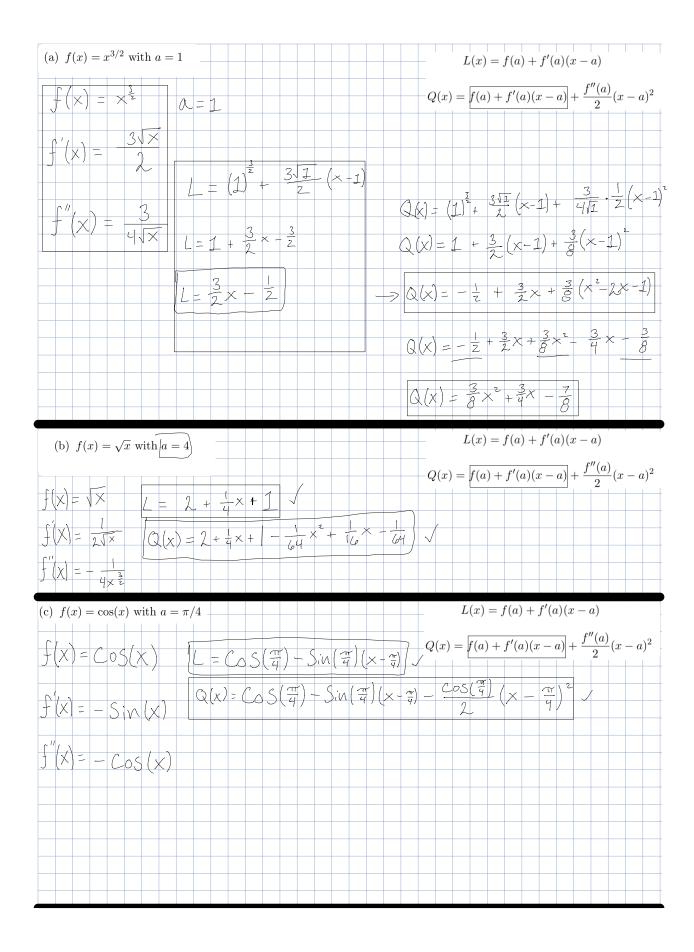
(a)
$$f(x) = x^{3/2}$$
 with $a = 1$

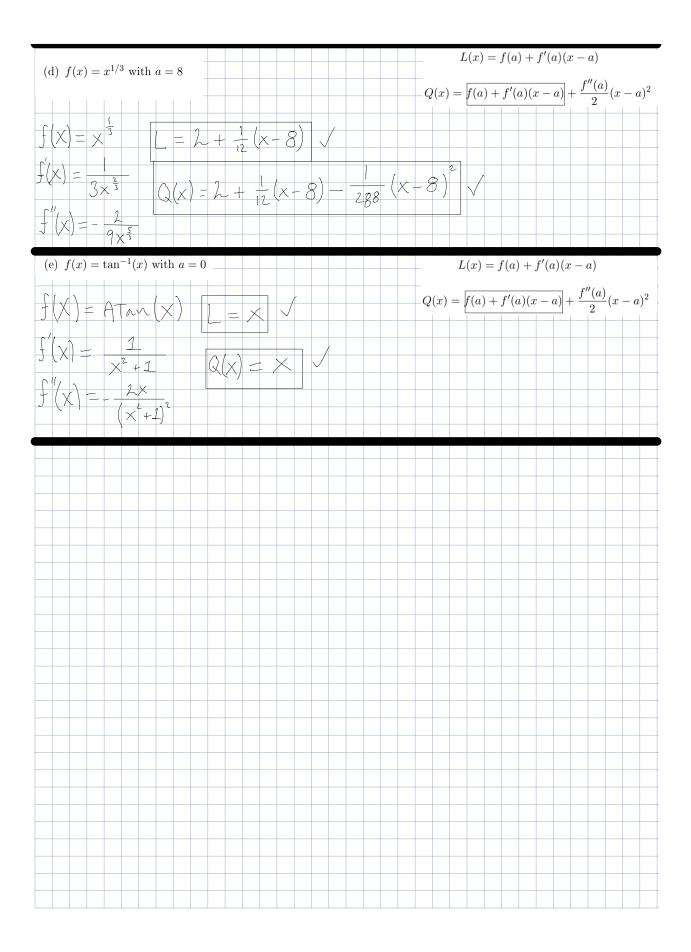
(b)
$$f(x) = \sqrt{x}$$
 with $a = 4$

(c)
$$f(x) = \cos(x)$$
 with $a = \pi/4$

(d)
$$f(x) = x^{1/3}$$
 with $a = 8$

(e)
$$f(x) = \tan^{-1}(x)$$
 with $a = 0$





2. TAYLOR POLYNOMIALS

Let f be a function whose derivatives $f', f'', \ldots, f^{(n)}$ are all defined at a. The nth order Taylor polynomial centered at a is defined as

$$p_n(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x - a)^n$$
$$= \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x - a)^k$$

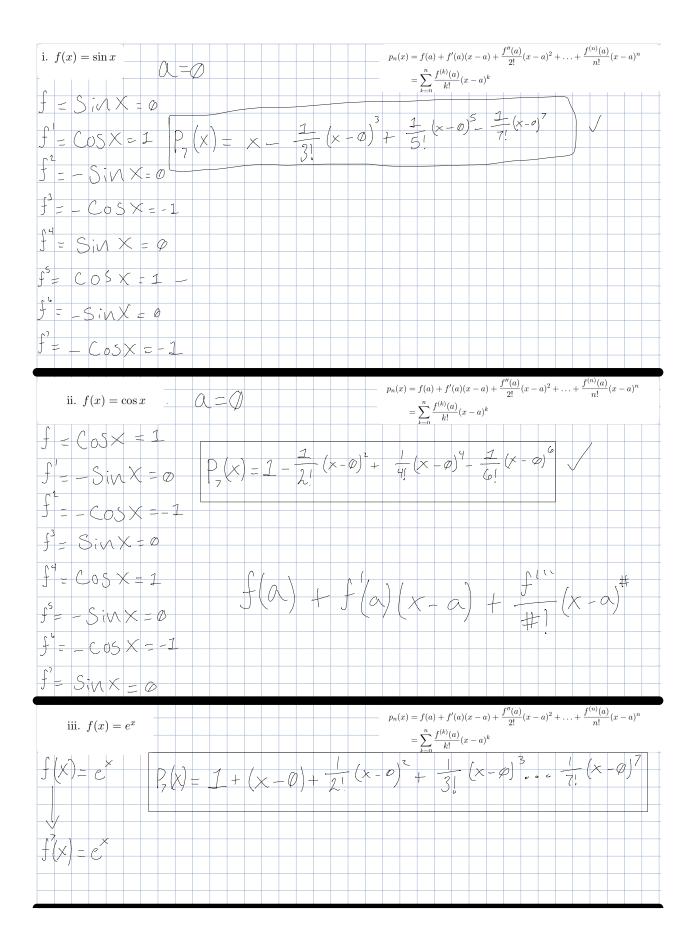
(a) Compute the 7th Taylor polynomial centered at a=0 for the functions

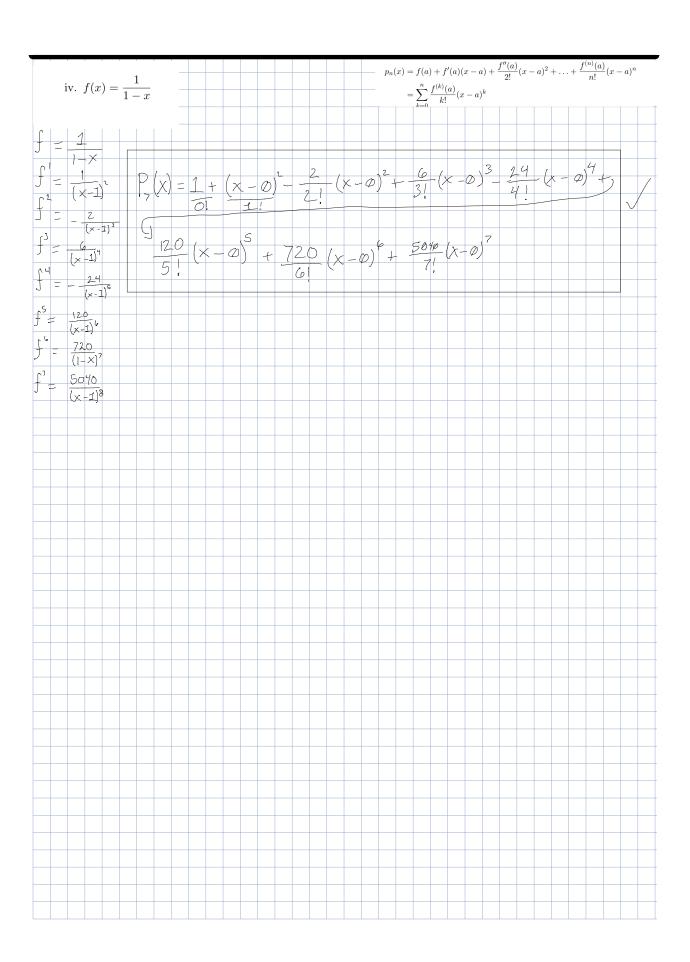
i.
$$f(x) = \sin x$$

ii.
$$f(x) = \cos x$$

iii.
$$f(x) = e^x$$

iv.
$$f(x) = \frac{1}{1-x}$$





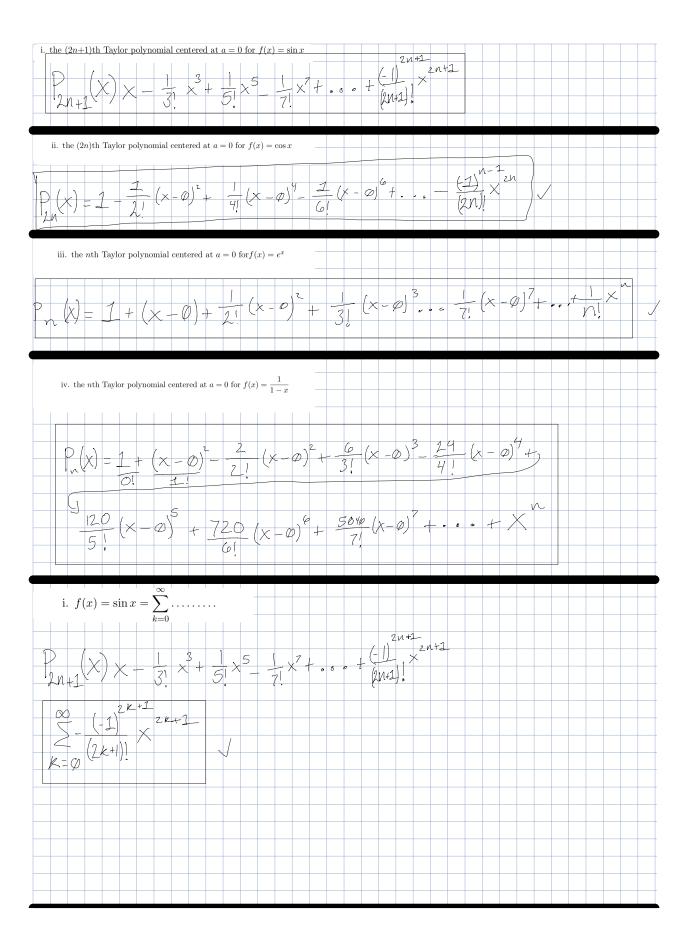
- (b) Following the patterns observed in the previous question, write out, i. the (2n+1)th Taylor polynomial centered at a=0 for $f(x)=\sin x$
 - ii. the (2n)th Taylor polynomial centered at a = 0 for $f(x) = \cos x$
 - iii. the *n*th Taylor polynomial centered at a = 0 for $f(x) = e^x$
 - iv. the nth Taylor polynomial centered at a=0 for $f(x)=\frac{1}{1-x}$
- (c) Let $n \to \infty$ in the expressions obtained in the previous question and write out the functions as a power series.

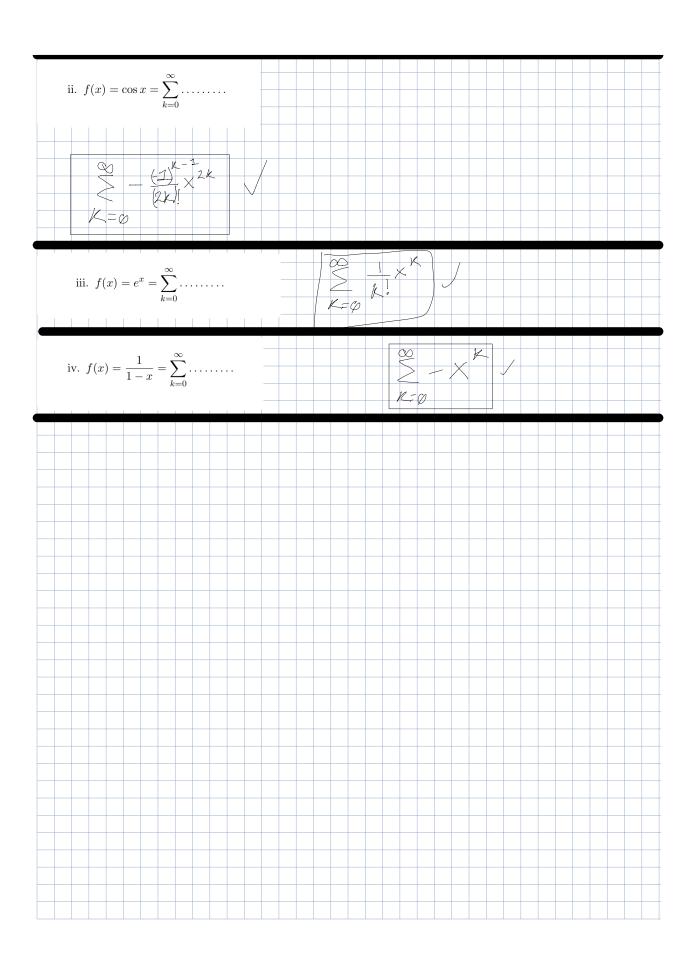
i.
$$f(x) = \sin x = \sum_{k=0}^{\infty} \dots$$

ii.
$$f(x) = \cos x = \sum_{k=0}^{\infty} \dots$$

iii.
$$f(x) = e^x = \sum_{k=0}^{\infty} \dots$$

iv.
$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} \dots$$





3. REMAINDER

The remainder R_n is the difference between p_n and the original function f, and if there exists a positive number M such that $|f^{(n+1)}(x)| \leq M$ then $|R_n(x)| \leq M \frac{|x-a|^{n+1}}{(n+1)!}$

(a) Evaluate the maximum error that can be generated if $\cos(0.1)$ is approximated using the Taylor polynomial $p_{10}(0.1)$.

(b) What is the minimal value of n required to make at most an error of 10^{-5} when approximating $\sin(0.05)$ by $p_n(0.05)$. Use a calculator.

