Math 76 Exercises – 2.5B Hydrostatic Force

1. What is the hydrostatic force exerted by water on a rectangular wall 60 m wide, 40 m tall which is submerged vertically so that the top is 10 m below the surface? (The density of water in the metric system is 1000 kg/m^3. Use 9.8 m/s^2 for the acceleration due to gravity.)

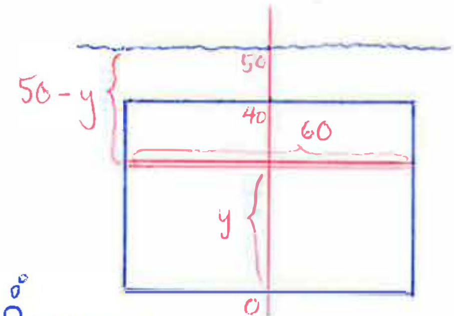
$$F = 1000 \cdot 9.8 \int_0^{40} 60(50-y)\,dy$$

$$= 9800 \cdot 60 \left(50y - \tfrac{1}{2}y^2\right)\Big|_0^{40}$$

$$= 588.000\left(50\cdot 40 - \tfrac{1}{2}(40)^2 - (0-0)\right)$$

$$= 588{,}000(2000 - 800)$$

$$= \boxed{105{,}600{,}000 \text{ N}}$$

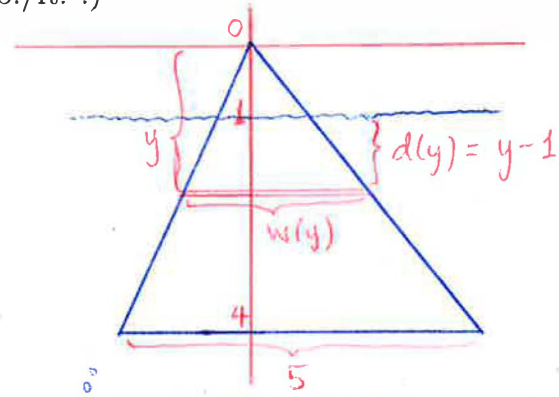


There are many ways to set a coordinate system for this problem. Here is one.

2. What is the hydrostatic force exerted by water on a triangular wall with base 5 ft. and height 4 ft., which is submerged vertically so that the top is 1 ft. above the surface? (In the English system, the weight density of water is 62.5 lb./ft.^{3}.)

Using the setup shown and similar triangles, we get

$$\frac{5}{4} = \frac{w(y)}{y}; \text{ so } w(y) = \frac{5}{4}y.$$



So the hydrostatic force is

$$62.5 \int_1^4 \frac{5}{4} y(y-1)\,dy$$

$$= 62.5\left(\frac{5}{4}\right)\int_1^4 (y^2 - y)\,dy$$

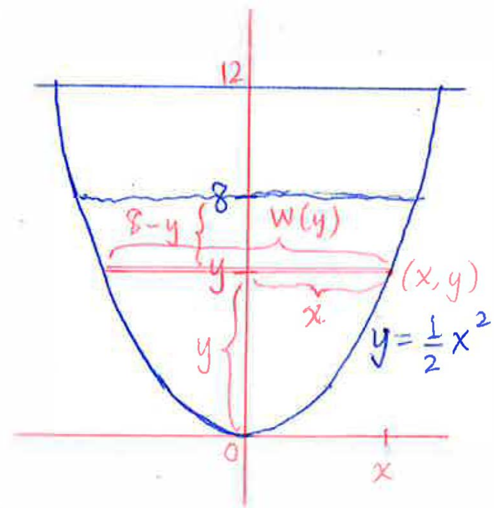
$$= \frac{625}{8}\left(\frac{1}{3}y^3 - \frac{1}{2}y^2\right)\Big|_1^4$$

$$= \frac{625}{8}\left(\frac{64}{3} - 8 - \left(\frac{1}{3} - \frac{1}{2}\right)\right) = \frac{16875}{16} \approx \boxed{1055 \text{ lb.}}$$

Again, multiple ways to set this up. Try more than one and check that you get the same answer!

3. A tank is designed with ends in the shape of the region between the curves $y = \frac{1}{2}x^2$ and $y = 12$, measured in feet. Find the hydrostatic force on one end of the tank if it is filled to a depth of 8 ft. with gasoline. (Use 42 lb./ft.^{3} for the weight density of gasoline.)

From the picture, we can see that since $y = \frac{1}{2}x^2$, we have $x^2 = 2y$, so $x = \sqrt{2y}$.
Therefore $w(y) = 2x = 2\sqrt{2y}$.
Also, depth at y is $8-y$. So



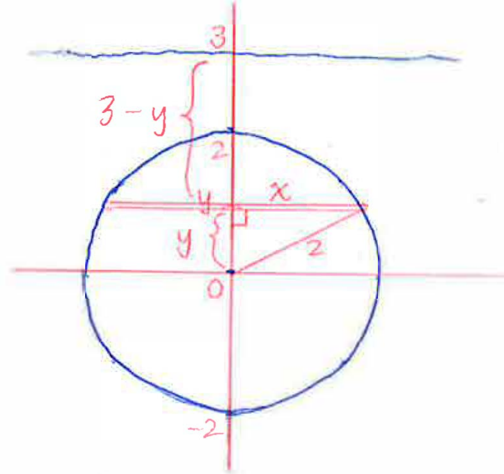
$$F = 42\int_0^8 (8-y)\cdot 2\sqrt{2y}\,dy$$

$$= 42\cdot 2\sqrt{2}\int_0^8 (8-y)\,y^{\frac{1}{2}}\,dy = 84\sqrt{2}\int_0^8 8y^{1/2} - y^{3/2}\,dy$$

$$= 84\sqrt{2}\left(\tfrac{2}{3}\cdot 8\cdot y^{3/2} - \tfrac{2}{5}y^{5/2}\right)\Big|_0^8 = 84\sqrt{2}\left(\tfrac{16}{3}8^{3/2} - \tfrac{2}{5}8^{5/2}\right) = \boxed{5734.4 \text{ lb}}$$

4. A diving pool has a circular observation window of radius 2 meters whose center is at a depth of 3 meters. Set up an integral for the hydrostatic force on the window. If you have time, evaluate the integral.

From the picture, we have $x = \sqrt{4-y^2}$, so $w(y) = 2\sqrt{4-y^2}$;
also depth at y is $3-y$. So



$$F = 9800\int_{-2}^{2} 2\sqrt{4-y^2}\,(3-y)\,dy$$

$$= 19600\int_{-2}^{2} 3\sqrt{4-y^2} - y\sqrt{4-y^2}\,dy$$

$$= 19600\left(3\boxed{\int_{-2}^{2}\sqrt{4-y^2}\,dy} - \boxed{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}\right)$$

Newtons

② Use a substitution with $u = 4-y^2$, or guess and check.

① This integral represents the area of half a circle of radius 2, so the value is $\frac{1}{2}\pi\cdot 2^2 = 2\pi$.