



Stack and Queue

CPE 112 Programming with Data Structures

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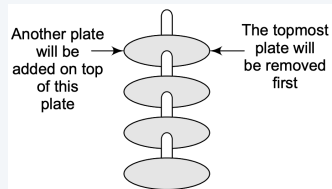
Overview

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8. Types of Queues
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Stack

Stack

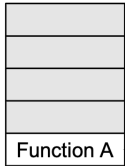
- Stack is an important data structure which stores its elements in an ordered manner.
- We will explain the concept of stacks using an analogy. You must have seen a pile of plates where one plate is placed on top of another as shown in Figure.
- Now, when you want to remove a plate, you remove the topmost plate first. Hence, you can add and remove an element (i.e., a plate) only at/from one position which is the topmost position.



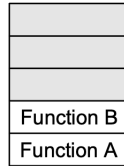
LIFO

Stack in Computer Science

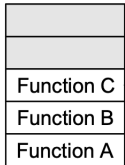
Function Calls



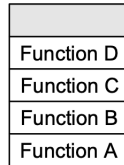
When A calls B, A is pushed on top of the system stack. When the execution of B is complete, the system control will remove A from the stack and continue with its execution.



When B calls C, B is pushed on top of the system stack. When the execution of C is complete, the system control will remove B from the stack and continue with its execution.



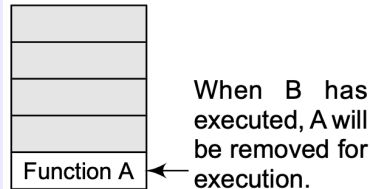
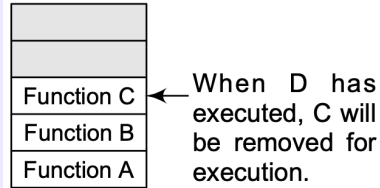
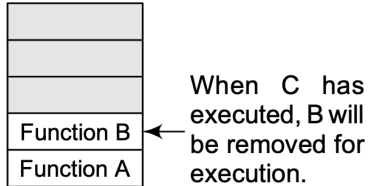
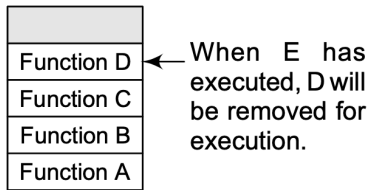
When C calls D, C is pushed on top of the system stack. When the execution of D is complete, the system control will remove C from the stack and continue with its execution.



When D calls E, D is pushed on top of the system stack. When the execution of E is complete, the system control will remove D from the stack and continue with its execution.

Stack in Computer Science

Function Calls



Array Representation of Stack

Array Representation of Stack

In the computer's memory, stacks can be represented as a linear array. Every stack has a variable called **TOP** associated with it, which is used to store the address of the topmost element of the stack. It is this position where the element will be added to or deleted from.

A	AB	ABC	ABCD	ABCDE					
0	1	2	3	TOP = 4	5	6	7	8	9

Operations on a Stack

- A stack supports three basic operations: **push, pop, and peek.**
- The push operation adds an element to the top of the stack.
- The pop operation removes the element from the top of the stack.
- The peek operation returns the value of the topmost element of the stack.

PUSH Operation

- The push operation is used to **insert an element into the stack**. The new element is **added at the topmost position** of the stack.
- However, before inserting the value, we must first check if **$TOP = MAX - 1$** , because **if that is the case, then the stack is full and no more insertions can be done**.
- If an attempt is made to insert a value in a stack that is already full, an **OVERFLOW** message is printed.

PUSH Operation

Insert an element with value 6

Check if $TOP = MAX - 1$

- If the condition is false,
 - Increment the value of TOP , and
 - Store the new element at the position given by $stack[TOP]$.

1	2	3	4	5					
0	1	2	3	TOP = 4	5	6	7	8	9



1	2	3	4	5	6				
0	1	2	3	4	TOP = 5	6	7	8	9

PUSH Operation

```
Step 1: IF TOP = MAX-1
        PRINT "OVERFLOW"
        Goto Step 4
    [END OF IF]
Step 2: SET TOP = TOP + 1
Step 3: SET STACK[TOP] = VALUE
Step 4: END
```

```
void push(int stack[], int val){
    if(top == MAX-1){
        printf("Overflow\n");
        return;
    }
    top++;
    stack[top] = val;
}
```

POP Operation

- The pop operation is used to delete the topmost element from the stack.
- However, before deleting the value, we must first check if $TOP=NULL$ because if that is the case, then it means the stack is empty and no more deletions can be done.
- If an attempt is made to delete a value from a stack that is already empty, an UNDERFLOW message is printed.

POP Operation

Delete the topmost element

Check if **TOP=NULL**

- If the condition is false,
 - Decrement the value pointed by **TOP**

1	2	3	4	5					
0	1	2	3	TOP = 4	5	6	7	8	9



1	2	3	4						
0	1	2	TOP = 3	4	5	6	7	8	9

POP Operation

```
Step 1: IF TOP = NULL
        PRINT "UNDERFLOW"
        Goto Step 4
    [END OF IF]
Step 2: SET VAL = STACK[TOP]
Step 3: SET TOP = TOP - 1
Step 4: END
```

```
int pop(int stack[]){
    int val;

    if(top == -1){
        printf("Underflow\n");
        return -1;
    } else{
        val = stack[top];
        top--;
        return val;
    }
}
```

PEEK Operation

- Peek is an operation that **returns the value of the topmost element of the stack without deleting it from the stack.**
- However, the Peek operation first checks if the stack is empty, i.e., if **TOP = NULL**, then an appropriate message is printed, else the value is returned.

1	2	3	4	5					
0	1	2	3	TOP = 4	5	6	7	8	9

PEEK Operation

```
Step 1: IF TOP = NULL
        PRINT "STACK IS EMPTY"
        Goto Step 3
Step 2: RETURN STACK[TOP]
Step 3: END
```

```
int peek(int stack[]){
    if(top == -1){
        printf("Stack is empty.\n");
        return -1;
    }
    return stack[top];
}
```

Linked List Representation of Stack

Linked List Representation of Stack

- The technique of creating a stack is easy, but the drawback is that the array must be declared to have some fixed size.
- In case the stack is a very small one or its maximum size is known in advance, then the array implementation of the stack gives an efficient implementation.
- But if the array size cannot be determined in advance, then the other alternative, i.e., linked representation, is used.

Linked List Representation of Stack

- The storage requirement of linked representation of the stack with n elements is $O(n)$, and the typical time requirement for the operations is $O(1)$.
- In a linked stack, every node has two parts—one that stores data and another that stores the address of the next node.
- The START pointer of the linked list is used as **TOP**. All insertions and deletions are done at the node pointed by **TOP**. If **TOP = NULL**, then it indicates that the **stack is empty**.

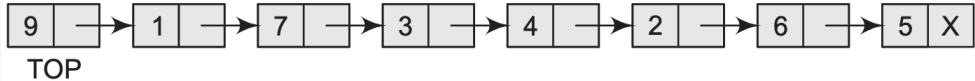
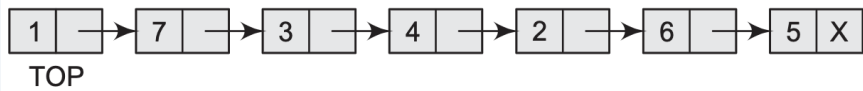


Operations on a Linked List

- A linked stack supports three basic operations: **push**, **pop**, and **peek**.
- The push operation adds an element to the top of the stack.
- The pop operation removes the element from the top of the stack.
- The peek operation returns the value of the topmost element of the stack.

PUSH Operation

Insert an element with value 9



PUSH Operation

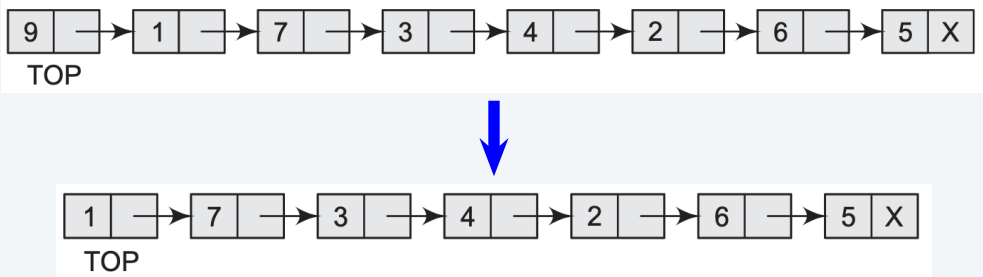
```
Step 1: Allocate memory for the new
        node and name it as NEW_NODE
Step 2: SET NEW_NODE->DATA = VAL
Step 3: IF TOP = NULL
        SET NEW_NODE->NEXT = NULL
        SET TOP = NEW_NODE
    ELSE
        SET NEW_NODE->NEXT = TOP
        SET TOP = NEW_NODE
    [END OF IF]
Step 4: END
```

```
void push(node **top,int val){
    node *newNode;

    newNode = (node*)malloc(sizeof(node));
    newNode->data = val;
    if(*top == NULL){
        newNode->next = NULL;
        *top = newNode;
    } else{
        newNode->next = *top;
        *top = newNode;
    }
}
```

POP Operation

Delete the topmost element



POP Operation

```
Step 1: IF TOP = NULL
        PRINT "UNDERFLOW"
        Goto Step 5
    [END OF IF]
Step 2: SET PTR = TOP
Step 3: SET TOP = TOP -> NEXT
Step 4: FREE PTR
Step 5: END
```

```
void pop(node **top){
    node *ptr;

    if(*top == NULL){
        printf("Underflow\n");
        return;
    }
    ptr = *top;
    *top = (*top)->next;
    free(ptr);
}
```

Applications of Stack

Applications of Stack

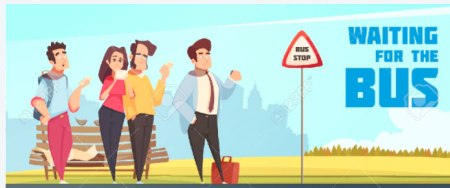
We will discuss typical problems where stacks can be easily applied for a simple and efficient solution. The topics that will be discussed in this section include the following:

- Reversing a list
- Parentheses checker
- Conversion of an infix expression into a postfix expression
- Evaluation of a postfix expression
- Conversion of an infix expression into a prefix expression
- Evaluation of a prefix expression
- Recursion
- Tower of Hanoi

Queue

Queue

- In all these examples, we see that the element at the first position is served first. Same is the case with queue data structure.
- A queue is a **FIFO** (First-In, First-Out) data structure in which the element that is inserted first is the first one to be taken out.
- The elements in a queue are added at one end called the **REAR** and removed from the other end called the **FRONT**. Queues can be implemented by using either **arrays** or **linked lists**. Next, we will see how queues are implemented using each of these data structures.



Array Representation of Queues

Array Representation of Queues

Queues can be easily represented using linear arrays. As stated earlier, every queue has front and rear variables that point to the position from where deletions and insertions can be done, respectively.

Queue

12	9	7	18	14	36				
0	1	2	3	4	5	6	7	8	9

Array Representation of Queues

Queue after insertion of a new element

12	9	7	18	14	36	45			
0	1	2	3	4	5	6	7	8	9

Queue after deletion of an element

	9	7	18	14	36	45			
0	1	2	3	4	5	6	7	8	9

Operations on Queues

- A queue supports three basic operations: **insert, and delete**.
- The insert operation adds an element to the rear of the queue.
- The delete operation removes the element from the front of the queue.

INSERT Operation

```
Step 1: IF REAR = MAX-1
        Write OVERFLOW
        Goto step 4
    [END OF IF]
Step 2: IF FRONT = -1 and REAR = -1
        SET FRONT = REAR = 0
    ELSE
        SET REAR = REAR + 1
    [END OF IF]
Step 3: SET QUEUE[REAR] = NUM
Step 4: EXIT
```

```
void insertQ(int queue[], int num){
    if(rear == MAX - 1){
        printf("OVERFLOW\n");
        return;
    }
    if(front == -1 && rear == -1){
        front = rear = 0;
    } else{
        rear = rear + 1;
    }
    queue[rear] = num;
}
```

DELETE Operation

```
Step 1: IF FRONT = -1 OR FRONT > REAR
        Write UNDERFLOW
    ELSE
        SET VAL = QUEUE[FRONT]
        SET FRONT = FRONT + 1
    [END OF IF]
Step 2: EXIT
```

```
int deleteQ(int queue[]){
    int val;

    if(front == -1 || front > rear){
        printf("UNDERFLOW\n");
        return -1;
    } else{
        val = queue[front];
        front++;
        return val;
    }
}
```

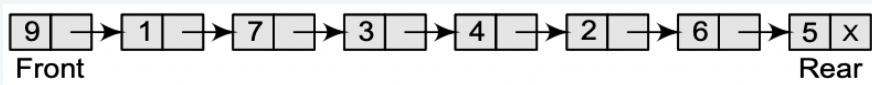
Linked List Representation of Queues

Linked List Representation of Queues

- We have seen how a queue is created using an array. Although this technique of creating a queue is easy, its drawback is that the array must be declared to have some fixed size.
- If we allocate space for 50 elements in the queue and it hardly uses 20– 25 locations, then half of the space will be wasted.
- And in case we allocate less memory locations for a queue that might end up growing large and large, then a lot of re-allocations will have to be done, thereby creating a lot of overhead and consuming a lot of time.
- In case the queue is a very small one or its maximum size is known in advance, then the array implementation of the queue gives an efficient implementation. But if the array size cannot be determined in advance, the other alternative, i.e., the linked representation is used.

Linked List Representation of Queues

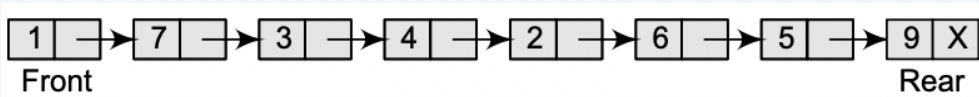
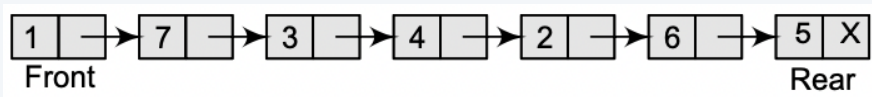
- The storage requirement of linked representation of a queue with n elements is $O(n)$ and the typical time requirement for operations is $O(1)$.
- In a linked queue, every element has two parts, one that stores the data and another that stores the address of the next element.



Operations on Linked Queues

- A queue has two basic operations: **insert** and **delete**.
- The **insert** operation adds an element to the end of the queue, and the **delete** operation removes an element from the front or the start of the queue.
- Apart from this, there is another operation **peek** which returns the value of the first element of the queue.

INSERT Operation

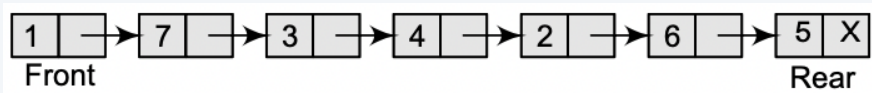
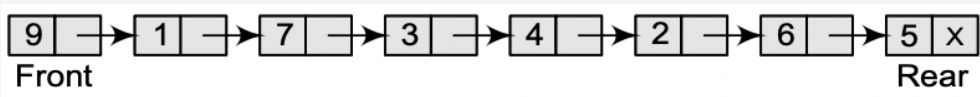


INSERT Operation

```
Step 1: Allocate memory for the new node and name  
        it as PTR  
Step 2: SET PTR->DATA = VAL  
Step 3: IF FRONT = NULL  
        SET FRONT = REAR = PTR  
        SET FRONT->NEXT = REAR->NEXT = NULL  
    ELSE  
        SET REAR->NEXT = PTR  
        SET REAR = PTR  
        SET REAR->NEXT = NULL  
    [END OF IF]  
Step 4: END
```

```
queue insertQ(queue q, int val){  
    node *ptr;  
  
    ptr = (node*)malloc(sizeof(node));  
    ptr->data = val;  
    if(q.front == NULL){  
        q.front = ptr;  
        q.rear = ptr;  
        q.front->next = q.rear->next = NULL;  
    } else{  
        q.rear->next = ptr;  
        q.rear = ptr;  
        q.rear->next = NULL;  
    }  
    return q;  
}
```

DELETE Operation



DELETE Operation

```
Step 1: IF FRONT = NULL
        Write "Underflow"
        Go to Step 5
    [END OF IF]
Step 2: SET PTR = FRONT
Step 3: SET FRONT = FRONT -> NEXT
Step 4: FREE PTR
Step 5: END
```

```
queue deleteQ(queue q){
    node *ptr;

    if(q.front == NULL)
        printf("Underflow\n");
    ptr = q.front;
    printf("Delete [%d].\n",q.front->data);
    q.front = q.front->next;
    free(ptr);
    return q;
}
```

Types of Queues

Types of Queues

A queue data structure can be classified into the following types:

- Circular Queue
- Deque
- Priority Queue
- Multiple Queue

Circular Queue

- In linear queues, we have discussed so far that insertions can be done only at one end called the REAR and deletions are always done from the other end called the FRONT.
- Suppose we want to insert a new element in the queue shown in figure. Even though there is space available, the overflow condition still exists because the condition $REAR = MAX - 1$ still hold true. This is a major drawback of a linear queues.

		7	18	14	36	45	21	99	72
0	1	2	3	4	5	6	7	8	9

Circular Queue

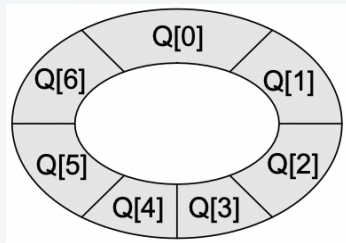
To resolve this problem, we have two solutions.

First Solution

Shift the elements to the left so that the vacant space can be occupied and utilized efficiently. But this can be very time-consuming, especially when the queue is quite large.

Second Solution

Use a circular queue. In the circular queue, the first index comes right after the last index. Conceptually, you can think of a circular queue as shown in figure.



Circular Queue

Full queue

90	49	7	18	14	36	45	21	99	72
FRONT = 01	2	3	4	5	6	7	8	REAR = 9	

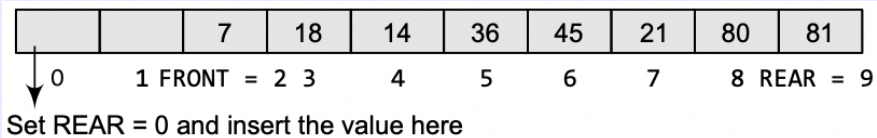
Queue with vacant locations

90	49	7	18	14	36	45	21	99	
FRONT = 01	2	3	4	5	6	7	REAR = 8	9	

Increment rear so that it points to location 9 and insert the value here

INSERT Operation

Inserting an element in a circular queue



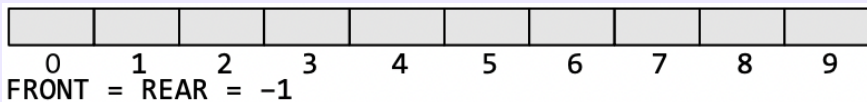
INSERT Operation

```
Step 1: IF FRONT = 0 and Rear = MAX - 1
        Write "OVERFLOW"
        Goto step 4
    [End OF IF]
Step 2: IF FRONT = -1 and REAR = -1
        SET FRONT = REAR = 0
    ELSE IF REAR = MAX - 1 and FRONT != 0
        SET REAR = 0
    ELSE
        SET REAR = REAR + 1
    [END OF IF]
Step 3: SET QUEUE[REAR] = VAL
Step 4: EXIT
```

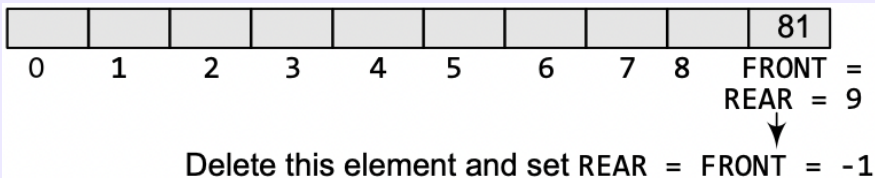
```
void insertQ(int queue[], int val){
    if(front == 0 && rear == MAX-1){
        printf("OVERFLOW");
        return;
    }
    if(front == -1 && rear == -1){
        front = rear = 0;
    } else if(rear == MAX - 1 && front != 0){
        rear = 0;
    } else {
        rear++;
    }
    queue[rear] = val;
}
```

DELETE Operation

Empty Queue

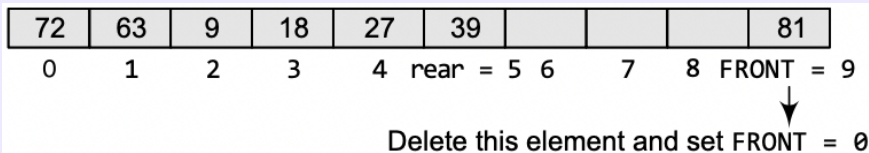


Queue with a single element



DELETE Operation

Queue where $\text{FRONT} = \text{MA}-1$ before deletion



DELETE Operation

```
Step 1: IF FRONT = -1
        Write "UNDERFLOW"
        Goto Step 4
    [END of IF]
Step 2: SET VAL = QUEUE[FRONT]
Step 3: IF FRONT = REAR
        SET FRONT = REAR = -1
    ELSE
        IF FRONT = MAX - 1
            SET FRONT = 0
        ELSE
            SET FRONT = FRONT + 1
        [END of IF]
    [END OF IF]
Step 4: EXIT
```

```
int deleteQ(int queue[]){
    int val;

    if(front == -1){
        printf("UNDERFLOW");
        return -1;
    }
    val = queue[front];
    if(front == rear)
        front = rear = -1;
    else{
        if(front == MAX - 1)
            front = 0;
        else
            front++;
    }
    return val;
}
```

Dequeues

- A deque (pronounced as 'deck' or 'dequeue') is a list in which **the elements can be inserted or deleted at either end.**
- It is also known as a **head-tail linked list** because elements can be added to or removed from either the front (head) or the back (tail) end.
- However, no element can be added and deleted from the middle. In the computer's memory, a deque is implemented using either a circular array or a circular doubly linked list.

			29	37	45	54	63		
0	1	2	LEFT = 3	4	5	6	RIGHT = 7	8	9

42	56						63	27	18
0	RIGHT = 1	2	3	4	5	6	LEFT = 7	8	9

Priority Queue

A priority queue is a data structure in which each element is assigned a priority. The priority of the element will be used to determine the order in which the elements will be processed. The general rules of processing the elements of a priority queue are

- An element with higher priority is processed before an element with a lower priority.
- Two elements with the same priority are processed on a first-come- first-served (FCFS) basis.

Multiple Queue

- When we implement a queue using an array, the size of the array must be known in advance. If the queue is allocated less space, then frequent overflow conditions will be encountered.
- To deal with this problem, the code will have to be **modified to reallocate more space for the array**.
- In case we allocate a large amount of space for the queue, it will result in sheer wastage of the memory. Thus, there lies a tradeoff between the frequency of overflows and the space allocated.
- So a better solution to deal with this problem is to have multiple queues or to have more than one queue in the same array of sufficient size.

Applicaitons of Queues

Applicaitions of Queues

- Queues are widely used as waiting lists for a single shared resource like printer, disk, CPU.
- Queues are used to transfer data asynchronously (data not necessarily received at same rate as sent) between two processes (IO buffers), e.g., pipes, file IO, sockets.
- Queues are used as buffers on MP3 players and portable CD players, iPod playlist.
- Queues are used in Playlist for jukebox to add songs to the end, play from the front of the list.
- Queues are used in operating system for handling interrupts. When programming are real-time system that can be interrupted.

