Backpropagation

In this assignment, you will implement Backpropagation from scratch. You will then verify the correctness of the your implementation using a "grader" function/cell (provided by us) which will match your implmentation.

The grader fucntion would help you validate the correctness of your code.

Please submit the final Colab notebook in the classroom ONLY after you have verified your code using the grader function/cell.

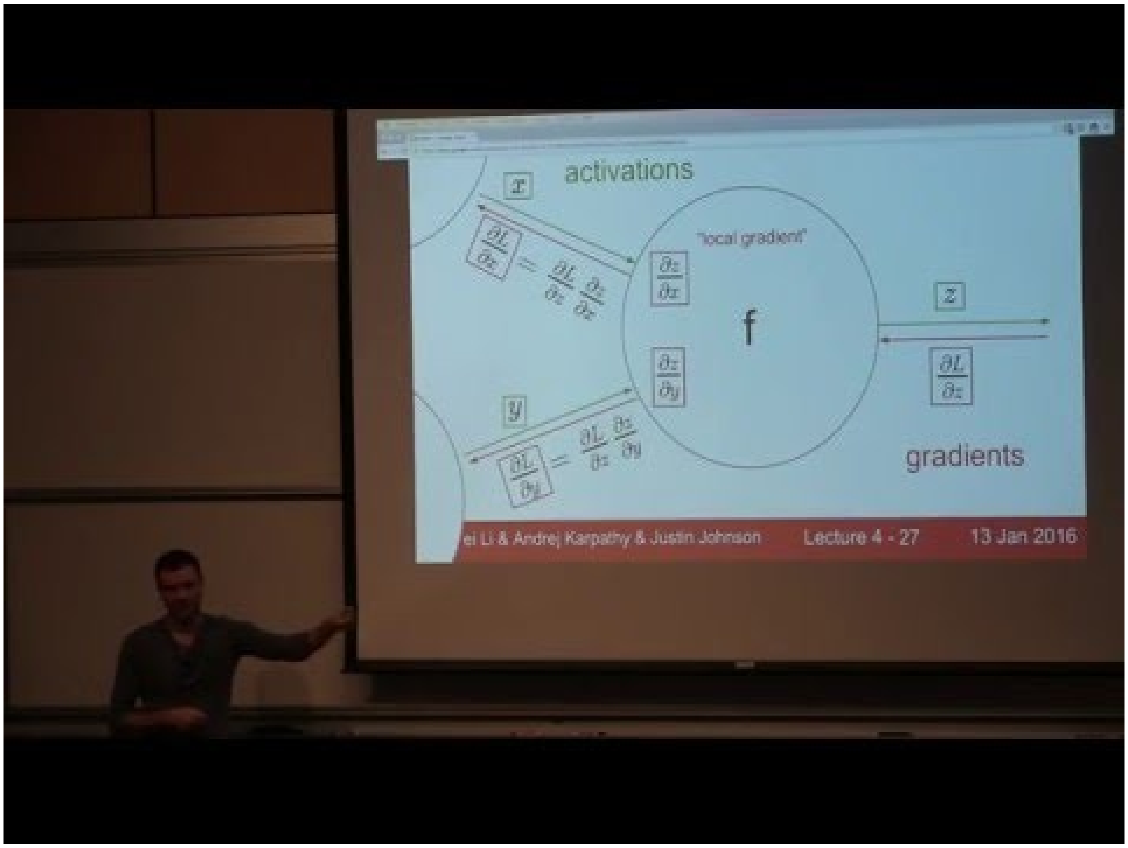
Loading data

```
import pickle
import numpy as np
from tqdm import tqdm
import matplotlib.pyplot as plt
import math
from sympy import *


with open('data.pkl', 'rb') as f:
    data = pickle.load(f)
print(data.shape)
X = np.array(data[:, :5])
y = np.array(data[:, -1])
print(X.shape, y.shape)
```

```
(506, 6)
(506, 5) (506,)
```

Check this video for better understanding of the computational graphs and back propagation

```
from IPython.display import YouTubeVideo
YouTubeVideo('i94OvYb6noo',width="1000",height="500")
```



Computational graph

- **If you observe the graph, we are having input features [f1, f2, f3, f4, f5] and 9 weights [w1, w2, w3, w4, w5, w6, w7, w8, w9].**
- **The final output of this graph is a value L which is computed as (Y-Y')^2**

Task 1: Implementing Forward propagation, Backpropagation and Gradient checking

Task 1.1

Forward propagation

- Forward propagation(Write your code in def forward_propagation()) For easy debugging, we will break the computational graph into 3 parts.

  Part 1 Part 2 Part 3

```
# sigmoid(logistic function) is a non-linear function which squishes
the output values within the range [0,1]
```

```
def sigmoid(z):
    '''In this function, we will compute the sigmoid(z)'''
    # we can use this function in forward and backward propagation
    # write the code to compute the sigmoid value of z and return that
value

    return  1/(1+np.exp(-z))

def grader_sigmoid(z):
  #if you have written the code correctly then the grader function
will output true
    val=sigmoid(z)
    assert(val==0.8807970779778823)
    return True
grader_sigmoid(2)
```

```
True
```

```
def tanh(z):
    return (np.exp(z)-np.exp(-z))/(np.exp(z)+np.exp(-z))

def forward_propagation(x, y, w):
        '''In this function, we will compute the forward propagation
'''
        # X: input data point, note that in this assignment you are
having 5-d data points
        # y: output varible
        # W: weight array, its of length 9, W[0] corresponds to w1 in
graph, W[1] corresponds to w2 in graph,..., W[8] corresponds to w9 in
graph.
        # you have to return the following variables
        # exp= part1 (compute the forward propagation until exp and
then store the values in exp)
        # tanh =part2(compute the forward propagation until tanh and
then store the values in tanh)
        # sig = part3(compute the forward propagation until sigmoid
and then store the values in sig)
        # we are computing one of the values for better understanding

        #calculating exponential
        val_1= (w[0]*x[0]+w[1]*x[1]) * (w[0]*x[0]+w[1]*x[1]) + w[5]
        part_1 = np.exp(val_1)

        #calculating tanh
        val_2 = part_1 + w[6]
        part_2 = np.tanh(val_2)

        #calculating sigmoid
        val_3 = (np.sin(w[2]*x[2]))* (w[3]*x[3]+w[4]*x[4])+ w[7]
        part_3 = sigmoid(val_3)
```

```python
        #calculating y_pred
        y_pred = part_2 + part_3*w[8]

        #calculating square loss
        loss = (y - y_pred)**2


        #calculating loss w.r.t y_pred
        dy_pred = -2*y + 2*y_pred




        # after computing part1,part2 and part3 compute the value of
y' from the main Computational graph using required equations
        # write code to compute the value of L=(y-y')^2 and store it
in variable loss
        # compute derivative of L  w.r.to y' and store it in dy_pred
        # Create a dictionary to store all the intermediate values
i.e. dy_pred ,loss,exp,tanh,sigmoid
        # we will be using the dictionary to find values in
backpropagation, you can add other keys in dictionary as well

        forward_dict={}
        forward_dict['exp']= part_1
        forward_dict['sigmoid'] = part_3
        forward_dict['tanh'] = part_2
        forward_dict['loss'] = loss
        forward_dict['dy_pred'] = dy_pred

        return forward_dict

w=np.ones(9)*0.1
d1=forward_propagation(X[0],y[0],w)
d1
```

```
{'exp': 1.1272967040973583,
 'sigmoid': 0.5279179387419721,
 'tanh': 0.8417934192562146,
 'loss': 0.9298048963072919,
 'dy_pred': -1.9285278284819143}
```

```python
def grader_forwardprop(data):
    dl = (data['dy_pred']==-1.9285278284819143)
    loss=(data['loss']==0.9298048963072919)
    part1=(data['exp']==1.1272967040973583)
    part2=(data['tanh']==0.8417934192562146)
    part3=(data['sigmoid']==0.5279179387419721)
    assert(dl and loss and part1 and part2 and part3)
    return True
```

```
w=np.ones(9)*0.1
d1=forward_propagation(X[0],y[0],w)
grader_forwardprop(d1)
```

```
True
```

Task 1.2

Backward propagation

```
def backward_propagation(x,y,w,forward_dict):
    '''In this function, we will compute the backward propagation '''
    # forward_dict: the outputs of the forward_propagation() function
    # write code to compute the gradients of each weight
[w1,w2,w3,...,w9]
    # Hint: you can use dict type to store the required variables

    # dw1 = # in dw1 compute derivative of L w.r.to w1
    dw1 = forward_dict['dy_pred']*(1-
(math.pow(forward_dict['tanh'],2)))*forward_dict["exp"]*2*((w[0]*x[0])
+(w[1]*x[1]))*x[0]

    # dw2 = # in dw2 compute derivative of L w.r.to w2
    dw2=forward_dict['dy_pred']*(1-
(math.pow(forward_dict['tanh'],2)))*forward_dict["exp"]*2*((w[0]*x[0])
+(w[1]*x[1]))*x[1]

    # dw3 = # in dw3 compute derivative of L w.r.to w3
    dw3 =forward_dict['dy_pred']*(forward_dict['sigmoid']*(1-
forward_dict['sigmoid']))*w[8]*((w[3]*x[3])+
(w[4]*x[4]))*math.cos(x[2]*w[2])*x[2]

    # dw4 = # in dw4 compute derivative of L w.r.to w4
    dw4 =forward_dict['dy_pred']*(forward_dict['sigmoid']*(1-
forward_dict['sigmoid']))*w[8]*math.sin(x[2]*w[2])*x[3]

    # dw5 = # in dw5 compute derivative of L w.r.to w5
    dw5 =forward_dict['dy_pred']*(forward_dict['sigmoid']*(1-
forward_dict['sigmoid']))*w[8]*math.sin(x[2]*w[2])*x[4]

    # dw6 = # in dw6 compute derivative of L w.r.to w6
    dw6 = forward_dict['dy_pred']*(1-
(math.pow(forward_dict['tanh'],2)))*forward_dict["exp"]

    # dw7 = # in dw7 compute derivative of L w.r.to w7
    dw7 =forward_dict['dy_pred']*(1-
(math.pow(forward_dict['tanh'],2)))

    # dw8 = # in dw8 compute derivative of L w.r.to w8
```

```
    dw8 =forward_dict['dy_pred']*(forward_dict['sigmoid']*(1-
forward_dict['sigmoid']))*w[8]

    # dw9 = # in dw9 compute derivative of L w.r.to w9
    dw9 = forward_dict['dy_pred']*forward_dict['sigmoid']


    backward_dict={}
    #store the variables dw1,dw2 etc. in a dict as
backward_dict['dw1']= dw1,backward_dict['dw2']= dw2...
    backward_dict['dw1']= dw1
    backward_dict['dw2']= dw2
    backward_dict['dw3']= dw3
    backward_dict['dw4']= dw4
    backward_dict['dw5']= dw5
    backward_dict['dw6']= dw6
    backward_dict['dw7']= dw7
    backward_dict['dw8']= dw8
    backward_dict['dw9']= dw9



    return backward_dict

def grader_backprop(data):
    dw1=(np.round(data['dw1'],6)==-0.229733)
    dw2=(np.round(data['dw2'],6)==-0.021408)
    dw3=(np.round(data['dw3'],6)==-0.005625)
    dw4=(np.round(data['dw4'],6)==-0.004658)
    dw5=(np.round(data['dw5'],6)==-0.001008)
    dw6=(np.round(data['dw6'],6)==-0.633475)
    dw7=(np.round(data['dw7'],6)==-0.561942)
    dw8=(np.round(data['dw8'],6)==-0.048063)
    dw9=(np.round(data['dw9'],6)==-1.018104)
    assert(dw1 and dw2 and dw3 and dw4 and dw5 and dw6 and dw7 and dw8
and dw9)
    return True
w=np.ones(9)*0.1
forward_dict=forward_propagation(X[0],y[0],w)
backward_dict=backward_propagation(X[0],y[0],w,forward_dict)
grader_backprop(backward_dict)
```

```
True
```

Task 1.3

Gradient clipping

Check this blog link for more details on Gradient clipping we know that the derivative of any function is

$$\lim_{\epsilon \to 0} \frac{f(x+\epsilon)-f(x-\epsilon)}{2\epsilon}$$

- The definition above can be used as a numerical approximation of the derivative. Taking an epsilon small enough, the calculated approximation will have an error in the range of epsilon squared.
- In other words, if epsilon is 0.001, the approximation will be off by 0.00001.

Therefore, we can use this to approximate the gradient, and in turn make sure that backpropagation is implemented properly. This forms the basis of gradient checking!

Gradient checking example

lets understand the concept with a simple example: $f(w1, w2, x1, x2) = w_1^2 . x_1 + w_2 . x_2$

from the above function , lets assume $w_1 = 1$, $w_2 = 2$, $x_1 = 3$, $x_2 = 4$ the gradient of f w.r.t w_1 is

\begin{array} {lcl} \frac{df}{dw_{1}} = dw_{1} &=&2.w_{1}.x_{1} \& = &2.1.3\& = &6 \end{array}

let calculate the aproximate gradient of w_1 as mentinoned in the above formula and considering $\epsilon = 0.0001$

\begin{array} {lcl} dw_1^{approx} & = & \frac{f(w1+\epsilon,w2,x1,x2)-f(w1-\epsilon,w2,x1,x2)}{2\epsilon} \ & = & \frac{((1+0.0001)^{2} . 3 + 2 . 4) - ((1-0.0001)^{2} . 3 + 2 . 4)}{2\epsilon} \ & = & \frac{(1.00020001 . 3 + 2 . 4) - (0.99980001. 3 + 2 . 4)}{2*0.0001} \ & = & \frac{(11.00060003) - (10.99940003)}{0.0002}\ & = & 5.99999999999 \end{array}

Then, we apply the following formula for gradient check: gradient_check = $\frac{\left\Vert\left (dW-dW^{approx}\rm\right) \right\Vert_2}{\left\Vert\left (dW\rm\right) \right\Vert_2+\left\Vert\left (dW^{approx}\rm\right) \right\Vert_2}$

The equation above is basically the Euclidean distance normalized by the sum of the norm of the vectors. We use normalization in case that one of the vectors is very small. As a value for epsilon, we usually opt for 1e-7. Therefore, if gradient check return a value less than 1e-7, then it means that backpropagation was implemented correctly. Otherwise, there is potentially a mistake in your implementation. If the value exceeds 1e-3, then you are sure that the code is not correct.

in our example: gradient_check $ = \frac{(6 - 5.999999999994898)}{(6 + 5.999999999994898)} = 4.2514140356330737e^{-13}$

you can mathamatically derive the same thing like this

\begin{array} {lcl} dw_1^{approx} & = & \frac{f(w1+\epsilon,w2,x1,x2)-f(w1-\epsilon,w2,x1,x2)}{2\epsilon} \ & = & \frac{((w_{1}+\epsilon)^{2} . x_{1} + w_{2} . x_{2}) -

((w_{1}-\epsilon)^{2} . x_{1} + w_{2} . x_{2})}{2\epsilon} \ & = & \frac{4. \epsilon.w_{1}. x_{1}}{2\epsilon} \ & = & 2.w_{1}.x_{1} \end{array}

Implement Gradient checking

(Write your code in def gradient_checking())

Algorithm

```
#https://github.com/Kulbear/deep-learning-coursera/blob/master/
Improving%20Deep%20Neural%20Networks%20Hyperparameter%20tuning%2C
%20Regularization%20and%20Optimization/Gradient%20Checking.ipynb
def gradient_checking(x,y,w,eps):
    # compute the dict value using forward_propagation()
    # compute the actual gradients of W using backword_propagation()
    forward_dict=forward_propagation(x,y,w)
    backward_dict=backward_propagation(x,y,w,forward_dict)

    #we are storing the original gradients for the given datapoints in
a list

    original_gradients_list=list(backward_dict.values())
    # make sure that the order is correct i.e. first element in the
list corresponds to  dw1 ,second element is dw2 etc.
    # you can use reverse function if the values are in reverse order

    approx_gradients_list=[]
    #now we have to write code for approx gradients, here you have to
make sure that you update only one weight at a time
    #write your code here and append the approximate gradient value
for each weight in  approx_gradients_list

    w_plus=[w[i]+eps for i in range(len(w))]
    loss_plus = forward_propagation(x,y,w_plus)['loss']

    w_sub =[w[i]-eps for i in range(len(w))]
    loss_sub  = forward_propagation(x,y,w_sub)['loss']

    approx_gradients_list.append((loss_plus - loss_sub) / (2. * eps))



    #performing gradient check operation
    original_gradients_list=np.array(original_gradients_list)
    approx_gradients_list=np.array(approx_gradients_list)
    gradient_check_value =(original_gradients_list-
approx_gradients_list)/(original_gradients_list+approx_gradients_list)
```

```
    return gradient_check_value


def grader_grad_check(value):
    print(value)
    assert(np.all(value <= 10**-3))
    return True

w=[ 0.00271756,  0.01260512,  0.00167639, -0.00207756,  0.00720768,
   0.00114524,  0.00684168,  0.02242521,  0.01296444]

eps=10**-7
value= gradient_checking(X[0],y[0],w,eps)
grader_grad_check(value)
```

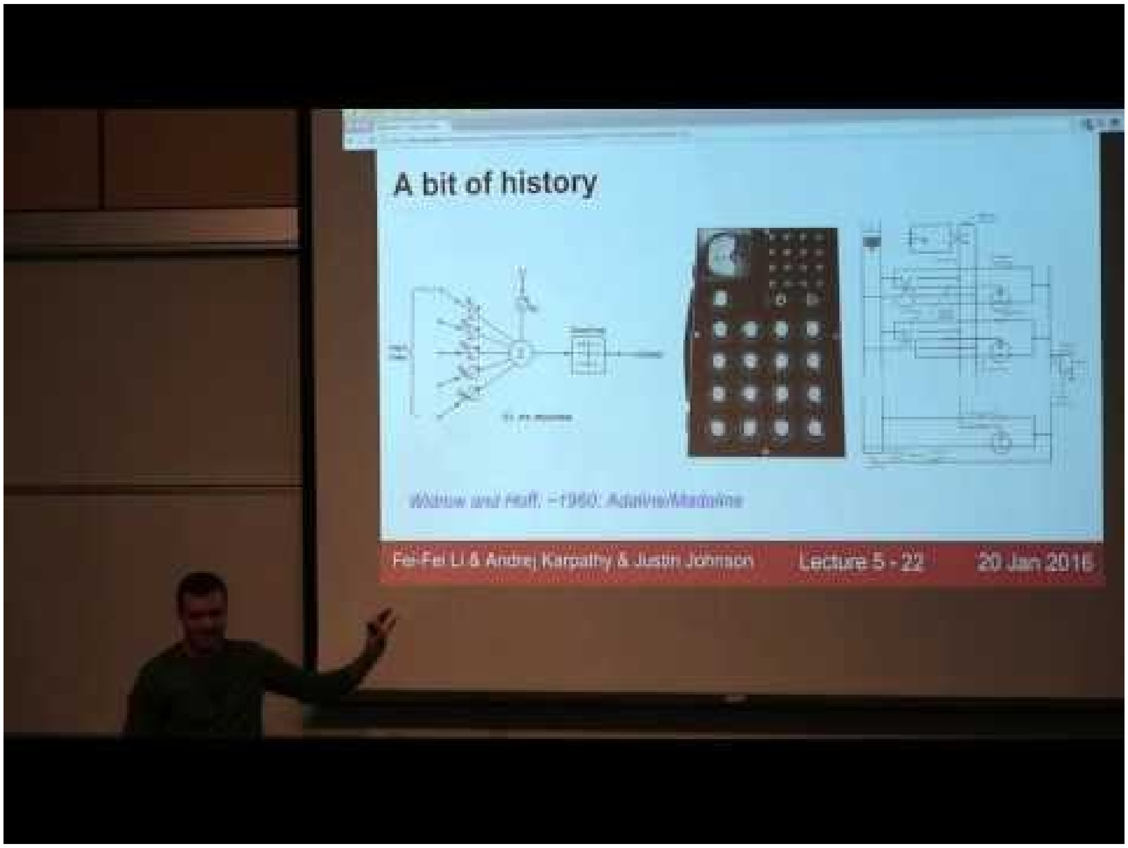
```
[-0.99205653 -0.99925711 -1.00000243 -0.99999214 -0.9999983  -
0.52808451
 -0.52850637 -0.99519247 -0.45355948]

True
```

Task 2 : Optimizers

- As a part of this task, you will be implementing 2 optimizers(methods to update weight)
- Use the same computational graph that was mentioned above to do this task
- The weights have been initialized from normal distribution with mean=0 and std=0.01. The initialization of weights is very important otherwiswe you can face vanishing gradient and exploding gradients problem.

Check below video for reference purpose

```
from IPython.display import YouTubeVideo
YouTubeVideo('gYpoJMlgyXA',width="1000",height="500")
```



Algorithm

Implement below tasks

- Task 2.1: you will be implementing the above algorithm with Vanilla update of weights
- Task 2.2: you will be implementing the above algorithm with Momentum update of weights
- Task 2.3: you will be implementing the above algorithm with Adam update of weights

Note : If you get any assertion error while running grader functions, please print the variables in grader functions and check which variable is returning False .Recheck your logic for that variable .

2.1 Algorithm with Vanilla update of weights

Vanilla update or Stochastic Gradient Descent: The simplest form of update is to change the parameters along the negative gradient direction (since the gradient indicates the direction of increase, but we usually wish to minimize a loss function).

Assuming a vector of parameters x and the gradient dx,

$$x += -learning\_rate * dx$$

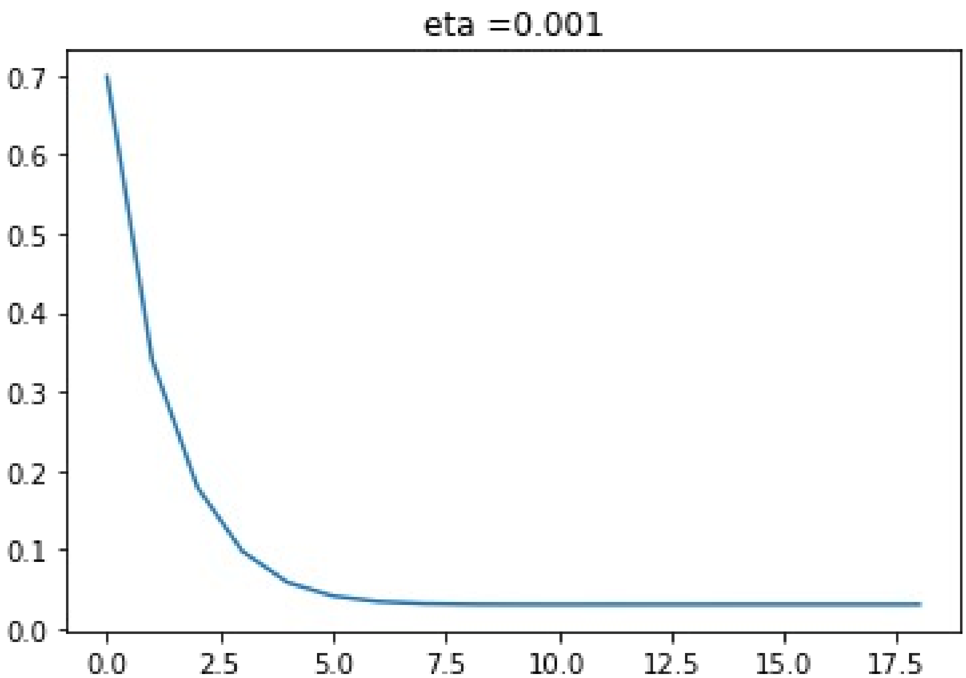
random intialization of weights with mean=0 and std=0.01

```
mean = 0
std  = 0.01
ran_w = np.random.normal(loc=mean, scale=std, size=9)

print(ran_w)
```

```
[-0.00430449  0.00838156 -0.00440015  0.0076267  -0.01167459 -
0.0046494
 -0.00494133  0.00222446  0.00191658]
```

```
def vanilla_weights(x,y,w,eta):
    mean_loss = []
    for epoch in range(1,20):
        loss_per_datapoint= 0
        for i in range(x.shape[0]):
            #calculate forward propagation
            forward_dict = forward_propagation(x[i],y[i],w)

           #adding loss for each datapoint
            loss_per_datapoint+=forward_dict['loss']

           #calculate gradient dict using backward propagation
            gradients = backward_propagation(x[i],y[i],w,forward_dict)
            dw = np.array(list(gradients.values()))

            #vanilla update of weights
            w = w-eta*dw

        mean_loss.append(loss_per_datapoint/x.shape[0])
    return mean_loss


mean_loss_vanilla=vanilla_weights(x=X,y=y,w=ran_w,eta=0.001)
```

```
import matplotlib.pyplot as plt

epoch_list = list(i for i in range(19))
xv = epoch_list
yv = mean_loss_vanilla
plt.plot(xv,yv)
plt.title("eta =0.001")
plt.show()
```



2.2 Algorithm with Momentum update of weights

Here Gamma referes to the momentum coefficient, eta is leaning rate and v_t is moving average of our gradients at timestep t

$$v = mu * v - learning_r ate * dx$$

$$x + ¿ v$$

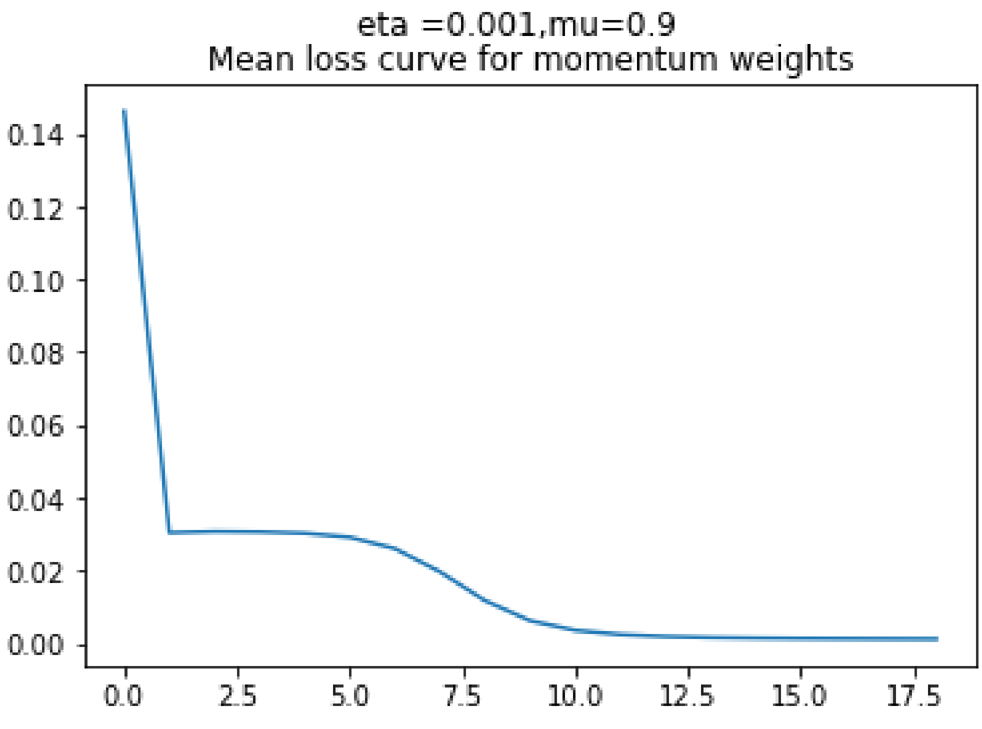
```
X[0]
```

```
array([-1.2879095 , -0.12001342, -1.45900038, -0.66660821, -
0.14421743])
```

```
def momentum_weights(x,y,w,eta,mu,v):
    mean_loss = []
    for epoch in range(1,20):
        loss_per_datapoint= 0
        for i in range(len(x)):
            #calculate forward propagation
            forward_dict = forward_propagation(x[i],y[i],w)

           #adding loss for each datapoint
            loss_per_datapoint+=forward_dict['loss']

           #calculate gradient dict using backward propagation
            gradients = backward_propagation(x[i],y[i],w,forward_dict)
            dw = np.array(list(gradients.values()))
```

```python
            #momentum update of weights
            v = mu*v + eta*dw
            w -=v
        mean_loss.append(loss_per_datapoint/len(x))
    return mean_loss
```

```python
mean_loss_momentum =
momentum_weights(x=X,y=y,w=ran_w,eta=0.001,mu=0.9,v=0)

epoch_list = list(i for i in range(19))
xm= epoch_list
ym= mean_loss_momentum
plt.plot(xm,ym)
plt.title("Mean loss curve for momentum weights")
plt.suptitle("eta =0.001,mu=0.9")
plt.show()
```



2.3 Algorithm with Adam update of weights

$$m=beta1*m+(1-beta1)*dx$$

$$mt=m/(1-beta1*¿t)$$

$$v=beta2*v+(1-beta2)*(dx*¿2)$$

$$vt = v / (1 - beta2 * ¿ t)$$

$$x + ¿ - learning_r ate * mt / (np.sqrt(vt) + eps)$$

where t is your iteration counter going from 1 to infinity

- recommended values :

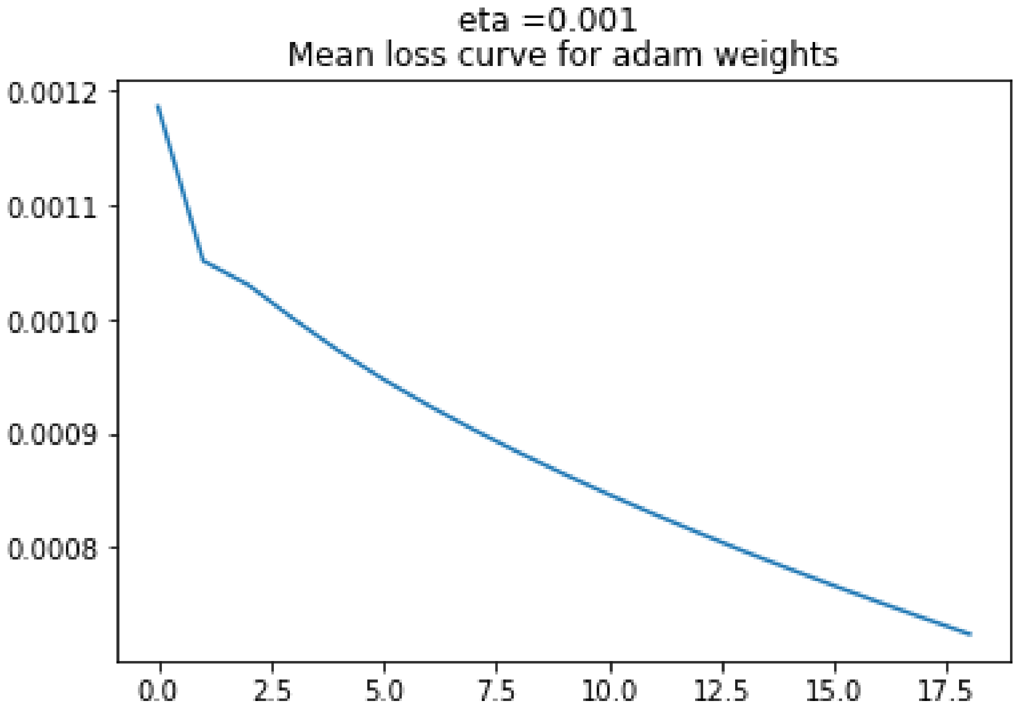
$$eps = 1e - 8$$

- ,

$$beta1 = 0.9,$$
$$beta2 = 0.999$$

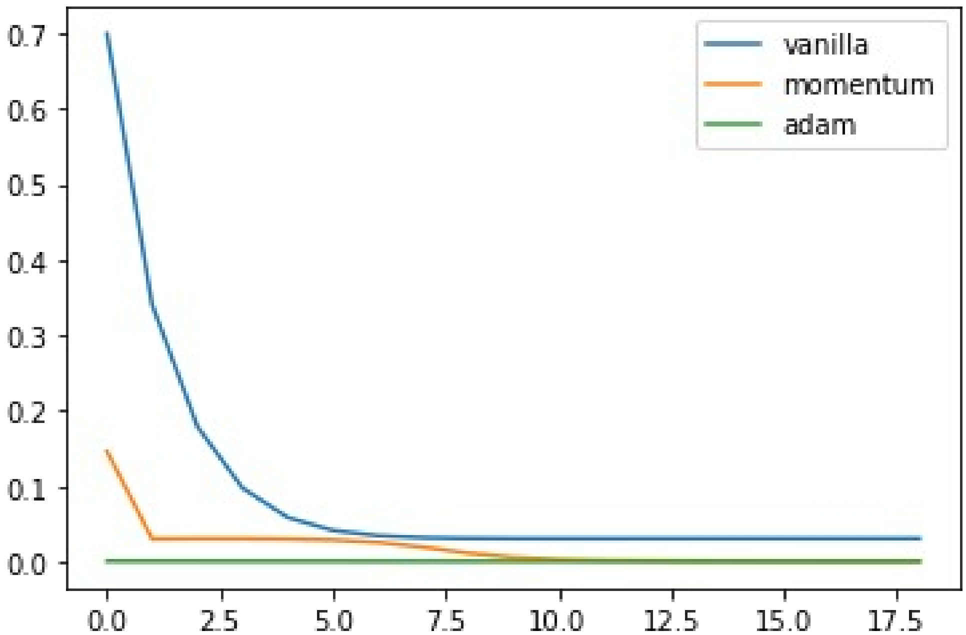
```
def adam_weights(x,y,w,eta,v,m,beta1,beta2,epsilon):
    mean_loss = []
    for epoch in range(1,20):
        loss_per_datapoint= 0
        for i in range(len(x)):
            #calculate forward propagation
            forward_dict = forward_propagation(x[i],y[i],w)

            #adding loss for each datapoint
            loss_per_datapoint+=forward_dict['loss']

            #calculate gradient dict using backward propagation
            gradients = backward_propagation(X[i],y[i],w,forward_dict)
            dw = np.array(list(gradients.values()))

            #calculating adam weights

            m = beta1*m + (1-beta1)*dw
            mt = m / (1-beta1**epoch)

            v = beta2*v + (1-beta2)*(dw**2)
            vt = v / (1-beta2**epoch)

            w += - eta * mt / (np.sqrt(vt) + epsilon)


        mean_loss.append(loss_per_datapoint/len(X))

    return mean_loss


mean_loss_adam  =
adam_weights(x=X,y=y,w=ran_w,eta=0.001,v=0,m=0,beta1=0.9,beta2=0.999,e
psilon=0.00000001)

epoch_list = list(i for i in range(epoch))
xa = epoch_list
ya = mean_loss_adam
```

```
plt.plot(xa,ya)
plt.title("Mean loss curve for adam weights")
plt.suptitle("eta =0.001")
plt.show()
```



Comparision plot between epochs and loss with different optimizers. Make sure that loss is conerging with increaing epochs

```
epoch_list = list(i for i in range(19))
epo = epoch_list
Yv = mean_loss_vanilla
Ym = mean_loss_momentum
Ya = mean_loss_adam
plt.plot(epo,Yv,label='vanilla')
plt.plot(epo,Ym,label ='momentum')
plt.plot(epo,Ya,label ='adam')
plt.legend()
plt.show()
```



You can go through the following blog to understand the implementation of other optimizers . Gradients update blog