Q.2 for Gaussian emission densities: we have

$$p(x|\phi_k) = \mathcal{N}(x|\mu_k, \Sigma_k)$$

$Q(\theta|\theta^{old})$ becomes

$$= \sum_{k=1}^{K} \gamma(z_{1k}) \ln \pi_k + \sum_{n=2}^{N} \sum_{j=1}^{K} \sum_{k=1}^{K} \xi(z_{n-1,j}, z_{nk}) \ln A_{jk}$$

$$+ \underbrace{\sum_{n=1}^{N} \sum_{k=1}^{K} \gamma(z_{nk}) \ln p(x_n|\phi_k)}$$

↓ can be written as

$$+ \cancel{\sum_{n=1}^{N} \sum_{1}}$$

$$\ln p(x_n|\phi_k) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu_k)^2}{\sigma_k^2}}\right) = \frac{-1}{2}\ln(2\pi\sigma_k^2) - \frac{1}{2}\frac{(x-\mu_k)^2}{\sigma_k^2}$$

$$-\frac{1}{2}\sum_{n=1}^{N} \sum_{k=1}^{K} \gamma(z_{nk}) \ln(2\pi\sigma_k^2) - \frac{1}{2}\sum_{n=1}^{N} \sum_{k=1}^{K} \gamma(z_{nk}) \frac{(x-\mu_k)^2}{\sigma_k^2}$$

so A and π will remain same.

$$\frac{d\, Q(\theta, \theta^{old})}{d\mu_k} = 0 + \frac{1}{\cancel{2}}\sum_{n=1}^{N} \cancel{\sum_{k=1}} \gamma(z_{nk}) \frac{\cancel{2}(x-\mu_k)}{\sigma_k^2} = 0$$

$$=$$

$$\mu_k = \frac{\sum_{n=1}^{N} \gamma(z_{nk}) x_n}{\sum_{n=1}^{N} \gamma(z_{nk})}$$

$$\frac{d\, Q(\theta, \theta^{old})}{d\sigma_k} = -\frac{1}{\cancel{2}}\sum_{n=1}^{N} \gamma(z_{nk}) \frac{\cancel{2}\,\cancel{4\pi}\,\sigma_k}{2\pi\sigma_k^{\cancel{2}}} + \frac{\cancel{2}}{\cancel{2}}\sum_{n=1}^{N} \gamma(z_{nk}) \frac{(x-\mu_k)^2}{\sigma_k^3}$$

$$= -\sum_{n=1}^{N} \frac{\gamma(z_{nk})}{\sigma_k} + \sum_{n=1}^{N} \gamma(z_{nk}) \frac{(x-\mu_k)(x-\mu_k)^T}{\sigma_k^2}$$

$$\sigma_k^2 = \frac{\sum_{n=1}^{N} \gamma(z_{nk}) (x_n-\mu_k)^2}{\sum_{n=1}^{N} \gamma(z_{nk})}$$

for multivariate it becomes

$$\Sigma_k = \frac{\sum_{n=1}^{N} \gamma(z_{nk}) (x_n-\mu_k)(x_n-\mu_k)^T}{\sum_{n=1}^{N} \gamma(z_{nk})}.$$