Q1 Since

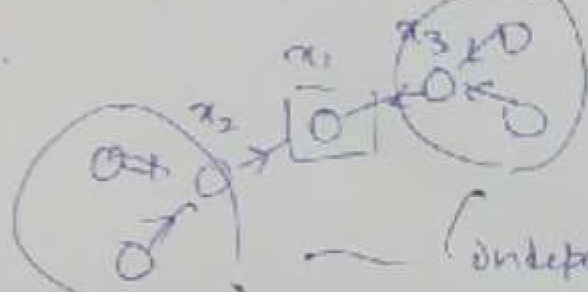
$$P(\underline{x}_{V_t} ; T_t) \propto \prod_{u \in V_t} \psi_u(x_u) \prod_{(v,w) \in E_t} \psi_{vw}(x_v, x_w)$$

$$\therefore \mu_s(x_s) = K \cdot \psi_s(x_s) \sum_{\substack{x_1, x_2 \ldots x_{s-1} \\ \cancel{t \in N(s)} \\ x_{s+1} \ldots x_n}} \prod_{t \in N(s)} P(x_{V_t} ; t) \prod_{t \in N(s)} \psi_{st}(x_s, x_t)$$

$$= K \psi_s(x_s) \sum_{x_1} \sum_{x_2} \sum_{x_3} \cdots \sum_{x_n} \prod_{t \in N(s)} \psi_{st}(x_s, x_t) P(\underline{x}_{V_t} ; T_t)$$

We observe that nodes in one subtree are independent of other (As discussed in class also)

eg.



— independent.

so it will be $\sum_{x_2} \sum_{x_3} \psi_{st}(x_s, x_t) P(\underline{x}_{V_t} ; T_t)$

$$= \psi_{st}(x_1, x_2) P(\underline{x}_2 ; T_2) \times \psi_{st}(x_1, x_3) P(\underline{x}_3 ; T_3)$$

Thus the quantity becomes

$$\mu_s(x_s) = K \psi_s(x_s) \prod_{t \in N(s)} \left[\sum_{x_{i_1}} \sum_{x_{i_2}} \cdots \sum_{x_{i_p} \in V_t} \psi_{st}(x_s, x_t) P(\underline{x}_{V_t}, T_{V_t}) \right]$$

$$= K \psi_s(x_s) \prod_{t \in N(s)} \left[\psi_{st}(x_s, x_t) \sum_{x_{i_1}} \sum_{x_{i_2}} \cdots \sum_{x_{i_p}} P(\underline{x}_{V_t} ; T_t) \right]$$

$$= K \psi_s(x_s) \prod_{t \in N(s)} \left[\psi_{st}(x_s, x_t) \sum_{x_{V_t}} P(\underline{x}_{V_t} ; T_t) \right.$$

$$= K \psi_s(x_s) \prod_{t \in N(s)} \mu_{ts}(x_s)$$

where $\mu_{ts}(x_s) = \psi_{st}(x_s, x_t) \sum_{x_{V_t}} p(x_{V_t} : T_t)$

or $\mu_{ts}(x_s) = \sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, p(\underline{x}_{V_t} ; T_t)$