Assignment - 1 (CS6840 - Causal Inference)

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Q.1 (a) $\{Y, W\}$

b) $\{Y, W, X\}$

c) $\{Y, Z\}$

d) $\{Y, Z, T\}$

e) $(X-Y-T)$, $(X-Y-Z-T)$, $(X-W-Z-T)$, $(X-W-Y-T)$
$(X-W-Y-Z-T)$, $(X-Y-W-Z-T)$, ~~$(X-W-Y-Z-T)$~~, $(X-W-Z-Y-T)$

f) $(X\rightarrow Y\rightarrow T)$, $(X\rightarrow W\rightarrow Y\rightarrow T)$, $(X\rightarrow W\rightarrow Z\rightarrow T)$, $(X\rightarrow Y\rightarrow Z\rightarrow T)$
~~$(X\rightarrow W\rightarrow Y\rightarrow T)$~~, $(X\rightarrow W\rightarrow Y\rightarrow Z\rightarrow T)$

g) No, as W is an ancestor of T, Both distributions are dependent.

h)

$$P(T,W,X,Y,Z) = P(X)\cdot P(W|X)\cdot P(Y|X,W)\cdot P(Z|Y,W)\cdot P(T|Y,Z)$$

i) No, W and T are not conditionally independent given Z, because there is a path from $W\rightarrow Y\rightarrow T$, hence, the path is unblocked by unobserved variable Y.

j) Yes.

$$P(W,T|Z,Y) = \frac{P(W,T,Z,Y)}{P(Z,Y)} = \sum_X \frac{P(X,W,T,Z,Y)}{P(Z,Y)}$$

$$= \sum_X \frac{P(X)\cdot P(W|X)\cdot P(Y|X,W)\cdot P(Z|Y,W)\cdot P(T|Y,Z)}{P(Z,Y)}$$

$$= \sum_X \frac{P(W,X,Y)\cdot P(Y,W,Z)\cdot P(T|Y,Z)}{P(Y,W)\cdot P(Z,Y)}$$

$$= \sum_X P(X|W,Y)\cdot P(W|Z,Y)\cdot P(T|Z,Y)$$

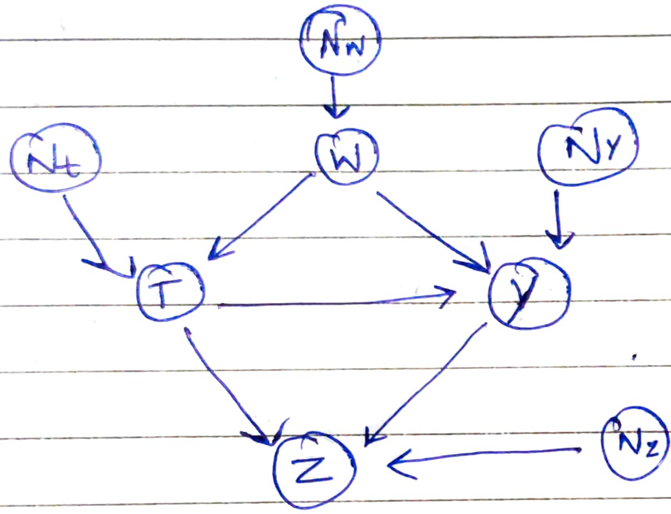
$$= P(W|Z,Y)\cdot P(T|Z,W) \sum_X P(X|W,Y)$$

Q.2

b) Y, Z set or {W, Y}
{~~X~~} is the only variable ~~that~~ d-seperates X and T.

X and T are ~~conditionatly~~ conditionally independent if Y, Z ~~is~~ are observed. or {W, Y} are observed.

Q.3 a)



(b), (c), (d), (e) in Jupyter notebook attached.

Q.2 a) For treatment unit 1

$D(1,3) = 108.7$
$D(1,4) = 4$
$D(1,5) = 2.5$
$D(1,6) = 9$
$D(1,7) = 10$
$D(1,8) = 101.5$

Match is control unit 5

For treatment unit 2

$D(2,3) = \cancel{4.2}\ 0.2$
$D(2,4) = 112.5$
$D(2,5) = 106$
$D(2,6) = 110.5$
$D(2,7) = 118.5$
$D(2,8) = 7$

Match is control unit 3.

b) Average Treatment Effect on treated is expected causal effect of the treatment for individuals in the treatment group. Mathematically,

$$ATT = E[Y(1) - Y(0) \mid T=1]$$
$$= E[Y(1) \mid T=1] - E[Y(0) \mid T=1]$$

Let p be the percentage of population receiving treatment. then

$$ATE = E[Y(1) - Y(0)] = p\, E[Y(1) - Y(0) \mid T=1] + (1-p)\, E[Y(1) - Y(0) \mid T=0]$$

$$= \{p\, E[Y(1) \mid T=1] + (1-p)\, E[Y(1) \mid T=0]\} - \{p\, E[Y(0) \mid T=1] + (1-p)\, E[Y(0) \mid T=0]\}$$

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when exchangability holds (for eg. in RCTs)

$E[Y(0)|T=0] = E[Y(0)|T=1]$ or

$E[Y(1)|T=0] = E[Y(1)|T=1]$

ie $ATE = E[Y(1)|T=1] - E[Y(0)|T=1]$

$= ATT.$

c) ~~AA~~ $ATT = E[Y(1) - Y(0) | T=1]$

we know $Y(1)$, ~~to~~ $Y(0)$ is counterfactual. So we will ~~or~~ approximate it with closest match. in control group.

$$ATT = \frac{Y_1(1) - Y_1(0)}{2} + \frac{Y_2(1) - Y_2(0)}{2}$$

$$= \frac{Y_1(1) - Y_5(0)}{2} + \frac{Y_2(1) - Y_3(0)}{2}$$

{ approximate with closest control unit }

$$= \frac{30 - 19.5}{2} + \frac{12.5 - 17}{2}$$

$$= 3$$

So, $ATT = 3$

Q.4

a) Mean independence assumption says that for a given random variable Y which mean independent of X, following is sufficent and necessary.

$$E[Y|X] = E[Y]$$

b) For β_1 and β_0 to be consistent and unbiased. $E[u|x]$ should be 0, ~~also ingeneral~~ as.

$$y = \beta_0 + \beta_1 x + u$$

$$u = y - \beta_0 - \beta_1 x \quad \text{or} \quad U = Y - \beta_0 - \beta_1 X$$

$$\underbrace{E[U|X]}_{0} = E[Y|X] - E[\beta_0|X] - E[\beta_1 X|X]$$

$$\Rightarrow \quad E[Y|X] = E[\beta_0|X] + E[\beta_1 X|X]$$

$$= \beta_0 + \beta_1 X$$

hence if $E[U|X] = 0$, β_0 and β_1 can be estimated consistently and unbiased

also $E[U|X] = 0 = E[U] \Rightarrow U$ is mean independent of X.

c) Regression is conditional mean functions and not causal function. ~~To use caus~~ when exchangability or conditional exchangability holds, we can represent causal quantities as conditional quantaties which allows us to use regression. If conditional exchangability holds then the Average treatment effect will be conditional average treatment effect.

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Q.5 $P(Y=y \mid do(T=t)) = \sum_x P(y \mid t, x) P(x)$ — (1)

$P(Y=y \mid T=t) = \sum_x P(y \mid t, x) P(x \mid t)$ — (2)

Now if distribution of X is same accross treatment groups. T.

ie $P(x \mid t=0) = P(x \mid t=1)$

$p(x) = p(t=0)\, p(x \mid t=0) + p(t=1)\, p(x \mid t=1)$

$= [p(t=0) + p(t=1)]\, p(x \mid t=0)$

$= 1 \cdot p(x \mid t=0)$

ie $p(x \mid t=0) = p(x) = p(x \mid t=1)$ or $p(x \mid t) = p(x)$

now Eq-(2) becomes

$P(y=y \mid T=t) = \sum_x p(y \mid t, x)\, p(x \mid t)$

$= \sum_x p(y \mid t, x)\, p(x) = P(y=y \mid do(T=t))$