Puneet Mangla - CS17BTECH11029

Q.1 a) $I = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ $F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

We add 1 layer of padding on top and left so that output has same dimension as I. I^p is the padded image

$$I^p = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$F * I^p = F' \oplus I^p$ where $\oplus$ is correlation and F' is flipped kernel

$$F' = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & - & - \\ - & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & - \\ - & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ - & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$$

$\downarrow$

Ans.

b) $F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$

so $F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $F_2 = [1, -1]$, we first pad I^p ~~horigond~~ on top

$F_1 * I^p = F_1' \oplus I^p$ here $F_1' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $I^p = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\cancel{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & - & - & - \\ - & - & - & - \end{bmatrix}}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & - & - \\ 0 & - & - & - \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & - & - \\ 0 & - & - & - \end{bmatrix}}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & - & - \\ 0 & 3 & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & - \\ 0 & 3 & - & - \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & - \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix}$$

$$F_1 * I^p = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix}$$

We pad $F_1 * I^p$ at left then

$F_2 * (F_1 * I^p) = F_2' \oplus (F_1 * I^p)$, $F_2' = [-1 \ 1]$

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & - & - \\ - & - & - \end{bmatrix}, \quad \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & - \\ - & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ - & - & - \end{bmatrix}, \quad \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & - \end{bmatrix}, \quad \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$$

so, $F_2 * (F_1 * I^p) = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$

c) Let F is $m \times m$ filter which seperable i.e $F_{m\times m} = F_{1_{m\times 1}} F_{2_{1\times m}}$

now $F_1 * I [i,j] = \sum_k I[i-k, j] \, F_1[k,1]$

now $F_2 * (F_1 * I)[i,j] = \sum_l \left(\sum_k I[i-k, j-l] \, F_1[k,1] \right) \cdot F_2[1,l]$

$$= \sum_l \sum_k I[i-k, j-l] \, F_1[k,1] \, F_2[1,l] \quad \text{—(1)}$$

now $F_{m\times m}[i,j] = F_1[i,1] \cdot F_2[1,j]$

so Eq -(1) becomes

$F_2 * (F_1 * I)[i,j] = \sum_{l,k} I[i-k, j-l] \, F[k,l]$

$= F * I \, [i,j]$

hence $F_2 * (F_1 * I) = F * I$

d) In part (a)

Image height x width after padding = 3x4

\# ops in one local op of F = 4 { since F is 2x2 filter }

\# times kernel gets convolved = 6 { as output has dimension 2x3 }

Total multiplications = 4x6 = 24

In part (b)

\# ops in one local op of F_1 = 2

" " " " F_2 = 2

\# times F_1 operates on I^p = ~~8~~ 6

\# " F_2 " on $(F_1 * I^p)$ = 6

Total multiplications = $2 \times \cancel{8}6 + 2 \times 6$

$= 2 \times (12)$

$= \cancel{28}\ 24$

Part (a) requires ~~fewer~~ same no of operations as part (b)

e) I: $M_1 \times N_1$ Image.
F: $M_2 \times N_2$ Seperable filter.

(i) # multiplications for direct 2D conv.

one local operation will require $\Rightarrow$ $M_2 N_2$ ops.

# of times F takes to gets convolved with I =
$(M_1 - M_2 + 1) \times (N_1 - N_2 + 1)$

Total multiplications:
$M_2 N_2 \times [(M_1 - M_2 + 1) \times (N_1 - N_2 + 1)]$

$\Rightarrow M_2 N_2 [M_1 N_1 - N_1 N_2 - M_2 N_1 - M_2 N_2 + N_1 - N_2 + 1$
$+ M_1 - M_2 \cancel{+}]$

(ii) $F = F_1 F_2$ F_1 : $M_2 \times 1$ filter.
F_2 : $1 \times N_2$ filter.

one local op. by F_1 takes = M_2 ops.
one " " " F_2 takes = N_2 ops.

times F_1 takes to convolve with I
$= (M_1 - M_2 + 1) \cdot N_1$

" F_2 " " with I =
$= (N_1 - N_2 + 1) \cdot (M_1 - M_2 + 1)$.

Total multiplication ops:

$(M_1 - M_2 + 1) \cdot N_1 M_2 + N_2 \cdot (N_1 - N_2 + 1) \cdot \cancel{M_1 N_2} (M_1 - M_2 + 1)$

(iii) direct 2D conv: $M_2 N_2 \times [(M_1 - N_2 + 1) \times (N_1 - N_2 + 1)]$

$$\equiv O(M_2 N_2 N_1 M_1)$$

using 2 successive 1D convs: $(M_1 - N_2 + 1) \cdot N_1 M_2 + (N_1 - N_2 + 1) \cdot M_1 N_2$

$$\equiv O(M_1 M_2 N_1) + O(M_1 N_1 N_2)$$

~~Generally $M_1 < N_1$ and $M_2 \leq N_2$~~

$\cancel{\equiv O(M_1 M_2}$

~~Generally M_2 and N_2 are smaller than M_1 and N_1~~

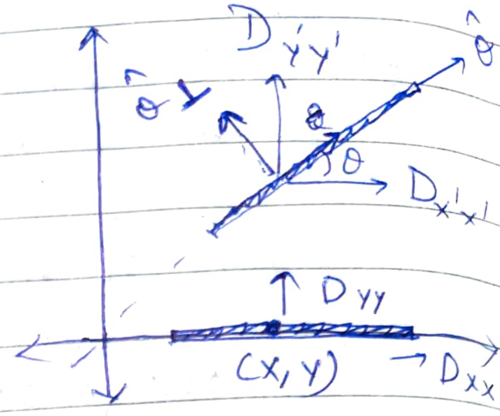
$\cancel{\equiv O(M_1 N_1 M_2)}$ ~~or~~ (

$$\equiv O(N_1 M_1 (M_2 + N_2))$$

Since $O(N_2 N_2 N_1 M_1) \gg O(M_1 N_1 (M_2 + N_2))$

2 1D convs are more efficient than a single 2D convs.

Q.2 a) Claim: Edge will be detected



Proof: Let at point (x, y)

$$D_{xx} = \frac{dI}{dx}, \quad D_{yy} = \frac{dI}{dy}.$$

here I is the intensity.

Now at rotated point (x', y') the magnitude along direction $\hat{\theta}_{\parallel} = (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$ will be same as D_{xx} $(D\hat{\theta}_{\parallel} = D_{xx})$ and magnitude along direction $\hat{\theta}_{\perp} = (-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$ will be same as D_{yy}. $(D\hat{\theta}_{\perp} = D_{yy})$

Now $D_{x'x'} = D\hat{\theta}_{\parallel}\cos\theta - D\hat{\theta}_{\perp}\sin\theta$

$D_{y'y'} = D\hat{\theta}_{\perp}\cos\theta + D\hat{\theta}_{\parallel}\sin\theta.$

Net magnitude $= \sqrt{D_{x'x'}^2 + D_{y'y'}^2}$

$$= \sqrt{(D\hat{\theta}_{\parallel}\cos\theta - D\hat{\theta}_{\perp}\sin\theta)^2 + (D\hat{\theta}_{\perp}\cos\theta + D\hat{\theta}_{\parallel}\sin\theta)^2}$$

$$= \sqrt{D\hat{\theta}_{\parallel}^2 + D\hat{\theta}_{\perp}^2}$$

$$= \sqrt{D_{xx}^2 + D_{yy}^2} \quad \left\{ \because \begin{array}{l} D\hat{\theta}_{\parallel} = D_{xx} \\ D\hat{\theta}_{\perp} = D_{xy} \end{array} \right\}$$

Hence magnitude of derivative will not change and edge will be detected.

b) Since there are long broken edges seperated by gaps. that means 'low' threshold is too high because of which edges are getting disconnected. Hence lowering the 'low' threshold will include more pixels of long edge. 'High' threshold is enough as parts of long edges are detected but to eliminate spurious edges, we need slightly higher 'high' threshold. Hence it should be increased with very little amount so as to avoid ~~di~~ complete disappearing of long edges.