

HOMEWORK 2 - Q4

Puneeth Kambhampati - z5164647

June 2020

1 Solution

1.1 Part A

To find the convolution of,

$$\langle 1, 0, 0 \dots 0, 0, 1 \rangle * \langle 1, 0, 0 \dots 0, 0, 1 \rangle$$

*Assume there's k number of 0s in between the 1s. We can easily multiply their polynomials and then revert them back to the list. Since, we know there are $k + 2$ elements in each list, their respective polynomials would be of degree $k + 1$. let,

$$P(x) = 1 + x^{k+1}$$

The question essentially requires us to find, $P(x)^2$

$$\begin{aligned} &= P(x)^2 \\ &= (1 + x^{k+1})^2 \\ &= 1^2 + (x^{k+1})^2 + 2 * (1) * (x^{k+1}) \\ &= 1 + (x^{2k+2}) + 2(x^{k+1}) \\ &= 1 + 2(x^{k+1}) + (x^{2k+2}) \end{aligned}$$

To represent this polynomial in the list form, There are k number of 0s in between the first 1 and second 1 and there are also k number of 0s in between the second 1 and the 2

$$A = \langle 1, 0, 0, \dots, 1, 0, 0 \dots 2 \rangle$$

1.2 Part B

DFT of a given polynomial $P(x)$ is just the list containing, $P(\omega)$ for every single root of unity ω for $P(x)$ where,

$$P(x) = 1 + x^{k+1}$$

Then we find the DFT for $P(x)$ that has a degree of $k + 1 + 1 = k + 2$,

$$\begin{aligned} DFT(P) &= \langle P(\omega_{k+2}^0), P(\omega_{k+2}^1) \dots P(\omega_{k+2}^{k+1}) \rangle \\ DFT(P) &= \langle 1 - \omega_{k+2}^{0*(k+1)}, 1 - \omega_{k+2}^{1*(k+1)}, 1 - \omega_{k+2}^{2*(k+1)} \dots 1 - \omega_{k+2}^{(k+1)*(k+1)} \rangle \\ DFT(P) &= \langle 0, 1 - \omega_{k+2}^{(k+1)}, 1 - \omega_{k+2}^{2*(k+1)} \dots, 1 - \omega_{k+2}^{(k+1)^2} \rangle \end{aligned}$$