HOMEWORK 1 - Q5

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1 Solution

To show that a function, g(n), is the asymptotic upper bound of another function, f(n), we have to prove that

$$f(n) <= c * g(n)$$

For a large enough n, $n > n_0$ and a constant c

1.1 Part A

We are given the two functions,

$$f(n) = (\log_2 n)^2$$
$$g(n) = \log_2(n^{\log_2 n})^2$$

i. Let's Prove that $f(n) = O(g(n))$,

$$g(n) = \log_2(n^{\log_2 n})^2$$
$$g(n) = \log_2(n^{2\log_2 n})$$
$$g(n) = 2\log_2 n * \log_2 n$$
$$g(n) = 2(\log_2 n)^2$$

We can notice that,

$$f(n) => (\log_2 n)^2 < 2(\log_2 n)^2$$
$$f(n) <= c * g(n)$$

Therefore,

$$f(n) = O(g(n))$$

ii. However, we also notice that,

$$g(n) <= c * f(n)$$

For a given c value that is greater than 2 like c = 3,

$$2(\log_2 n)^2 <= 3 * (\log_2 n)^2$$

We can hence deduce that,

$$g(n) = O(f(n))$$

iii. This in turn shows us that,

$$f(n) = \theta(g(n))$$

1.2 Part B

We are given the functions,

$$f(n) = n^{10}$$

$$g(n) = 2^{n^{1/10}}$$

i. Once again, let's start by trying to prove,

$$f(n) <= c * g(n)$$

Hence we are trying to show,

$$n^{10} <= c * 2^{n^{1/10}}$$

To simplify this inequality, we apply log to both sides

$$\log(n^{10}) <= \log(c * 2^{n^{1/10}})$$

$$10\log(n) <= \log(c) + n^{1/10} * \log(2)$$

$$10\log(n) <= \log(c) + n^{1/10}$$

Since, the constant is only contributing through addition, we can remove it with $c = 1$ and get,

$$10\log(n) <= n^{1/10}$$

We know that the polynomial $n^{1/10}$ will be larger than $log(n)$, Therefore, We can say that for some large value of $n >> 1$

$$f(n) <= c * g(n)$$

ii. Unfortunately, from the above inequality function we can also safely say that, $f(n)$ can not be an upper asymptotic bound for $g(n)$

iii. This also means that we can not say that,

$$f(n) = \theta(g(n))$$

1.3 Part C

We are given the functions,

$$f(n) = n^{1+(-1)^n}$$

$$g(n) = n$$

i. On last time, to show that $f(n) = O(g(n))$ let's start by trying to prove,

$$f(n) <= c * g(n)$$

Since $f(n)$ behaves in different ways when n is either even or odd, we'll consider both parts of the piecewise function.

- IF n is even:

$$f(n) = n^{1+1}$$

$$f(n) = n^2$$

We can clearly see that, For any given constant c.

$$f(n) >= c * g(n)$$

$$n^2 >= c * n$$

Therefore, we cannot deduce that $g(n)$ is an asymptotic upper bound for $f(n)$.

ii. The previous equation may point at the fact that, $g(n) = O(f(n))$, but we must check $f(n)$ for the other half of its piecewise behaviour.

- IF n is odd:

$$f(n) = n^{1-1}$$

$$f(n) = n^0$$

$$f(n) = 1$$

We can clearly see that, For any given constant c.

$$g(n) >= c * f(n)$$

$$n >= c * 1$$

Therefore, we cannot deduce that $f(n)$ is an asymptotic upper bound for $g(n)$.

Hence, for these two examples $f(n)$ and $g(n)$, we have to say that it is neither $f(n) = O(g(n))$ or $g(n) = O(f(n))$