Problems

1. Construct a truth table for the following compound proposition:

$(p \wedge q) \vee \neg r$

Solution:

- p, q, r : Variables
- $(p \wedge q)$: p AND q
- $\neg r$: NOT r
- $(p \wedge q) \vee \neg r$: given compound statement

Truth Table

p	q	r	p ∧ q	¬r	(p ∧ q) ∨ ¬r
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

- The columns under p, q, and r list all possible combinations of truth values for these variables.
- The p∧q column is true only when both p and q are true.
- The ¬r column is true when r is false.
- The final column (p∧q)∨¬r is true when either p∧q is true, or ¬r is true, or both.

2. Construct a truth table for the compound proposition

$(p \vee q) \wedge \neg p$

Solution:

- p, q : Variables
- $(p \vee q)$: p OR q
- $\neg p$: NOT p
- $(p \vee q) \wedge \neg p$: given compound statement

Truth Table

p	q	p ∨ q	¬p	(p∨q)∧¬p
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

- p∨q: This disjunction is true if either p or q is true, or both.
- ¬p: This negation flips the truth value of p
- (p∨q)∧¬p: The final conjunction is true only when both p∨q is true, and ¬p is true.

3. Construct a truth table for the compound proposition

$$\neg (p \wedge q) \vee (p \rightarrow r)$$

Solution:

- p, q, r : Variables
- (p∧q): p AND q
- ¬(p∧q) : NOT p AND q
- (p→r) : p IMPLIES r
- ¬(p∧q)∨(p→r) : given compound statement

- The columns under p, q, and r list all possible combinations of truth values for these variables.
- The p∧q column shows the result of p AND q.
- The ¬(p∧q) column shows the negation of p∧q.
- **The p→r column is true except when p is true and r is false.**
- The final column ¬(p∧q)∨(p→r) shows the result of the entire expression, which is true if either ¬(p∧q) is true, or p→r is true, or both.

Truth Table

p	q	r	p ∧ q	¬(p∧q)	(p→r)	¬(p∧q)∨(p→r)
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Construct a truth table for the compound proposition

- $(p \rightarrow q) \vee (\neg q \wedge r)$
- $(p \wedge \neg q) \rightarrow r$
- $(\neg p \vee q) \wedge r$
- $\neg (p \wedge q) \vee (r \wedge \neg q)$
- $(p \wedge \neg (q \vee r)) \vee q$

Problem

Represent using Propositional Logic:

P: The system is online.

Q: The server is responsive.

R: The network is stable.

a. If the system is online, then the server is responsive.

b. If the network is unstable, then the system is not online.

c. If the server is responsive, then the system is online.

d. If the network is stable and the system is online, then the server is responsive.

Solution:

a. $P \rightarrow Q$

b. $\neg R \rightarrow \neg P$

c. $Q \rightarrow P$

d. $(R \wedge P) \rightarrow Q$

Problem

Convert the following sentence into a first-order logic expression:

i. “All students in the class passed the exam.”
ii. “Some students study in the library.”
iii. "There exists a student who has never missed a class."
iv. "Not all students stay in hostel.“
v. "There is a student who is both an athlete and a top scholar."

Solution:

i. "All students in the class passed the exam.“
FOL expression:$\forall x(S(x) \rightarrow P(x))$ where
S(x) means "x is a student in the class"
P(x) means "x passed the exam.“
This states that for every student, if they are in the class, they passed the exam.

ii. "Some students study in the library.“
FOL expression: $\exists x(S(x) \wedge L(x))$ where
S(x) means "x is a student"
L(x) means "x studies in the library.“
This states that there exists at least one student who studies in the library.

iii. "There exists a student who has never missed a class.“

FOL expression:$\exists x(S(x) \wedge \neg M(x))$ where

S(x) means "x is a student“

M(x) means "x missed a class.“

This states that there exists a student who has not missed any class.

iv. "Not all students stay in the hostel.“

FOL expression:$\neg \forall x(S(x) \rightarrow H(x))$ where

S(x) means "x is a student“

H(x) means "x stays in the hostel.“

This states that not every student stays in the hostel, meaning some do not.

v. "There is a student who is both an athlete and a top scholar.“

FOL expression:$\exists x(S(x) \wedge A(x) \wedge T(x))$ where

S(x) means "x is a student,“

A(x) means "x is an athlete,“

T(x) means "x is a top scholar.“

This states that there exists a student who is both an athlete and a top scholar.

Activity

1. Explain procedural and declarative knowledge with example. Also provide the difference between them.
2. Explain AI forward and backward reasoning. Compare and contract between them.
3. Convert the following sentence into a first-order logic expression:
    a. “Every program that has a syntax error fails to compile.”
    b. “There exists a machine learning model that performs better on unseen data than on training data.”
    c. "Not all sorting algorithms are stable."
    d. "Some operating systems are open-source.“
    e. "All AI models require training data to learn."

4. Represent using Propositional Logic:

P: The light switch is on.

Q:The room is bright.

R: The bulb is working.

a. If the light switch is on, then the room is bright.

b. If the bulb is not working, then the room is not bright.

c. If the room is bright, then the bulb is working.

d. If the light switch is on and the bulb is working, then the room is bright

5. Represent using Propositional Logic:

P: The code is error-free.

Q: The program runs successfully.

R: The compiler is functioning.

a. If the code is error-free, then the program runs successfully.
b. If the compiler is not functioning, then the program does not run successfully.
c. If the program runs successfully, then the code is error-free.
d. If the code is error-free and the compiler is functioning, then the program runs successfully.