

### Problem Statement 1:

A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

```
In [3]: # Importing libraries
from scipy.stats import binom
# No. of Trails = No.of multiple choice questions
n = 20

# Number of questions answered wrong = 5
success = 5

# Probability of getting a question right = 1 - (No. of correct answers per question/Total number of possible answers per question)
p = 1 - (1/4)
print(p)
```

0.75

```
In [4]: # PMF is the function that we can use to get the probability of exact values
print("The probability that a person undertaking the test answered exactly 5 questions wrong is : {:.10f}".format(binom.pmf(success, n, p)))
```

The probability that a person undertaking the test answered exactly 5 questions wrong is : 0.0000034265

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Step 1: Determine the Formula: Binomial Distribution: Bernoulli trials where  $n$  = number of trials attempted,  $k$  = number of successes in ' $n$ ' trials  $n - k$  = number of failures  $s$  = probability of success in a trial  $(1 - s)$  = probability of failure

$P$  (' $k$ ' successes in ' $n$ ' trials) =  $C(n, k)(s^k)((1-s)^{(n-k)})$  where,  $C(n, k) = n!/(k!(n-k)!)$

Step 2: Formula substitution:

$n = 20$   $n - k = 5$   $k = 20 - 5 = 15$

$s = (1/4)$ , where probability of success or the right answer  $1 - s = 1 - (1/4) = (3/4)$ , where probability of failure or the wrong answer

Therefore the binomial distribution is  $P$  (exactly 5 out of 20 answers incorrect) =  $C(20, 5) ((1/4)^{15}) ((3/4)^5)$

=  $P$  (5 out of 20) =  $(20 \times 19 \times 18 \times 17 \times 16) / (5 \times 4 \times 3 \times 2 \times 1) (1/4)^{15} (3/4)^5$

=  $3.4 \times 10^{-6} = 0.0000034$

### Problem Statement 2:

A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

```
In [6]: # Importing libraries
from scipy.stats import binom

# No. of Trails = No. of times the die is rolled i.e. 50
n=50

# The die has to roll a "D" exactly 5 times
success = 5

# Probability of getting "D" when rolled once is = 1/5
p = 1/5
print("p: ", p)
p: 0.2

# PMF is the function that we can use to get the probability of exact values
print("The probability of getting a "D" exactly 5 times is : {:.10f}".format(binom
.pmf(success,n,p) ) )

p: 0.2
The probability of getting a "D" exactly 5 times is : 0.0295312043
```

Here,  $n = 50$ ,  $k = 5$ ,  $n - k = 4/5$ .

The probability of success = probability of getting a "D" =  $s = 1/5$

Hence, the probability of failure = probability of not getting a "D" =  $1 - s = 4/5$ .

In [ ]:

Problem Statement 3:

Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes.

```
In [7]: # Importing libraries# Impor
from scipy.stats import binom

# Initializing values
r = 4 # No. of red balls
b = 6 # No. of black balls
t = r + b # Total number of balls
```

```
In [8]: #There are only four possible combinations when the two balls are drawn at random:
# (red, red),(red,black),(black, red), (black, black)

# Probability of (red, red)
prob_rr = ((r/t)*((r-1)/(t-1)))
print("Probability of (red, red): ", prob_rr)

# Probability of (red, black)
prob_rb = ((r/t)*((b)/(t-1)))
print("Probability of (red,black): ", prob_rb)

# Probability of (black, red)
prob_br = ((b/t)*((r)/(t-1)))
print("Probability of (black, red): ", prob_br)

# Probability of (black, black)
prob_bb = ((b/t)*((b-1)/(t-1)))
print("Probability of (black, black): ", prob_bb)

Probability of (red, red):  0.13333333333333333
Probability of (red,black):  0.26666666666666666
Probability of (black, red):  0.26666666666666666
Probability of (black, black):  0.33333333333333333
```

```
In [9]: # The probabily of all possible outcomes
prob_all = ((r/t)*((r-1)/(t-1))) + ((r/t)*((b)/(t-1))) + ((b/t)*((r)/(t-1)))
+ ((b/t)*((b-1)/(t-1)))
# OR
# prob_all = prob_rr + prob_rb + prob_br + prob_bb
prob_all
```

Out[9]: 1.0

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**First determine the probabilities of the events.**

Table of Probability of events	
Events	Probability
RR	$(4/10)(3/9) = 2/15$
RB	$(4/10)(6/9) = 4/15$
BR	$(6/10)(4/9) = 4/15$
BB	$(6/10)(5/9) = 1/3$

The probability of 0 black balls (RR) is  $2/15$

The probability of 1 black ball is (RB or BR) is  $4/15 + 4/15 = 8/15$

The probability of 2 black balls (BB) is  $1/3$

If Z is the random variable representing the number black balls. The probability distribution will be :

**First determine the probabilities of the events.**

Table of Probability of events	
Events	Probability
RR	$(4/10)(3/9) = 2/15$
RB	$(4/10)(6/9) = 4/15$
BR	$(6/10)(4/9) = 4/15$
BB	$(6/10)(5/9) = 1/3$

Notice that the sum of the probabilities =  $2/15 + 4/15 + 4/15 + 1/3 = 1$

In [ ]: