

Statistics - 4

Additional Exercise - 14.11442

Sol 1

Target diameter to achieve = 12.5 mm

$$f(d) = 20 e^{-20(d-12.5)}, d \geq 12.5$$

if $d > 12.6 \Rightarrow$

$$\text{let } x = 20(d - 12.5)$$

$$d(x) = d(20)(d - 12.5) \\ = 20dd - 0$$

$$\Rightarrow dd = \frac{d(x)}{20}$$

$$\begin{aligned} \int_{12.5}^{12.6} f(d) dd &= \int_{12.5}^{12.6} 20(e^{-20(d-12.5)}) \times \frac{dx}{20} \\ &= \int_{12.5}^{12.6} e^{-x} d(x) \end{aligned}$$

Changing limits,

$$\text{let } x = 12.5 = 20(12.5 - 12.5) = 0$$

$$12.6 = 20(12.6 - 12.5) = 2$$

$$\Rightarrow \int_0^2 e^{-x} dx = \left[-e^{-x} + e^{-0} \right] = \underline{0.8647}$$

$$\text{proportion to scrapped} = 1 - 0.8647 = 0.1353$$

$$\text{CDF}(x_1 = 11) = 0$$

as per continuous distribution probability at point is 0

\therefore CDF is also 0.

Sol 2

(a) (i) $P(Z > 1.26)$

$$\Rightarrow 1 - P(Z \leq 1.26) \\ = 1 - 0.8962 \\ = \underline{0.1036}$$

(iii) $P(Z > -1.37)$

$$= 1 - P(Z \leq -1.37) \\ = 1 - 0.0853 \\ = \underline{0.9147}$$

(ii) $P(Z < -0.86)$

$$= 1 - P(Z \geq -0.86) \\ = 1 - 0.8051 \\ = \underline{0.1949}$$

(iv) ~~$P(Z \leq \dots)$~~

$$P(-1.25 < Z < 0.39)$$

$$P(Z \leq 0.39) = 0.6443$$

$$P(Z > -1.25) = 0.1056$$

$$\therefore P(-1.25 < Z < 0.39) = 0.6443 - 0.1056 \\ = \underline{0.5387}$$

(v) $P(Z \leq -4.6) \rightarrow$ Maximum Z value at -3.49 is 0.0002

So $P(Z \leq -4.6)$ will be approx 0

(b) $P(Z > 2) = 0.05$

$$\Rightarrow P(Z < 2) = \underline{0.95}$$

(c) $P(-2 < Z < 2) = 0.99$

$$\Rightarrow P(Z < 2) = 0.99 \approx 2.33$$

$$P(Z \leq -2) = 0.01 = \underline{0.233}$$

Sol 3)

$$\mu = 10 \text{ mA} ; \sigma^2 = 4 \Rightarrow \sigma = 2 \quad \left\{ \text{given} \right\}$$

$$P(X > 13)$$

$$Z = \frac{13-10}{2} = 1.5 \Rightarrow P(Z) = 0.9332$$

$$\Rightarrow 1 - 0.9332$$

$$= \underline{\underline{0.0668}}$$

$$P(9 < X < 11)$$

$$P(X < 11) = \frac{11-10}{2} = 0.5 \Rightarrow P(Z) = 0.6915$$

$$P(X > 9) = \frac{9-10}{2} = -0.5 \Rightarrow P(Z) = 0.3085$$

$$\Rightarrow P(9 < X < 11) = 0.6915 - 0.3085 = \underline{\underline{0.383}}$$

$$P(Z < 2.06) \text{ as } P(0.98) = 2.06$$

$$Z = 2.06$$

$$\therefore X = 2(2.06) + 10 = \underline{\underline{14.12}}$$

Sol 4/

$$\mu = 0.2508$$

$$\sigma = 0.0005$$

$$\text{Specifications} = 0.2500 \pm 0.0015$$

(i) $0.2508 < x < 0.2515$

$$z = \frac{0.2515 - 0.2508}{0.0005} = \underline{1.4}$$

$$0.2508 > x > 0.2485$$

$$z(0.2485) = \frac{0.2485 - 0.2508}{0.0005} = -4.6$$

$$p(z) = p(1.4) = 0.9192 \Rightarrow \underline{\underline{91.92\%}}$$

(ii)

$$\mu = 0.25 \text{ (given)}$$

$$\Rightarrow z = \frac{0.2515 - 0.25}{0.0005} = 3$$

$$z = \left(\frac{0.2485 - 0.25}{0.0005} \right) = -3$$

$$p(z=3) = 0.9987$$

$$p(z=-3) = 0.0013$$

$$\left\{ \begin{array}{l} p(z) = 0.9987 - 0.0013 \\ = \underline{\underline{99.74\%}} \end{array} \right.$$